7주차(1/3)

# 순방향 신경망

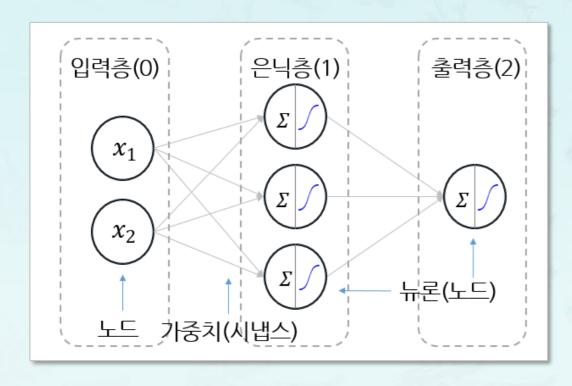
파이썬으로배우는기계학습

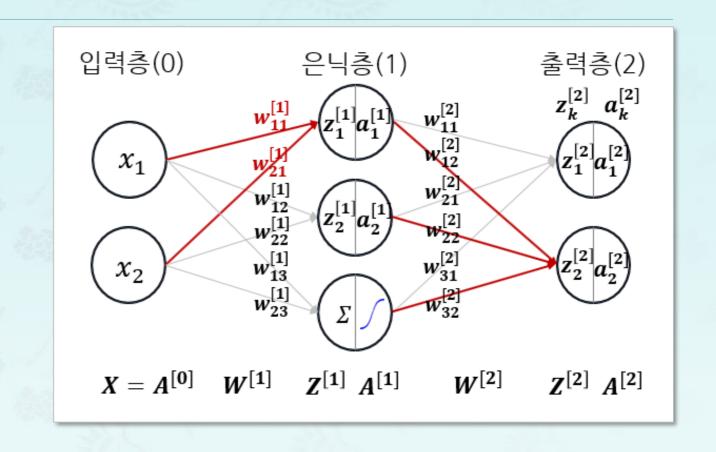
한동대학교 김영섭교수

### 순방향 신경망

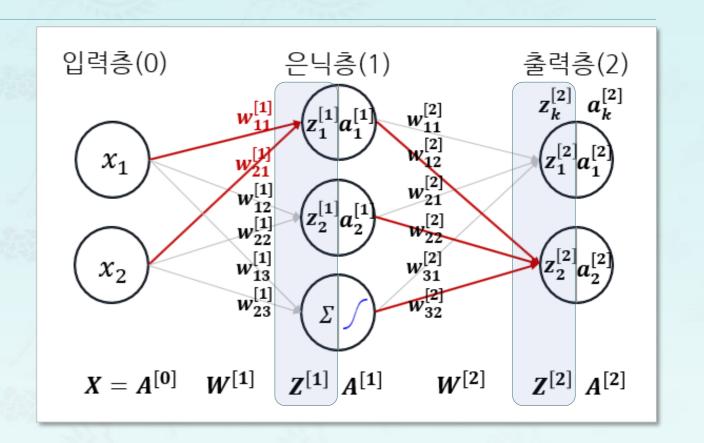
- 학습 목표
  - 순방향 신경망의 신호를 처리한다.
- 학습 내용
  - 순방향 신경망 신호표기
  - 순방향 신경망 신호처리
  - 가중치 표기법
  - 순방향 신경망 예제

■ 다층 신경망





- **Z**: 뉴론의 입력
- A: 뉴론의 출력
- L: 전체 층의 수
- I: 각 층 번호(소문자 엘)

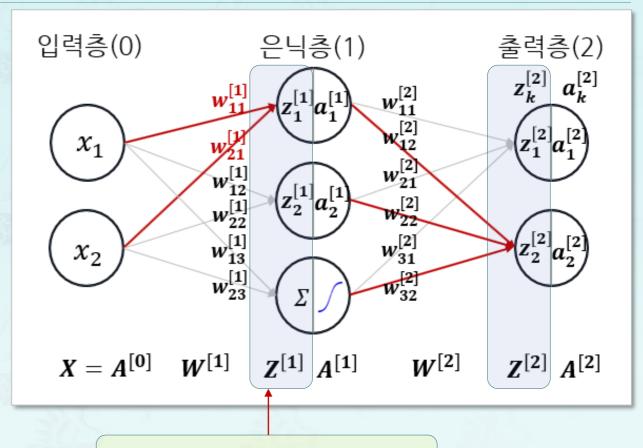


■ Z: 뉴론의 입력

■ A: 뉴론의 출력

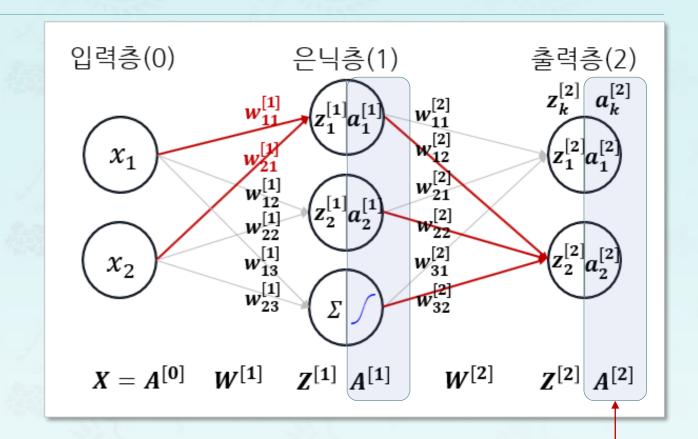
■ L: 전체 층의 수

■ I: 각 층 번호(소문자 엘)



■ Z<sup>[1]</sup>: 은닉층(1)의 입력

- Z: 뉴론의 입력
- A: 뉴론의 출력
- L: 전체 층의 수
- I: 각 층 번호(소문자 엘)



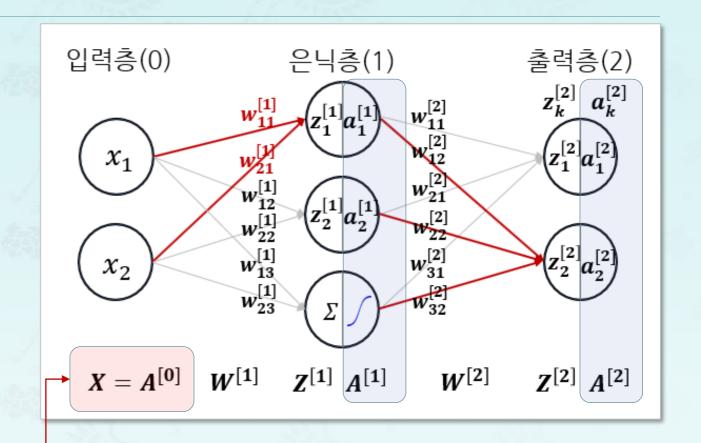
- **Z**<sup>[1]</sup>: 은닉층(1)의 입력
- A<sup>[2]</sup>: 출력층(2)의 출력

■ Z: 뉴론의 입력

■ A: 뉴론의 출력

■ L: 전체 층의 수

■ I: 각 층 번호(소문자 엘)

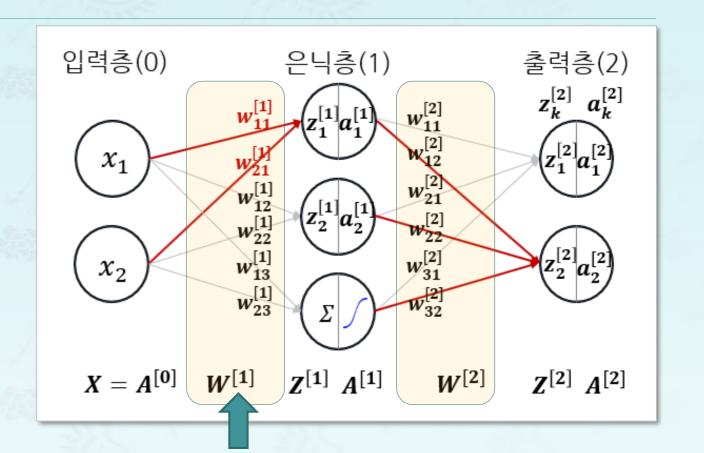


■ **Z**<sup>[1]</sup>: 은닉층(1)의 입력

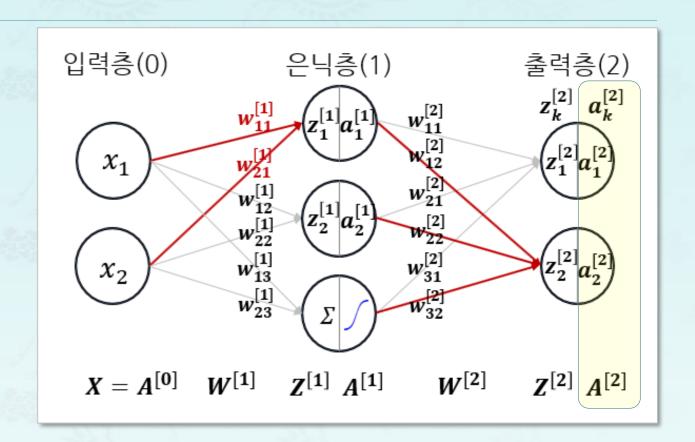
■ A<sup>[2]</sup>: 출력층(2)의 출력

A<sup>[0]</sup>: 입력층(0)의 출력

- **Z**: 뉴론의 입력
- A: 뉴론의 출력
- L: 전체 층의 수
- **!**: 각 층 번호
- W: 가중치

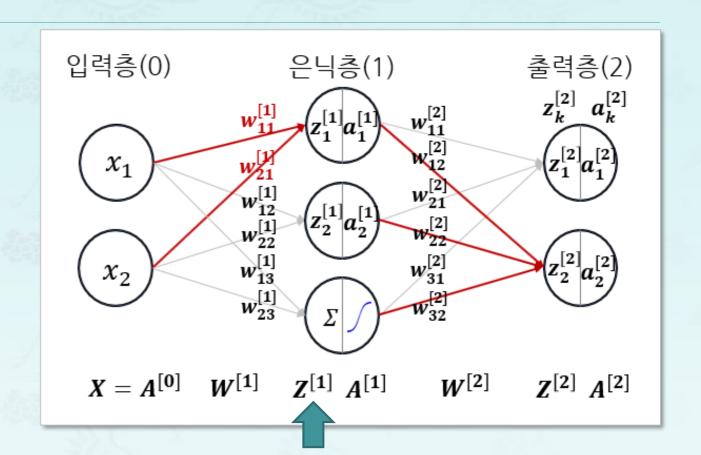


- Z: 뉴론의 입력
- A: 뉴론의 출력
- L: 전체 층의 수
- **!**: 각 층 번호
- W: 가중치
- ŷ: 최종 출력

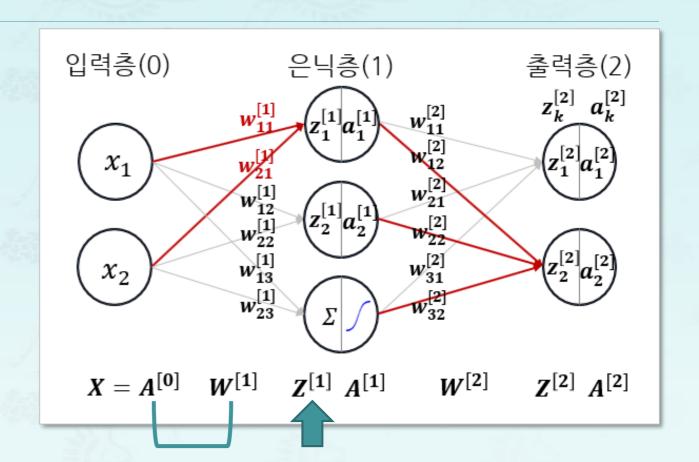


$$\widehat{y} = A^{[2]}$$

- Z: ∑(가중치 \* 입력)
  - 순입력
  - net input 혹은 weighted sum

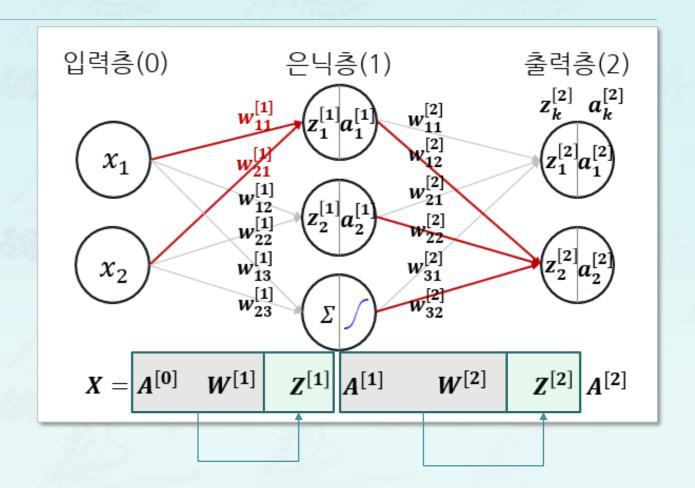


- Z: ∑(가중치 \* 입력)
  - 순입력
  - net input 혹은 weighted sum



- Z: ∑(가중치 \* 입력)
  - 순입력
  - net input 혹은 weighted sum

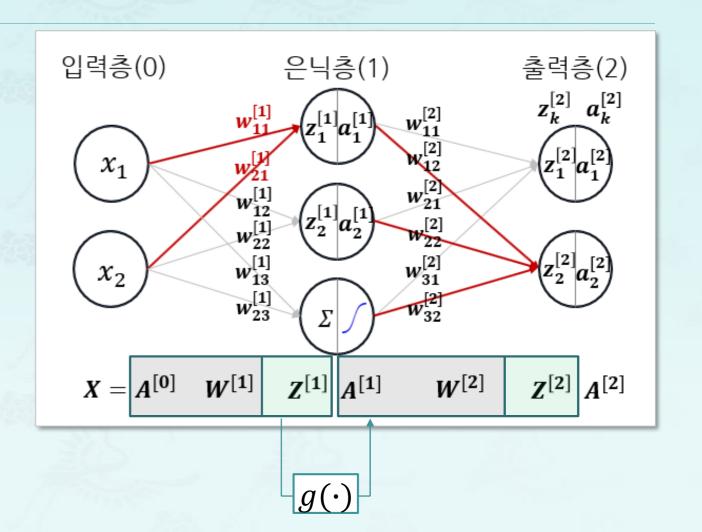




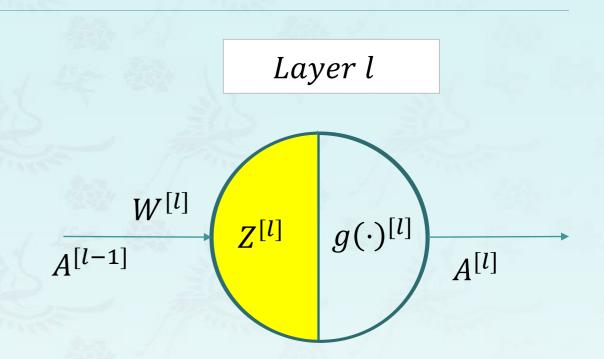
- Z: ∑(가중치 \* 입력)
  - 순입력
  - net input 혹은 weighted sum

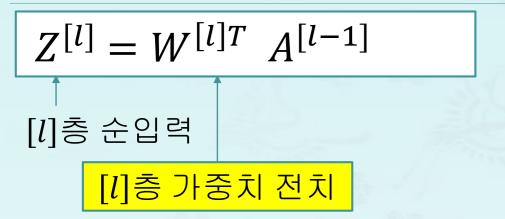
$$Z^{[l]} = W^{[l]T}A^{[l-1]}$$

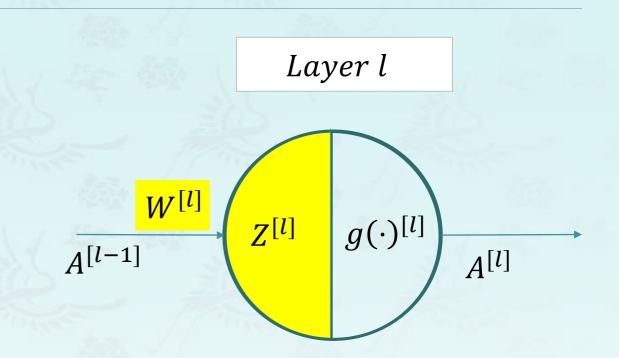
$$A^{[l]} = g(Z^{[l]})$$
활성화 함수

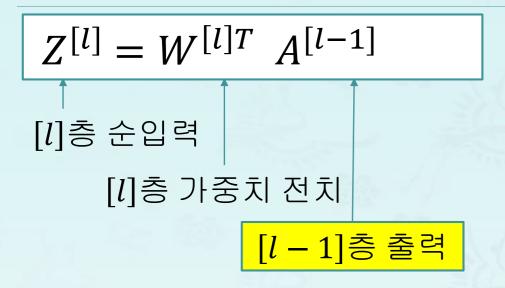


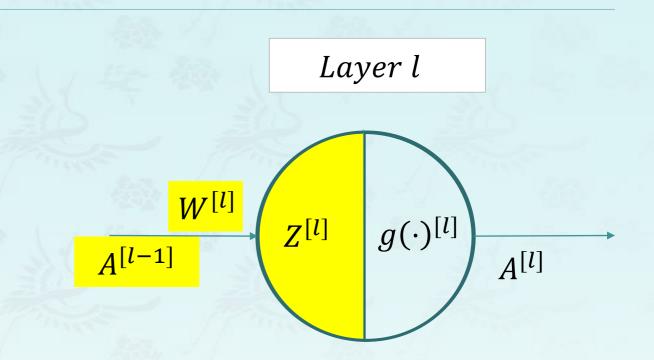
$$Z^{[l]} = W^{[l]T} A^{[l-1]}$$
[기층 순입력









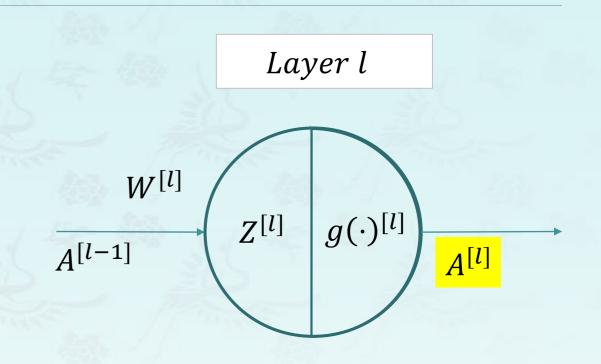


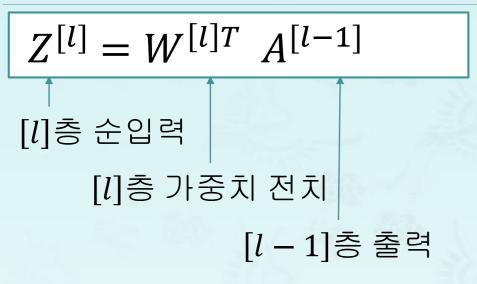
$$Z^{[l]} = W^{[l]T} A^{[l-1]}$$

$$[l] \stackrel{\wedge}{=} 2^{l} + 2^{l} +$$

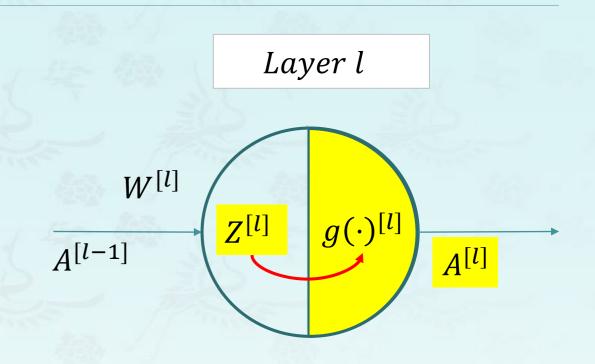
$$A^{[l]} = g(Z^{[l]})$$

[*l*]층 출력

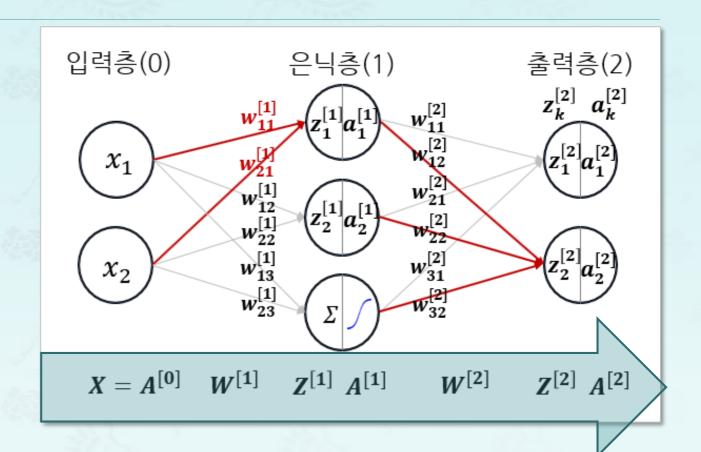




$$A^{[l]} = g(Z^{[l]})$$
[ $l$ ]층 출력
활성화 함수
[ $l$ ]층 순입력



- Z: ∑(가중치 \* 입력)
  - 순입력
  - net input 혹은 weighted sum



$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{Z}^{[l]} = W^{[l]T}A^{[l-1]}$$
 
$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
 엘흥가중치

$$\mathbf{Z}^{[l]} = W^{[l]T}A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
엘 층 가중치 
$$\begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
은닉층 가중치

$$\mathbf{Z}^{[l]} = W^{[l]T}A^{[l-1]}$$

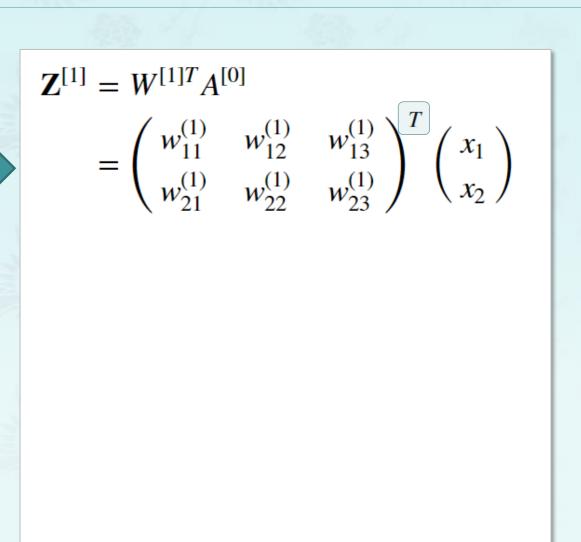
$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
엘 층 가중치 
$$\begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
은닉층 가중치



$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$\mathbf{Z}^{[l]} = W^{[l]T}A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
엘충가중치 
$$\begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
은닉총가중치



$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$
 
$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
 엘총 가중치

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
 은닉층 가중치

$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
엘총가중치

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
은닉총 가중치

$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{12}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{13}^{(1)} x_1 + w_{23}^{(1)} x_2 \end{pmatrix} = \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$

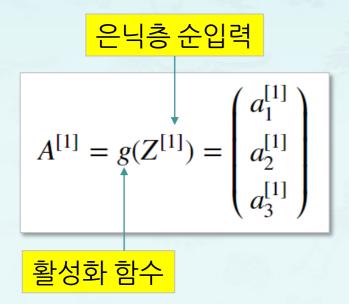
$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$

은닉층 순입력 
$$\mathbf{Z}^{[1]} = W^{[1]T}A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)}x_1 + w_{21}^{(1)}x_2 \\ w_{12}^{(1)}x_1 + w_{22}^{(1)}x_2 \\ w_{13}^{(1)}x_1 + w_{23}^{(1)}x_2 \end{pmatrix} = \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$
은닉층 순입력

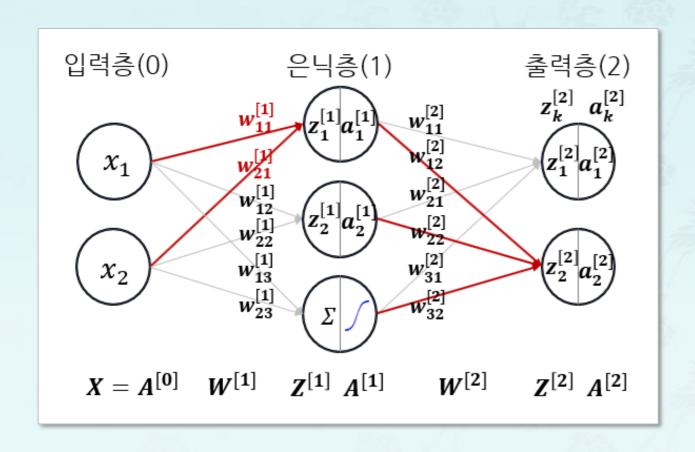


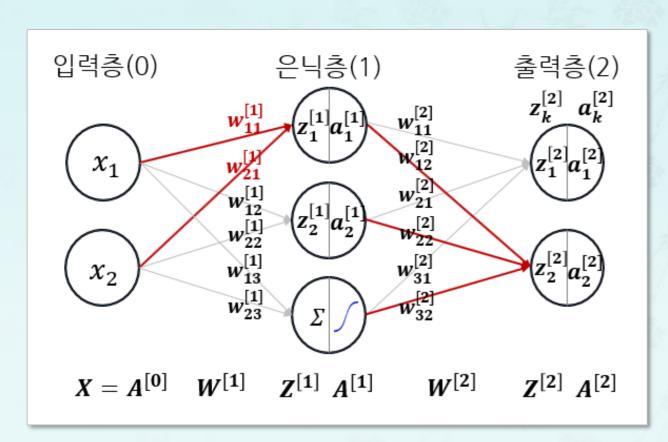
$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

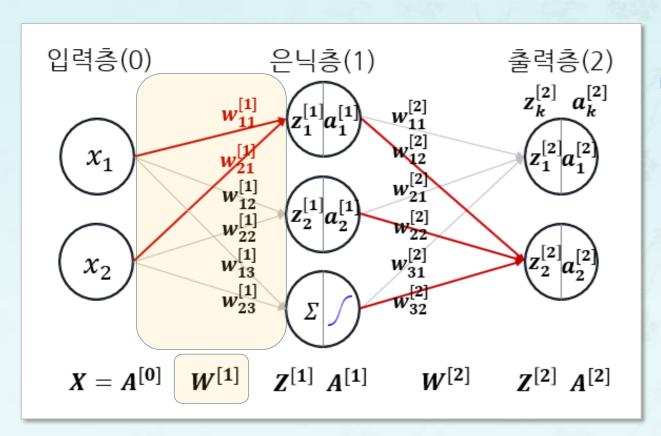
$$= \begin{pmatrix} w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{12}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{13}^{(1)} x_1 + w_{23}^{(1)} x_2 \end{pmatrix} = \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$





•  $W_{ij}^T$  형상

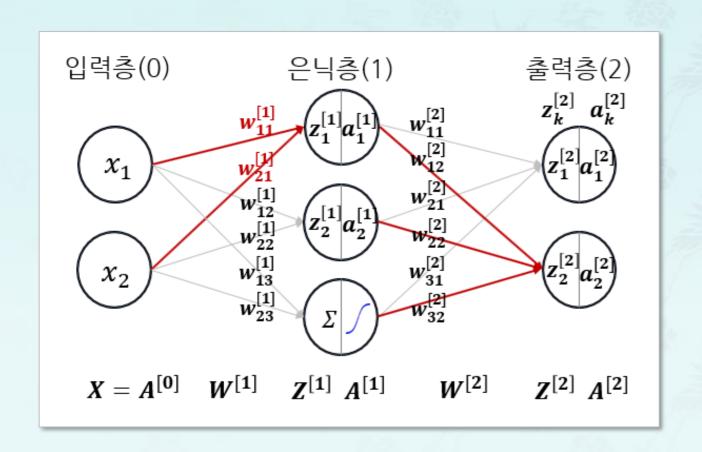
l 층의 노드 +  $\times$  (l-1)층의 노드 +



- $W_{ij}^T$  형상
  - l 층의 노드 수 x (l 1)층의 노드 수
- $W^1$ .shape = (3,2)

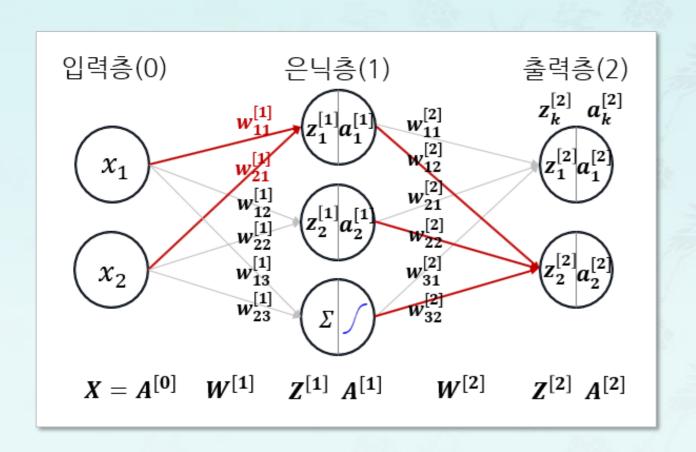
$$W^{[1]} = \begin{pmatrix} w_{11}^{[1]} & w_{21}^{[1]} \\ w_{12}^{[1]} & w_{22}^{[1]} \\ w_{13}^{[1]} & w_{23}^{[1]} \end{pmatrix}$$

$$W^{[2]} = \begin{pmatrix} w_{11}^{[2]} & w_{21}^{[2]} & w_{31}^{[2]} \\ w_{12}^{[2]} & w_{22}^{[2]} & w_{32}^{[2]} \end{pmatrix}$$



$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$

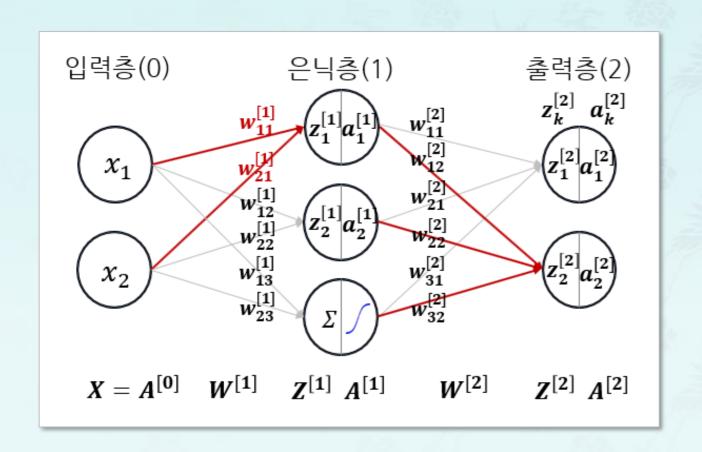
$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$



$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$

$$\mathbf{Z}^{[l]} = W^{[l]}A^{[l-1]}$$

$$\mathbf{Z}^{[1]} = W^{[1]}A^{[0]}$$

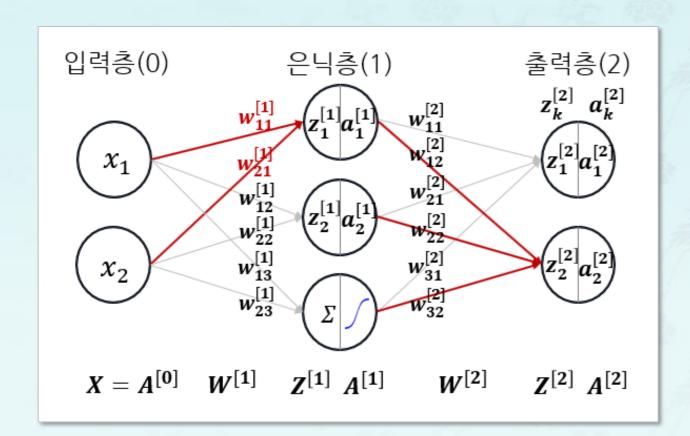




$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$





$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

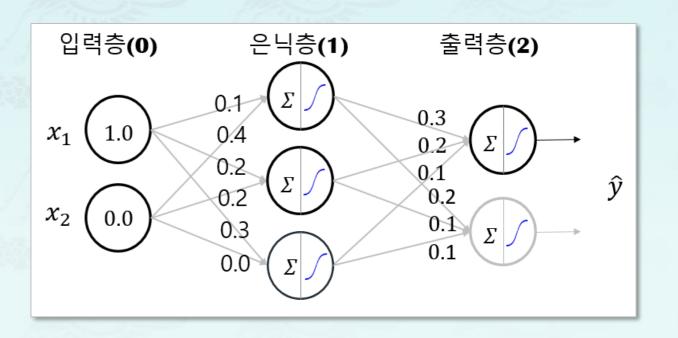
$$= \begin{pmatrix} z_{1}^{(1)} \\ z_{2}^{(1)} \\ z_{3}^{(1)} \end{pmatrix}$$

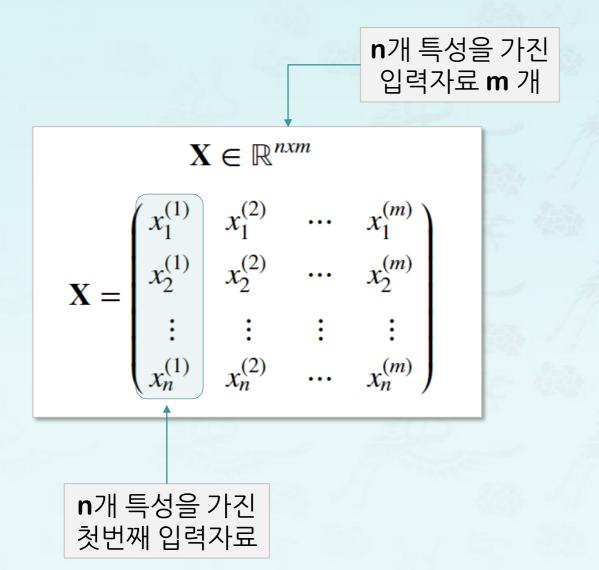
# 2. 가중치 표기법: $W_{ij}^T$ 방식(혹은 $W_{ji}$ 방식)

•  $W_{ij}$  와  $W_{ij}^T$  표기법

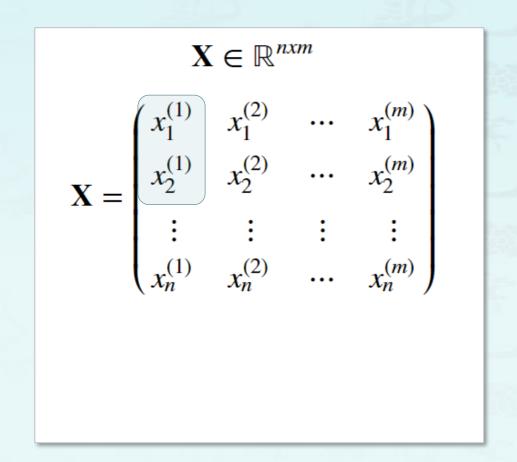
## 3. 순방향 신경망 예제: $W_{ij}^{T}$ 표기법

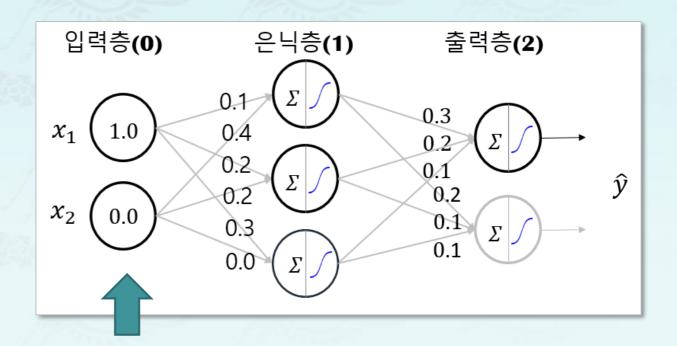
•  $W_{ij}$ 와  $W_{ij}^T$ 표기법





■ 입력 X: m = 1, n = 2





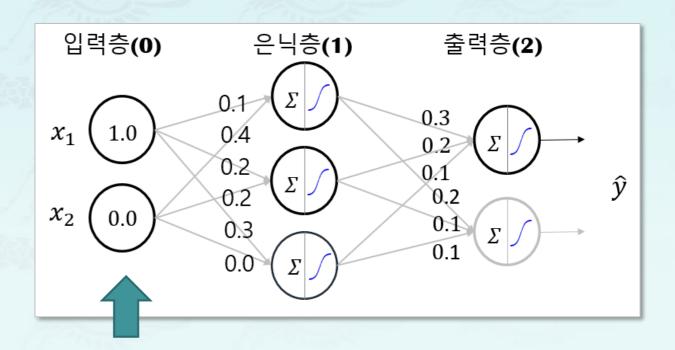
■ 입력 X: m = 1, n = 2

$$\mathbf{X} \in \mathbb{R}^{nxm}$$

$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{bmatrix}$$

$$\mathbf{x}^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

■ 가중치 초기화



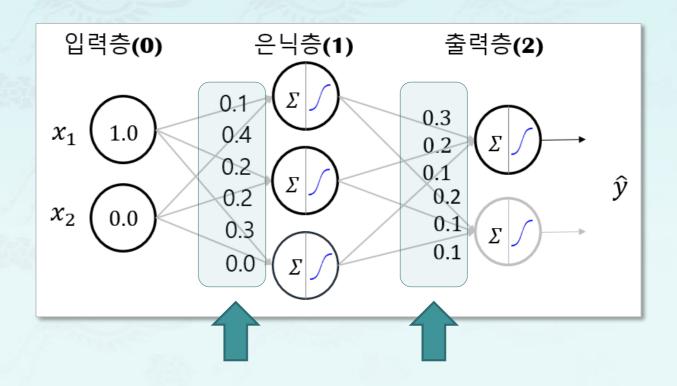
■ 입력 X: m = 1, n = 2

$$\mathbf{X} \in \mathbb{R}^{n \times m}$$

$$\mathbf{X} = \begin{pmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{pmatrix}$$

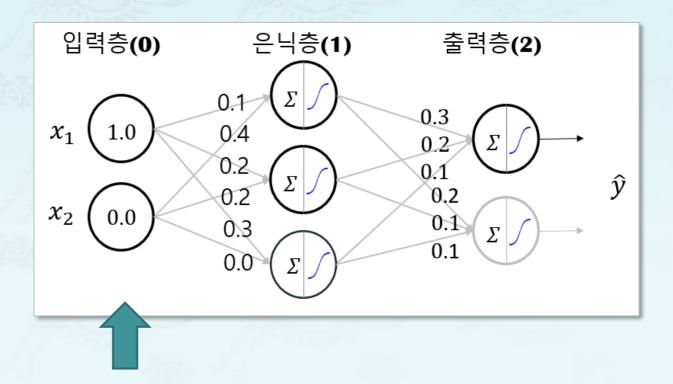
$$\mathbf{x}^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

■ 가중치 초기화



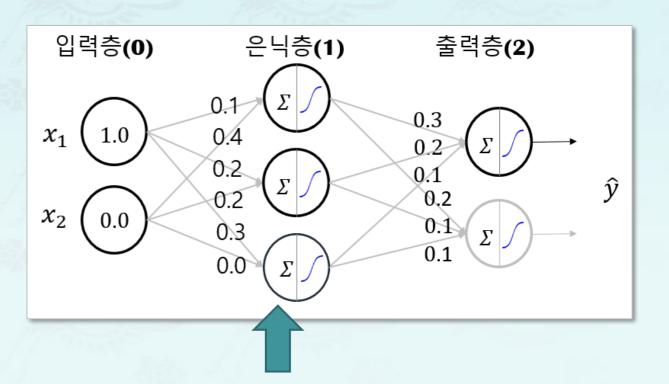
## 4. 순방향 신경망 계산: 입력층

•  $A^{[0]} = X$ 

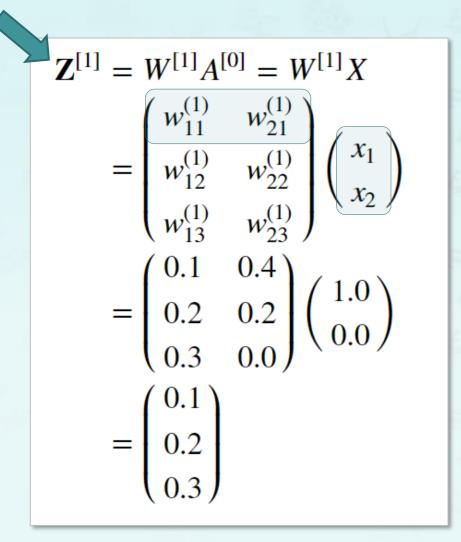


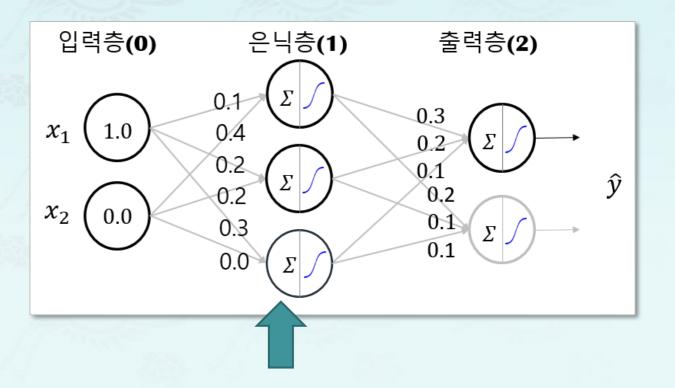
### 4. 순방향 신경망 계산: 은닉층

- $Z^{[l]} = W^{[l]}A^{[l-1]}$
- $A^{[i]} = g(Z^{[i]})$

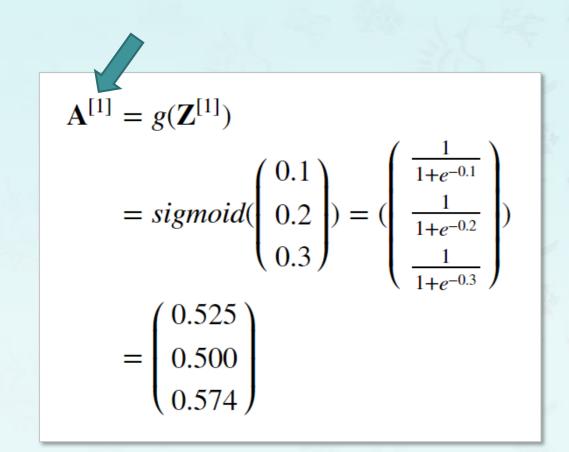


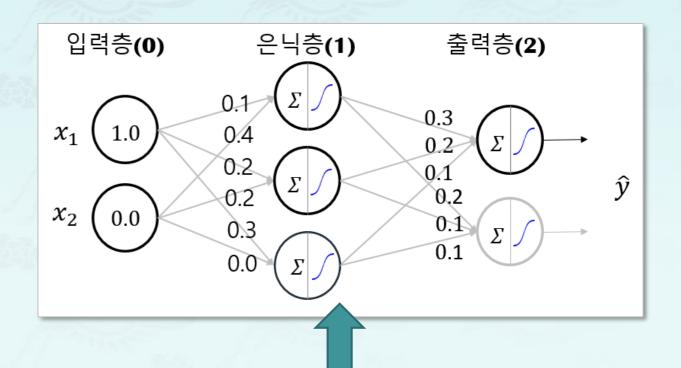
#### 4. 순방향 신경망 계산: 은닉층



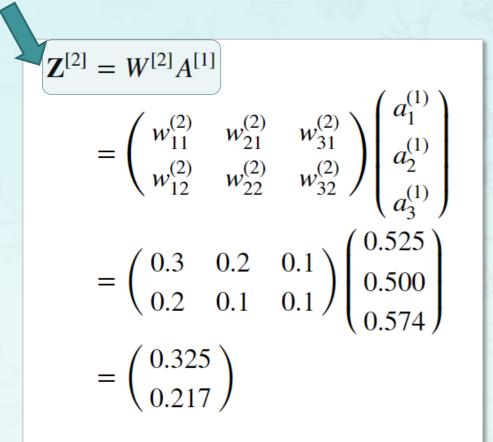


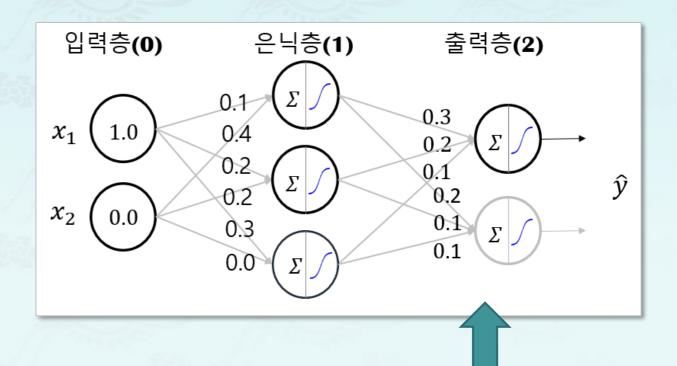
#### 4. 순방향 신경망 계산: 은닉층



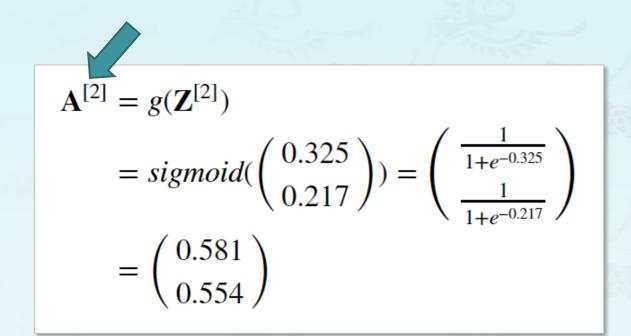


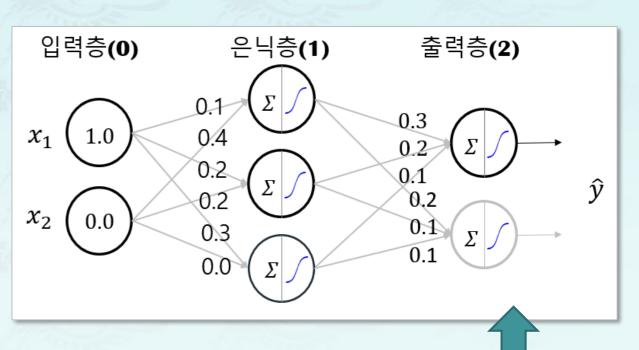
#### 4. 순방향 신경망 계산: 출력층





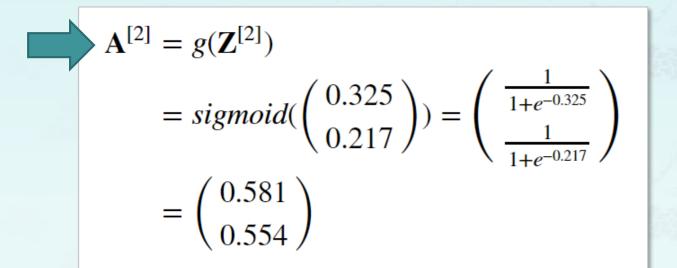
#### 4. 순방향 신경망 계산: 출력층

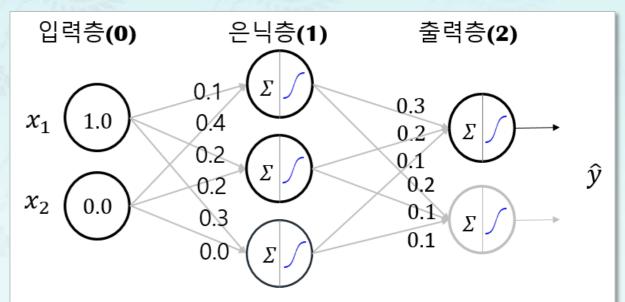




#### 4. 순방향 신경망 계산: 출력층

#### 출력층(2)







$$\hat{y} = \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \begin{pmatrix} 0.581 \\ 0.554 \end{pmatrix}$$

#### 순방향 신경망

- 학습 정리
  - 순방향 신경망 신호 표기
  - 순방향 신경망 신호 처리
  - 가중치  $W_{iJ}$  과  $W_{ij}^T$  방식
  - 순방향 신경망 예제와 계산

■ **7-2** 순방향 신경망 예제

7주차(1/3)

# 순방향 신경망

파이썬으로배우는기계학습

한동대학교 김영섭교수

여러분 곁에 항상 열려 있는 K-MOOC 강의실에서 만나 뵙기를 바랍니다.