

# Report 2: Geiger-Müller Counter Properties

*Author:*

Håkon FOSSHEIM

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We determine the optimal operational voltage of our Geiger Müller counter to be at around 500 V by locating its voltage plateau. Using this applied voltage, half-lives of the metastable states of Barium and Protactinium were estimated at  $\tau_{1/2}(^{137}_{56}\text{Ba}^m) = 154.7\text{ s} \pm 1.0\text{ s}$  and  $\tau_{1/2}(^{234}_{92}\text{Pa}^m) = 71.2\text{ s} \pm 2.6\text{ s}$  respectively.

## I. Introduction

As  $\alpha$ ,  $\beta$  or  $\gamma$  particles of sufficient energy enter a gaseous "chamber", it may ionize the atoms of this gas. With no applied potential, the free electron will simply recombine with the ion and thus no signal is detected. As we start applying a voltage difference, more and more of these ions and electrons will be collected. This corresponds to region I in Fig.(1). At some point, all ionization products<sup>1</sup> will be collected. This corresponds to region II.

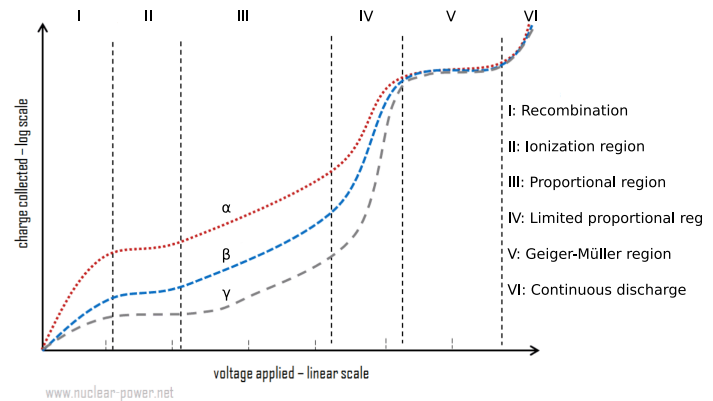


FIG. (1) Detector charge collection as a function of the applied voltage for gaseous detectors.

Increase the voltage further, and the electrons acquire sufficient energy to generate localized cascades close to the anode. The size of these cascades<sup>2</sup> increase in proportion to the applied voltage. This corresponds to region III.

<sup>1</sup> By which we mean the positively charged atoms and electrons

<sup>2</sup> And thus the signal yield

Next, we enter region IV, viz. the limited proportional region. Here the electric field is sufficient to generate space charge around the anode, which results in loss of the aforementioned proportionality.

In region V, the Geiger-Müller region, the voltage is sufficient to cause chain reactions of cascades along the anode, thereby completely saturating the output current. A quenching gas is used to stop these avalanches. As seen in Fig.(1), the detector is not sensitive to radiation type in this region.

In region VI our detector is no longer operational and we have a continuous discharge in the absence of radiation. This can damage the detector and so this region is best avoided.

## II. Theory

### A. Radioactive decay

We make the assumption that the probability for any atom to decay in an infinitesimal time period  $dt$  is a constant as a function of time and that this constant is the same for all atoms of the same type in our sample<sup>3</sup>.

This entails that the fractional number of atoms that decay during  $dt$  should not change with time, i.e.

$$\frac{dN/N}{dt} = C \quad (1)$$

Where  $C$  is a constant in time. Letting  $N$  denote the number of atoms in our sample that have not yet decayed, we see that  $dN/dt \leq 0$  since the number of "undecayed" atoms can only decrease<sup>4</sup> with time, and thus we set  $C = -\lambda$  where  $\lambda \geq 0$  is called the *decay constant*. We therefore get

$$\frac{dN}{N} = -\lambda dt \quad (2)$$

or

$$\int_{N(t_0)}^{N(t)} \frac{1}{N'} dN' = - \int_{t_0}^t \lambda dt' \quad (3)$$

Which yields

$$N(t) = N_0 e^{-\lambda t} \quad (4)$$

Where  $N(t_0) = N_0$  and  $t_0 \equiv 0$ .

Since the time it takes for a sample to fully decay cannot be determined accurately due to the quantum mechanical nature of radioactive decay, we instead use the

*half-life* as the characteristic measure for how fast an isotope decays.

The half-life of a radionuclide is defined as the time it takes for a sample to decay to half its original size, viz.

$$N(\tau_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda \tau_{1/2}} \quad (5)$$

And thus

$$\begin{aligned} \ln(2^{-1}) &= -\lambda \tau_{1/2} \\ \tau_{1/2} &= \frac{\ln 2}{\lambda} \end{aligned} \quad (6)$$

Likewise, after  $k$  half-lives, i.e. after a time period of  $t = k\tau_{1/2}$ , the sample will have been reduced to  $\frac{1}{2^k}$  of its original size. We therefore get

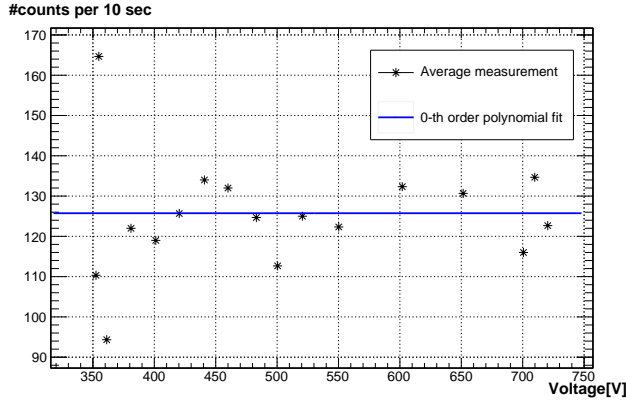
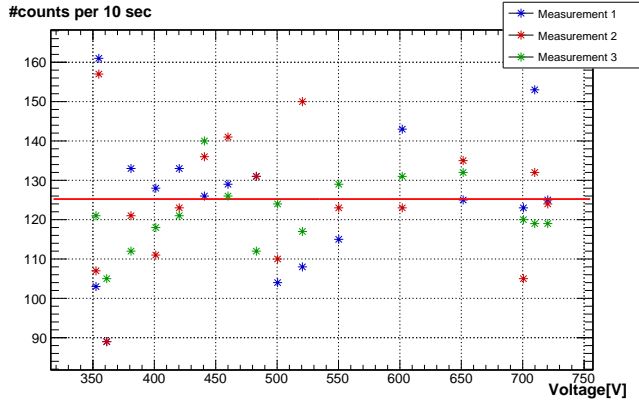
$$\begin{aligned} N(k\tau_{1/2}) &= \frac{N_0}{2^k} = \frac{N_0}{2^k} e^{-\lambda(k\tau_{1/2})} \\ -k \ln 2 &= -\lambda(k\tau_{1/2}) \\ k\tau_{1/2} &= k \frac{\ln 2}{\lambda} \end{aligned} \quad (7)$$

## III. Experiment 1, Geiger Müller plateau

Due to the cylindrical shape of the Geiger Müller counter, the electric field goes as  $E \propto \frac{1}{r}$  where  $r$  is the distance from the central anode. This allows for the creation of cascades at much lower applied voltages than what would be required to e.g. make a parallel plate set-up operational.

<sup>3</sup> In the case of e.g. alpha decay, this makes sense, since the probability of tunneling through the potential barrier of the nucleus is constant as a function of time.

<sup>4</sup> Or stay constant for non-radioactive atoms.



(a) The first plot shows our decay rate measurements, color coded according to the order in which they were taken. The bottom plot shows the averages and the associated 0-th order polynomial fit.

## IV. Experiment 2, Barium half life

For radioisotopes of short half-lives, storage and shipping to a customer from the production site poses a problem since most of the sample will have decayed by the time it arrives.

Therefore, so-called "generators" are used to produce such isotopes on-site. So to measure the half-life of  $^{137}\text{Ba}^m$ , we flush it out<sup>5</sup> as it is produced through the decay of  $^{137}_{55}\text{Cs}$ . The decay chain is shown in Fig.(3).

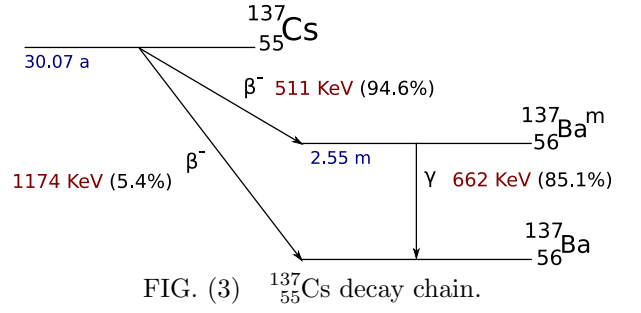


FIG. (3)  $^{137}_{55}\text{Cs}$  decay chain.

We had our Scaler 77 set to "manual", which counts the total number of decays as a function of time, i.e. the cumulative radioactive decay rate,  $r_c(t)$ . At any given time, this quantity will be equal to the number of atoms that have decayed, which again should be equal to the original number of undecayed atoms ( $N_0$ ) minus the current number of undecayed atoms ( $N(t)$ ) and so

$$r_c(t) = N_0 - N(t) = N_0(1 - e^{-\lambda t}) \quad (8)$$

In our experiment, we had three different samples of  $^{137}_{56}\text{Ba}^m$ . We also performed a measurement of the background radiation. It was measured to be  $r(bgr) = \frac{61+63}{60 \times 2}$  counts/s. The background is then subtracted from the radiation measurements, which are plotted in Fig.(4). The reason these curves separate can most readily be explained by variations in the original sample sizes<sup>6</sup>, which we denote as  $N_1(0)$ ,  $N_2(0)$  and  $N_3(0)$ . From Eq.8, we see that as  $t \rightarrow \infty$ , the cumulative decay rates will approach these original sample sizes.

We can now perform  $\chi^2$  fits to determine the decay rates and the initial sample sizes.

<sup>5</sup> By pushing saline water through the generator via a syringe. The liquid that comes out then contains  $^{137}_{56}\text{Ba}^m$  and no Cesium.

<sup>6</sup> It could also correspond to different source-to-detector distances, but this was set to a fixed value of 5.5 cm.

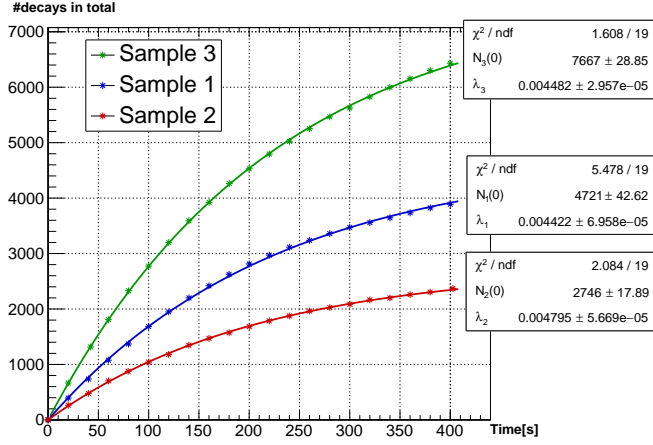


FIG. (4) Datapoints are marked with stars. They are the result of subtracting the measured background from raw data (i.e. signal+background). The solid curves are the fitted functions, and the fit results are shown in the boxes next to them. We also include the fit results in Tab.(I) for better readability.

TABLE (I) Fit results for Barium

Fit results of $^{137}\text{Ba}^m$ decay			
	Sample 1	Sample 2	Sample 3
$\chi^2/\text{ndf}$	5.478/19	2.084/19	1.608/19
$N_0$	$4721 \pm 42.62$	$2746 \pm 17.89$	$7667 \pm 28.85$
$\lambda[10^{-5}/\text{s}]$	$442.2 \pm 6.958$	$479.5 \pm 5.669$	$448.2 \pm 2.957$
$\tau_{1/2}[\text{s}]$	$156.756 \pm 2.428$	$144.545 \pm 1.689$	$154.667 \pm 1.014$

We can now compare the experimentally found half life to the reference value

$$\tau_{1/2}(\text{ref.}) = 2.552\text{m} = 153.1\text{s}$$

$$\tau_{1/2}(\text{exp.}) = 154.7\text{s} \pm 1.0\text{s}$$

## V. Experiment 3, Protactinium half life

Next, we aim to find the half-life of  $^{234}\text{Pa}^m$ . This was done by use of a Protactinium generator. It is produced through the  $^{238}\text{U}$  decay chain depicted in fig(5).

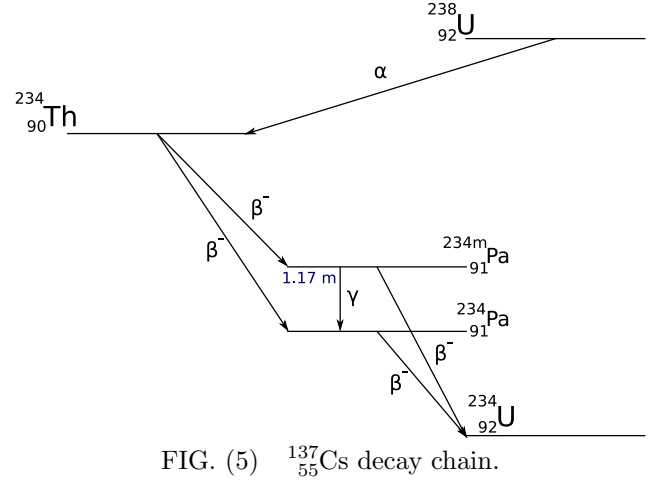


FIG. (5)  $^{137}\text{Cs}$  decay chain.

The protactinium generator was shook for about 10 seconds in order to let the Protactinium settle in a layer on top of the rest of the solution. Because the Geiger-counter doesn't distinguish  $\alpha$ ,  $\beta$  and  $\gamma$  radiation, the rest of the solution can yield a signal contribution<sup>7</sup> and so we measure the background by placing the unshook generator at the same distance from the detector as when the signal will be measured. The background measurements were taken over a period of 60 seconds, and our two measurements were 63 and 61 counts. Figure 6 shows the result of subtracting the average integrated background from our total measurements. These measurements are then fitted according to Eq.(8).

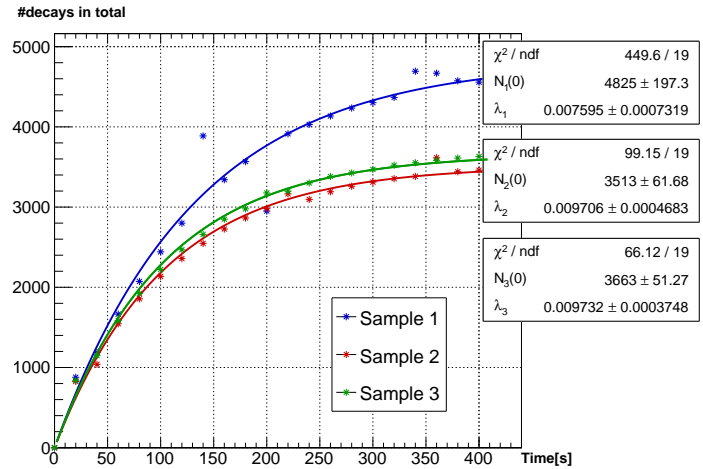


FIG. (6) Caption

As seen, the blue curve (corresponding to sample 1) reaches its asymptote at a much higher value. This is might be due to not giving the solution sufficient time

<sup>7</sup> Letting the Protactinium settle on top before starting the measurement is therefore important.

to separate before starting the measurements. The fit results of our measurements are shown in Tab.(II).

TABLE (II) Fit results for Protactinium after subtracting background

Fit results of $^{234}_{91}\text{Pa}^m$ decay			
	Sample 1	Sample 2	Sample 3
$\chi^2/\text{ndf}$	449.6/19	99.15/19	66.12/19
$N_0$	$4825 \pm 197.3$	$3531 \pm 61.68$	$3663 \pm 51.27$
$\lambda[10^{-4}/\text{s}]$	$75.95 \pm 7.32$	$97.06 \pm 4.68$	$97.32 \pm 3.75$
$\tau_{1/2}[\text{s}]$	$91.26 \pm 8.03$	$71.41 \pm 3.29$	$71.22 \pm 2.64$

We pick the fit with the smallest  $\chi^2$ , viz. sample 3. As seen in Tab.(II), the half life from this fit is  $71.22\text{s} \pm 2.64\text{s}$ . Comparing with the reference value:

$$\begin{aligned}\tau_{1/2}(\text{reference}) &= 1.17\text{m} = 70.2\text{s} \\ \tau_{1/2}(\text{experimental}) &= 71.22\text{s} \pm 2.64\text{s}\end{aligned}\tag{9}$$

As seen, the reference value lies well within the 68% confidence interval of our measured half-life for  $^{234}_{91}\text{Pa}^m$ .

## VI. Conclusion

In this experiment, we first found the voltage plateau of our Geiger-Müller counter in order to estimate its optimal operating voltage. This voltage was picked to be at the center of our plateau, since this corresponds to the smallest slope and thus our counter will be the least sensitive to voltage fluctuations in this region. This voltage was found to be 500 V.

We then estimated the half-life of  $^{137}_{56}\text{Ba}^m$  to be  $154.7\text{s} \pm 1.0\text{s}$  and that of  $^{234}_{91}\text{Pa}^m$  to be  $71.22\text{s} \pm 2.64\text{s}$ .