

Foundations of Software Fall 2022

Week 7

1

Plan

PREVIOUSLY: unit, sequencing, let, pairs, tuples

TODAY:

1. options, variants
2. recursion
3. state

NEXT: exceptions?

NEXT: polymorphic (not so simple) typing

2

Records

| | |
|---------------------------------|------------------------|
| $t ::= \dots$ | <i>terms</i> |
| $\{l_i = t_i \mid i \in 1..n\}$ | <i>record</i> |
| $t.l$ | <i>projection</i> |
| $v ::= \dots$ | <i>values</i> |
| $\{l_i = v_i \mid i \in 1..n\}$ | <i>record value</i> |
| $T ::= \dots$ | <i>types</i> |
| $\{l_i : T_i \mid i \in 1..n\}$ | <i>type of records</i> |

3

Evaluation rules for records

$$\{l_i = v_i \mid i \in 1..n\}.l_j \longrightarrow v_j \quad (\text{E-PROJRCD})$$

$$\frac{t_1 \longrightarrow t'_1}{t_1.l \longrightarrow t'_1.l} \quad (\text{E-PROJ})$$

$$\frac{t_j \longrightarrow t'_j}{\{l_i = v_i \mid i \in 1..j-1, l_j = t_j, l_k = t_k \mid k \in j+1..n\} \longrightarrow \{l_i = v_i \mid i \in 1..j-1, l_j = t'_j, l_k = t_k \mid k \in j+1..n\}} \quad (\text{E-RCD})$$

4

Typing rules for records

$$\frac{\text{for each } i \quad \Gamma \vdash t_i : T_i}{\Gamma \vdash \{l_i = t_i \mid i \in 1..n\} : \{l_i : T_i \mid i \in 1..n\}} \quad (\text{T-RCD})$$

$$\frac{\Gamma \vdash t_1 : \{l_i : T_i \mid i \in 1..n\}}{\Gamma \vdash t_1.l_j : T_j} \quad (\text{T-PROJ})$$

5

Sums and variants

6

Sums – motivating example

```
PhysicalAddr = {firstlast:String, addr:String}
VirtualAddr  = {name:String, email:String}
Addr         = PhysicalAddr + VirtualAddr
inl  : "PhysicalAddr → PhysicalAddr+VirtualAddr"
inr  : "VirtualAddr  → PhysicalAddr+VirtualAddr"
```

```
getName = λa:Addr.
  case a of
    inl x ⇒ x.firstlast
  | inr y ⇒ y.name;
```

7

New syntactic forms

| | |
|-----------------------------|----------------------|
| $t ::= \dots$ | <i>terms</i> |
| inl t | tagging (left) |
| inr t | tagging (right) |
| case t of inl x⇒t inr x⇒t | case |
| $v ::= \dots$ | <i>values</i> |
| inl v | tagged value (left) |
| inr v | tagged value (right) |
| $T ::= \dots$ | <i>types</i> |
| T+T | sum type |

T_1+T_2 is a *disjoint union* of T_1 and T_2 (the tags `inl` and `inr` ensure disjointness)

8

New evaluation rules

$t \rightarrow t'$

$\text{case } (\text{inl } v_0) \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2 \rightarrow [x_1 \mapsto v_0]t_1$ (E-CASEINL)

$\text{case } (\text{inr } v_0) \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2 \rightarrow [x_2 \mapsto v_0]t_2$ (E-CASEINR)

$\frac{t_0 \rightarrow t'_0}{\text{case } t_0 \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2 \rightarrow \text{case } t'_0 \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2}$ (E-CASE)

$\frac{t_1 \rightarrow t'_1}{\text{inl } t_1 \rightarrow \text{inl } t'_1}$ (E-INL)

$\frac{t_1 \rightarrow t'_1}{\text{inr } t_1 \rightarrow \text{inr } t'_1}$ (E-INR)

9

New typing rules

$\Gamma \vdash t : T$

$\frac{\Gamma \vdash t_1 : T_1}{\Gamma \vdash \text{inl } t_1 : T_1 + T_2}$ (T-INL)

$\frac{\Gamma \vdash t_1 : T_2}{\Gamma \vdash \text{inr } t_1 : T_1 + T_2}$ (T-INR)

$\frac{\Gamma \vdash t_0 : T_1 + T_2 \quad \Gamma, x_1 : T_1 \vdash t_1 : T \quad \Gamma, x_2 : T_2 \vdash t_2 : T}{\Gamma \vdash \text{case } t_0 \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2 : T}$ (T-CASE)

10

Sums and Uniqueness of Types

Problem:

If t has type T , then $\text{inl } t$ has type $T+U$ for every U . i.e.,

we've lost uniqueness of types.

Possible solutions:

- "Infer" U as needed during typechecking
- Give constructors different names and only allow each name to appear in one sum type (requires generalization to "variants," which we'll see next) — OCaml's solution
- Annotate each `inl` and `inr` with the intended sum type.

For simplicity, let's choose the third.

11

New syntactic forms

$t ::= \dots$
 $\text{inl } t \text{ as } T$
 $\text{inr } t \text{ as } T$

terms
tagging (left)
tagging (right)

$v ::= \dots$
 $\text{inl } v \text{ as } T$
 $\text{inr } v \text{ as } T$

values
tagged value (left)
tagged value (right)

Note that `as T` here is not the ascription operator that we saw before — i.e., not a separate syntactic form: in essence, there is an ascription "built into" every use of `inl` or `inr`.

12

New typing rules

$\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1}{\Gamma \vdash \text{inl } t_1 \text{ as } T_1+T_2 : T_1+T_2} \quad (\text{T-INL})$$

$$\frac{\Gamma \vdash t_1 : T_2}{\Gamma \vdash \text{inr } t_1 \text{ as } T_1+T_2 : T_1+T_2} \quad (\text{T-INR})$$

13

Evaluation rules ignore annotations:

$\boxed{t \longrightarrow t'}$

$$\frac{\text{case } (\text{inl } v_0 \text{ as } T_0) \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2}{\longrightarrow [x_1 \mapsto v_0] t_1} \quad (\text{E-CASEINL})$$

$$\frac{\text{case } (\text{inr } v_0 \text{ as } T_0) \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2}{\longrightarrow [x_2 \mapsto v_0] t_2} \quad (\text{E-CASEINR})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{inl } t_1 \text{ as } T_2 \longrightarrow \text{inl } t'_1 \text{ as } T_2} \quad (\text{E-INL})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{inr } t_1 \text{ as } T_2 \longrightarrow \text{inr } t'_1 \text{ as } T_2} \quad (\text{E-INR})$$

14

Variants

Just as we generalized binary products to labeled records, we can generalize binary sums to labeled *variants*.

15

New syntactic forms

| | |
|---|-------------------------|
| $t ::= \dots$ | <i>terms</i> |
| $\langle l=t \rangle \text{ as } T$ | <i>tagging</i> |
| $\text{case } t \text{ of } \langle l_i=x_i \rangle \Rightarrow t_i \quad i \in 1..n$ | <i>case</i> |
| $T ::= \dots$ | <i>types</i> |
| $\langle l_i:T_i \rangle \quad i \in 1..n$ | <i>type of variants</i> |

16

New evaluation rules

$$t \longrightarrow t'$$

case $\langle l_j = v_j \rangle$ as T of $\langle l_i = x_i \rangle \Rightarrow t_i \quad i \in 1..n$ (E-CASEVARIANT)
 $\longrightarrow [x_j \mapsto v_j] t_j$

$$\frac{t_0 \longrightarrow t'_0}{\text{case } t_0 \text{ of } \langle l_i = x_i \rangle \Rightarrow t_i \quad i \in 1..n \longrightarrow \text{case } t'_0 \text{ of } \langle l_i = x_i \rangle \Rightarrow t_i \quad i \in 1..n} \quad (\text{E-CASE})$$

$$\frac{t_i \longrightarrow t'_i}{\langle l_j = t_j \rangle \text{ as } T \longrightarrow \langle l_j = t'_j \rangle \text{ as } T} \quad (\text{E-VARIANT})$$

17

New typing rules

$$\Gamma \vdash t : T$$

$$\frac{\Gamma \vdash t_j : T_j}{\Gamma \vdash \langle l_j = t_j \rangle \text{ as } \langle l_i : T_i \quad i \in 1..n \rangle : \langle l_i : T_i \quad i \in 1..n \rangle} \quad (\text{T-VARIANT})$$

$$\frac{\begin{array}{c} \Gamma \vdash t_0 : \langle l_i : T_i \quad i \in 1..n \rangle \\ \text{for each } i \quad \Gamma, x_i : T_i \vdash t_i : T \end{array}}{\Gamma \vdash \text{case } t_0 \text{ of } \langle l_i = x_i \rangle \Rightarrow t_i \quad i \in 1..n : T} \quad (\text{T-CASE})$$

18

Example

```
Addr = <physical:PhysicalAddr, virtual:VirtualAddr>;
a = <physical=pa> as Addr;
getName = λa:Addr.
  case a of
    <physical=x> ⇒ x.firstlast
  | <virtual=y> ⇒ y.name;
```

19

Options

Just like in OCaml...

```
OptionalNat = <none:Unit, some:Nat>;

Table = Nat → OptionalNat;

emptyTable = λn:Nat. <none=unit> as OptionalNat;

extendTable =
  λt:Table. λm:Nat. λv:Nat.
    λn:Nat.
      if equal n m then <some=v> as OptionalNat
      else t n;

x = case t(5) of
  <none=u> ⇒ 999
  | <some=v> ⇒ v;
```

20

Enumerations

```
Weekday = <monday:Unit, tuesday:Unit, wednesday:Unit,  
          thursday:Unit, friday:Unit>;  
  
nextBusinessDay = λw:Weekday.  
  case w of <monday=x>   ⇒ <tuesday=unit> as Weekday  
    | <tuesday=x>       ⇒ <wednesday=unit> as Weekday  
    | <wednesday=x>    ⇒ <thursday=unit> as Weekday  
    | <thursday=x>     ⇒ <friday=unit> as Weekday  
    | <friday=x>       ⇒ <monday=unit> as Weekday;
```