11주차(1/3)

다층 신경망의 행렬 모델링

파이썬으로 배우는 기계학습

한동대학교
김영섭 교수

다층 신경망의 행렬 모델링

- 학습 목표
  - 미분의 연쇄법칙을 학습한다.
  - 오차함수의 행렬 표기에서 미분하는 방법을 학습한다.
  - 다층 인공 신경망의 행렬 모델을 학습한다.

- 학습 내용
  - 미분의 연쇄법칙
  - 오차함수의 행렬 미분
  - 다층 인공 신경망 행렬 모델

연쇄법칙: 연쇄법칙이란?

- “연쇄”

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$$F(x) = f(g(x))$$

연쇄법칙: 연쇄법칙이란?

- "연쇄"

$$F(x) = f(g(x))$$

$$F'(x) = f'\big(g(x)\big)\boxed{g'(x)}$$

연쇄법칙: 연쇄법칙이란?

- “연쇄”

$$F(x) = f(g(x))$$

$$F'(x) = f'\big(g(x)\big)g'(x)$$

$y = f(u), u = g(x)$ 일 때,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

(1)번 (2)번

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

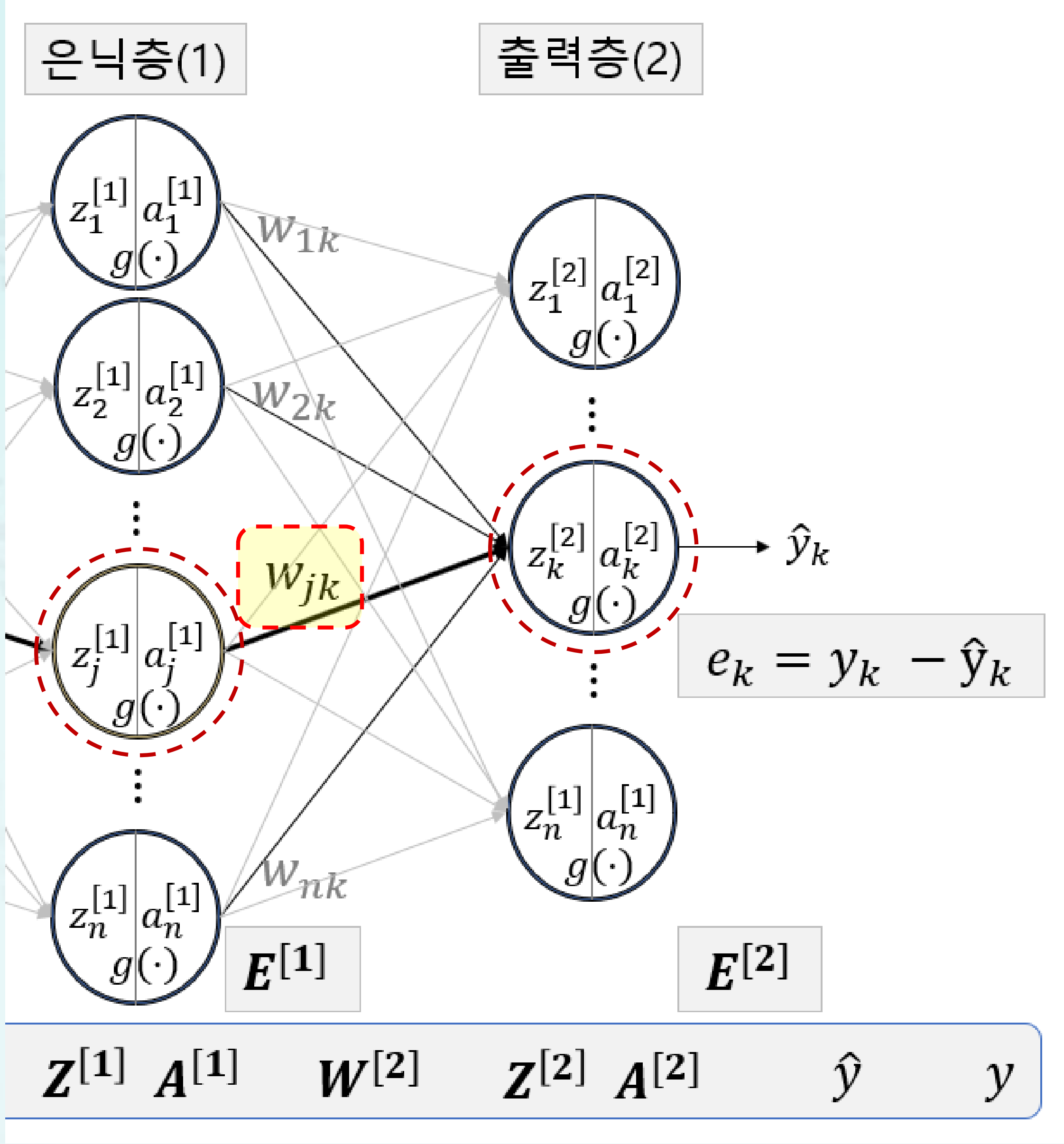
- 3단계

$$\frac{\partial E}{\partial w_{jk}} = -(y_k - \hat{y}_k) \cdot \frac{\partial}{\partial w_{jk}} g(z_k)$$

$$= -(y_k - \hat{y}_k) \cdot g'(z_k) \frac{\partial z_k}{\partial w_{jk}}$$

$$= -(y_k - \hat{y}_k) \cdot g'(z_k) \frac{\partial}{\partial w_{jk}} (\sum_j w_{jk} \cdot$$

$$= -(y_k - \hat{y}_k) \cdot g'(z_k) \cdot a_j$$



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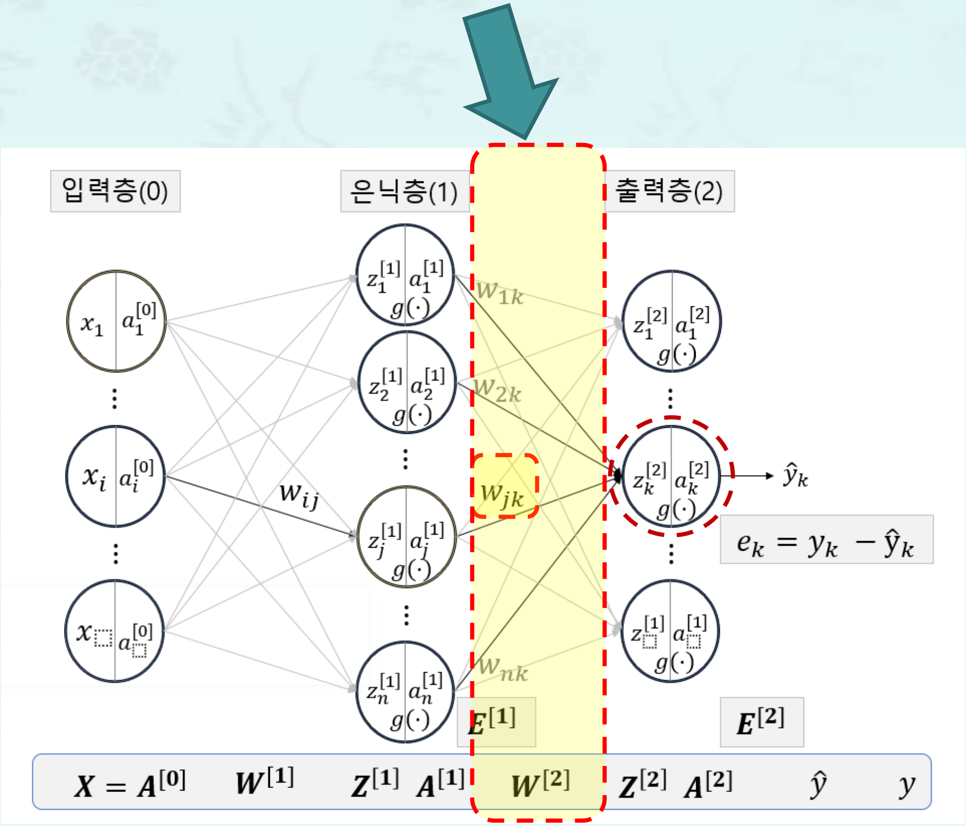
$$= -(y_k - \hat{y}_k) \cdot g'(z_k) \frac{\partial}{\partial w_{jk}} (\sum_j w_{jk} \cdot a_j)$$

$$= -(y_k - \hat{y}_k) \cdot g'(z_k) \cdot a_j$$

$$\boxed{\frac{\partial E}{\partial W^{[2]}} = -E^{[2]} \cdot g'(Z^{[2]}) \cdot A^{[1]T}}$$



오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

- **1단계**

$$W^{[2]} := W^{[2]} - \alpha\,\Delta W^{[2]}$$
$$= W^{[2]} - \alpha \frac{\partial E}{\partial W^{[2]}}$$

- **2단계**

$$w_{jk}^{[2]} := w_{jk}^{[2]} - \alpha \Delta w_{jk}^{[2]}$$
$$= w_{jk}^{[2]} - \alpha \frac{\partial E}{\partial w_{jk}^{[2]}}$$

- **3단계**

$$\frac{\partial E}{\partial w_{jk}} = -(y_k - \hat{y}_k) \cdot \frac{\partial}{\partial w_{jk}} g(z_k)$$

- **4단계**

$$\Delta w_{jk}^{[2]} = \frac{\partial E}{\partial w_{jk}} = -(y_k - \hat{y}_k) \cdot g'(z_k) \cdot a_j$$

$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}} = -E^{[2]} \cdot g'(Z^{[2]}) \cdot A^{[1]T}$$

- **결론**

$$W^{[2]} := W^{[2]} - \alpha \Delta W^{[2]}$$
$$= W^{[2]} - \alpha \frac{\partial E}{\partial W^{[2]}}$$
$$= \boxed{\mathrm{W}^{[2]} + E^{[2]} \cdot g'(Z^{[2]}) \cdot A^{[1]T}}$$

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

- **1단계**

$$W^{[2]} := W^{[2]} - \alpha \boxed{\Delta W^{[2]}}$$

$$= W^{[2]} - \alpha \frac{\partial E}{\partial W^{[2]}}$$

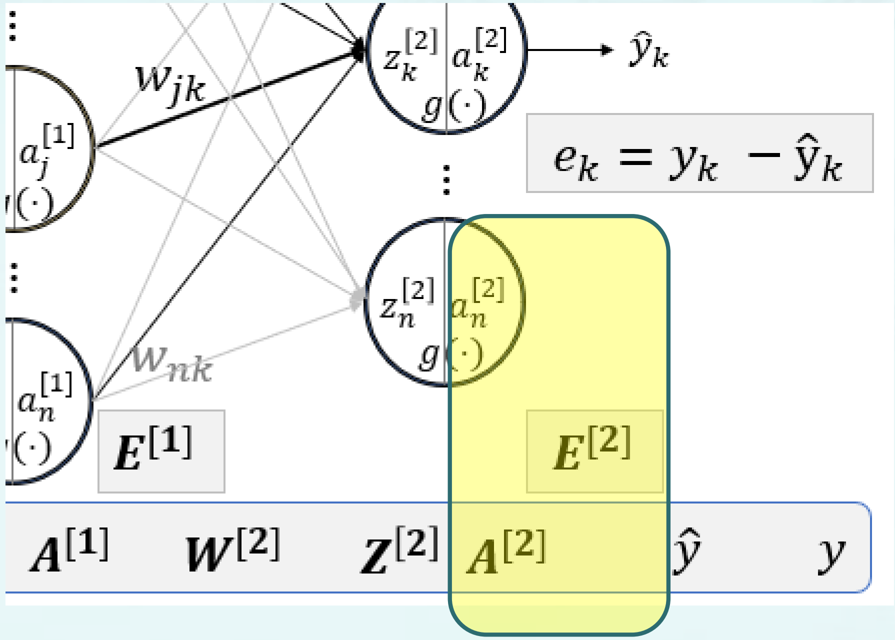
$$\boxed{\Delta W^{[2]}} = \frac{\partial E}{\partial W^{[2]}}$$

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

- 1단계

$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}}$$

$$= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial W^{[2]}}$$



오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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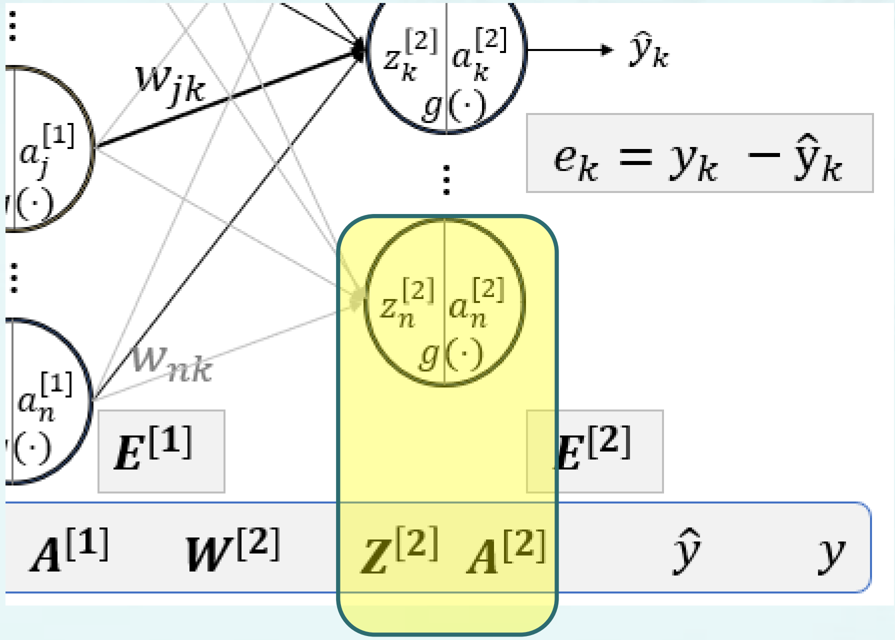
$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}}$$
$$= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial W^{[2]}}$$
$$= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$



오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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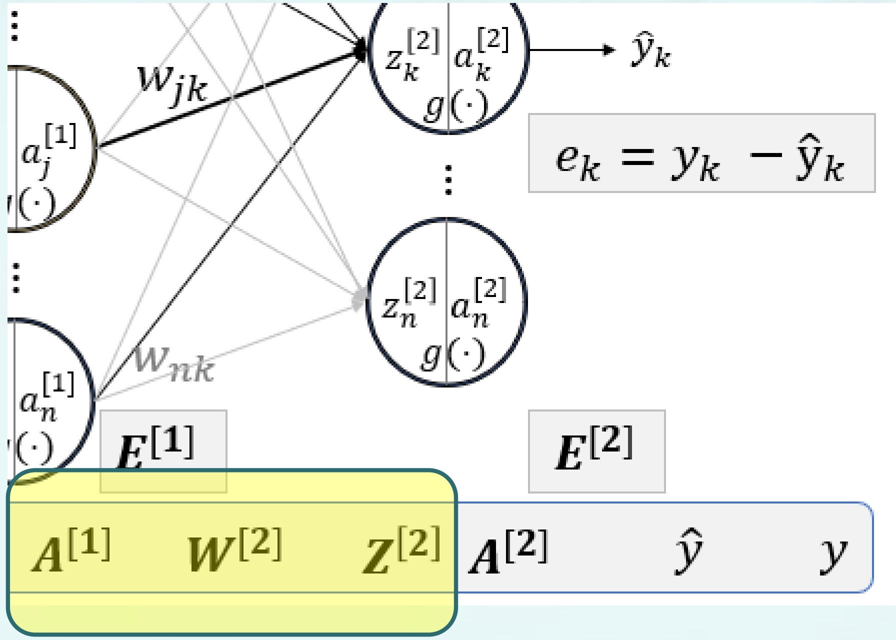

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}}$$

$$= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial W^{[2]}}$$

$$= \boxed{\frac{\partial E}{\partial A^{[2]}}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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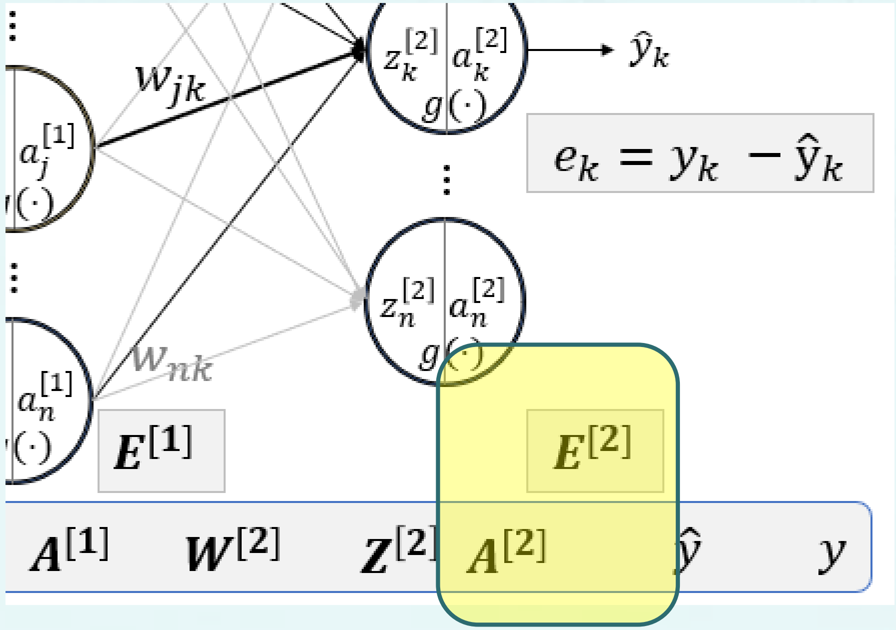
$$= \boxed{\frac{\partial E}{\partial A^{[2]}}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$= E^{[2]} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$E = \frac{1}{2}\left(A^{[2]} - Y\right)^2$$

$$E^{[2]} := \left(A^{[2]} - Y\right)$$



오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}}$$
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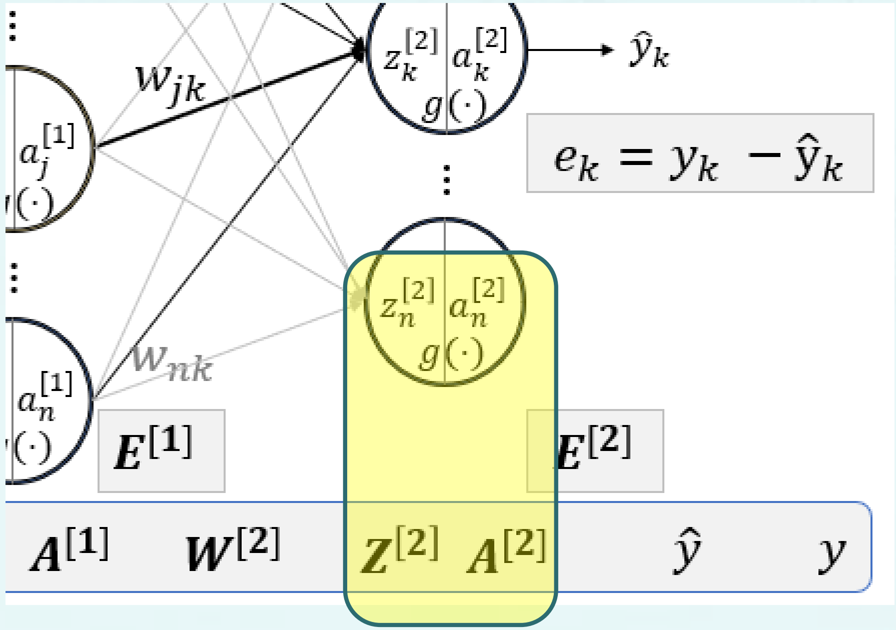
$$= E^{[2]} \cdot \boxed{\frac{\partial A^{[2]}}{\partial Z^{[2]}}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$



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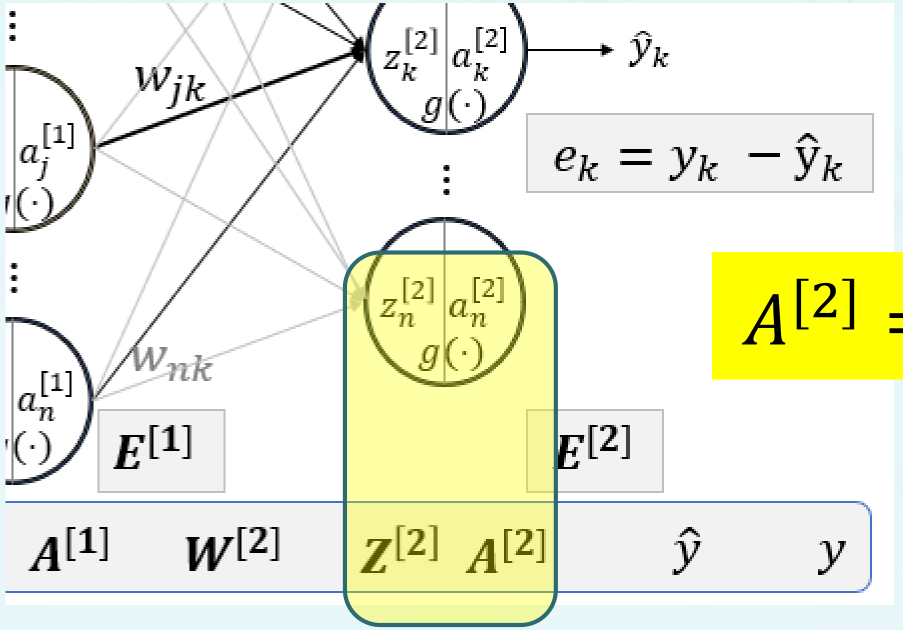
$$= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$= E^{[2]} \cdot \boxed{\frac{\partial A^{[2]}}{\partial Z^{[2]}}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$= E^{[2]} \cdot g'(Z^{[2]}) \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$



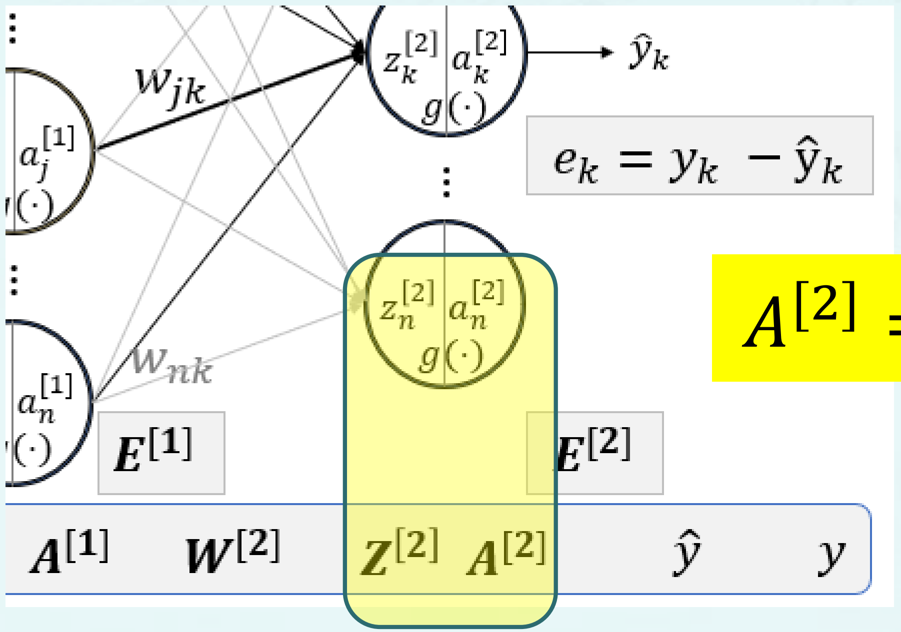
$$A^{[2]} = g(Z^{[2]})$$

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

- 1단계

$$E^{[2]} \cdot \boxed{\sigma(Z^{[2]})(1-\sigma(Z^{[2]}))} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$\begin{aligned}
\Delta W^{[2]} &= \frac{\partial E}{\partial W^{[2]}} \\
&= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial W^{[2]}} \\
&= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}} \\
&= E^{[2]} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}} \\
&= E^{[2]} \cdot \boxed{g'(Z^{[2]})} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}
\end{aligned}$$



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오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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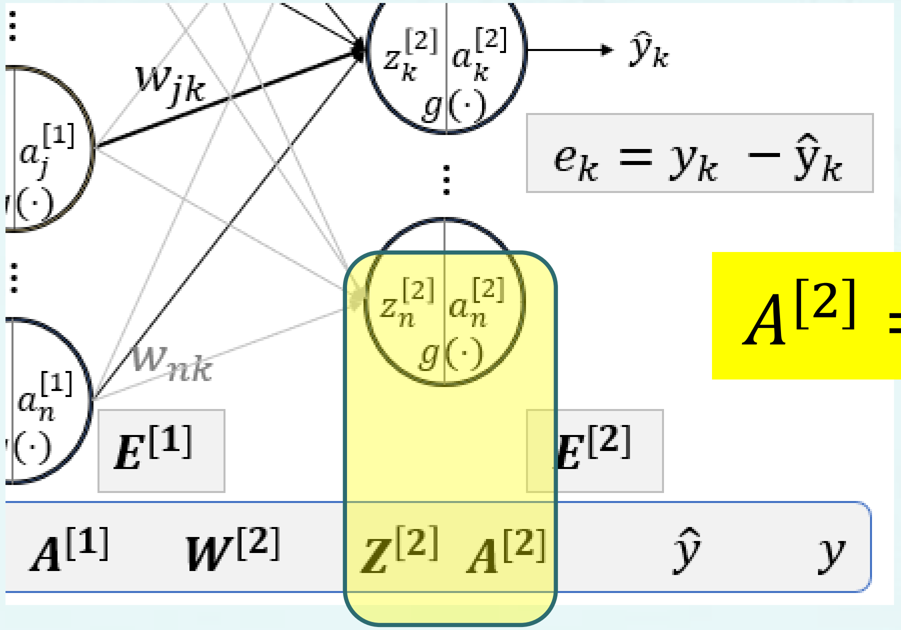
$$E^{[2]} \cdot \boxed{\sigma(Z^{[2]})(1-\sigma(Z^{[2]}))} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$$= E^{[2]} \cdot \boxed{A^{[2]}\,(1-A^{[2]})} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$



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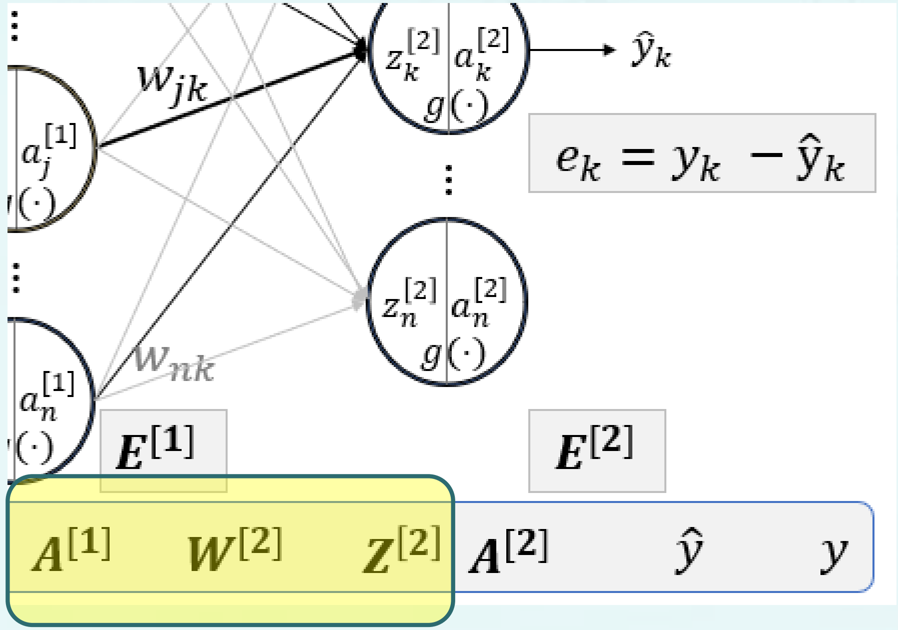
$$= E^{[2]} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$
$$= E^{[2]} \cdot g'(Z^{[2]}) \cdot \boxed{\frac{\partial Z^{[2]}}{\partial W^{[2]}}}$$



오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

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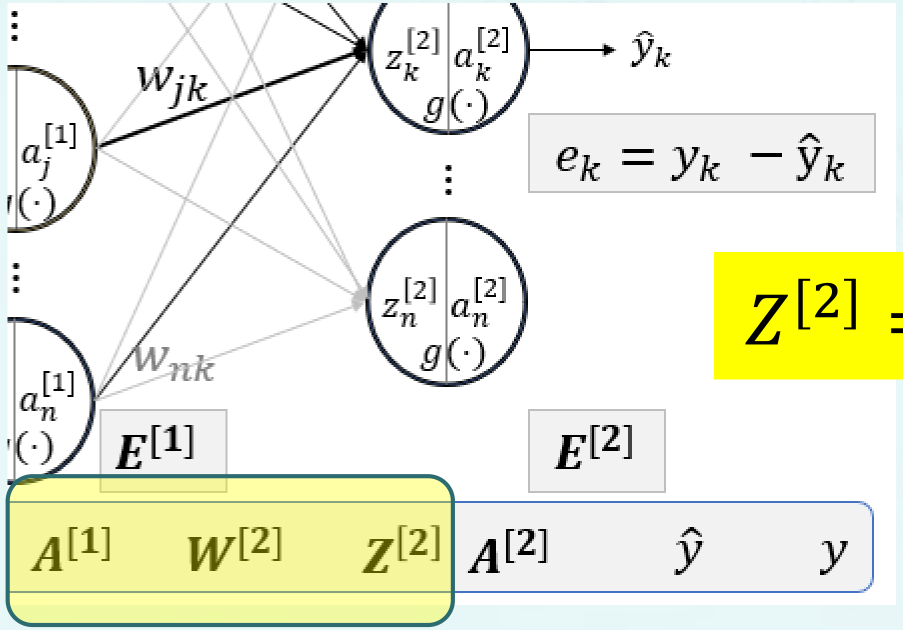


$$Z^{[2]} = W^{[2]} A^{[1]}$$

오차함수의 행렬 미분: $W^{[2]}$의 오차함수 미분

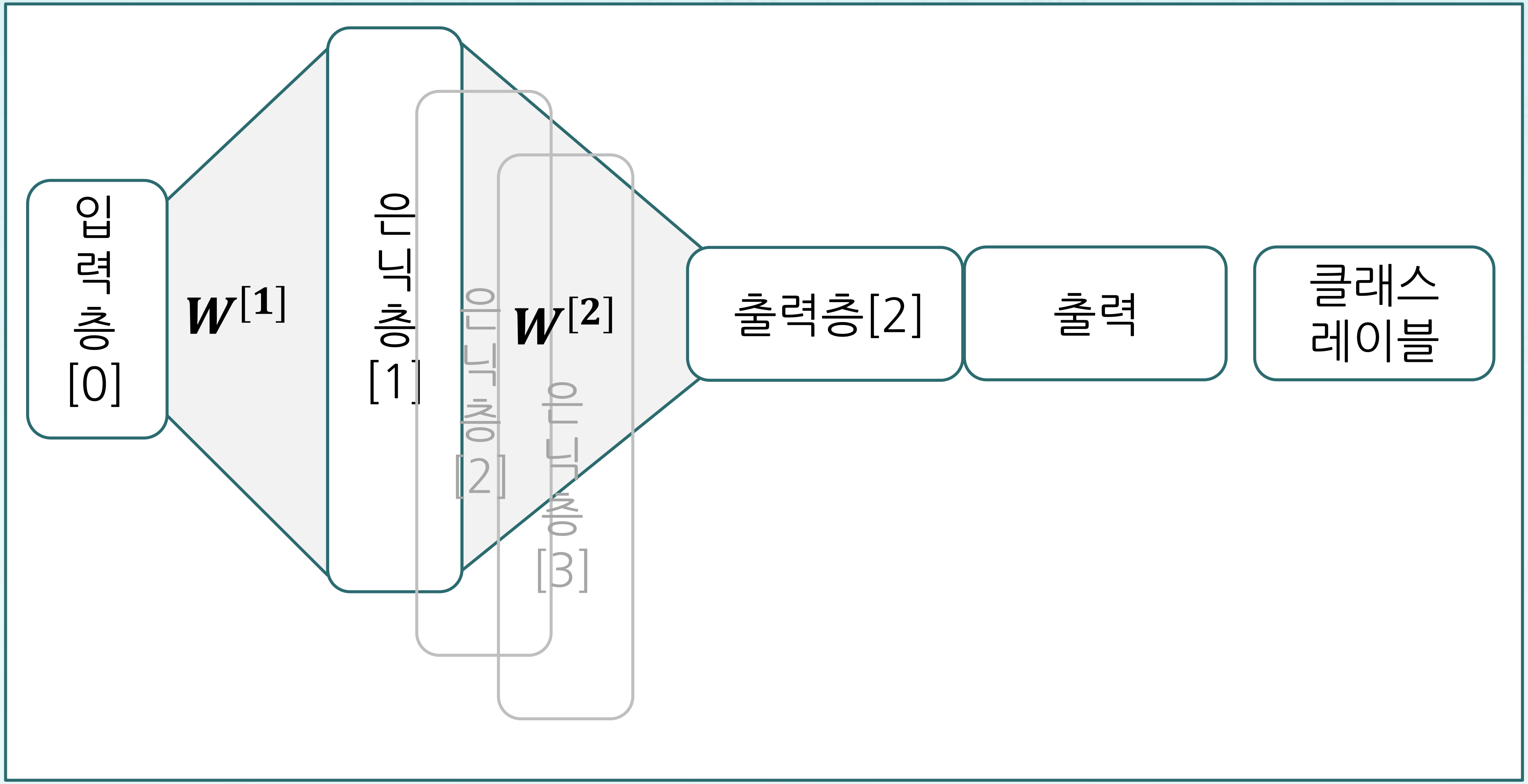
- 1단계

$$
\begin{aligned}
\Delta W^{[2]} &= \frac{\partial E}{\partial W^{[2]}} \\
&= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial W^{[2]}} \\
&= \frac{\partial E}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}} \\
&= E^{[2]} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}} \\
&= E^{[2]} \cdot g'(Z^{[2]}) \cdot \frac{\partial Z^{[2]}}{\partial W^{[2]}} \\
&= E^{[2]} \cdot g'(Z^{[2]}) \cdot \boxed{A^{[1]\cdot T}}
\end{aligned}
$$

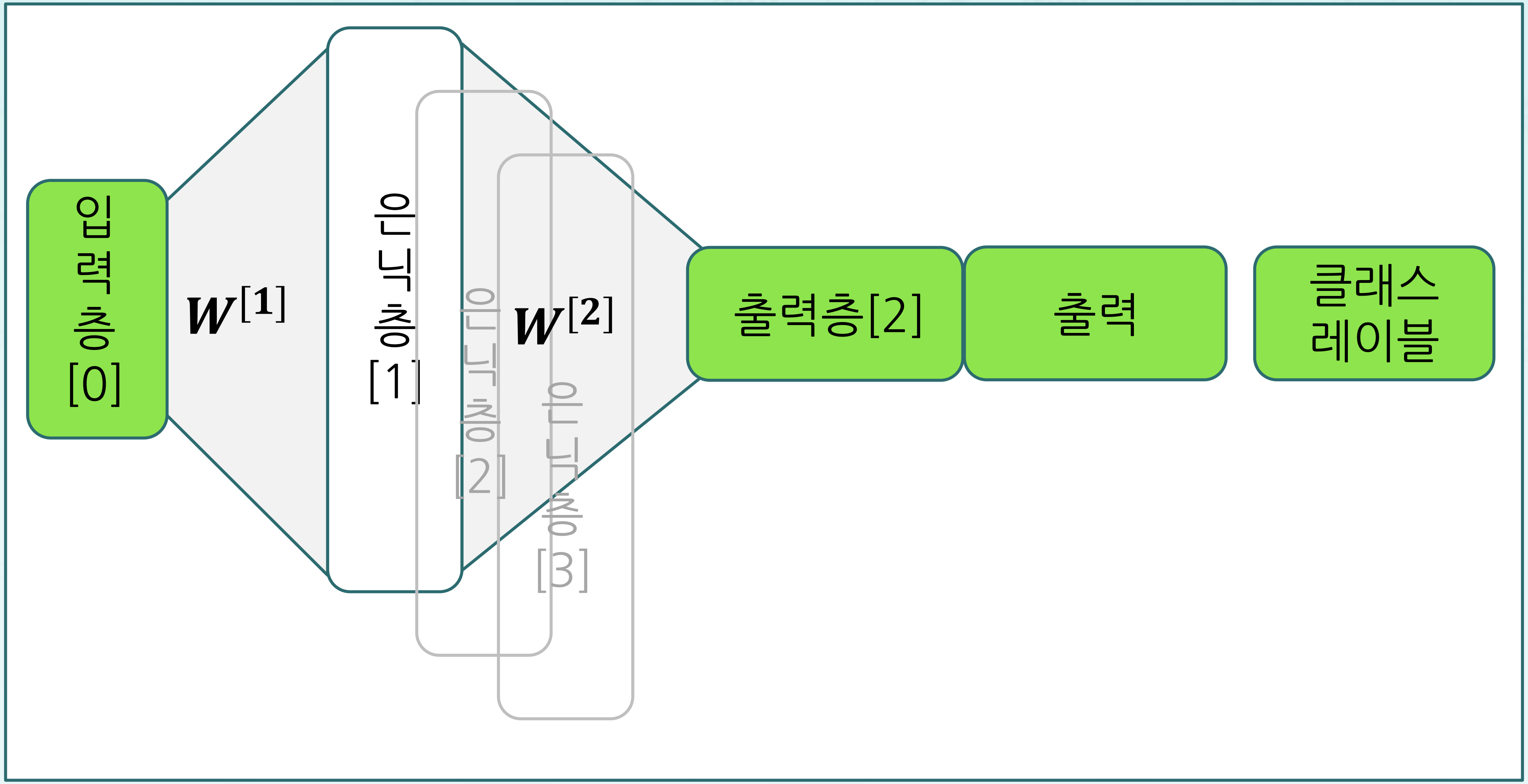
다층 인공신경망 행렬 모델:

- 다층 신경망의 구조

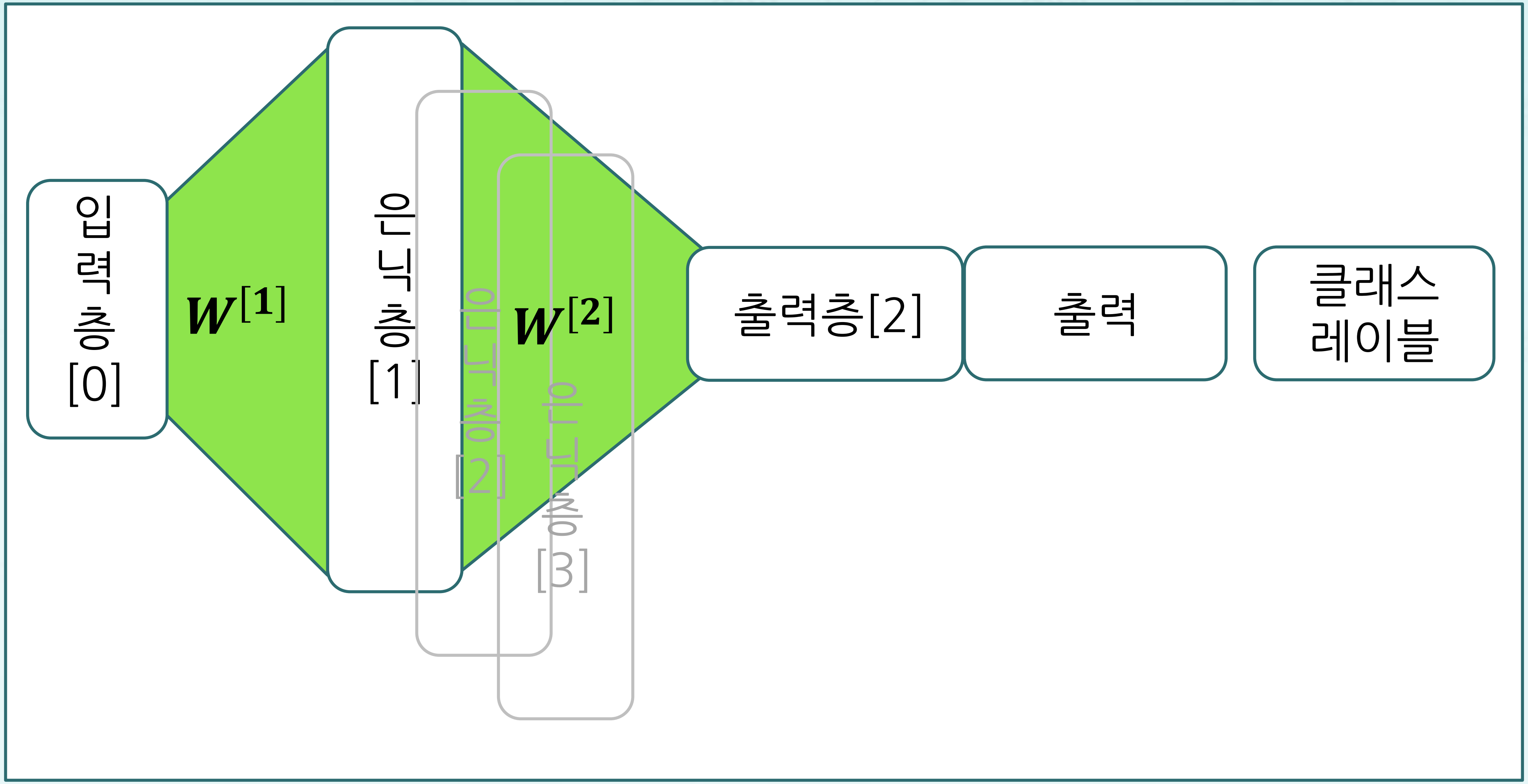
다층 인공신경망 행렬 모델:



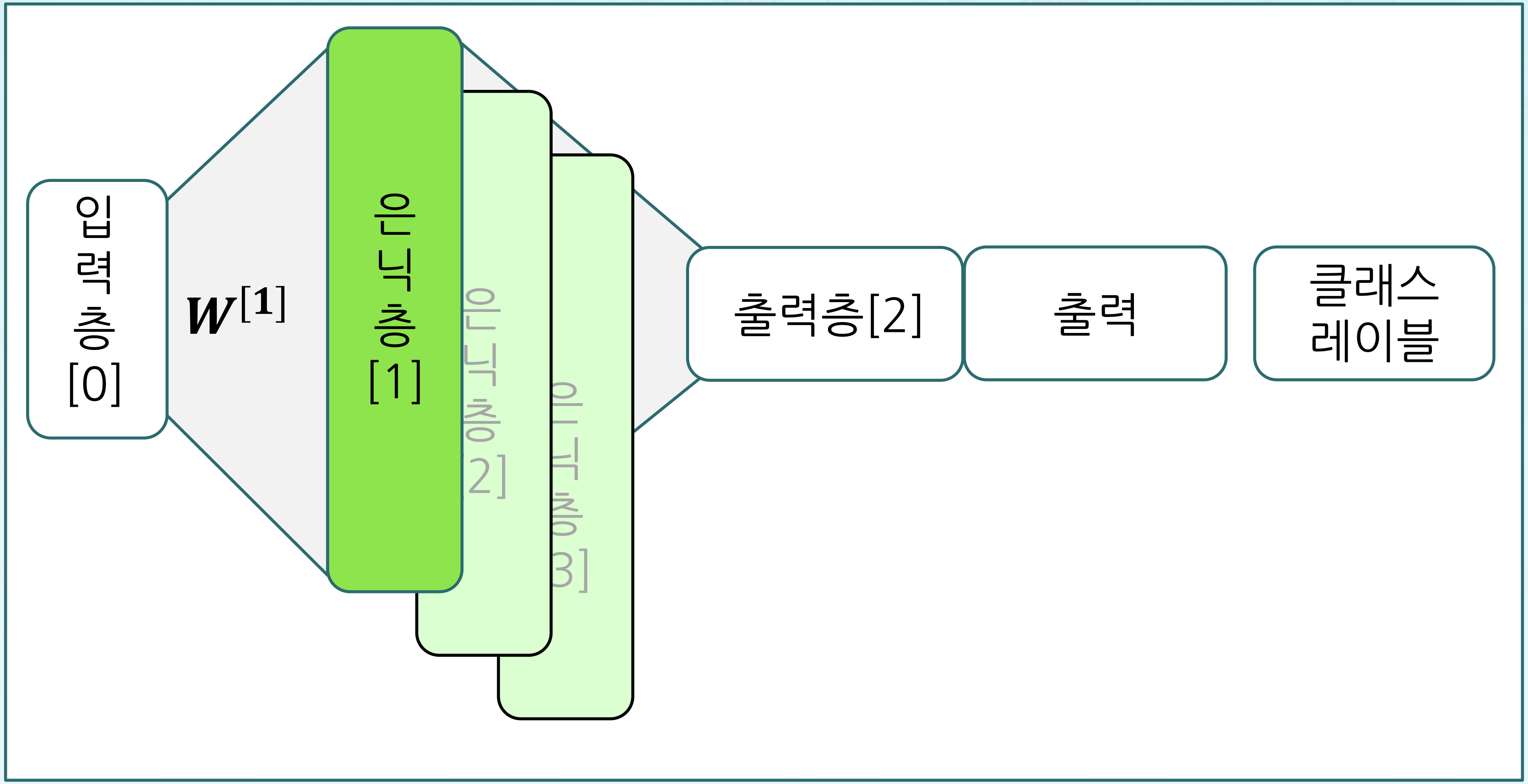
다층 인공신경망 행렬 모델: 입력층, 출력층



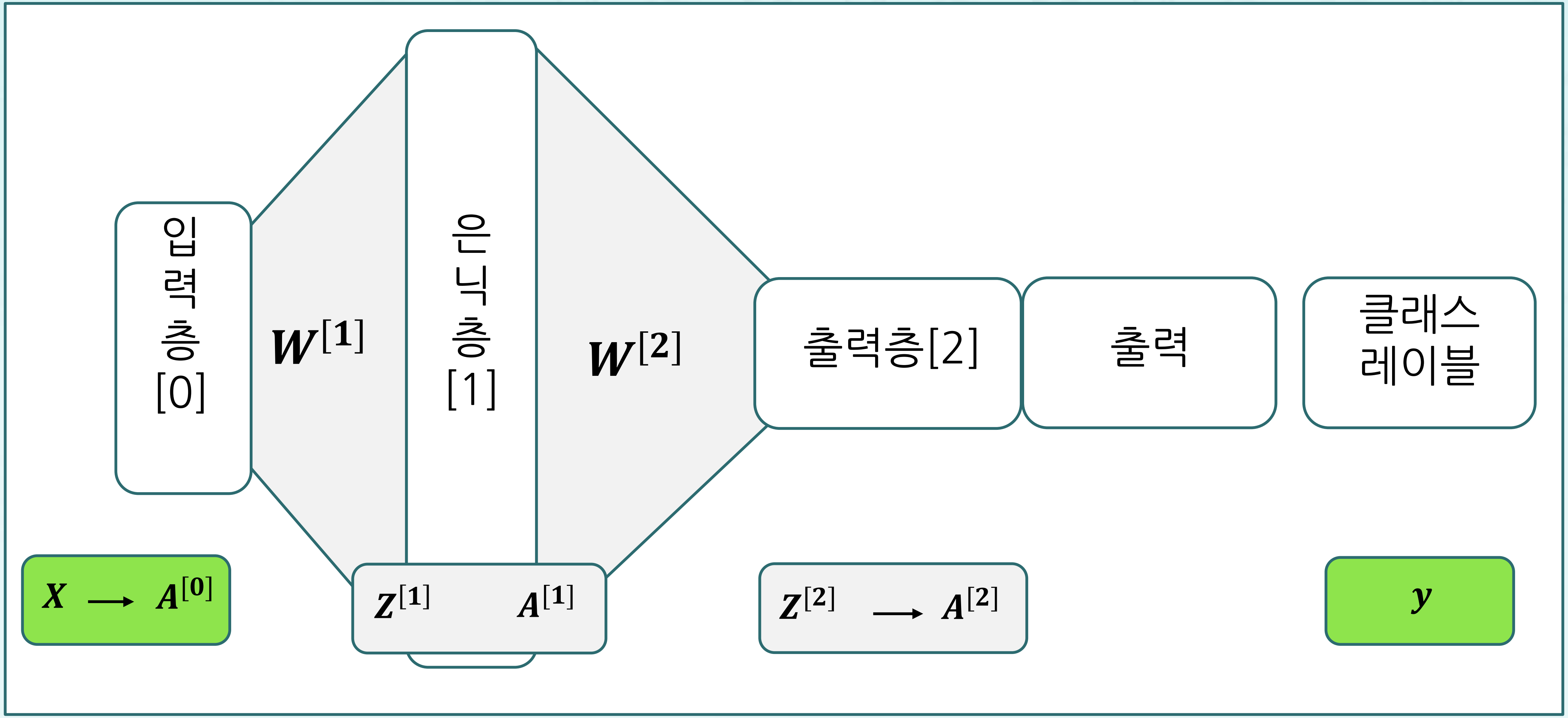
다층 인공신경망 행렬 모델: 가중치



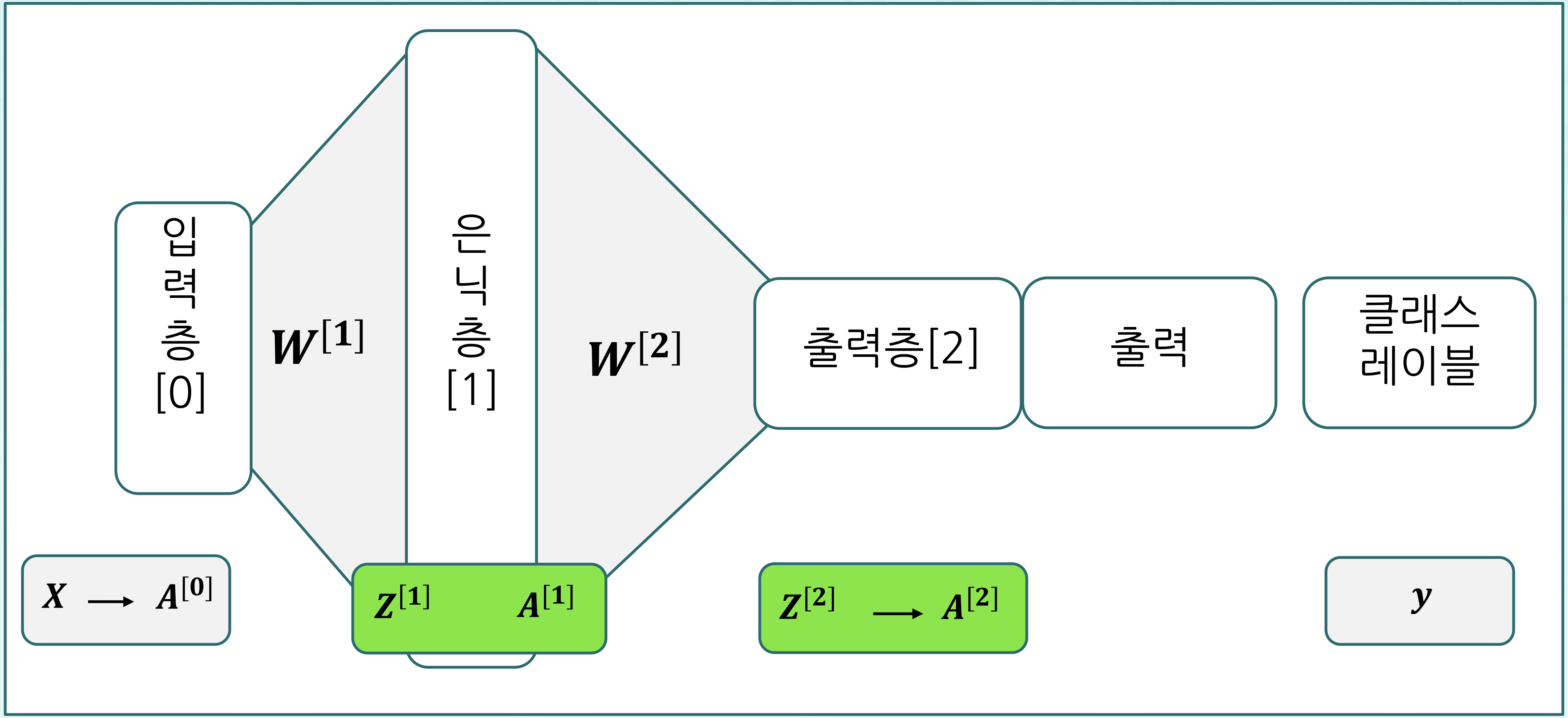
다층 인공신경망 행렬 모델: 은닉층



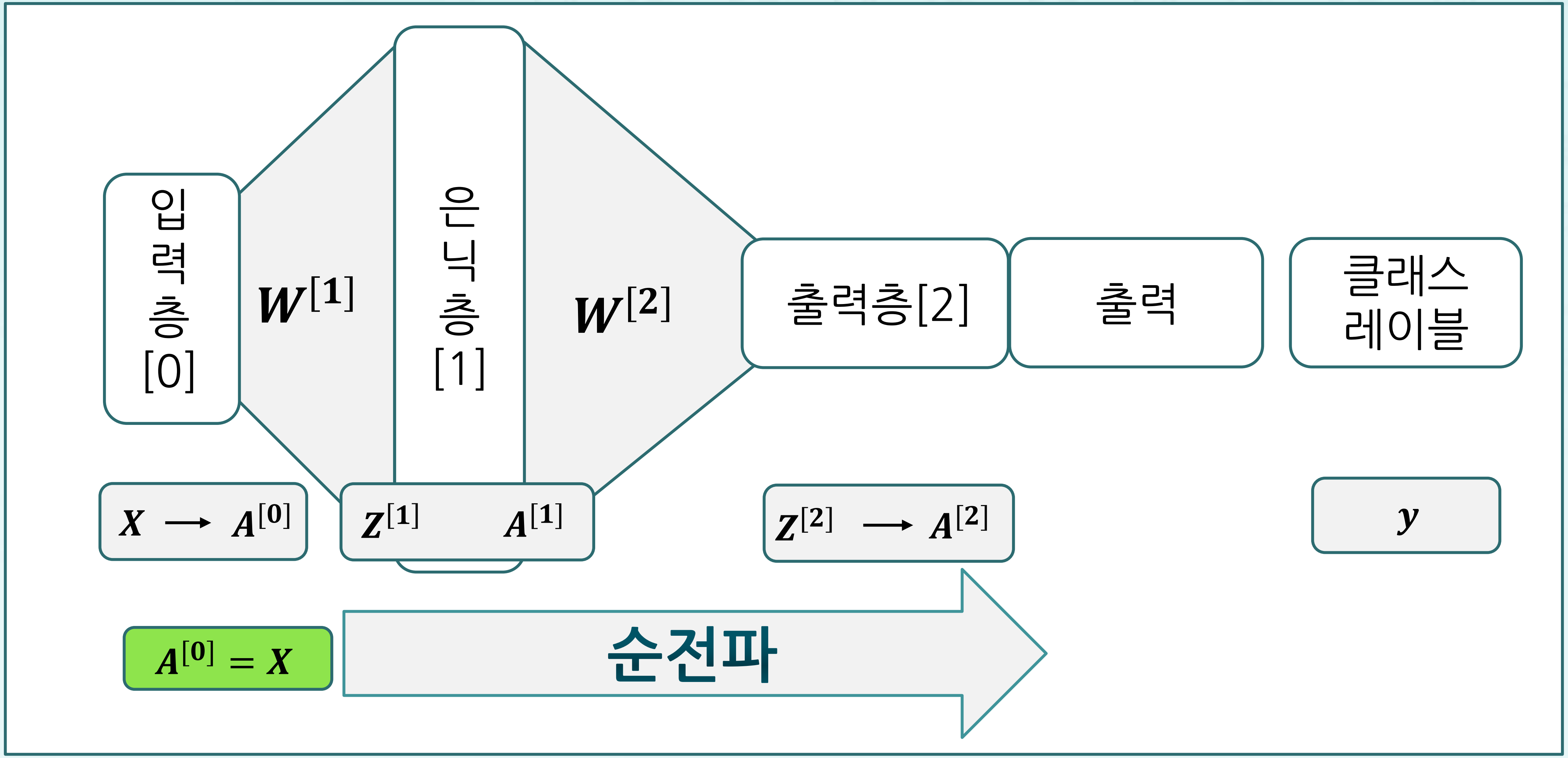
다층 인공신경망 행렬 모델: 입력과 출력



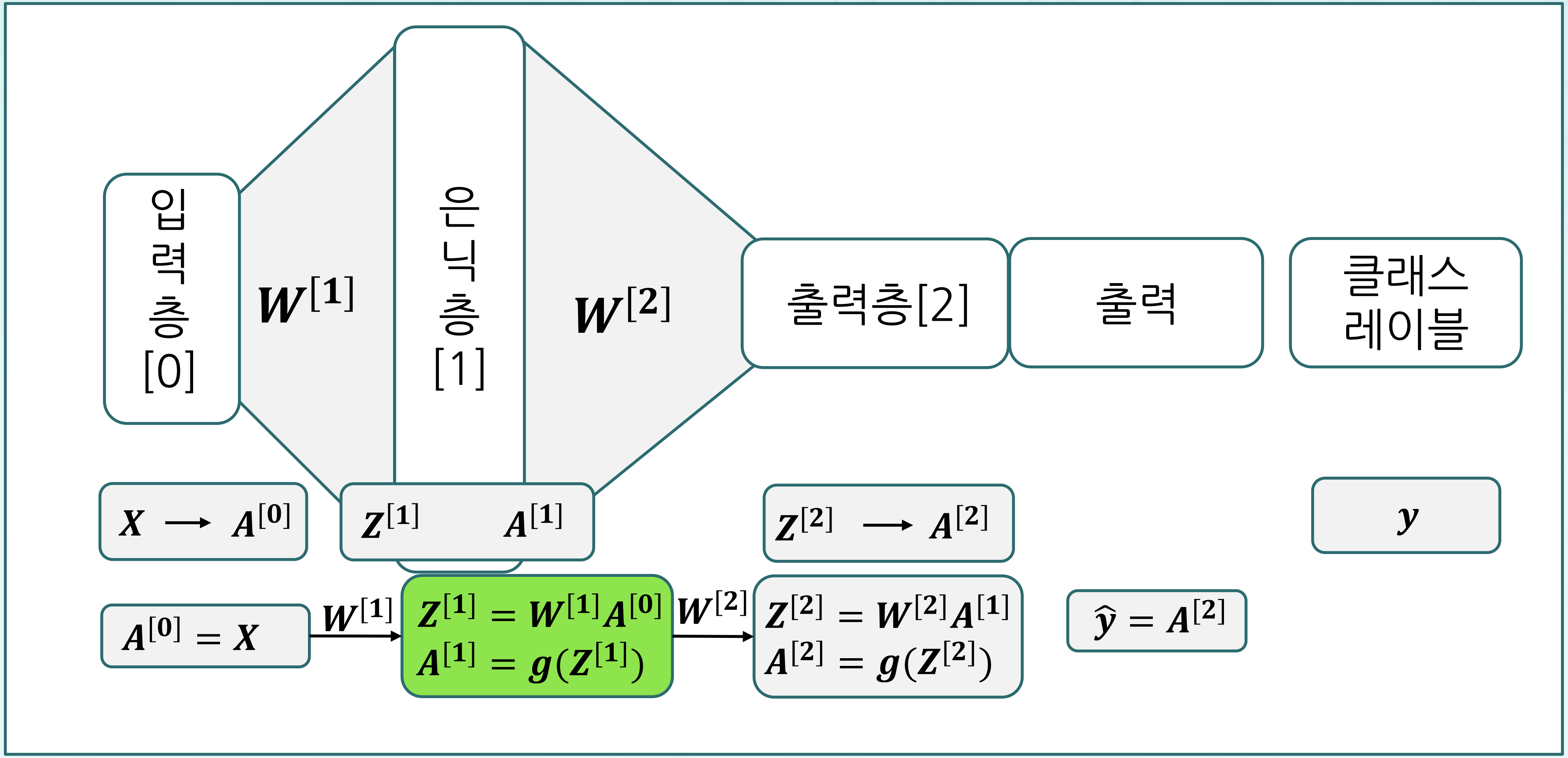
다층 인공신경망 행렬 모델: 입력과 출력



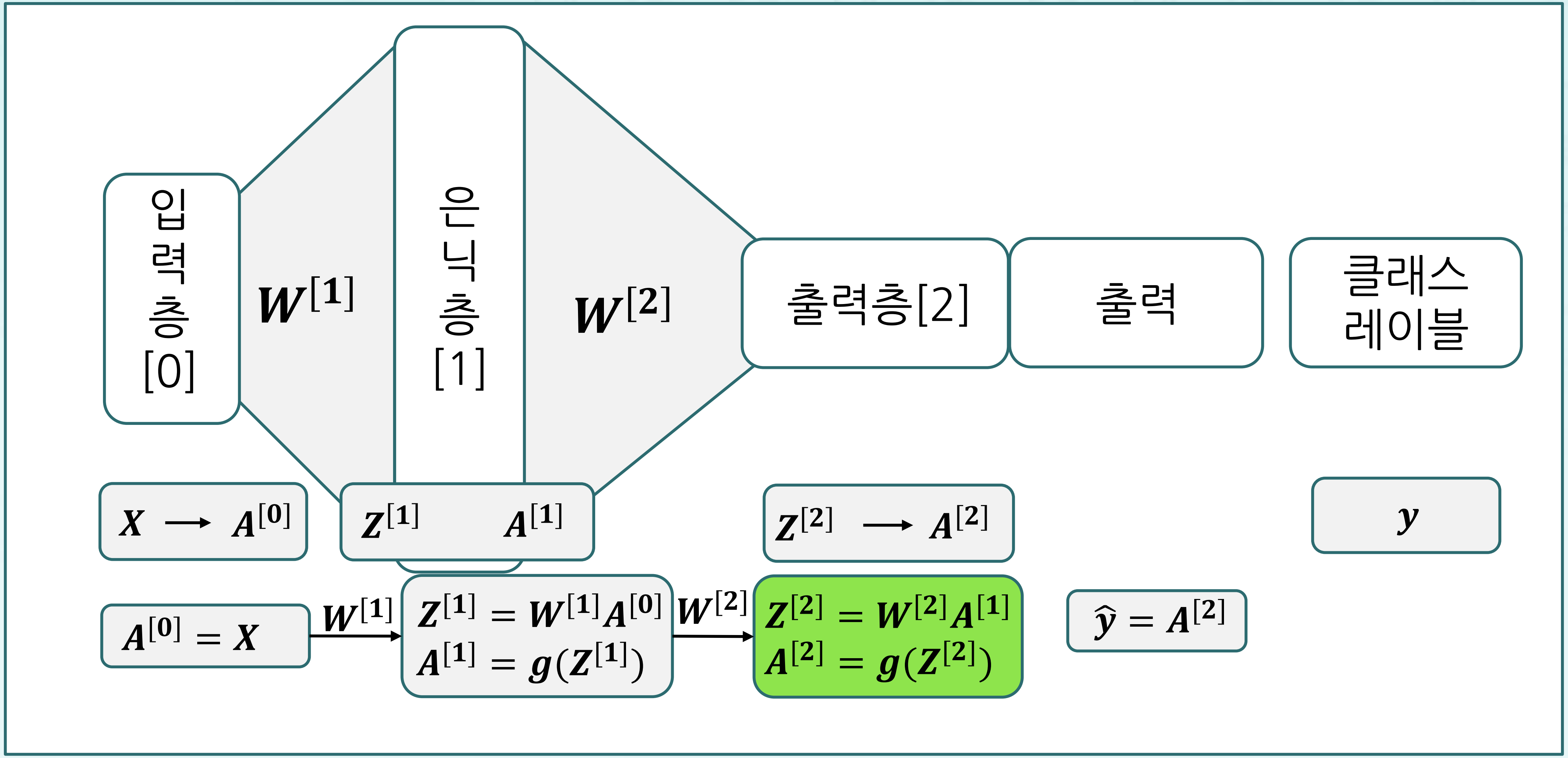
다층 인공신경망 행렬 모델: 순전파



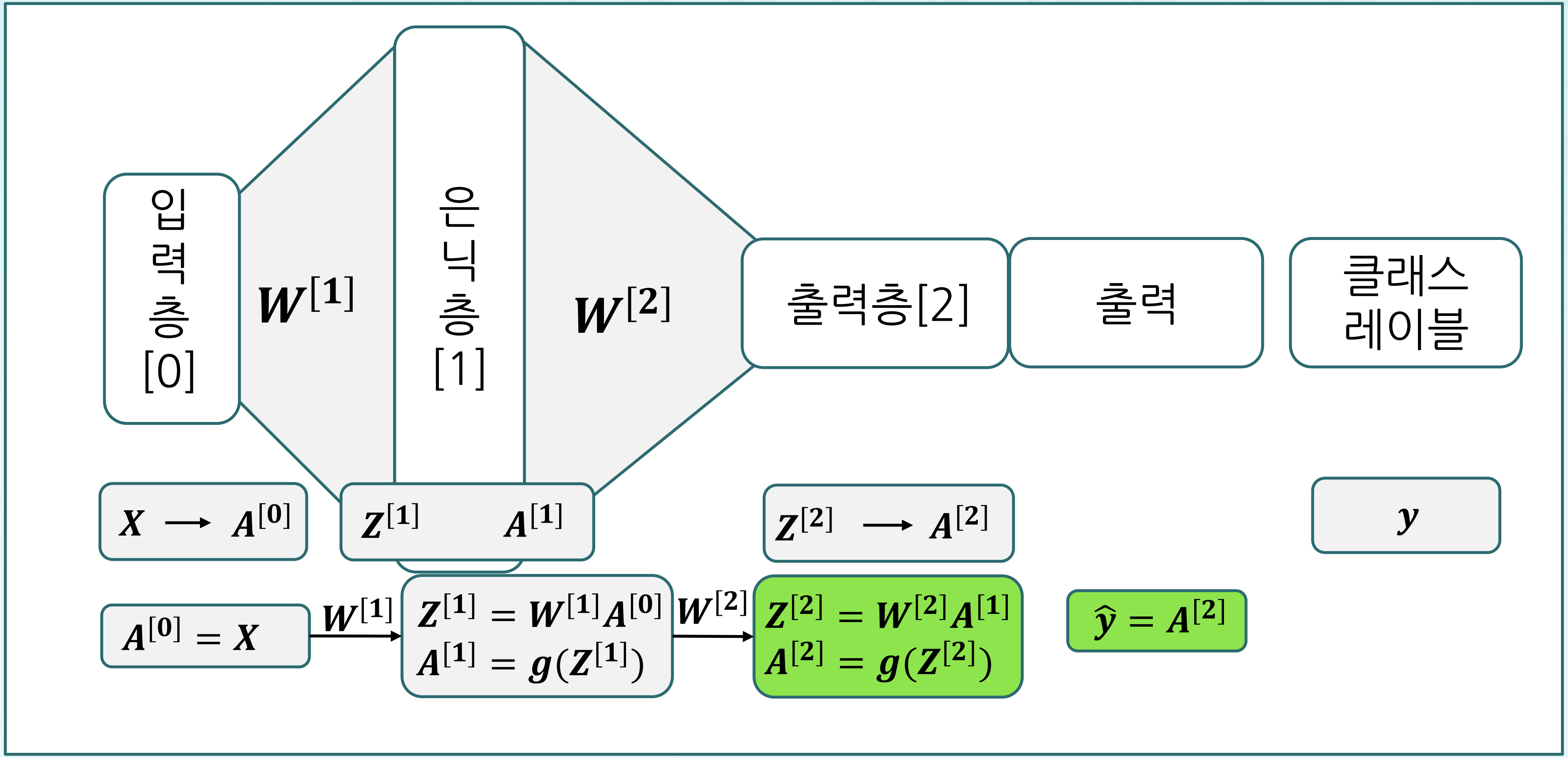
다층 인공신경망 행렬 모델: 순전파



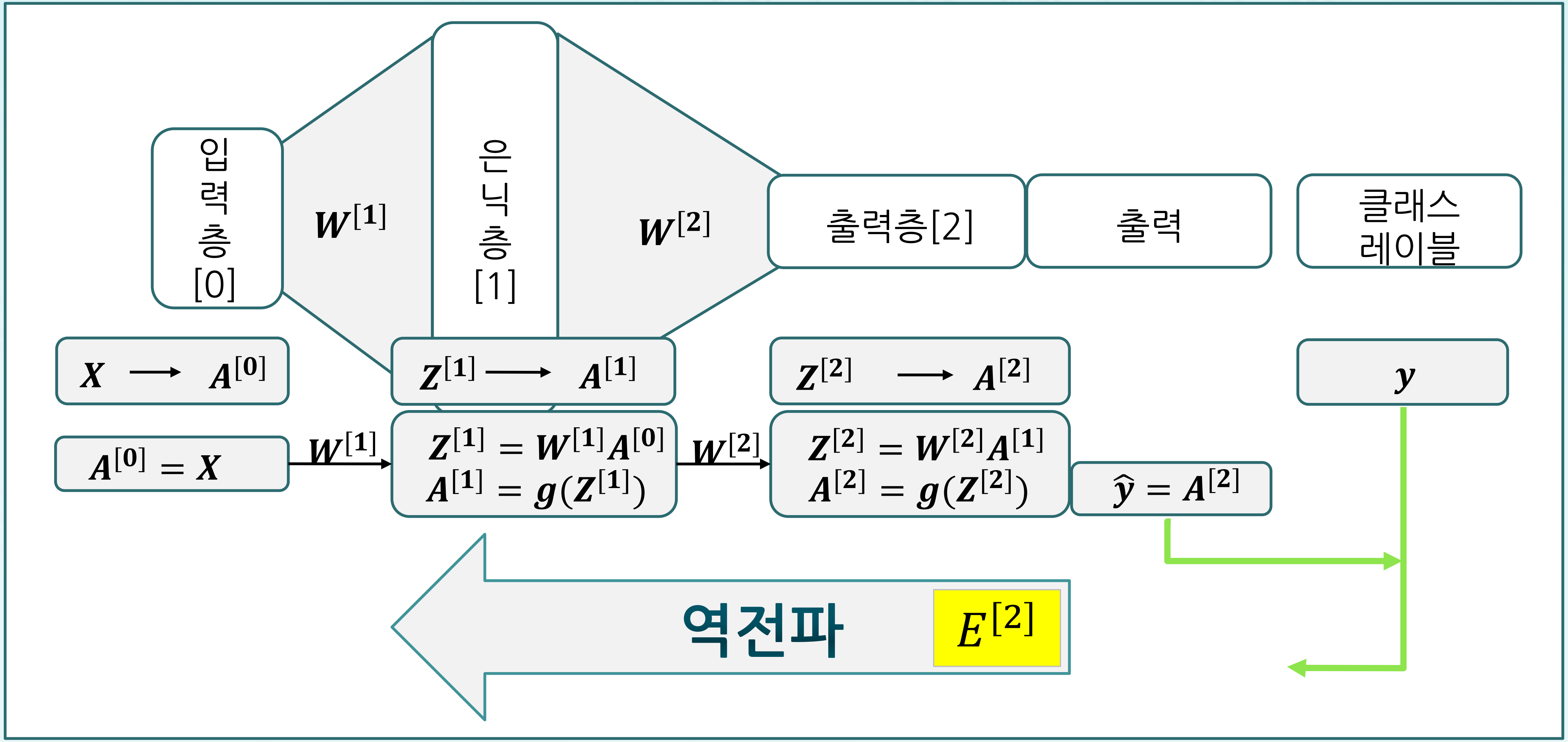
다층 인공신경망 행렬 모델: 순전파



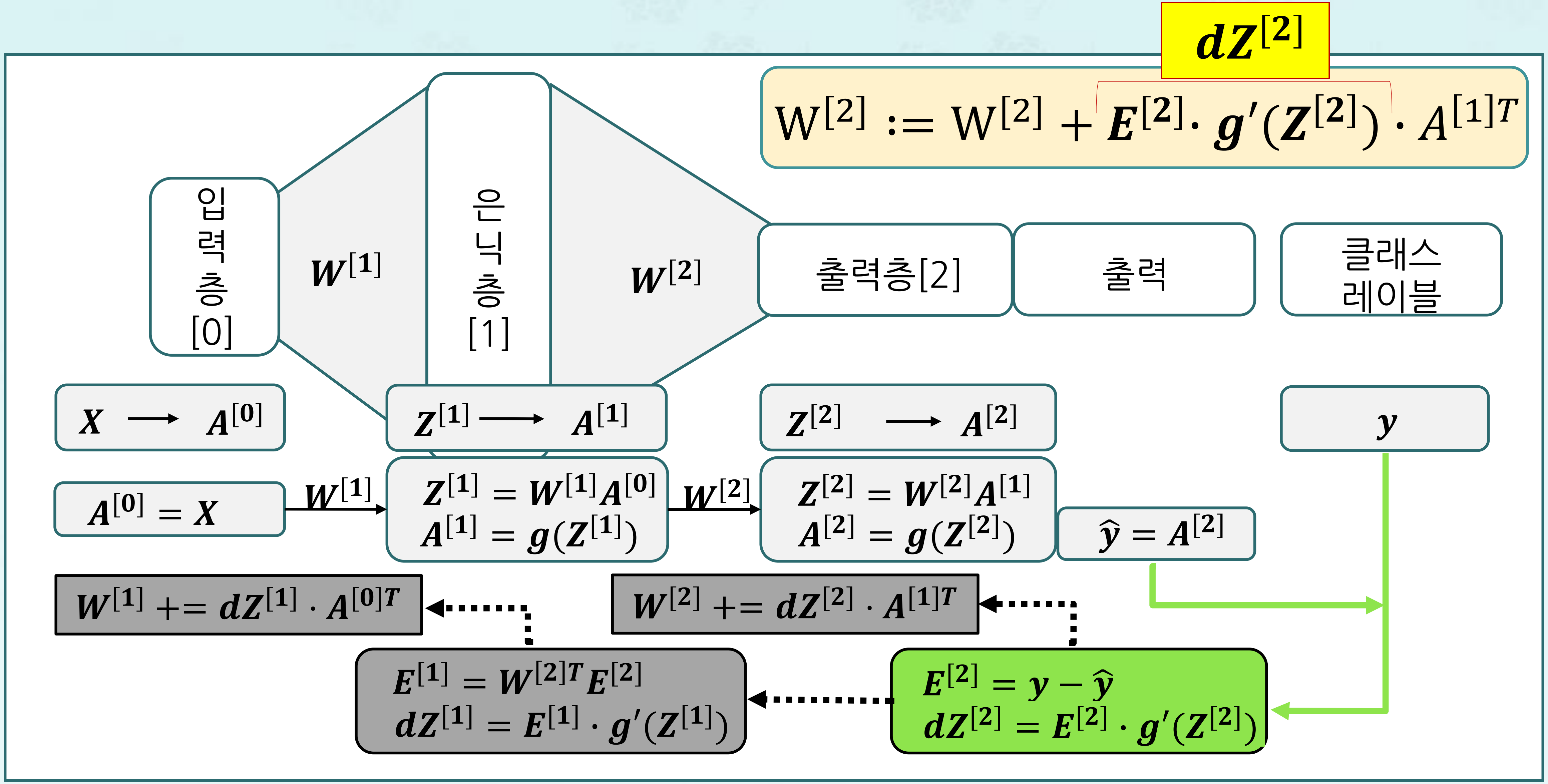
다층 인공신경망 행렬 모델: 순전파



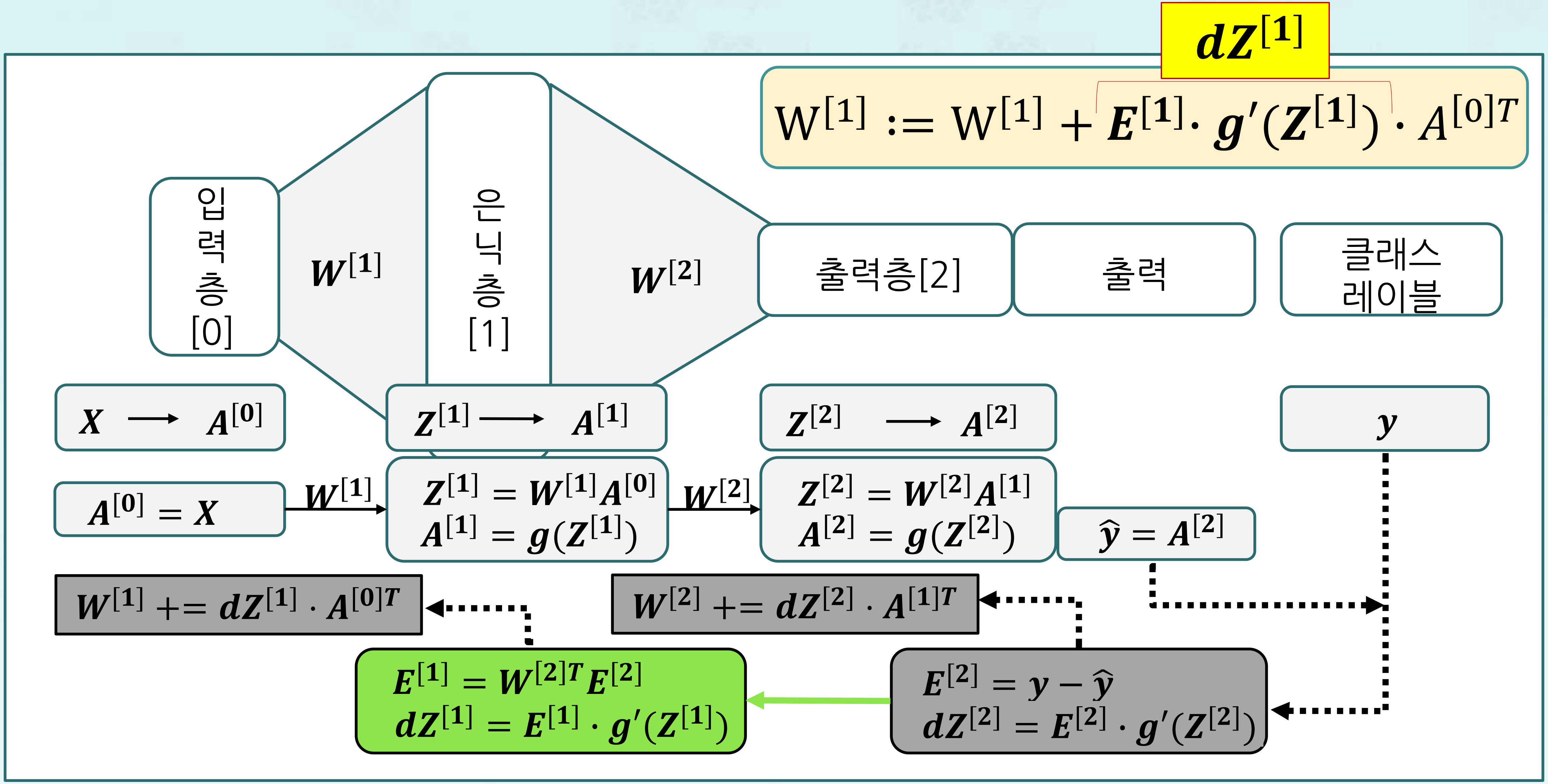
다층 인공신경망 행렬 모델: 역전파



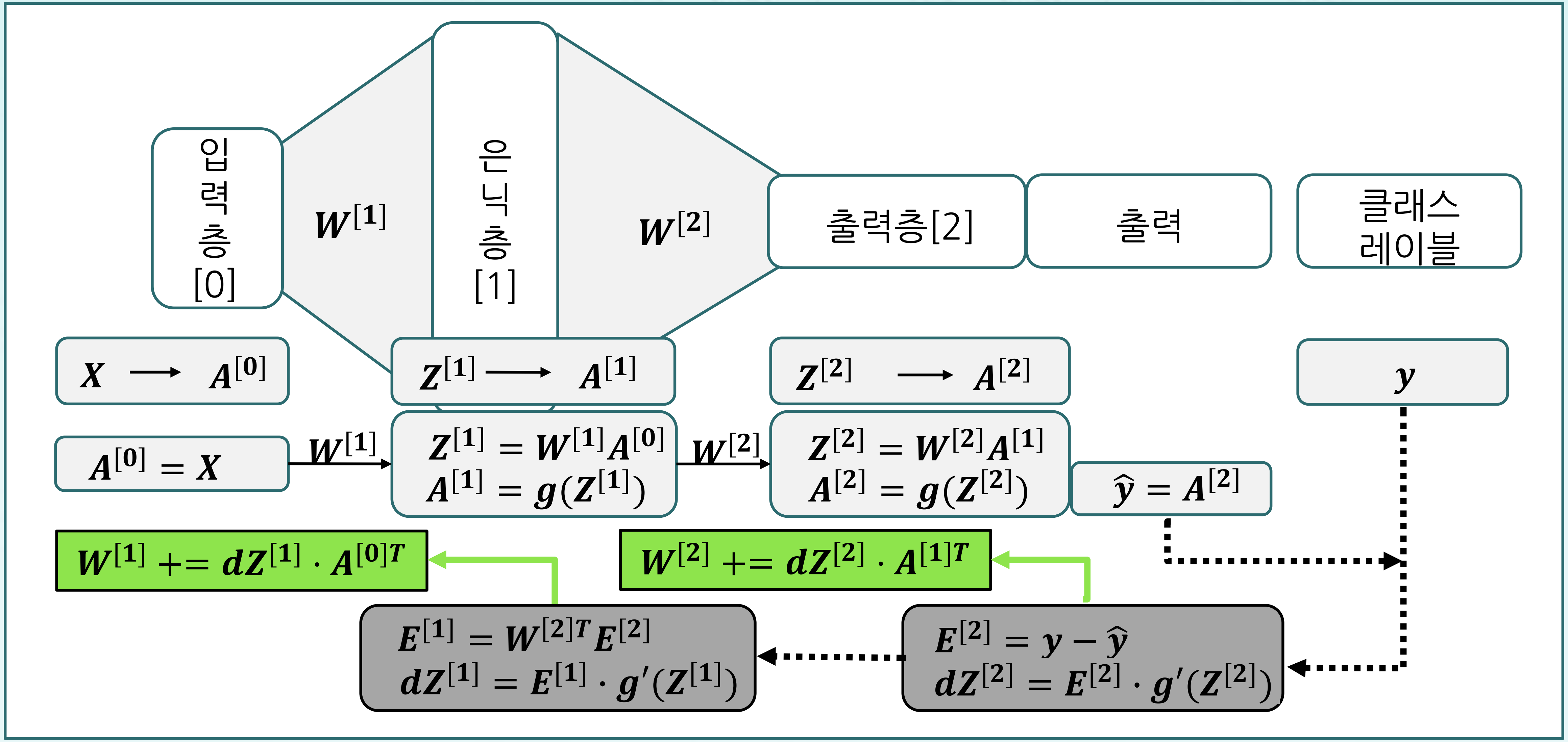
다층 인공신경망 행렬 모델: 역전파



다층 인공신경망 행렬 모델: 역전파



다층 인공신경망 행렬 모델: 역전파

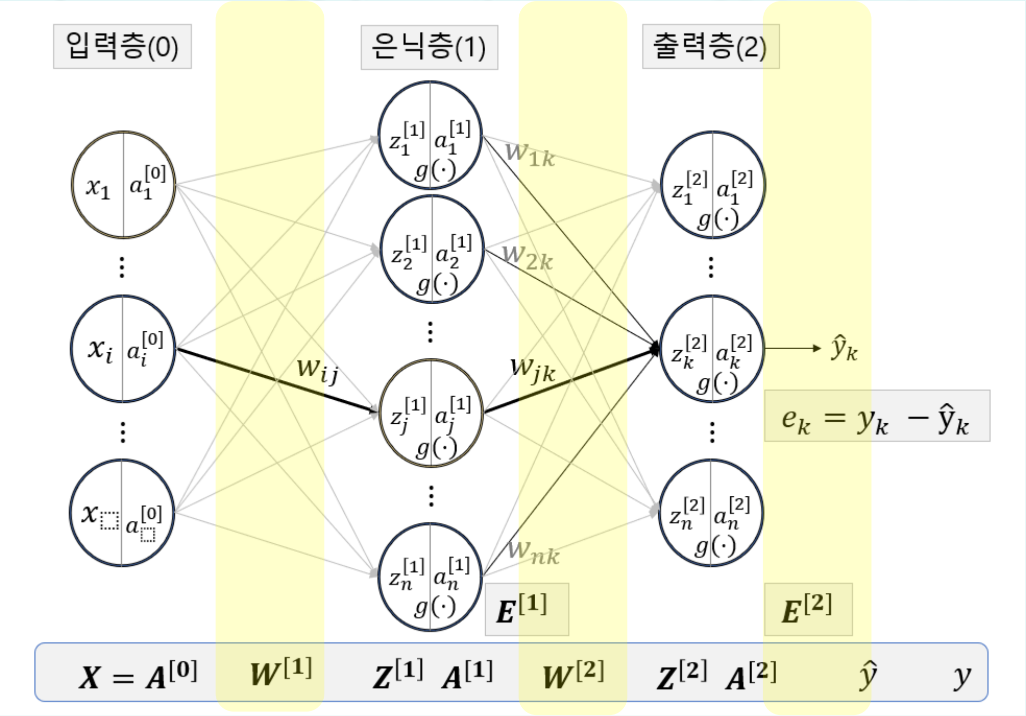


역전파 2: 역전파의 가중치 조정

- 최종 결과

$$
\begin{aligned}
W^{[2]} &:= W^{[2]} - \alpha \Delta W^{[2]} \\
&= W^{[2]} - \alpha \frac{\partial E}{\partial W^{[2]}} \\
&= \mathrm{W}^{[2]} + \boxed{\alpha E^{[2]} \cdot g'(Z^{[2]}) \cdot A^{[1]T}}
\end{aligned}
$$

$$
\begin{aligned}
W^{[1]} &:= W^{[1]} - \alpha \Delta W^{[1]} \\
&= W^{[1]} - \alpha \frac{\partial E}{\partial W^{[1]}} \\
&= \mathrm{W}^{[1]} + \boxed{\alpha E^{[1]} \cdot g'(Z^{[1]}) \cdot A^{[0]T}}
\end{aligned}
$$



다층 신경망의 행렬 모델링

- 학습 정리
  - 미분에서 연쇄법칙이 무엇인지 이해하기
  - 오차함수의 행렬로 미분하기
  - 다층 인공 신경망의 행렬 모델을 학습하기

- 차시 예고
  - **10-2** 로지스틱 회귀

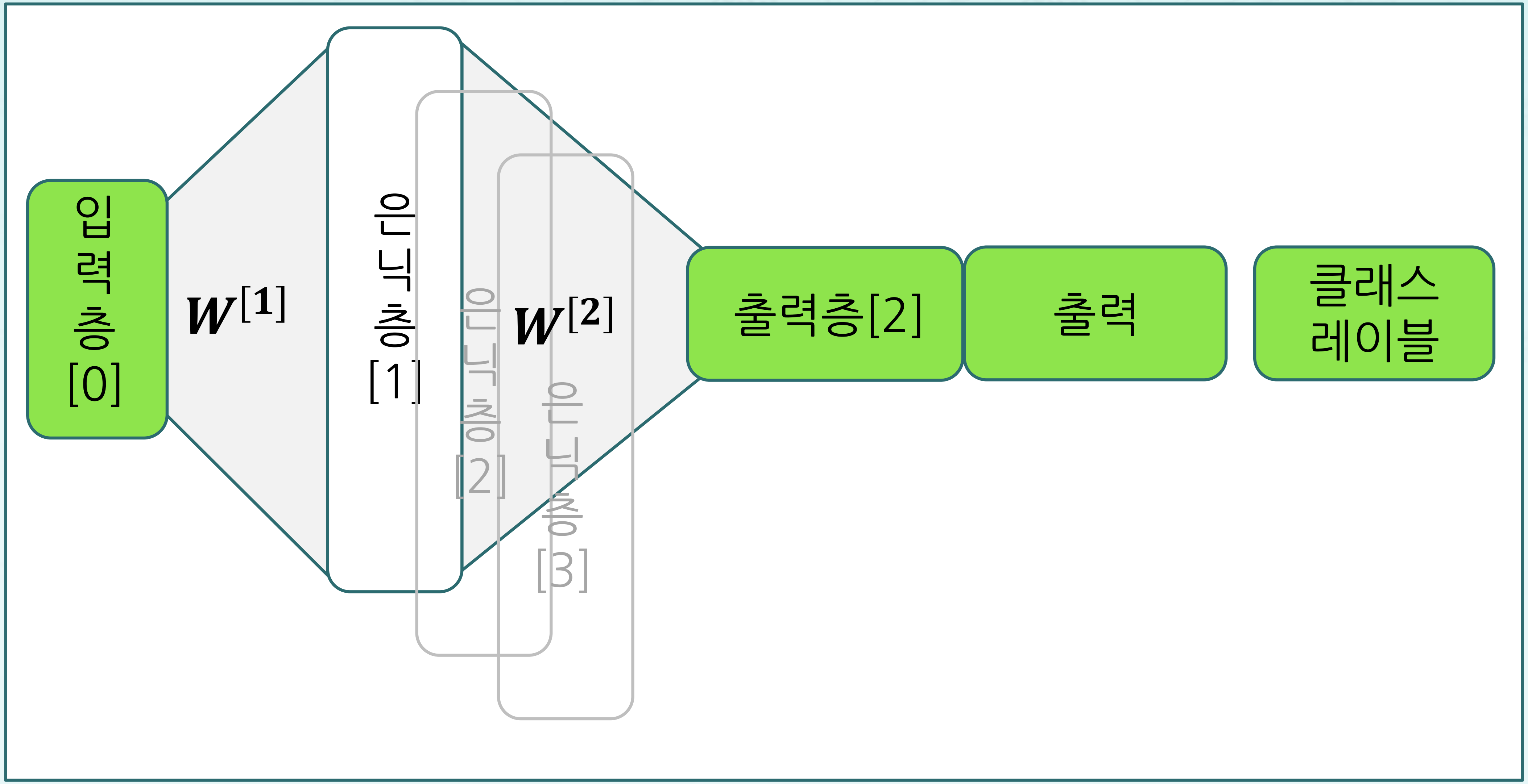
11주차(1/3)

다층 신경망의 행렬 모델링

파이썬으로 배우는 기계학습

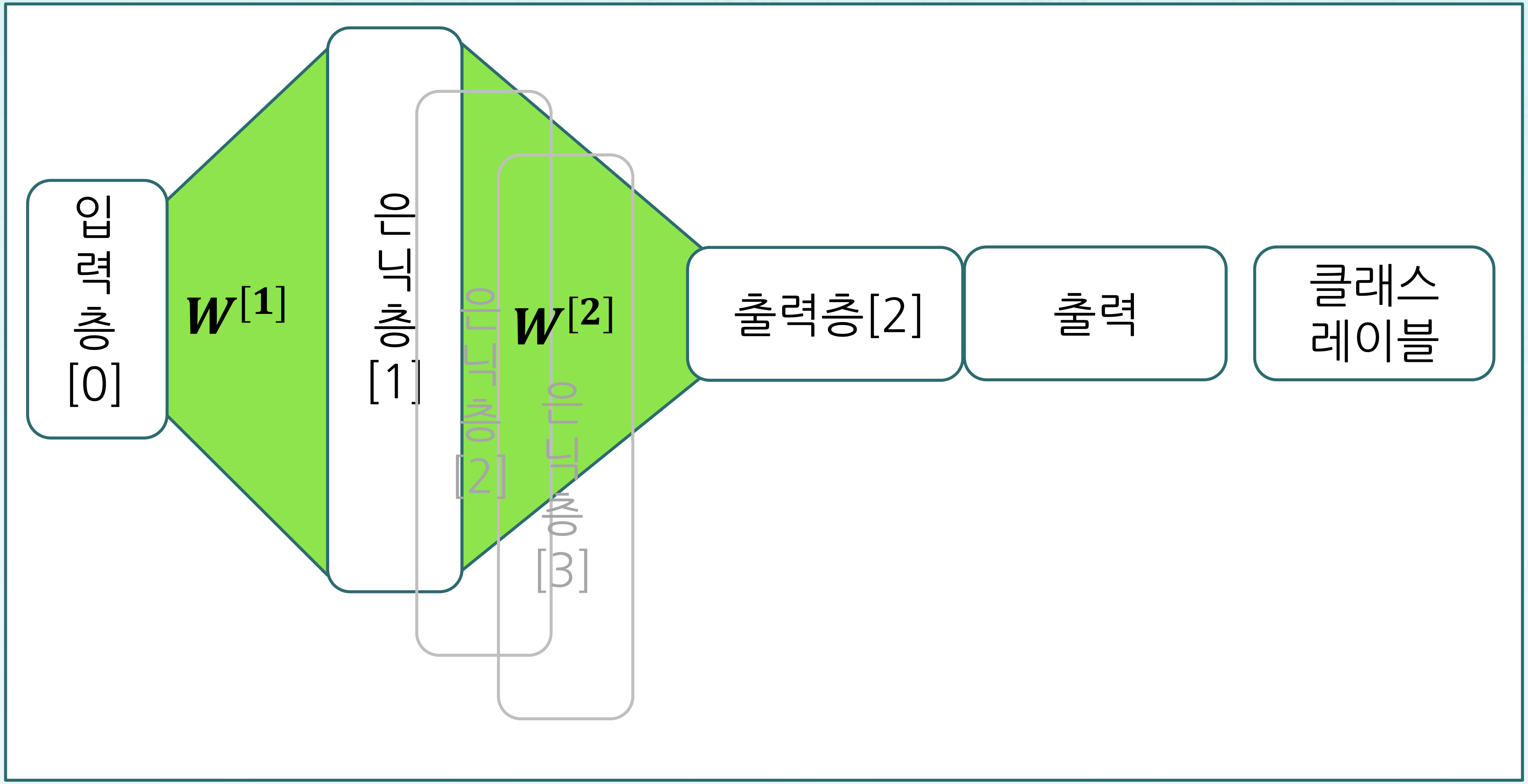
한동대학교
김영섭 교수

- 다층 신경망의 구조

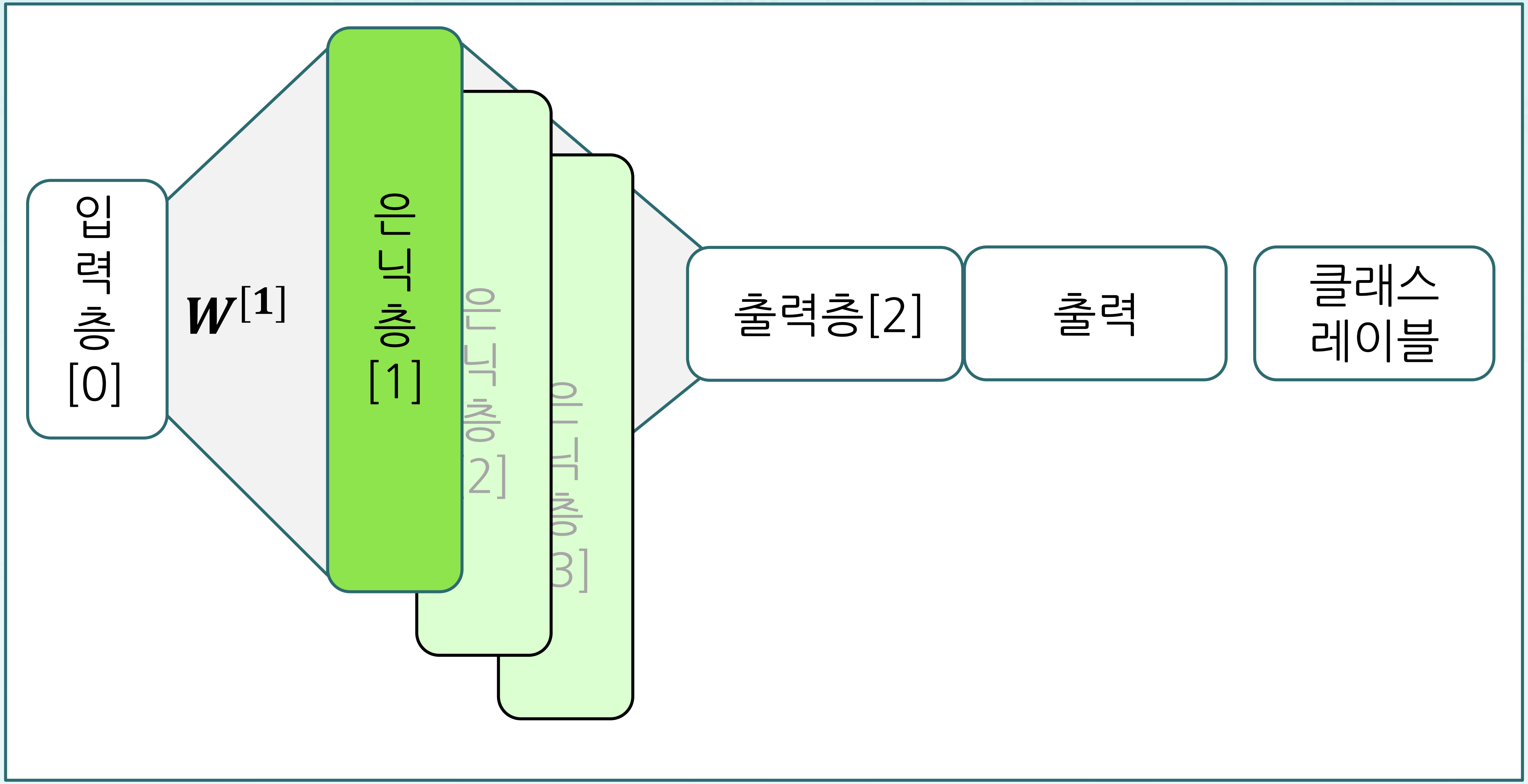


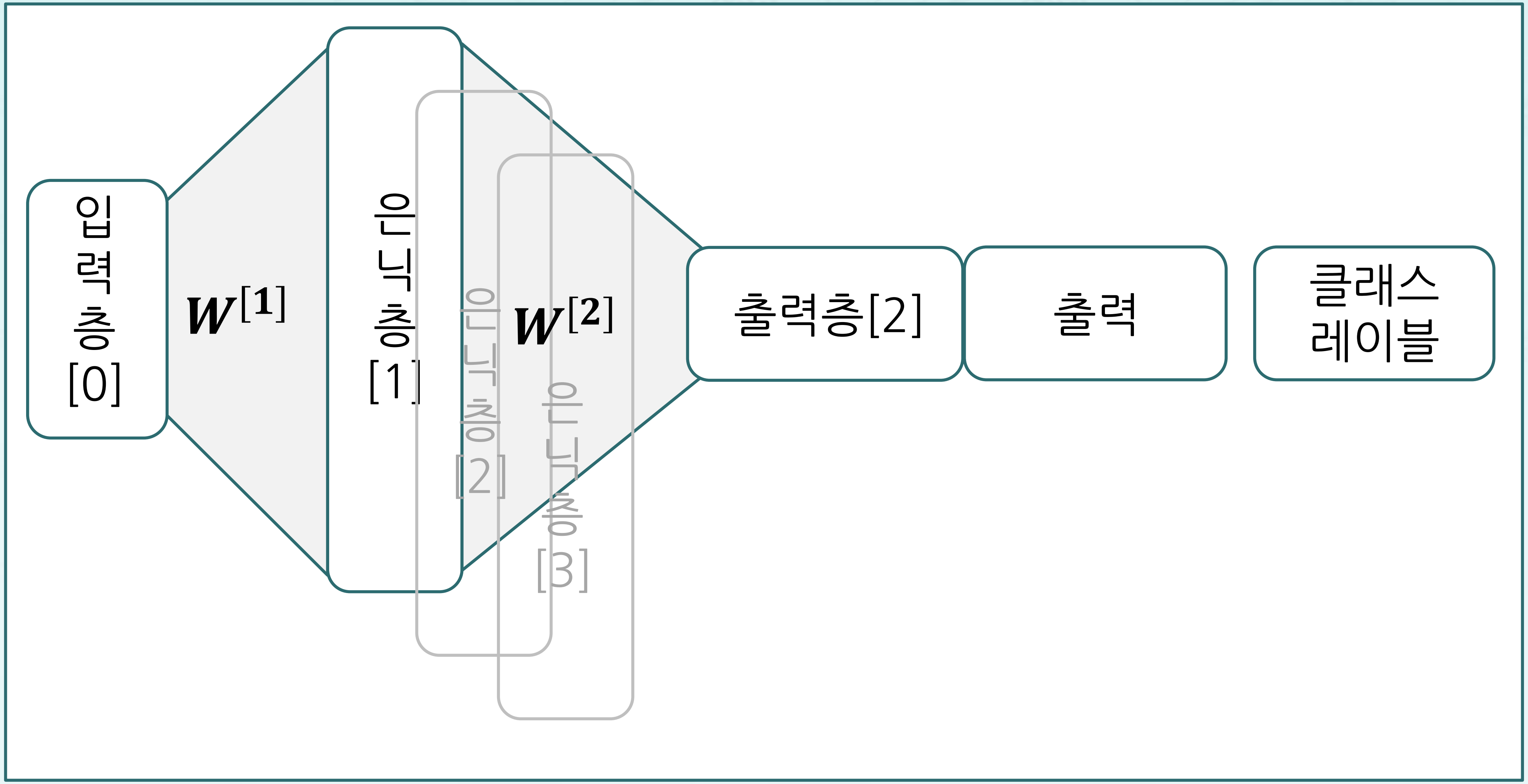
입력층 [0]
$\boldsymbol{W}^{[1]}$
은닉층 [1]
은닉층 [2]
$\boldsymbol{W}^{[2]}$
은닉층 [3]
출력층[2]
출력
클래스 레이블

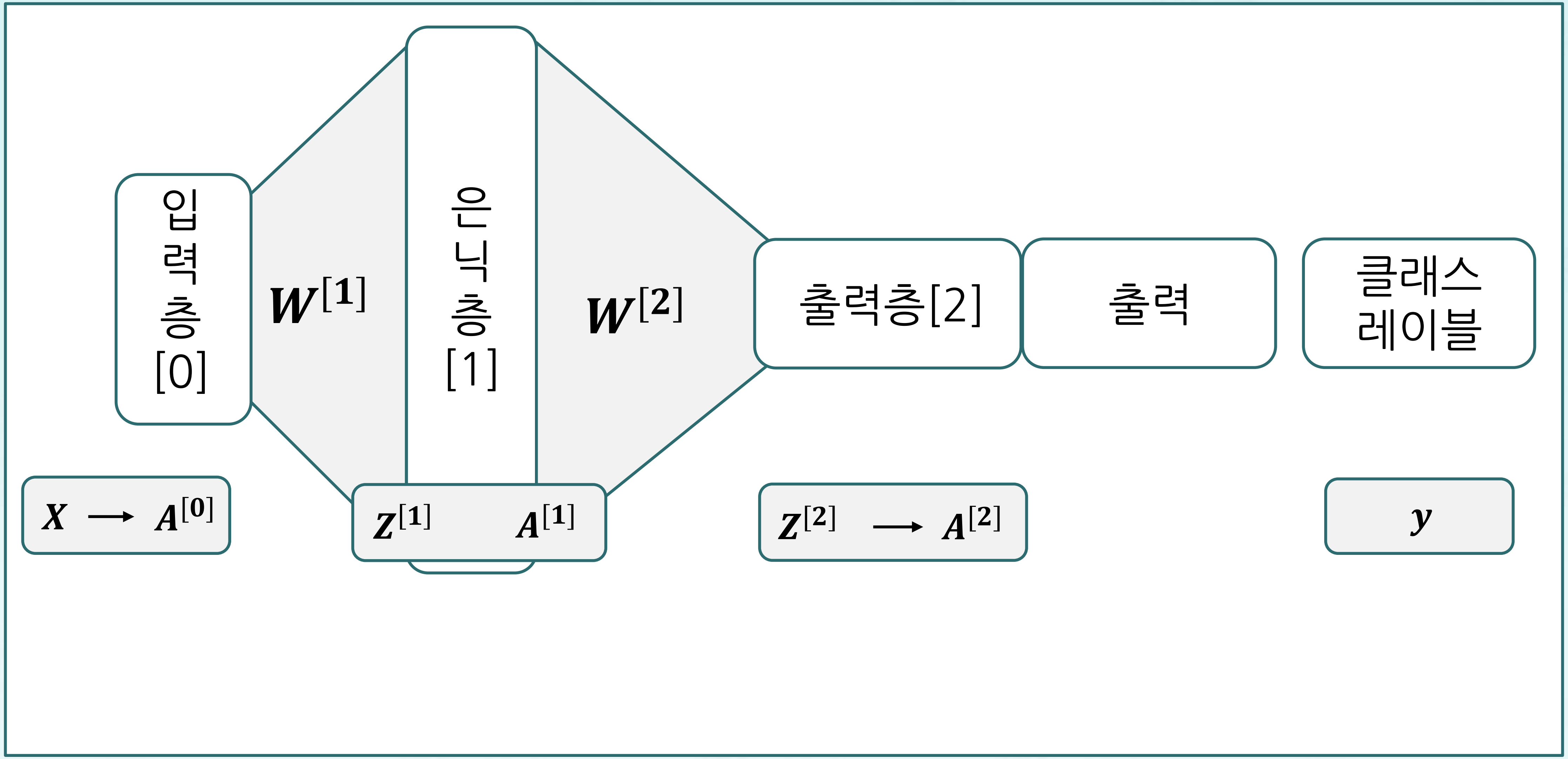
▪ 다층 신경망의 구조



- 다층 신경망의 구조

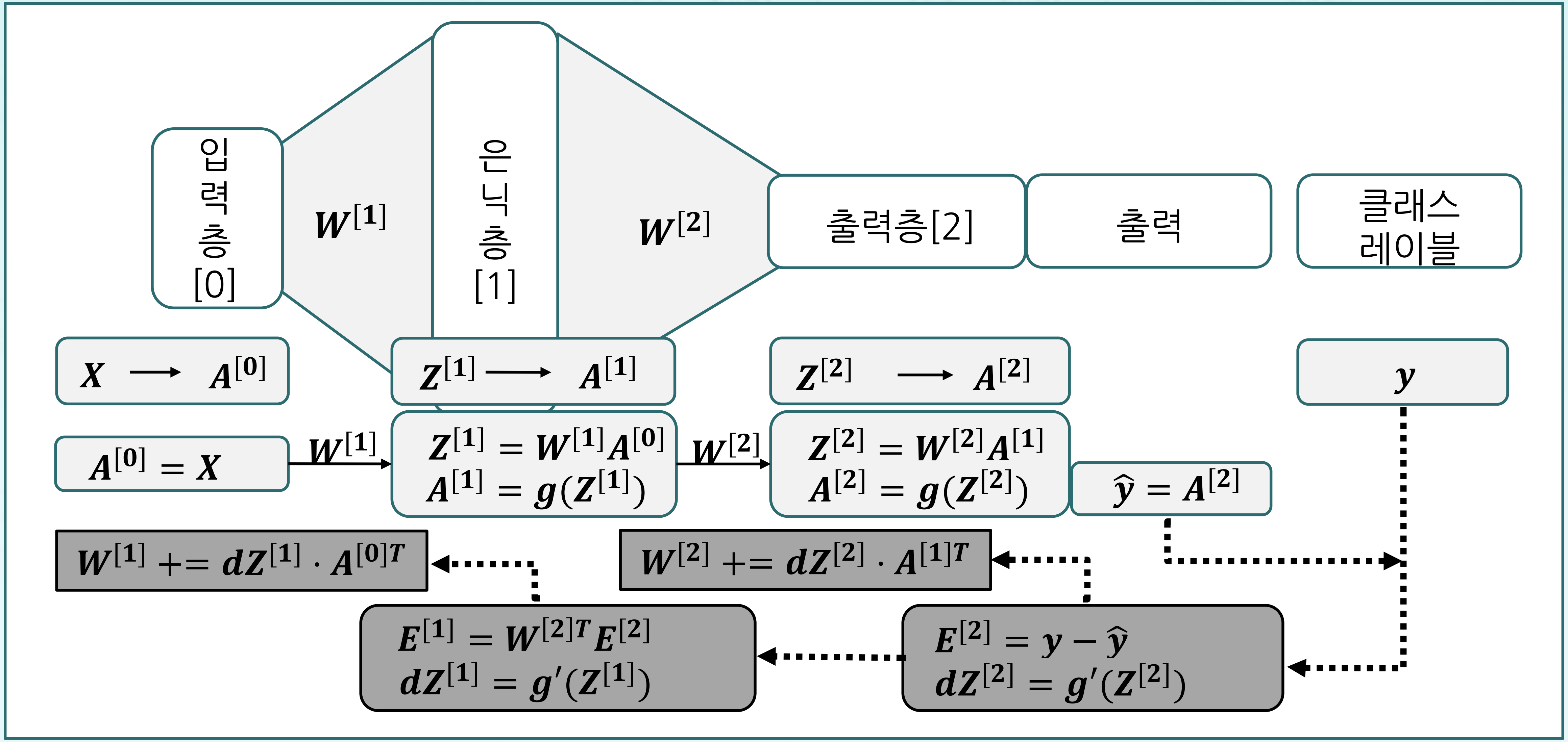



입력층[0]
W[1]
은닉층[1]
은닉층[2]
W[2]
은닉층[3]
출력층[2]
출력
클래스 레이블

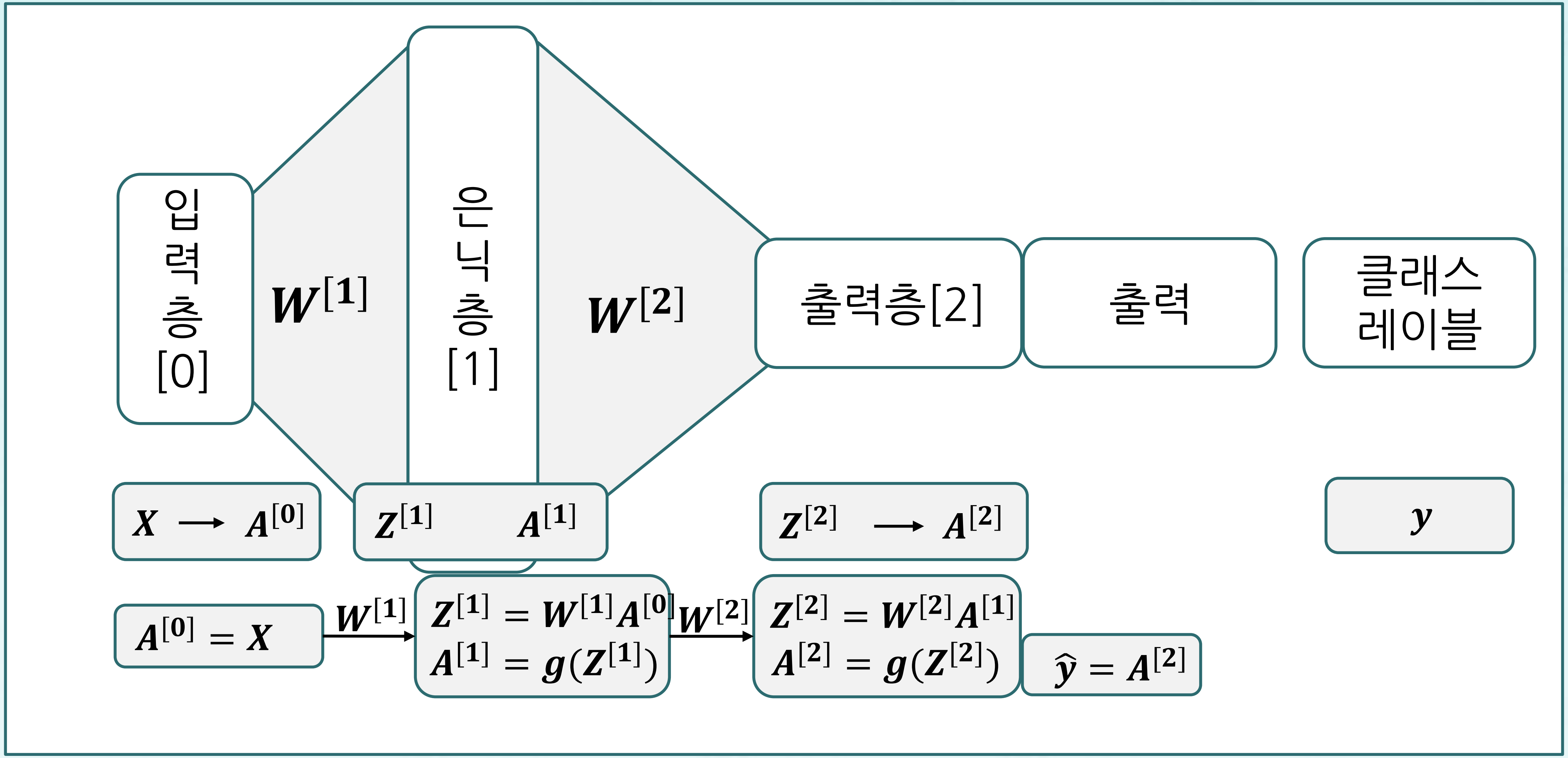


입력층[0]
$W^{[1]}$
은닉층[1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]}$ $A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y

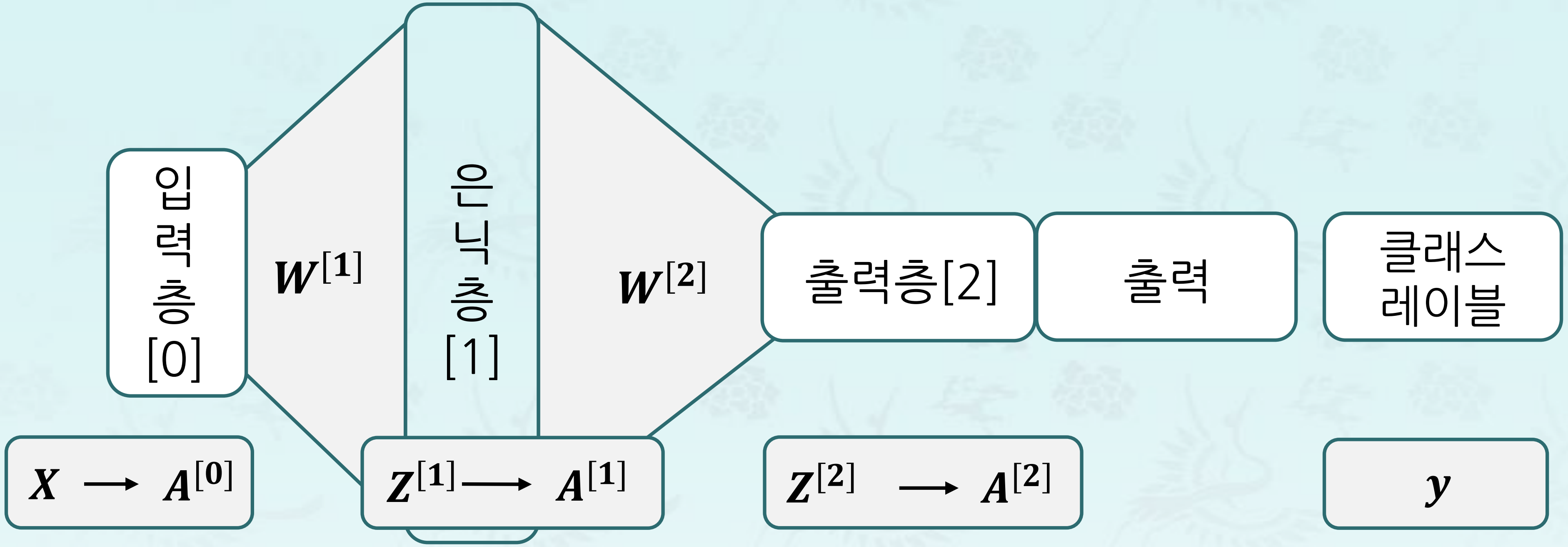
▪ 역전파 모델링

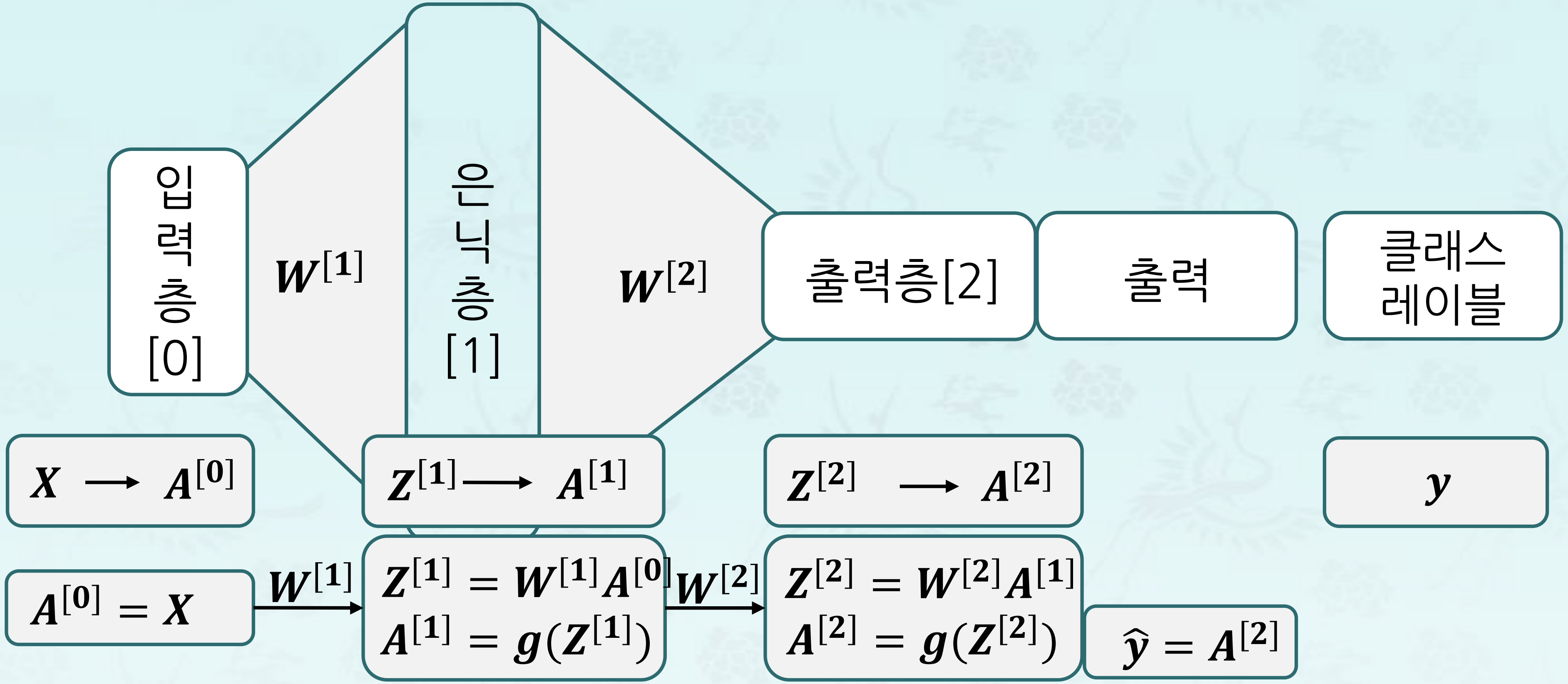


입력층 [0]
$W^{[1]}$
은닉층 [1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]} \longrightarrow A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y
$A^{[0]} = X$
$W^{[1]}$
$Z^{[1]} = W^{[1]}A^{[0]}$
$A^{[1]} = g(Z^{[1]})$
$W^{[2]}$
$Z^{[2]} = W^{[2]}A^{[1]}$
$A^{[2]} = g(Z^{[2]})$
$\hat{y} = A^{[2]}$
$W^{[1]} += dZ^{[1]} \cdot A^{[0]T}$
$W^{[2]} += dZ^{[2]} \cdot A^{[1]T}$
$E^{[1]} = W^{[2]T}E^{[2]}$
$dZ^{[1]} = g'(Z^{[1]})$
$E^{[2]} = y - \hat{y}$
$dZ^{[2]} = g'(Z^{[2]})$

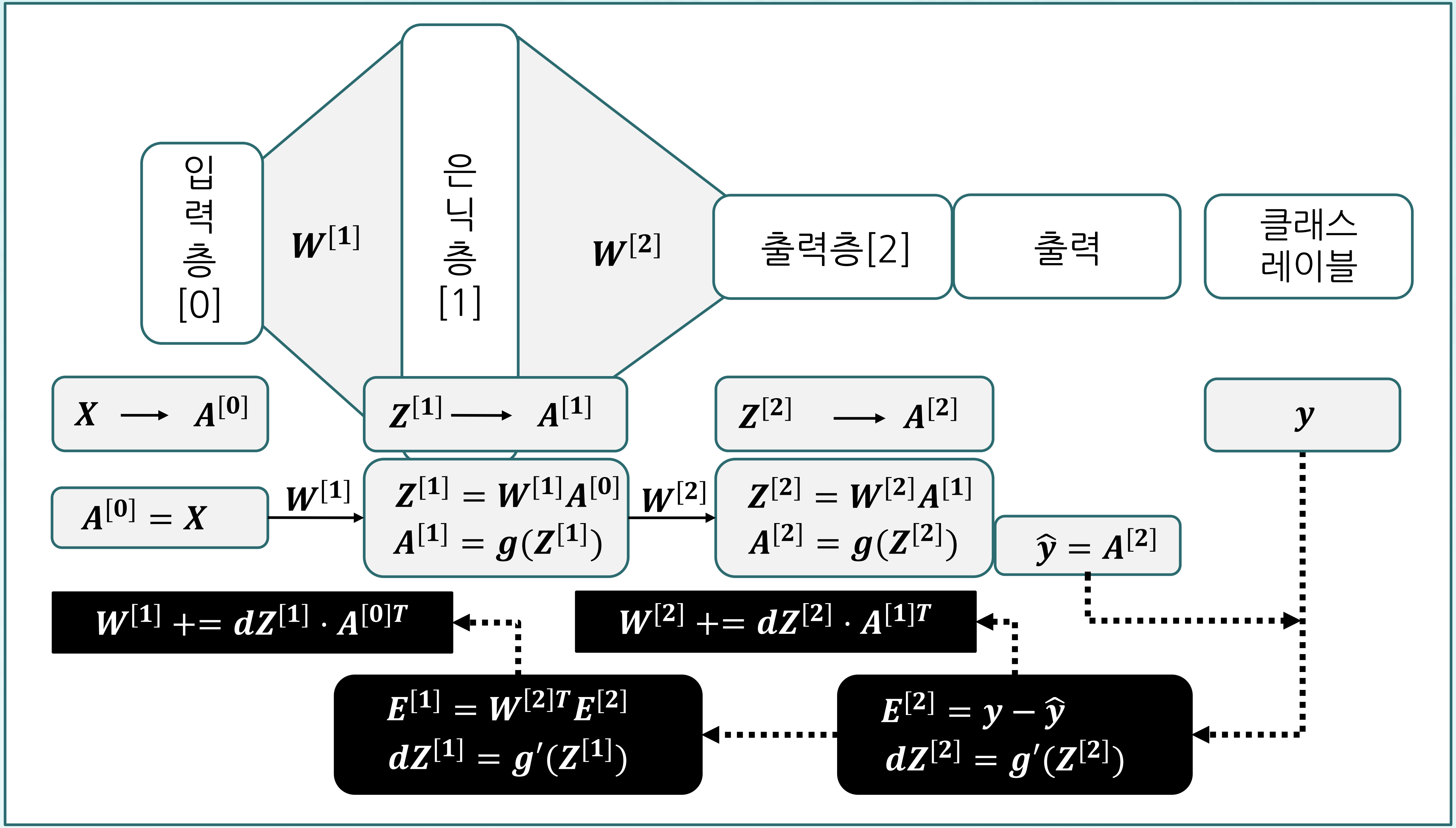


입력층[0]
$W^{[1]}$
은닉층[1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]}$ $A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y
$A^{[0]} = X$
$W^{[1]}$
$Z^{[1]} = W^{[1]}A^{[0]}$
$A^{[1]} = g(Z^{[1]})$
$W^{[2]}$
$Z^{[2]} = W^{[2]}A^{[1]}$
$A^{[2]} = g(Z^{[2]})$
$\hat{y} = A^{[2]}$


입력층[0]
$W^{[1]}$
은닉층[1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]} \longrightarrow A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y



입력층[0]
$W^{[1]}$
은닉층[1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]} \longrightarrow A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y
$A^{[0]} = X$
$W^{[1]}$
$Z^{[1]} = W^{[1]}A^{[0]}$
$A^{[1]} = g(Z^{[1]})$
$W^{[2]}$
$Z^{[2]} = W^{[2]}A^{[1]}$
$A^{[2]} = g(Z^{[2]})$
$\hat{y} = A^{[2]}$



입력층[0]
$W^{[1]}$
은닉층[1]
$W^{[2]}$
출력층[2]
출력
클래스 레이블
$X \longrightarrow A^{[0]}$
$Z^{[1]} \longrightarrow A^{[1]}$
$Z^{[2]} \longrightarrow A^{[2]}$
y
$A^{[0]} = X$
$W^{[1]}$
$Z^{[1]} = W^{[1]}A^{[0]}$
$A^{[1]} = g(Z^{[1]})$
$W^{[2]}$
$Z^{[2]} = W^{[2]}A^{[1]}$
$A^{[2]} = g(Z^{[2]})$
$\hat{y} = A^{[2]}$
$W^{[1]} += dZ^{[1]} \cdot A^{[0]T}$
$W^{[2]} += dZ^{[2]} \cdot A^{[1]T}$
$E^{[1]} = W^{[2]T}E^{[2]}$
$dZ^{[1]} = g'(Z^{[1]})$
$E^{[2]} = y - \hat{y}$
$dZ^{[2]} = g'(Z^{[2]})$