Mathematical Modeling

Lesson 3 –

Continuous and Discrete Models

Can anyone remind us what a deterministic model is?

deterministic

- Deterministic models operate under the assumption that the outcome is predictable if the initial conditions are known.
- There's no randomness involved—if you know the starting point and the rules, you can predict the outcome with certainty.

What about stochastic models?

stochastic

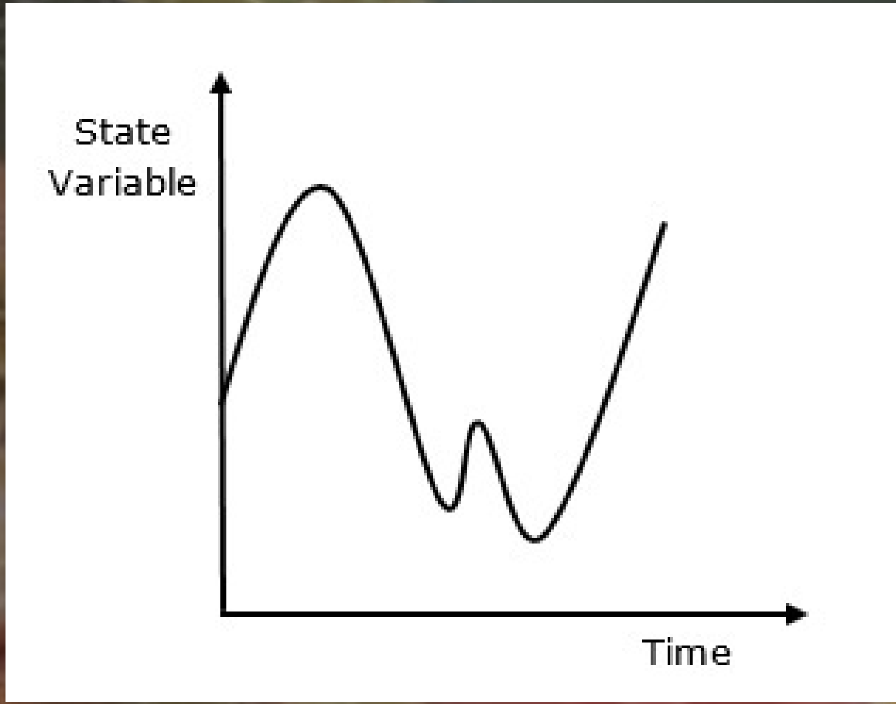
- stochastic models incorporate randomness, meaning that even with the same initial conditions, the outcomes can vary.

Why is it important to understand whether a system should be modeled deterministically or stochastically?

It helps us choose the right tools and approaches for our predictions and analyses.

Continuous model

- is one where the variables change smoothly over time, without any jumps or interruptions.
- ideal for representing processes that evolve gradually and can be described using differential equations



graphical representation of its behavior.

Why Use Continuous Simulation?

- use continuous simulation as it depends on differential equation of various parameters associated with the system and their estimated results known to us.

Application Areas

- **Civil engineering** for the construction of dam embankment and tunnel constructions
- **Military applications** for simulation of missile trajectory, simulation of fighter aircraft training, and designing & testing of intelligent controller for underwater vehicles.
- **Logistics** for designing of toll plaza, passenger flow analysis at the airport terminal, and proactive flight schedule evaluation.
- **Business development** for product development planning, staff management planning, and market study analysis

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Step 1: Understand the Biological Process

Step 2: Assumptions

Step 3: Define the Variables

Step 4: Formulate the Differential Equation

Step 5: Solve the Differential Equation

Step 6: Interpretation of the Model

Step 7: Real-World Application

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Step 1: Understand the Biological Process

When a drug is introduced into the bloodstream, it begins to be metabolized by the liver and other organs. The rate of this metabolism can often be described by a continuous, time-dependent process where the concentration of the drug in the bloodstream decreases gradually over time.

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Step 2: Assumptions

- **First-Order Kinetics:** We assume that the metabolism of the drug follows first-order kinetics. This means that the rate of change in the drug concentration is proportional to the concentration of the drug in the bloodstream at any given time.
- **Constant Rate of Metabolism:** The body metabolizes the drug at a rate that is constant relative to the current concentration.
- **Homogeneous Distribution:** The drug is evenly distributed throughout the bloodstream.

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Step 3: Define the Variables

- Let $C(t)$ represent the concentration of the drug in the bloodstream at time t.
- Let k represent the rate constant for the metabolism of the drug. This is a positive constant that determines how quickly the drug is metabolized.

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Step 4: Formulate the Differential Equation

Given the assumptions of first-order kinetics, the rate of change of the drug concentration over time can be expressed as:

$$\frac{dC(t)}{dt} = -k \cdot C(t)$$

This is a differential equation where:

- $\frac{dC(t)}{dt}$ is the rate of change of the drug concentration over time.
- $-k \cdot C(t)$ indicates that the concentration decreases at a rate proportional to the current concentration.

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Step 5: Solve the Differential Equation

To find the solution to the differential equation, we separate the variables and integrate:

$$\frac{dC(t)}{C(t)} = -k \cdot dt$$

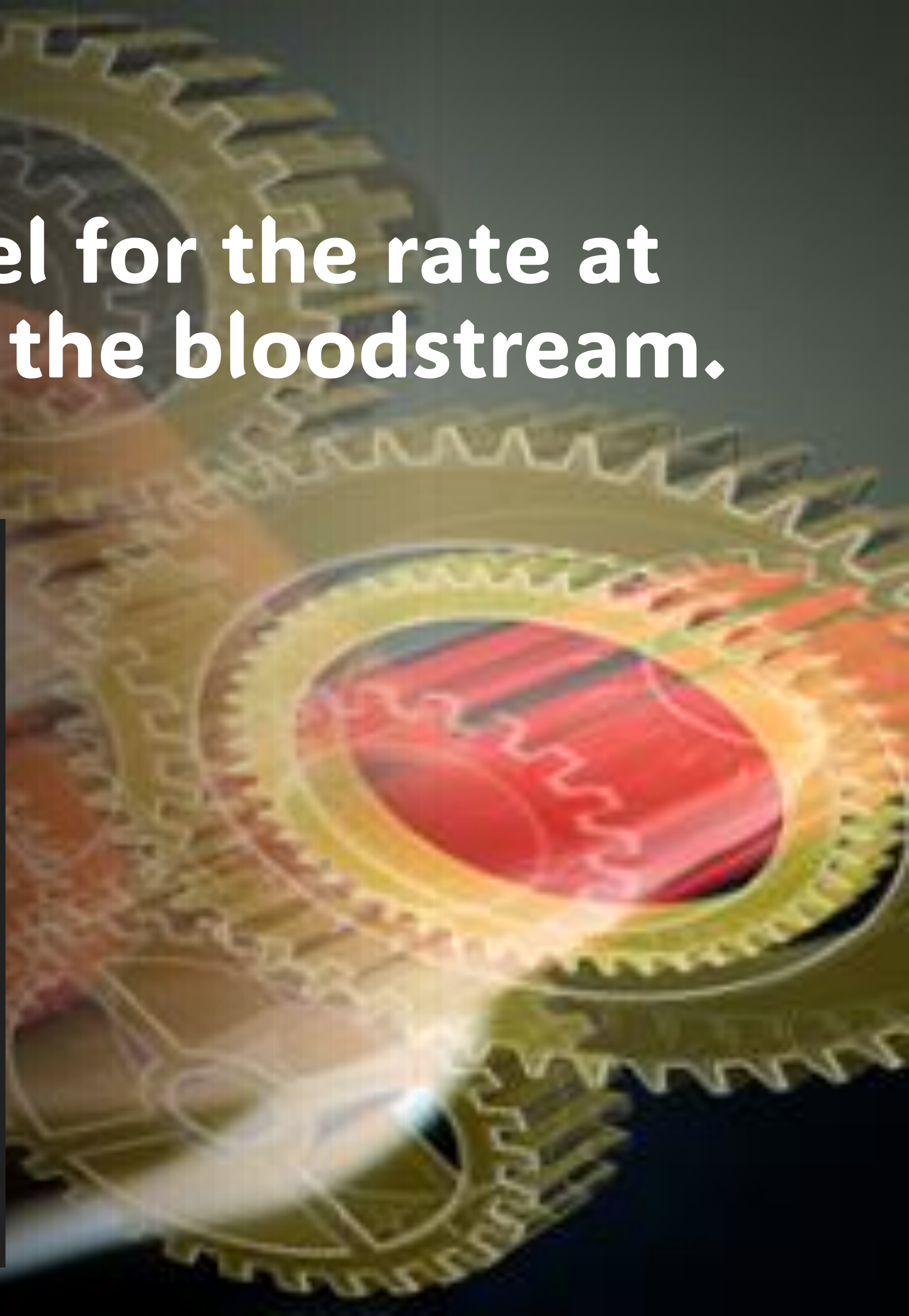
Integrating both sides:

$$\ln(C(t)) = -kt + \ln(C_0)$$

Where C_0 is the initial concentration of the drug in the bloodstream at time $t = 0$.

To solve for $C(t)$, we exponentiate both sides:

$$C(t) = C_0 \cdot e^{-kt}$$



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Step 6: Interpretation of the Model

The solution $C(t) = C_0 \cdot e^{-kt}$ describes how the drug concentration decreases exponentially over time. The rate constant k determines the speed of this decline:

- If k is large, the drug is metabolized quickly, and the concentration drops rapidly.
- If k is small, the drug is metabolized slowly, and the concentration decreases more gradually.

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Step 7: Real-World Application

This model is used in pharmacokinetics to predict how long a drug will remain in a patient's bloodstream at therapeutic levels. For instance, if a doctor knows the value of k for a particular drug, they can estimate how frequently a patient needs to take their medication to maintain a certain drug concentration and achieve the desired therapeutic effect.

Formulate a continuous model for the rate at which a drug is metabolized in the bloodstream.

Example:

Suppose a patient is administered a drug with an initial concentration of 50 mg/L in the bloodstream, and the rate constant k is found to be 0.1 per hour. The concentration of the drug in the bloodstream after 5 hours can be calculated as:

$$C(5) = 50 \cdot e^{-0.1\times5} \approx 50 \cdot e^{-0.5} \approx 50 \cdot 0.6065 \approx 30.3 \text{ mg/L}$$

This tells us that after 5 hours, the concentration of the drug in the bloodstream will have decreased to approximately 30.3 mg/L.

Assignment
Formulating a Continuous Model

Scenario:

A population of bacteria grows in a controlled laboratory environment. The population grows exponentially over time and can be modeled using a continuous function. However, due to limited resources, the growth rate decreases over time as the population approaches the environment's carrying capacity.

1. **Formulate a continuous model that describes the growth of the bacterial population.**
   - Start with the basic exponential growth model and modify it to account for the decreasing growth rate.
   - Define all variables and parameters used in your model.

2. **Solve the differential equation for your model.**
   - Find the general solution for the population at any time t.

3. **Interpret the solution of your model.**
   - Explain what your solution means in terms of the population's behavior over time. Discuss how the population changes as it approaches the carrying capacity.

Reflection

Reflect on the challenges of working with continuous models.

Discuss any difficulties you encountered while formulating, solving, or interpreting continuous models. How did you overcome these challenges? What did you learn from the process?

Thank You.