

# Foundations of Software

Martin Odersky, EPFL

Slides in part adapted from:  
University of Pennsylvania CIS 500: Software Foundations - Fall 2006  
by Benjamin Pierce

1

## Course Overview

2

## What is “software foundations”?

---

Software foundations (or “theory of programming languages”) is the mathematical study of the [meaning](#) of programs.

The goal is finding ways to describe program behaviors that are both [precise](#) and [abstract](#).

- ▶ [precise](#) so that we can use mathematical tools to formalize and check interesting properties
- ▶ [abstract](#) so that properties of interest can be discussed clearly, without getting bogged down in low-level details

## Why study software foundations?

---

- ▶ To prove specific properties of particular programs (i.e., program verification)
  - ▷ Important in some domains (safety-critical systems, hardware design, security protocols, inner loops of key algorithms, ...), but still quite difficult and expensive
- ▶ To develop intuitions for *informal* reasoning about programs
- ▶ To prove general facts about all the programs in a given programming language (e.g., safety or isolation properties)
- ▶ To understand language features (and their interactions) deeply and develop principles for better language design  
([PL is the “materials science” of computer science...](#))

## What you can expect to get out of the course

---

- ▶ A more sophisticated perspective on programs, programming languages, and the activity of programming
  - ▷ See programs and whole languages as formal, mathematical objects
  - ▷ Make and prove rigorous claims about them
  - ▷ Detailed knowledge of a variety of core language features
- ▶ Deep intuitions about key language properties such as type safety
- ▶ Powerful tools for language design, description, and analysis

Most software designers are language designers!

## Greenspun's Tenth Rule Of Programming

---

Any sufficiently complicated C or Fortran program contains an ad-hoc, informally-specified, bug-ridden, slow implementation of half of Common Lisp.

– Philip Greenspun

## What this course is not

---

- ▶ An introduction to programming
- ▶ A course on functional programming (though we'll be doing some functional programming along the way)
- ▶ A course on compilers (you should already have basic concepts such as lexical analysis, parsing, abstract syntax, and scope under your belt)
- ▶ A comparative survey of many different programming languages and styles

## Approaches to Program Meaning

---

- ▶ [Denotational semantics](#) and [domain theory](#) view programs as simple mathematical objects, abstracting away their flow of control and concentrating on their input-output behavior.
- ▶ [Program logics](#) such as [Hoare logic](#) and [dependent type theories](#) focus on logical rules for reasoning about programs.
- ▶ [Operational semantics](#) describes program behaviors by means of abstract machines. This approach is somewhat lower-level than the others, but is extremely flexible.
- ▶ [Process calculi](#) focus on the communication and synchronization behaviors of complex concurrent systems.
- ▶ [Type systems](#) describe approximations of program behaviors, concentrating on the shapes of the values passed between different parts of the program.

## Overview

---

This course will concentrate on operational techniques and type systems.

- ▶ Part I: Modeling programming languages
  - ▷ Syntax and parsing
  - ▷ Operational semantics
  - ▷ Inductive proof techniques
  - ▷ The lambda-calculus
  - ▷ Syntactic sugar; fully abstract translations
- ▶ Part II: Type systems
  - ▷ Simple types
  - ▷ Type safety
  - ▷ References
  - ▷ Subtyping

## Overview

---

- ▶ Part III: Object-oriented features (case study)
  - ▷ A simple imperative object model
  - ▷ An analysis of core Java
  - ▷ An analysis of core Scala

## Organization of the Course

11

### Staff

---

Instructor:	Martin Odersky <code>martin.odersky@epfl.ch</code>
Teaching Assistants:	Aleksander Boruch-Gruszecki <b>Contact him about the mailing list!</b> <code>aleksander.boruch-gruszecki@epfl.ch</code>  Charly Castes <code>charly.castes@epfl.ch</code>

12

## Information

---

Textbook: Types and Programming Languages,  
Benjamin C. Pierce, MIT Press, 2002

Webpage: <https://fos2021.github.io>

## Elements of the Course

---

- ▶ The Foundations of Software course consists of
  - ▷ lectures (Tue 14:15-16:00, INF 1)
  - ▷ exercises and project work (Wed 11:15-13:00, INJ 218)
- ▶ The lecture will follow in large parts the textbook.
- ▶ For lack of time, we cannot treat all essential parts of the book in the lectures, that's why the [textbook is required reading](#) for participants of the course.

## Homework and Projects

---

You will be asked to

- ▶ solve and hand in some written exercise sheets,
- ▶ do a number of programming assignments, including
  - ▷ parsers,
  - ▷ interpreters and reduction engines,
  - ▷ type checkersfor a variety of small languages.
- ▶ The recommended implementation language for these assignments is [Scala](#).

## Scala

---

- ▶ Scala is a functional and object-oriented language that is closely interoperable with Java.
- ▶ It is very well suited as an implementation language for type-checkers, in particular because it supports:
  - ▷ pattern matching,
  - ▷ higher-order functions,
  - ▷ inheritance and mixins.



## Learning Scala

---

If you don't know Scala yet, there's help:

- ▶ The Scala web site:  
[www.scala-lang.org](http://www.scala-lang.org)
- ▶ On this site, the documents:
  - ▷ *A Brief Scala Tutorial - an introduction to Scala for Java programmers.* (short and basic).
  - ▷ *An Introduction to Scala* (longer and more comprehensive).
  - ▷ *An Overview of the Scala Programming Language* (high-level).
  - ▷ *Scala By Example* (long, comprehensive, tutorial style).
- ▶ The assistants.

17

## Grading and Exams

---

Final course grades will be computed as follows:

- ▶ Homework and project: 40%
- ▶ Final exam: 60%

Exams:

1. Final exam: to be announced

18

## Collaboration

---

- ▶ Collaboration on homework is [strongly encouraged](#).
- ▶ Studying with other people is the best way to internalize the material
- ▶ Form pair programming and study groups!  
2-3 people is a good size. 4 is too many for all to have equal input.

"You never really misunderstand something  
until you try to teach it...  
" – Anon.

## Plagiarism

---

- ▶ A single group will of course share code.
- ▶ But plagiarizing [code](#) by [other groups](#) as part of a project is unethical and will not be tolerated, whatever the source.

# Part I

## Modelling programming languages

21

### Syntax and Parsing

---

- ▶ The first-level of modeling a programming language concerns its **context-free syntax**.
- ▶ Context free syntax determines a set of legal **phrases** and determines the **(tree-)structure** of each of them.
- ▶ It is often given on two levels:
  - ▷ **concrete**: determines the exact (character-by-character) set of legal phrases
  - ▷ **abstract**: concentrates on the tree-structure of legal phrases.
- ▶ We will be mostly concerned with abstract syntax in this course.
- ▶ But to be able to write complete programming tools, we need a convenient way to map character sequences to trees.

22

## Approaches to Parsing

---

There are two ways to construct a parser:

- ▶ **By hand** Derive a parser program from a grammar.
- ▶ **Automatic** Submit a grammar to a tool which generates the parser program.

In the second approach, one uses a special **grammar description language** to describe the input grammar.

## Domain-Specific Languages

---

- ▶ The grammar description language is an example of a **domain-specific language (DSL)**.
- ▶ The parser generator acts as a processor (**“compiler”**) for this language — that's why it's sometimes called grandly a **“compiler-compiler”**.
- ▶ Example of a “program” in the grammar description DSL:

```
Expr ::= Term { '+' Term | '-' Term }.  
Term  ::= Factor { '*' Factor | '/' Factor }.  
Factor ::= Number | '(' Expr ')'
```

## Embedded Domain Specific Languages

---

- ▶ An alternative to a stand-alone DSL is an [Embedded DSL](#).
- ▶ Here, the DSL does not exist as a separate language but as an API in a [host language](#).
- ▶ The host language is usually a general purpose programming language.

We will now develop this approach for grammar description languages.

25

## An EDSL for Parsing in Scala

---

```
def expr : Parser[Any] = term ~ rep("+" ~ term | "-" ~ term)
def term : Parser[Any] = factor ~ rep("*" ~ factor | "/" ~ factor)
def factor: Parser[Any] = "(" ~ expr ~ ")" | numericLit
```

A small Perl hack could derive the above code from the informal grammar definition above:

- ▶ Insert a **def** at the beginning of each production.
- ▶ The “`::=`” becomes “`: Parser[Any] =`”.
- ▶ Sequential composition is now expressed by a `~`.
- ▶ Repetition `{...}` is now expressed by `rep(...)`.
- ▶ Option `[...]` is now expressed by `opt(...)`.

26

## Parser Combinators

---

- ▶ The differences between Grammar A and Grammar B are fairly minor.  
(Note in particular that existing DSLs for grammar descriptions also tend to add syntactic complications to the idealized Grammar A we have seen).
- ▶ The important difference is that Grammar B is a valid Scala program, when combined with an API that defines the necessary primitives.
- ▶ These primitives are called [parser combinators](#).

27

## The Basic Idea

---

For each language (identified by grammar symbol  $S$ ), define a function  $f_S$  that, given an input stream  $i$ ,

- ▶ if a prefix of  $i$  is in  $S$ , return `Success(Pair( $x$ ,  $i'$ ))` where  $x$  is a result for  $S$  and  $i'$  is the rest of the input.
- ▶ otherwise, return `Failure(msg,  $i$ )` where `msg` is an error message string.

The first behavior is called [success](#), the second [failure](#).

28

## The Basic Idea in Code

---

Assume:

```
class StandardTokenParsers {  
  type Parser = Input ⇒ ParseResult
```

where

```
type Input = Reader[Token]  // a stream of Tokens with positions.
```

and we assume a class `Token` with subclasses

- ▷ case class `Keyword(chars: String)` for keywords,
- ▷ case class `NumericLit(chars: String)` for numbers,
- ▷ case class `StringLit(chars: String)` for strings,
- ▷ case class `Identifier(chars: String)` for identifiers.

Also assume a class `ParseResult[T]` with subclasses

```
case class Success[T](result: T, in: Input)  
extends ParseResult[T]  
  
case class Failure(msg: String, in: Input)  
extends ParseResult[Nothing]
```

## Object-Oriented Parser Combinators

---

- ▶ In fact, we will also need to express `|` and `~` as methods of parsers.
- ▶ That's why we extend the function type of parsers as follows:

```
abstract class Parser[T] extends (Input ⇒ ParseResult[T]) {  
  // An unspecified method that defines the parser function.  
  def apply(in: Input): ParseResult  
  
  // A parser combinator for sequential composition  
  def ~ ...  
  
  // A parser combinator for alternative composition  
  def | ...  
}
```

It remains to define concrete combinators that implement this class (see below).

31

## A Generic Single-Token Parser

---

- ▶ The following parser succeeds if the first token in the input satisfies a given predicate `p`.
- ▶ If it succeeds, it reads the token string and returns it as a result.

```
def token(kind: String)(p: Token ⇒ boolean) = new Parser[String] {  
  def apply(in: Input) =  
    if (p(in.head)) Success(in.head.chars, in.tail)  
    else Failure(kind+" expected", in)  
}
```

32



## Specific Single-Token Parsers

---

- ▶ The following parser succeeds if the first token in the input is the given keyword `chars`:
- ▶ If it succeeds, it returns a keyword token as a result.

```
implicit def keyword(chars: String) = token("'" + chars + "'") {  
  case Keyword(chars1) => chars == chars1  
  case _ => false  
}
```

- ▶ Note that `keyword` is marked `implicit`, so that you can usually write `"if"` instead of `keyword("if")`.

- ▶ The following parsers succeed if, respectively, the first token in the input is a numeric or string literal, or an identifier.

```
def numericLit = token("number")(_._1.isInstanceOf[NumericLit])  
def stringLit = token("string literal")(_._1.isInstanceOf[StringLit])  
def ident = token("identifier")(_._1.isInstanceOf[Identifier])
```

## The Sequence Combinator

- ▶ The sequence combinator  $P \sim Q$  succeeds if  $P$  and  $Q$  succeed sequentially, and returns the concatenation of their results.
- ▶  $\sim$  is implemented as a method of class `Parser`.

```
abstract class Parser[T] {  
  def ~ [U](q: Parser[U]) = new Parser[T ~ U] {  
    def apply(in: Input) = Parser.this(in) match {  
      case Success(x, in1) =>  
        q(in1) match {  
          case Success(y, in2) => Success(new ~ (x, y), in2)  
          case failure => failure  
        }  
      case failure => failure  
    }  
  }  
}
```

35

## Concatenating Results

Normally, the  $\sim$  combinator returns a the results of the two parsers that are run sequentially wrapped in a  $\sim$ -object.

The case class  $\sim$  is similar to `Pair` (but  $\sim$  turns out to be nicer – see below).

```
case class ~[T, U](_1: T, _2: U) {  
  override def toString = "(" + _1 + " ~ " + _2 + ")"  
}
```

There are also two variants of  $\sim$  which return only the left or only the right operand:

```
P <~ Q    // returns only P's result  
P ~> Q    // returns only Q's result
```

36

## The Alternative Combinator

---

- ▶ The alternative combinator  $P \mid Q$  succeeds if either  $P$  or  $Q$  succeeds.
- ▶ It returns the result of  $P$  if  $P$  succeeds, or the result of  $Q$ , if  $Q$  succeeds.
- ▶ The alternative combinator is implemented as a method of class `Parser`.

```
def | (q: => Parser[T]) = new Parser[T] {  
  def apply(in: Input) = Parser.this(in) match {  
    case s1 @ Success(_, _) => s1  
    case failure => q(in)  
  }  
}
```

37

## Failure And Success Parsers

---

- ▶ The parser `failure(msg)` always fails with the given error message. It is implemented as follows:

```
def failure(msg: String) = new Parser[Nothing] {  
  def apply(in: Input) = Failure(msg, in)  
}
```

- ▶ The parser `success(result)` always succeeds with the given result. It does not consume any input. It is implemented as follows:

```
def success[T](result: T) = new Parser[T] {  
  def apply(in: Input) = Success(result, in)  
}
```

38

## Result Conversion

The parser  $P \hat{\hat{}} f$  succeeds iff  $P$  succeeds. In that case it returns the result of applying  $f$  to the result of  $P$ .

```
def ^^ [U](f: T => U) = new Parser[U] {  
  def apply(in: Input) = Parser.this(in) match {  
    case Success(x, in1) => Success(f(x), in1)  
    case f => f  
  }  
}
```

A variant  $\hat{\hat{\hat{}}}$  takes a value  $V$  as right hand side argument.

It returns  $V$  if the left hand parser succeeds:

```
def ^^^ [U](r: U): Parser[U] = ^^ (x => r)
```

## Option and Repetition Combinators

- ▶ The  $\text{opt}(P)$  combinator always succeeds and returns an **Option** result.  
It returns  $\text{Some}(R)$  iff  $P$  succeeds with  $R$ . In case of failure, the result is **None**.
- ▶ The  $\text{rep}(P)$  combinator applies  $P$  zero or more times until it fails, the result is a list of  $P$ 's results.

The two combinators are implemented as follows:

```
def opt[T](p: Parser[T]): Parser[Option[T]] =  
  p ^^ Some | success(None)  
def rep[T](p: Parser[T]): Parser[List[T]] =  
  p ~ rep(p) ^^ { case x ~ xs => x :: xs } | success(List())
```

Note that neither of these combinators can fail!

## The Interleaved Repetition Combinator

---

The `repsep(P, Q)` parser parses a (possibly empty) sequence

*P Q P ... Q P*

It returns a list of all results returned by *P*.

**Example:** `repsep(ident, ",")` parses a list of identifiers separated by commas.

The `repsep` combinator is implemented as follows:

```
def repsep[T, U](p: Parser[T], q: Parser[U]): Parser[List[T]] =  
  p ~ rep(q ~> p) ^^ { case r ~ rs => r :: rs } | success(List())
```

## Other Combinators

---

More combinators can be defined if necessary.

**Exercise:** Implement the `rep1(P)` parser combinator, which applies *P* one or more times.

**Exercise:** Define `opt` and `rep` directly, without making use of `~`, `|`, and `empty`.

## An Example: JSON

---

JSON, or JavaScript Object Notation, is a popular data interchange format.

JSON data essentially consists of objects {...}, arrays [...], numbers, and strings.

Here is an example of a JSON value:

```
{ "address book": {  
  "name": "John Smith",  
  "address": { "street": "10 Market Street",  
               "city"  : "San Francisco, CA",  
               "zip"   : 94111 },  
  "phone numbers": ["408 338-4238", "408 111-6892"]  
}
```

43

## A JSON parser

---

```
package examples.parsing  
  
// import the standard parser class  
import scala.util.parsing.combinator.syntactical.StandardTokenParsers  
  
object JSON extends StandardTokenParsers {  
  // fix some delimiter symbols ...  
  lexical.delimiters += ("{" , "}" , "[" , "]" , ":" )  
  // ... and some reserved words  
  lexical.reserved += ("null" , "true" , "false")  
  
  // here are the four productions making up the JSON grammar  
  def obj  : Parser[Any] = "{" ~ repsep(member, ",") ~ "}"  
  def arr  : Parser[Any] = "[" ~ repsep(value, ",") ~ "]"  
  def member : Parser[Any] = stringLit ~ ":" ~ value  
  def value : Parser[Any] = stringLit | numericLit | obj | arr |  
    "null" | "true" | "false"
```

44

## Testing the JSON Parser

---

Add a method `main` that can be used to test the parser.

```
def main(args: Array[String]) {  
  val tokens = new lexical.Scanner(args(0))  
  println(args(0))  
  println(phrase(value)(tokens))  
}
```

Here are two test runs:

```
>java examples.parsing.JSON "{ \"x\": true, \"y\": [1, 2, 3] }"  
{ x: true, y: [1, 2, 3] }  
[1.26] parsed: ((({ ~ List(((x ~ :) ~ true), ((y ~ :) ~ (([ ~ List(1,  
2, 3)) ~ ])))) ~ })
```

```
>java examples.parsing.JSON "{ \"x\": true \"y\": [1, 2] }"  
{ x: true y: [1, 2] }  
[1.13] failure: unexpected token string literal y  
{ \"x\": true ^ \"y\": [1, 2] }
```

## Getting Better Output

- ▶ The result of the previous JSON parser was a tree containing all input tokens (slightly chaotically).
- ▶ We transform the productions using the `^^` combinator to get more structured output:

```
def obj : Parser[Any] = // return a Map
  "{" ~> rep(member) <~ "}" ^^ (ms => Map() ++ ms)

def arr : Parser[Any] = // return a List
  "[" ~> rep{value} <~ "]"

def member: Parser[Any] = // return a name/value pair
  stringLit ~ ":" ~ value ^^
  { case name ~ ":" ~ value => (name, value) }
```

**Question:** What are more precise types for `obj`, `arr`, and `member`?

47

```
def value : Parser[Any] = (
  | obj
  | arr
  | stringLit
  | numericLit ^^ (...toInt) // return an Int
  | "null" ^^^ null          // return 'null'
  | "true" ^^^ true           // return 'true'
  | "false" ^^^ false        // return 'false'
)
```

If we run the test now, we get:

```
>java examples.parsing.JSON1 "{ \"x\": true, \"y\": [1, 2, 3] }"
{ x: true, y: [1, 2, 3] }
[1.30] parsed: Map(x -> true, y -> List(1, 2, 3))
```

48



## Table of Parser Combinators

ident	identifier
keyword(...)	keyword or special symbol (implicit)
numericLit	integer number
stringLit	string literal
P ~ Q	sequential composition
P <~ Q, P ~> Q	sequential composition; keep left/right only
P   Q	alternative
opt(P)	option
rep(P)	repetition
repsep(P, Q)	interleaved repetition
P ^^ f	result conversion
P ^^ v	constant result

49

## Arithmetic Expressions Again

Here is the full parser for arithmetic expressions:

```
object Arithmetic extends StandardTokenParsers {  
  lexical.delimiters += List("(", ")", "+", "-", "*", "/")  
  def expr: Parser[Any] = term ~ rep("+ ~ term | - ~ term)  
  def term = factor ~ rep("* ~ factor | / ~ factor)  
  def factor: Parser[Any] = "(" ~ expr ~ ")" | numericLit
```

**Question:** How can we make it evaluate the parsed expression?

50

## A Problem with Top-Down Parsing

---

As parser combinators work top-down, they don't allow left-recursion.

A production like

```
def expr = expr ~ "-" ~ term
```

would go into an infinite recursion when executed.

The alternative:

```
def expr = term ~ rep("-" ~ term)
```

produces a “right-leaning” tree:  $X - Y - Z$  parses

```
X ~ List("-" ~ Y, "-" ~ Z)
```

But the correct reduction/evaluation of  $+, -, *, /$  is left-leaning!

51

## Evaluate by FoldLeft

---

We can solve this problem by delaying reduction until all elements of a repetition have been parsed and then performing a fold left on the list:

```
def expr : Parser[Int] =  
  term ~ rep ("+" ~ term | "-" ~ term) ^^ reduceList  
def term : Parser[Int] =  
  factor ~ rep ("*" ~ factor | "/" ~ factor) ^^ reduceList  
def factor: Parser[Int] =  
  "(" ~> expr <~ ")" | numericLit ^^ (_.toInt)
```

Here, `reduceList` is defined in terms of the fold-left operation `/:` ...

```
val reduceList: Expr ~ List[String ~ Expr] => Expr = {  
  case i ~ ps => (i /: ps)(reduce)  
}
```

52

... and `reduce` is defined as follows:

```
def reduce(x: Int, r: String ~ Int) = r match {  
  case "+" ~ y => x + y  
  case "-" ~ y => x - y  
  case "*" ~ y => x * y  
  case "/" ~ y => x / y  
  case _ => throw new MatchError("illegal case: "+r)  
}
```

With this, we get:

```
java examples.parsing.ArithmeticParsers1 "2 * (3 + 7)"  
2 * (3 + 7)  
[1.12] parsed: 20
```

(Note: the `chain` combinator in the standard library does this for you.)

## Conclusion

---

- ▶ Combinator parsers give a provide a concise, flexible, and high-level way to construct parsers.
- ▶ The token classes of a context free grammar are modeled as primitive parsers.
- ▶ The combination forms are modeled as higher-order parsers.
- ▶ Combinator parsers are an example of an embedded DSL.
- ▶ By contrast, classical parser generators are stand-alone DSLs.
- ▶ Advantage of an embedded DSL over a parser generator: It's easier to connect the results of combinator parsers with the environment.
- ▶ Disadvantage: lower efficiency – but this can be overcome (using packrat parsing)