Module-3

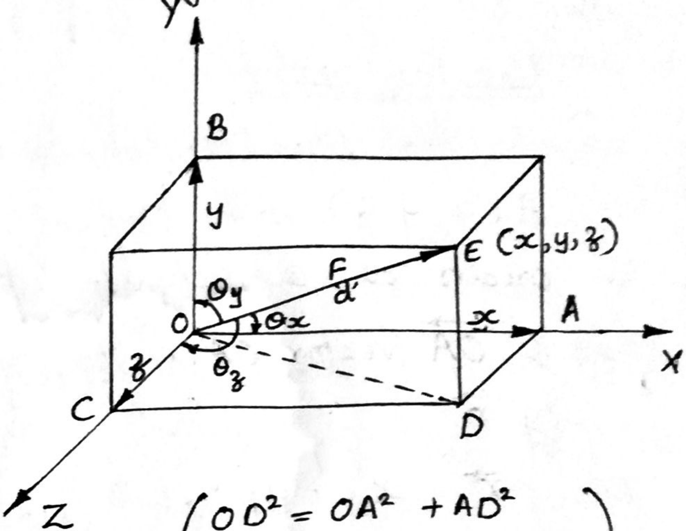
Force systems in space: { Ref EM- Benjamin, D.S. Kumar

Consider a force 'F' acting at the origin-O of the system of rectangular co-ordinates X, Y and Z as shown in figure. The angles θ_x, θ_y & θ_z that the force 'F' makes with the X, Y & Z axis define the direction of the force 'F'. The components of the force F along X, Y & Z directions are given by;

$$F_X = F\cos\theta_x$$

$$F_Y = F\cos\theta_y$$

$$F_Z = F\cos\theta_z$$



From fig,

$$(OE)^2 = (OD)^2 + (DE)^2$$

$$= (OA)^2 + (AD)^2 + (DE)^2$$

$$= (OA)^2 + (OC)^2 + (OB)^2$$

$\left(OD^2 = OA^2 + AD^2 \quad AD = OC \ \& \ DE = OB \right)$

$$F^2 = F_X^2 + F_Z^2 + F_Y^2$$

$$\therefore \boxed{F = \sqrt{F_X^2 + F_Y^2 + F_Z^2}}$$

Force vector $F = F_x i + F_y j + F_z k$

Let the co-ordinates of E be (x, y, z) & $OE = d$.

From fig, $OA = x$; $OB = y$; $OC = z$

$$OE^2 = OD^2 + ED^2$$

$$= OA^2 + AD^2 + OB^2$$

$$= OA^2 + OC^2 + OB^2$$

$\{ ED = OB$, $OD^2 = OA^2 + AD^2$, $AD = OC$

i.e, $d = \sqrt{x^2 + y^2 + z^2}$

Direction cosines of the force;

where, $x = d\cos\theta_x$; $l = \cos\theta_x = \dfrac{x}{d}$

$y = d\cos\theta_y$; $m = \cos\theta_y = \dfrac{y}{d}$

$z = d\cos\theta_z$; $n = \cos\theta_z = \dfrac{z}{d}$

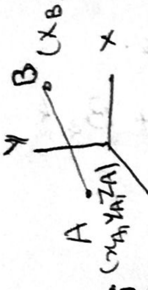
{ Check: $l^2 + m^2 + n^2 = 1$

$$\therefore F_X = F\cos\theta_x = F\cdot\frac{x}{d}$$

$$F_Y = F\cos\theta_y = F\cdot\frac{y}{d}$$

$$F_Z = F\cos\theta_z = F\cdot\frac{z}{d}$$

Note: Let A & B are two points on the line of action of a Force 'F', with co-ordinates (x_A, y_A, z_A) & (x_B, y_B, z_B) resply. Then $x = x_B - x_A$; $y = y_B - y_A$; $z = z_B - z_A$. The distance between A & B, $d = \sqrt{x^2 + y^2 + z^2}$



Unit Vector-

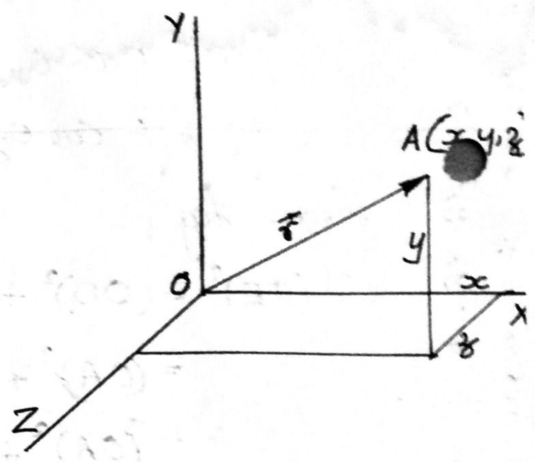
The vector having a unit length is known as unit vector. If 'F' is a force vector then its magnitude is $|F|$.

$$\therefore \text{Unit vector} = \frac{\text{Force vector- } F}{\text{Magnitude of force vector}} = \frac{F}{|F|}$$

Unit vectors along X, Y & Z directions are denoted by i, j and k respectively.

Position vector-

The position vector of a point $A(x, y, z)$ with respect to the origin of co-ordinates X, Y & Z is $\vec{OA}$ (vector OA). It is denoted by $\vec{r}$.



$\vec{r} = xi + yj + zk$; where i, j & k are the unit vectors along X, Y and Z directions and the magnitude of position vector,

$$r = \sqrt{x^2 + y^2 + z^2}$$

Vector operations:

Vector addition & subtraction-

The sum of number of vectors can be obtained by adding the respective components of the vectors.

If $A = A_x i + A_y j + A_z k$ and $B = B_x i + B_y j + B_z k$ then

$$A + B = (A_x + B_x)i + (A_y + B_y)j + (A_z + B_z)k$$

$$B + A = (B_x + A_x)i + (B_y + A_y)j + (B_z + A_z)k$$

$$\therefore A + B = B + A$$

IIIrly, $A - B = (A_x - B_x)i + (A_y - B_y)j + (A_z - B_z)k$

Dot product-

The dot product of vectors $\vec{A}$ and $\vec{B}$ is a scalar quanti[ty] and is defined as the product of the magnitude of the vectors and cosine of their included angle.

$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$; where θ - angle between the vectors A and B.

$$\cos\theta = \frac{A \cdot B}{|A|\,|B|}$$

If $A = A_x i + A_y j + A_z k$ & $B = B_x i + B_y j + B_z k$ then,

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

Cross product-

The cross product of two vectors A and B is denoted by $A \times B$ and is a vector whose;

i) magnitude is equal to the magnitude of A times the magnitude of B times the sine of the smaller angle between the two vectors.

ii) direction is perpendicular to the plane containing the vectors A and B

iii) sense is given by right hand rule.

ie, $|A \times B| = |A|\,|B| \sin\theta$

If $A = A_x i + A_y j + A_z k$ & $B = B_x i + B_y j + B_z k$ then in terms of rectangular components,

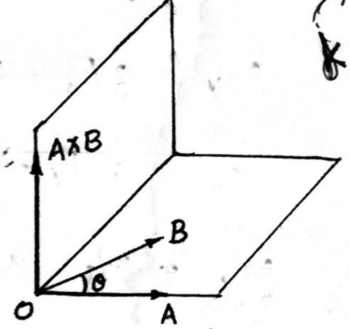
$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y) i + (A_z B_x - A_x B_z) j + (A_x B_y - A_y B_x) k$$

$i \times i = 0$; $j \times j = 0$; $k \times k = 0$

$i \times j = k$; $j \times i = -k$

$j \times k = i$; $k \times j = -i$

$k \times i = j$; $i \times k = -j$



Resultant of concurrent forces in space:

Resultant of concurrent force in space can be obtained by summing their rectangular components.

$R_x = \Sigma F_x$, $R_y = \Sigma F_y$ & $R_z = \Sigma F_z$

Resultant force, $R = R_x i + R_y j + R_z k$

Magnitude of resultant, $|R| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Let θ_x, θ_y & θ_z be the angles resultant makes with the co-ordinate axes.

ie, $\cos\theta_x = \frac{R_x}{R}$; $\cos\theta_y = \frac{R_y}{R}$; $\cos\theta_z = \frac{R_z}{R}$

<u>Equations of equilibrium of forces in space:</u>

A particle will be in equilibrium if the resultant of all the forces acting on it is zero. For the equilibrium the components R_x, R_y & R_z must be zero.

ie, $\Sigma F_x = R_x = 0$; $\Sigma F_y = R_y = 0$; $\Sigma F_z = R_z = 0$.

<u>Moment of a force in space:</u>

Moment of a force - F about a point is given by,

$M = r \times F$; where $r = xi + yj + zk$.

The moment is a vector directed perpendicular to the plane containing the vectors r and F.

$$M = r \times F = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)i + (zF_x - xF_z)j + (xF_y - yF_x)k$$

ie, $M_x = yF_z - zF_y$

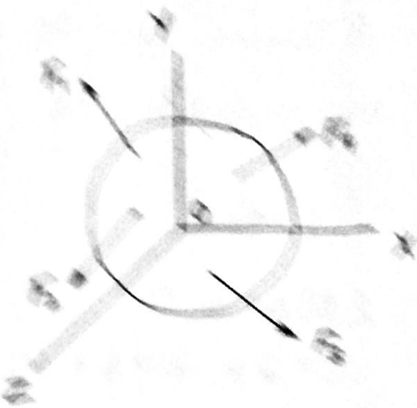
$M_y = zF_x - xF_z$

$M_z = xF_y - yF_x$

M_x, M_y & M_z are the moment about the axes X, Y & Z respectively. The moment about an axis is due to those forces lying in a plane perpendicular to the axis of the moments.

Resultant of a system of force in space

Consider a body subjected to several forces F_1, F_2, F_3, F_4 as shown. This force system may be replaced by a single force acting through the point O along with a couple.



Let R be the resultant force acting through the point O and M_R be the resultant moment which is equal to the sum of the moments of the forces F_1, F_2, F_3 & F_4 about O. Then,

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$R_x = \Sigma F_x \quad ; \quad R_y = \Sigma F_y \quad ; \quad R_z = \Sigma F_z$$

$$\therefore R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Also, $\Sigma M_O = (M_R)_O$

where ΣM_O - sum of the moments of all forces

$(M_R)_O$ - moment of resultant about O

ie, $\Sigma M_x = (M_R)_x \quad ; \quad \Sigma M_y = (M_R)_y \quad ; \quad \Sigma M_z = (M_R)_z$

$$(M_R)_O = \sqrt{\Sigma M_x^2 + \Sigma M_y^2 + \Sigma M_z^2}$$

1. A force of magnitude 5 kN makes 30°, 50° and 100° with X, Y & Z axis respectively. Find the force vector and magnitude of its components along X-Y and Z axis

$F = 5\text{ kN}, \ \theta_x = 30°, \ \theta_y = 50°, \ \theta_z = 100°$

$F_x = F\cos\theta_x = 5\cos 30 = 4.33\text{ kN}$

$F_y = F\cos\theta_y = 5\cos 50 = 3.2\text{ kN}$

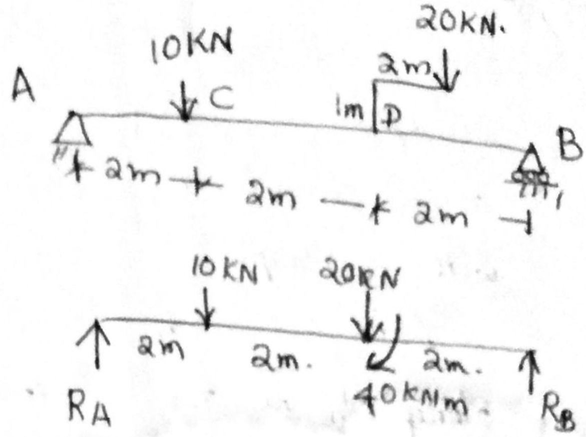
$F_z = F\cos\theta_z = 5\cos 100 = -0.87\text{ kN}$

The force vector $F = F_x i + F_y j + F_z k = 4.33i + 3.2j - 0.87k$

Magnitude of components along X, Y and Z directions are 4.33 kN, 3.2 kN and −0.87 kN respectively.

Beam

12) A beam 6m long is loaded as shown in fig. Calculate the reactions at A & B.



$\Sigma M = 0$

$\Sigma M_A = 0$

$10 \times 2 + 20 \times 4 + 40 - R_B \times 6 = 0$

$R_B = 23.33\ kN$

$\Sigma f_y = 0$

$R_A + R_B - 10 - 20 = 0$

$R_A = 6.667\ kN$

---

2) The magnitude of components of a force along X, Y and Z are −4.5 kN, 5 kN and −3 kN respectively. Find the magnitude of the force and its inclination with X–Y and Z axis

$F_x = -4.5\ kN$, $F_y = 5\ kN$, $F_z = 3\ kN$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 7.37\ kN$

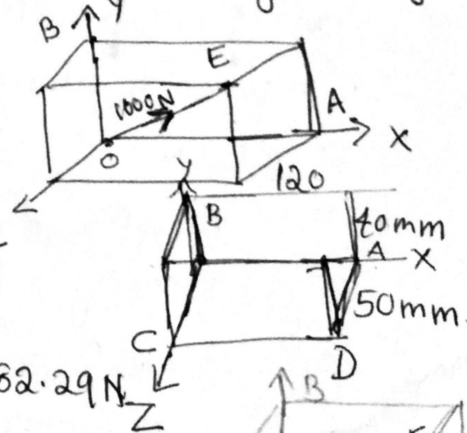
$\cos\theta_x = \frac{F_x}{F} = \frac{-4.5}{7.37}$ ∴ $\theta_x = 127.63°$ with x axis

$= \frac{5}{7.37}$ $\theta_y = 47.28°$ with Y axis

$= \frac{-3}{7.37}$ $\theta_z = 114.02°$ with Z axis

The inclination of force F

3) Calculate the components of the force 1000N shown in fig. along with X, Y and Z directions.



$x = 120mm$, $y = 40mm$, $z = 50mm$.

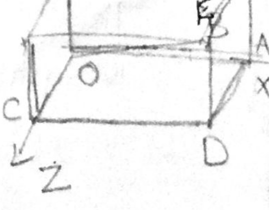
$d = \sqrt{x^2 + y^2 + z^2} = \sqrt{120^2 + 40^2 + 50^2}$

$= 136.01\ mm$.

The components $F_x = F \times \frac{x}{d} = \frac{1000 \times 120}{136.01} = 882.29\ N$

$F_y = F \times \frac{y}{d} = \frac{1000 \times 40}{136.01} = 294.10\ N$

$F_z = F \times \frac{z}{d} = \frac{1000 \times 50}{136.01} = 367.62\ N$



: Vector operations - Vector addition

1. Find the unit vector in the direction of the resultant of vectors $A = 2i - j + k$, $B = i + j + 2k$, $C = 2i + 2j + 4k$

Resultant $R = A + B + C$

$$= (2+1+2)i + (-1+1+2)j + (1+2+4)k$$

$$= 5i + 2j + 7k$$

Unit vector in the direction of R is $\frac{R}{|R|}$

$$= \frac{5i + 2j + 7k}{\sqrt{5^2 + 2^2 + 7^2}} = 0.57i + 0.23j + 0.79k$$

Dot product

2. Find the projection of vector $A = 4i - 3j + k$ on the line joining the points $C(2,3,-1)$ and $D(-2,-4,3)$

Soln : Line joining C and D is

$$B = (-2-2)i + (-4-3)j + (3-(-1))k$$

$$= -4i - 7j + 4k$$

The projection of vector A on vector B is $|A|\cos\theta$, where θ is the angle b/w vectors A and B

$$A \cdot B = |A||B|\cos\theta$$

$$\therefore |A|\cos\theta = \frac{A \cdot B}{|B|}$$

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

$$= (4 \times (-4) + -3 \times (-7) + 1 \times 4) = 9$$

$$|B| = \sqrt{(-4)^2 + (-7)^2 + (4)^2} = 9$$

$$\therefore |A|\cos\theta = \frac{A \cdot B}{|B|} = \frac{9}{9} = 1$$

The point of application of a force $F = 5i + 10j - 15k$ is displaced from the point $A(i + 0j + 3k)$ to the point

Unit Vector.

4. A force has line of action that goes through point A (4, 2, 5) and point B(12, 4, 6). If the magnitude of the force in the direction of A to B is 100N, find the force vector in terms of unit vectors.

$x = x_B - x_A = 12 - 4 = 8$

$y = y_B - y_A = 4 - 2 = 2$

$z = z_B - z_A = 6 - 5 = 1$

Distance b/w the points A and B, $d = \sqrt{8^2 + 2^2 + 1^2} = 8.31$

Unit vector in direction of AB $= \dfrac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{8i + 2j + 1k}{8.31}$

$= 0.9627i + 0.2407j + 0.1203k$

The Force vector = F x unit vector

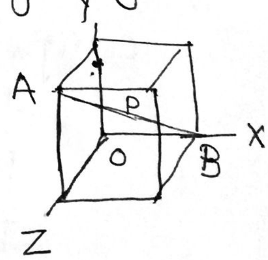
$= 100(0.9627i + 0.2407j + 0.1203k)$

$= 96.27i + 24.07j + 12.03k$

5. A force P acts as shown in fig. Express the force P as vector. Magnitude of force P = 100N. Length of cube is 2cm.

Co-ordinates of B are (2,0,0)

A are (0,2,2)



$x = x_B - x_A = 2 - 0 = 2$

$y = y_B - y_A = 0 - 2 = -2$

$z = z_B - z_A = 0 - 2 = -2$

The vector joining A and B $= xi + yj + zk$

$= 2i - 2j - 2k$

The unit vector in the direction of AB is $= \dfrac{AB}{|AB|} = \dfrac{2i - 2j - 2k}{\sqrt{2^2 + 2^2 + 2^2}}$

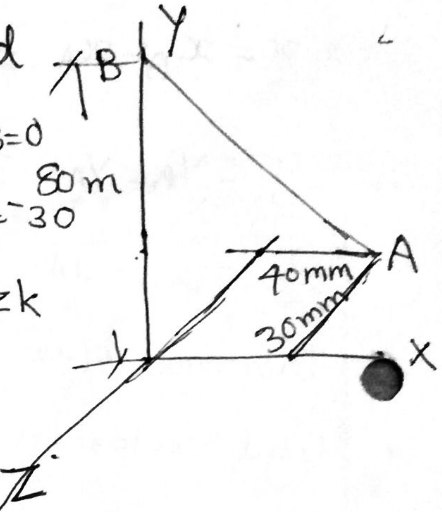
$= \dfrac{2(i - j - k)}{2\sqrt{3}} = \dfrac{(i - j - k)}{\sqrt{3}}$

The Force vector P is given by, $P = 100\left[\dfrac{i - j - k}{\sqrt{3}}\right]$

$= 57.74i - 57.74j - 57.74k$

HOD-CEO

6. A vertical post guy wire is anchored by means of a bolt A as shown in fig. The tension in the wire is is 2000 N. Determine i) the force vector
ii) the angles defining the direction of the force and
iii) the components of the force acting on the bolt.
The co-ordinates of B are (0, 80, 0) and that of A (40, 0, −30)



$X_B = 0,\ Y_B = 80,\ Z_B = 0$

$x_A = 40,\ Y_A = 0,\ Z_A = -30$

Distane b/w A & B

$x = 0 - 40 = -40,\ y = 80 - 0 = 80,\ z = 0 - {-30} = 30$

Vector joining A and B is $= xi + yj + zk$
B and A

$= -40i + 80j + 30k$

Unit vector $= \dfrac{-40i + 80j + 30k}{\sqrt{(-40)^2 + 80^2 + 30^2}} = \dfrac{-40i + 80j + 30k}{94.34}$

$= -0.424i + 0.848j + 0.318k$

i) The force vector $= F \times$ unit vector

$= 2000(-0.424i + 0.848j + 0.318k)$

$= -848i + 1696j + 636k$

ii) The components of the force acting on the bolt,

$F_x = -848\ N$

$F_y = 1696\ N$

$F_z = 636\ N$

iii) Direction of force,

$F_x = F\cos\theta_x,\quad \cos\theta_x = \dfrac{F_x}{F} = \dfrac{-848}{2000}$ l

$\theta_x = 115.09^\circ$

$F_y = F\cos\theta_y,\quad \cos\theta_y = \dfrac{F_y}{F} = \dfrac{1696}{2000}$ m

$\theta_y = 32^\circ$

Check,

$l^2 + m^2 + n^2 = 1$

$F_z = F\cos\theta_z,\quad \cos\theta_z = \dfrac{F_z}{F} = \dfrac{636}{2000}$ n

$\theta_z = 71.4^\circ$

: Vector operations - Vector addition

1. Find the unit vector in the direction of the resultant of vectors $A = 2i - j + k$, $B = i + j + 2k$, $C = 2i + 2j + 4k$

Resultant $R = A + B + C$

$= (2+1+2)i + (-1+1+2)j + (1+2+4)k$

$= 5i + 2j + 7k$

Unit vector in the direction of R is $\frac{R}{|R|}$

$$= \frac{5i + 2j + 7k}{\sqrt{5^2 + 2^2 + 7^2}} = 0.57i + 0.23j + 0.79k$$

Dot product

2. Find the projection of vector $A = 4i - 3j + k$ on the line joining the points $C(2,3,-1)$ and $D(-2,-4,3)$

Soln : Line joining C and D is

$B = (-2-2)i + (-4-3)j + (3-(-1))k$

$= -4i - 7j + 4k$

The projection of vector A on vector B is $|A|\cos\theta$, where θ is the angle b/w vectors A and B

$A \cdot B = |A||B|\cos\theta$

$\therefore |A|\cos\theta = \frac{A \cdot B}{|B|}$

$A \cdot B = A_x B_x + A_y B_y + A_z B_z$

$= (4 \times (-4) + -3 \times (-7) + 1 \times 4) = 9$

$|B| = \sqrt{(-4)^2 + (-7)^2 + (4)^2} = 9$

$\therefore |A|\cos\theta = \frac{A \cdot B}{|B|} = \frac{9}{9} = 1$

3. The point of application of a force $F = 5i + 10j - 15k$ is displaced from the point $A(i + 0j + 3k)$ to the point

$B(3i - j - 6k)$. Find the workdone by the force.

$F = 5i + 10j - 15k$

Co-ordinates of $A(1, 0, 3)$

$B(3, -1, 6)$

$\text{Workdone} = F \cdot d$

Distance A and D, $d = (3-1)i + (-1+0)j + (-6-3)k$

$= 2i - j - 9k$.

Workdone $F \cdot d = (5i + 10j - 15k)(2i - j - 9k)$

$= -10 - 10 + 135 = 135$

Cross product

4. Determine the cross product of vectors $A = 2i - 6j - 3k$ and $B = 4i + 3j - k$ and the angle between these two vectors.

$$A \times B = \begin{vmatrix} i & j & k \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = \begin{aligned} &[(-6 \times -1) - (-3 \times 3)]\, i \\ &+ [(-3 \times 4)(-2 \times -1)]\, j \\ &+ [(2 \times 3) - (-6 \times 4)]\, k \end{aligned}$$

$= 15i - 10j + 30k$

Unit vector along the direction of $A \times B$,

$$= \frac{A \times B}{|A \times B|} = \frac{15i - 10j + 30k}{\sqrt{15^2 + (-10)^2 + 30^2}}$$

$= 0.43i - 0.29j + 0.86k$

Angle between vectors A and B

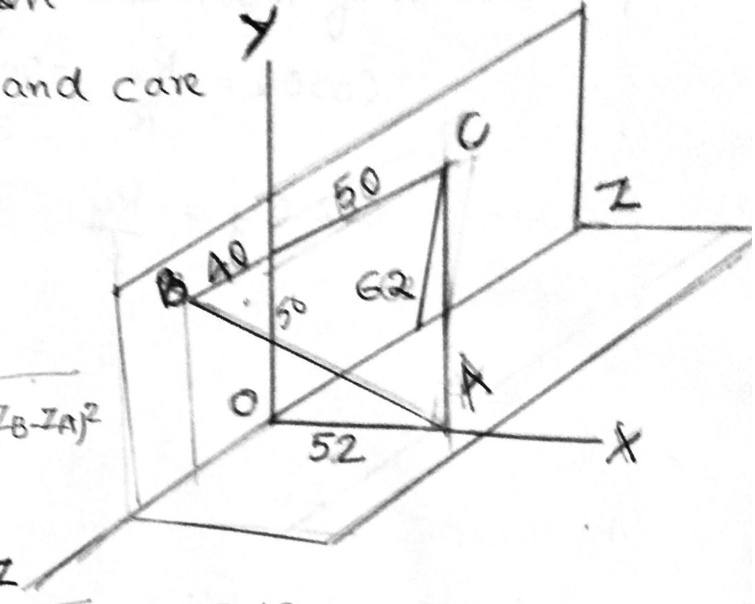
$$\sin\theta = \frac{|A \times B|}{|A||B|} = \frac{35}{7 \times \sqrt{26}} = 0.98$$

$\theta = 78.52°$ or $101.48°$

5. Two cables AB and AC are attached at A as in fig.

a) Determine the resultant of the forces exerted at A by the two cables, if the tension is 2000N in the cable AB and 1500N in the cable AC.

The coordinates of A, B and C are



A(52, 0, 0)

B(0, 50, 40) and

C(0, 62, -50)

$d_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$

$= (-52)^2 + 50^2 + 40^2$

$= 82.5m.$

$d_{AC} = \sqrt{(-52)^2 + 62^2 + -50^2} = 95.12m.$

Unit vector in the direction of AB

$$= \frac{(0-52)i + (50-0)j + (40-0)k}{\sqrt{(-52)^2 + (50)^2 + (40)^2}} = 2000\frac{(-52i + 50j + 40k)}{82.49}$$

Force vector along $AB = 2000\left(\frac{-52i + 50j + 40k}{82.49}\right)$

$= -1260.75i + 1212.27j + 969.81k$

Unit vector in the direction of AC,

$$= \frac{(0-52)i + (62-0)j + (-50-0)k}{\sqrt{(-52)^2 + 62^2 + (-50)^2}} = \frac{-52i + 62j - 50k}{95.12}$$

Force vector along $AC = 1500\left(\frac{-52i + 62j - 50k}{95.12}\right)$

$= -820.02i + 977.71j - 788.48k$

Resultant force at A, $R = F_{AB} + F_{AC}$

$= -1260.75i + 1212.27j + 969.81k - 820.02i + 977.71j - 788.48k$

$= -2080.78i + 2189.98j + 181.33k$

The magnitude of resultant at A,

$$R = \sqrt{(-2080.78)^2 + (2189.88)^2 + (181.33)^2}$$

$$= 3026.431\text{N}$$

Direction of resultant is given by,

$$\cos\theta_x = \frac{R_x}{R} = \frac{-2080.78}{3026.31}, \quad \theta_x = 133.44^\circ$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{2189.98}{3026.31}, \quad \theta_y = 43.64^\circ$$

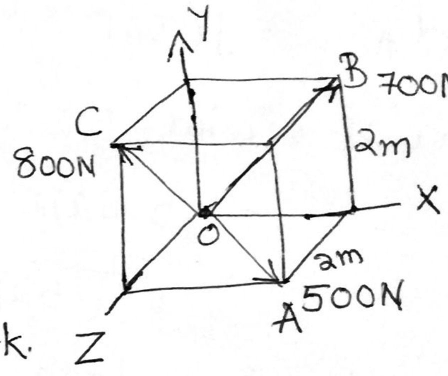
$$\cos\theta_z = \frac{R_z}{R} = \frac{181.33}{3026.31}, \quad \theta_z = 86.56^\circ$$

6. Three forces 500N, 700N and 800N are acting along the three diagonals of adjacent ~~faces~~ faces of a cube of side 2m as shown in fig 1.157. Determine the resultant of the forces

Co-ordinates of A (2, 0, 2)

B (2, 2, 0)

C (0, 2, 2)



Unit vector along OA $= \frac{2i + 0j + 2k}{\sqrt{2^2 + 2^2}}$

$= 0.71i + 0j + 0.7k.$

Force vector along OA $= 500(0.71i + 0j + 0.71k) = 355i + 0j + 355k$

Unit vector along OB $= \frac{2i + 2j + 0k}{\sqrt{2^2 + 2^2 + 0^2}} = 0.71i + 0.71j + 0k.$

Force vector along OB $= 700(0.71i + 0.71j + 0k)$

$= 497i + 497j + 0k$

Unit vector along OC $= \frac{0i + 2j + 2j}{\sqrt{0^2 + 2^2 + 2^2}} = 0i + 0.71j + 0.71k$

Force vector along OC $= 800(0i + 0.71j + 0.71k) = 0i + 569j + 569k$

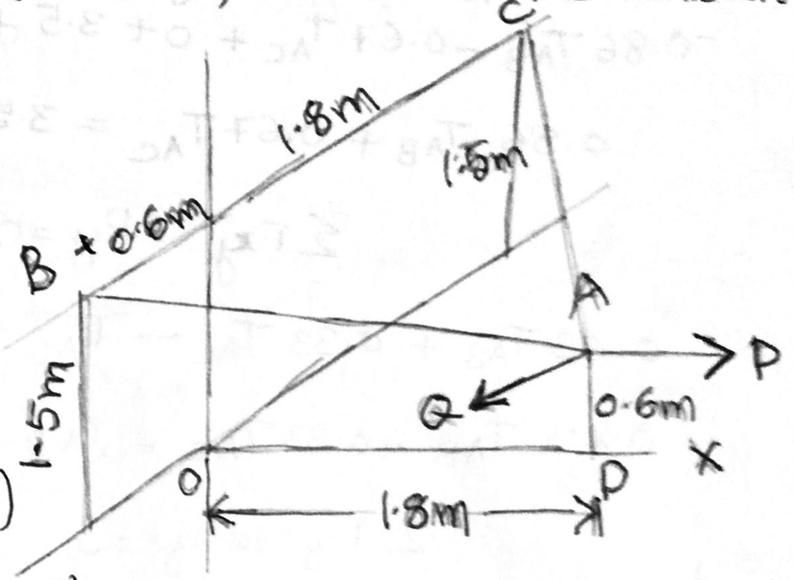
Resultant R $= F_{OA} + F_{OB} + F_{OC} = 852i + 1066j + 924k$

$R = \sqrt{852^2 + 1066^2 + 924^2} = 1648\text{N}$

$\cos\theta_x = \frac{\Sigma F_x}{\Sigma F} = \frac{852}{1648}$ $\theta_x = 58.87^\circ$, $\theta_y = 49.7^\circ$, $\theta_z = 55.9^\circ$

Equilibrium of forces

3) Three cables are joined at A, where two forces P and Q are applied in x and ~~x~~ z directions as in figure. If $P = 3.5\,kN$ and $Q = 1.5\,kN$, determine the tension in each cable.



Solution

Co-ordinates of A are $(1.8, 0.6, 0)$

Co-ordinates of B are $(0, 1.5, 0.6)$.

Co-ordinates of C $(0, 1.5, -1.8)$

Co-ordinates of D $(1.8, 0, 0)$

$$d_{AB} = \sqrt{(-1.8)^2 + 0.9^2 + 0.6^2} = \cancel{2.7m}.$$

$$d_{AC} = \sqrt{(0-1.8)^2 + (1.5-0.6)^2 + (-1.8-0)^2} = 2.7m$$

$$d_{AD} = \sqrt{(1.8-1.8)^2 + (0-0.6)^2 + (0-0)^2} = 0.6m$$

Unit vector in the direction of AB,

$$= \frac{(0-1.8)i + (1.5-0.6)j + (0.6-0)k}{\sqrt{1.8^2 + 0.9^2 + 0.6^2}} = \frac{-1.8i + 0.9j - 0.6k}{2.1}$$

$$= T_{AB}(-0.86i + 0.43j + 0.29k)$$

Unit vector in the direction of AC,

$$= \frac{(0-1.8)i + (1.5-0.6)j + (0-1.8)k}{\sqrt{(-1.8)^2 + 0.9^2 + 1.8^2}} = \frac{-1.8i + 0.9j - 1.8k}{2.7}$$

$$\text{Force vector along AC} = T_{AC}\frac{(-1.8i + 0.9j - 1.8k)}{2.7}$$

$$= T_{AC}(-0.76i + 0.33j - 0.67k)$$

Unit vector in the direction of AD,

$$= \frac{(0-0)i + (0-0.6)j + (0-0)k}{\sqrt{0^2 + (-.6)^2 + 0^2}} = \frac{0i - 0.6j + 0k}{0.6}$$

$$\text{Force vector along AD} = T_{AD}\frac{(0i - 0.6j + 0k)}{0.6}$$

$$= 0i - T_{AD}\,j + 0k$$

Force along P $= 3.5i + 0j + 0k$

Force along Q $= 0i + 0j + 1.5k$

For the equillibrium of point A, $\Sigma F_x = R_x = 0$

$-0.86\,T_{AB} - 0.67\,T_{AC} + 0 + 3.5 + 0 = 0$

$0.86\,T_{AB} + 0.67\,T_{AC} = 3.5 \ldots (i)$

$\Sigma F_y = R_y = 0$

$0.43\,T_{AB} + 0.33\,T_{AC} - T_{AD} + 0 + 0 = 0$

$0.43\,T_{AB} + 0.33\,T_{AC} - T_{AD} = 0 \quad —(ii)$

$\Sigma F_z = R_z = 0$

$0.29\,T_{AB} - 0.67\,T_{AC} + 0 + 1.5 = 0$

$0.29\,T_{AB} - 0.67\,T_{AC} + 0 + 1.5 = 0$

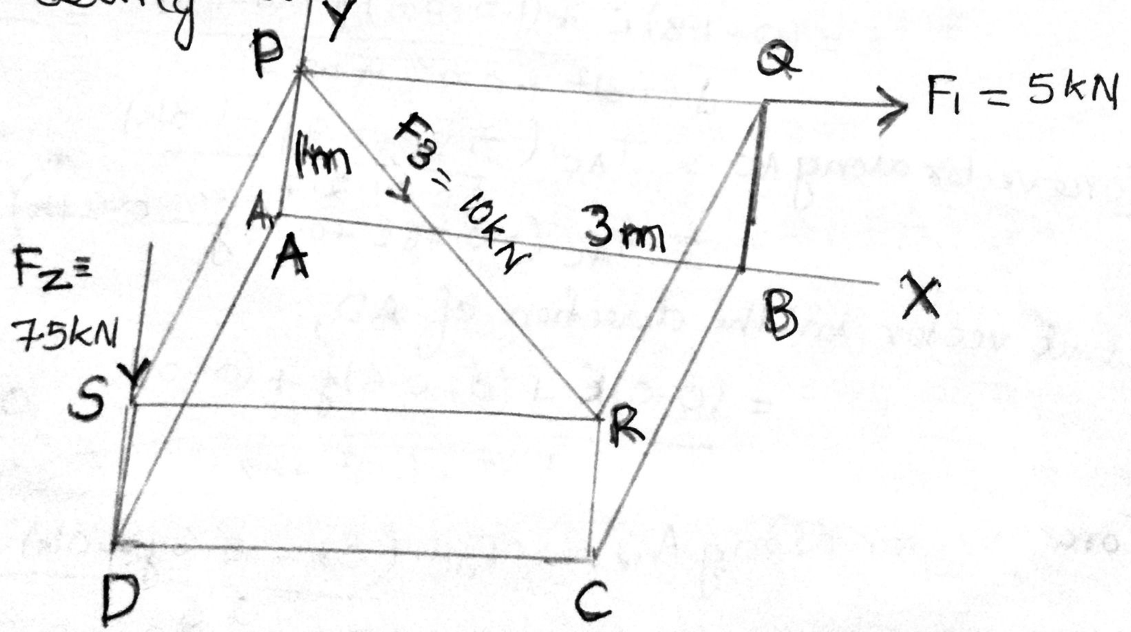
$0.29\,T_{AB} - 0.67\,T_{AC} = -1.5 \ldots (iii)$

Solving eqns (i), (ii) and (iii) $T_{AB} = 1.74$ kN

$T_{AC} = 3$ kN

$T_{AD} = 1.74$ kN

A rectangular block is subjected to three forces as in fig. Reduce them to an equivalent force couple system acting at A.



∴ Co-ordinates of various points are

$A(0,0,0)$; $P(0,1,0)$; $Q(3,1,0)$; $R(3,1,2)$; $S(0,1,2)$;

$D(0,0,+2)$

Unit vector in the direction of PQ is,

$$= \frac{(3-0)i + (1-1)j - 0k}{\sqrt{3^2 + (1-1)^2 + 0^2}} = \frac{3i}{3} = i$$

$$F_1 = 5(i + 0j + 0k)$$

Unit vector in the direction of PR,

$$= \frac{(3-0)i + (1-1)j + (2-0)k}{\sqrt{3^2 + 0^2 + 2^2}} = \frac{3i + 0j + 2k}{\sqrt{13}}$$

$$= 0.832i + 0j + 0.554k$$

$$F_3 = 10(0.832i + 0j + 0.554k) = 8.32i + 0j + 5.54k$$

Unit ~~force~~ in the direction of SD,

$$= \frac{(0-0)i + (0-1)j + (2-2)k}{\sqrt{0^2 + 1^2 - 0^2}} = 0i - j + 0k$$

$$F_2 = 7.5(0i - j + 0k) = 0i - 7.5j + 0k$$

Resultant $R = F_1 + F_2 + F_3$

$$= (5 + 0 + 8.32)i + (0 - 7.5 + 0)j + (0 + 5.54 + 0)$$

$$R = 13.32i - 7.5j + 5.54k$$

The resultant moment about A is obtained by summing the moments of F_1, F_2, and F_3 about A.

$$M_A = r_{AP} \times F_1 + r_{AP} \times F_3 + r_{AD} \times F_2$$

$$r_{AP} = (0-0)i + (1-0)j + (0-0)z = 0i + j + 0k.$$

$$r_{AD} = (0-0)i + (0-0)j + (2-0)k = 0i + 0j + 2k$$

$$r_{AP} \times F_1 = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 5 & 5 & 0 \end{vmatrix} = 0i + 0j - 5k$$

$$r_{AP} \times F_3 = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 8.32 & 0 & 5.54 \end{vmatrix} = 5.54i + 0j - 8.32k$$

$$r_{AD} \times F_2 = \begin{vmatrix} i & j & k \\ 0 & 0 & 2 \\ 0 & -7.5 & 0 \end{vmatrix} = 15i + 0j - 0k$$

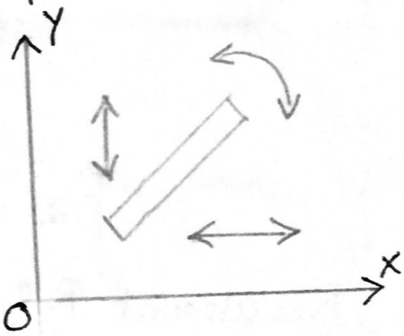
$$M_A = r_{AP} \times F_1 + r_{AP} \times F_3 + r_{AD} \times F_2$$
$$= (0i + 0j - 5k) + (5.54i + 0j - 8.32k) + (15i + 0j - 0k)$$
$$= (20.54i + 0j + 13.32k) \text{ kNm.}$$

Degree of Freedom of a Rigid Body (DOF) - Ref. wikipedia

The degree of freedom of a rigid body is defined as the number of independant movements it has.

Degrees of freedom of a rigid body in a plane.

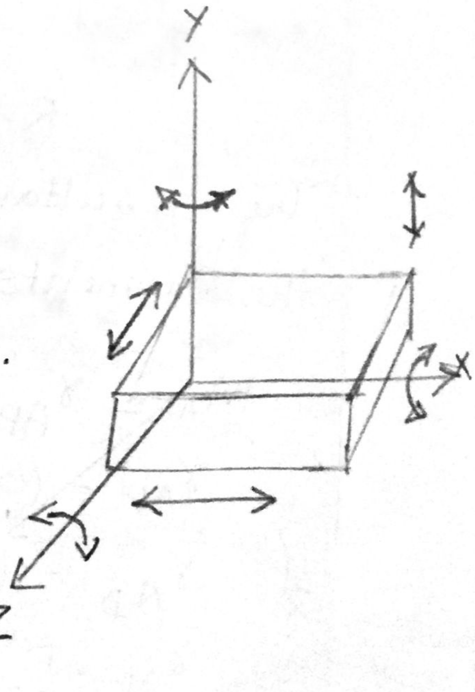
A rigid body in a plane has 3 DOF - 2 translations + 1 rotation, Translation along x & y axis & rotation about z axis.



Degrees of freedom of a rigid body in space

An unrestrained rigid body in space has 6 DOF → 3 translations + 3 rotations.

- translating motions along x, y, & z axes.
- rotating (rotary) motions around x, y & z axes.



✓ Forces 30kN, 20kN, 25kN and 100kN are respectively directed through pts whose cordinates are A(2,1,5), B(3,-1,4), C(-3,-2,1) & D(4,1,-2). If these forces are concurrent, at the origin O, make calculations for the resultant of the system

$\vec{OA} = 2i + j + 5k$

$\vec{OB} = 3i - j + 4k$

$\vec{OC} = -3i - 2j + k$

$\vec{OD} = 4i + j - 2k.$

Unit vector along $\vec{OA} = \dfrac{2i + j + 5k}{\sqrt{2^2+1^2+5^2}}$

$= 0.3651 i + 0.1826 j + 0.9129 k$

Force vector along $\vec{OA}$, $F_1 = 30(0.365 i + 0.1826 j + 0.9129 k)$

$= 10.95 i + 5.48 j + 27.39 k.$

Unit vector along $\vec{OB}$, $F_2 = 20(0.5884 i - 0.196 j + 0.7845 k$

$= 11.77 i - 3.92 j + 15.69 k.$

Force vector along $\vec{OC}$, $\dfrac{-3i - 2j + k}{\sqrt{9+4+1}} = -0.8018 i - 0.5345 j + 0.2637 k)$

Force vector along OC, $F_3 = 25(-0.8018 i - 0.5345 j + 0.2637 k)$

$= -20.05 i - 13.36 j + 6.68 k$

Unit vector along $\vec{OD} = \dfrac{4i + j - 2k}{\sqrt{16+1+4}}$

Force vector along OD, $F_4 = 100(0.8729 i + 0.2182 j - 0.436 k)$

$= 87.29\,i + 21.82\,j - 43.64\,k$

$\vec{R} = F_1 + F_2 + F_3 + F_4$

$= 89.96\,i + 10.02\,j + 6.12\,k.$

$|R| = \sqrt{89.96^2 + 10.02^2 + 6.12^2}$

$= 90.72\ kN.$

$\underset{(l)}{\cos\theta_x} = \frac{R_x}{R} = \frac{89.96}{90.72} = 0.9916 \quad , \theta_x = 7.42$

$\underset{(m)}{\cos\theta_y} = \frac{R_y}{R} = \frac{10.02}{90.72} = 0.1105 \, , \theta_y = 83.66$

$\underset{(n)}{\cos\theta_z} = \frac{R_z}{R} = \frac{6.12}{9.72} = 0.0675$

$\theta_z = 86.13^\circ$

Check $l^2 + m^2 + n^2 = 1$

ie $0.9916^2 + 0.1105^2 + 0.0675^2 = 1.0000$