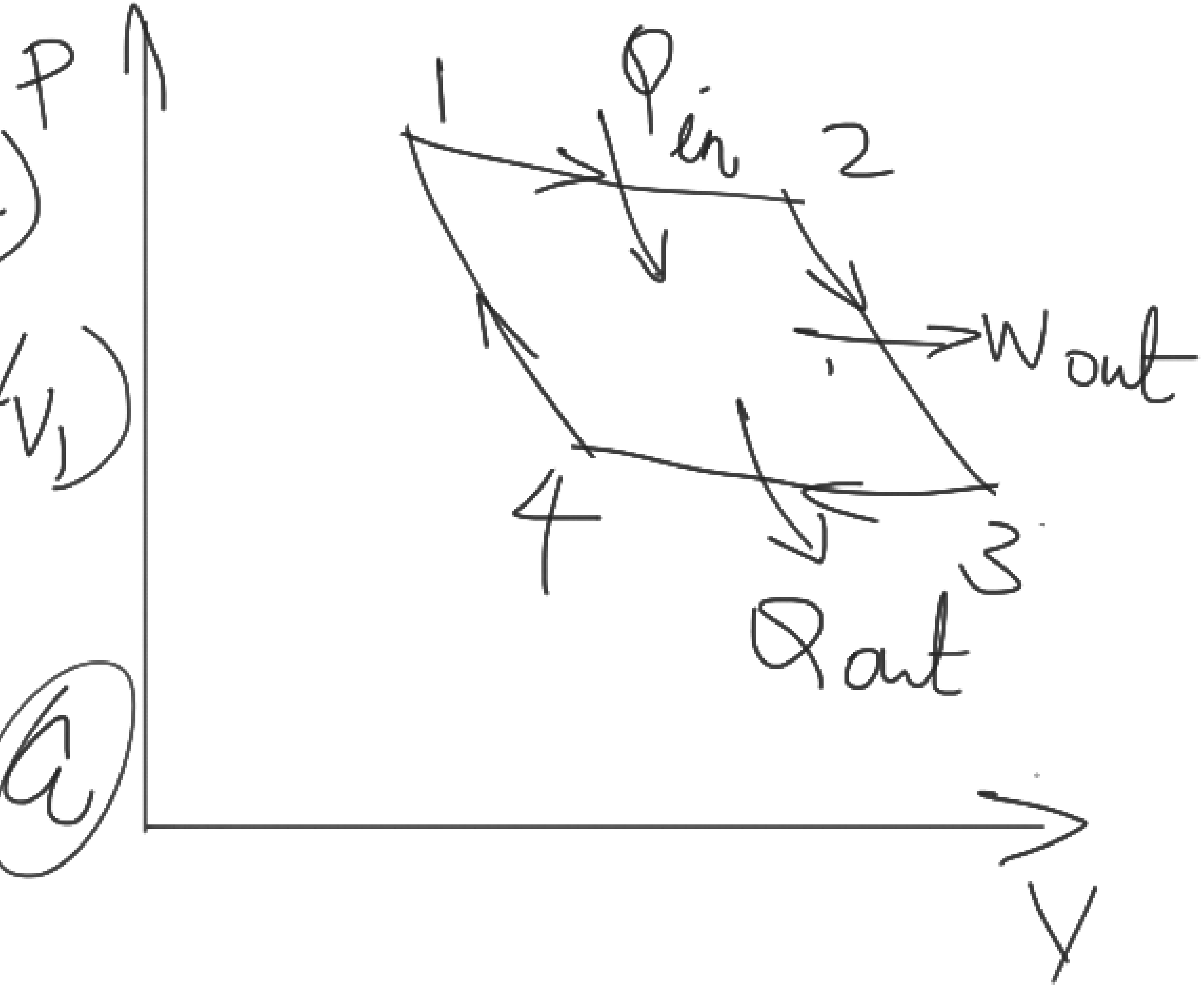
Carnot Cycle (continues - - -)

04-12-'21

$$\eta = 1 - \frac{Q_R}{Q_S} = 1 - \frac{mRT_3 \ln(V_3/V_4)}{mRT_1 \ln(V_2/V_1)}$$



$$\therefore \eta = 1 - \frac{T_3}{T_1} \times \frac{\ln(V_3/V_4)}{\ln(V_2/V_1)} \quad \text{--- (a)}$$

Process 2-3

V-T relation.

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{--- (i)}$$

Process 4-1

V-T relation

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\frac{T_2}{T_3} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{--- (ii)}$$

$T_1 = T_2$

$T_3 = T_4$

Comparing (i) and (ii)

$$\Rightarrow \left(\frac{V_3}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \quad \text{--- (iii)}$$

Adiabatic expansion ratio = Adiabatic compression ratio or $\dfrac{V_3}{V_4} = \dfrac{V_2}{V_1}$ —— (iv).

ie isothermal compression ratio = isothermal expansion ratio

$\therefore \eta = 1 - \dfrac{T_3 \times \ln\left(\frac{V_3}{V_4}\right)}{T_1 \times \ln\left(\frac{V_2}{V_1}\right)}$ (from (iv))

$= 1 - \dfrac{T_3 \times \ln(V_3/V_4)}{T_1 \times \ln(V_3/V_4)}$

ie $\boxed{\eta = 1 - \dfrac{T_3}{T_1}}$

$\boxed{\eta = 1 - \dfrac{T_{cold\ body}}{T_{hot\ body}}}$

If T_H - temp. of hot body

T_L - temp. of cold body.

$$\boxed{\eta = 1 - \frac{T_L}{T_H}}$$

- Practically Carnot cycle is impossible
- of all the theoretical cycles, Carnot cycle is having maximum efficiency - to compare with other cycles

Alternate isothermal & adiabatic processes are difficult to carry out.

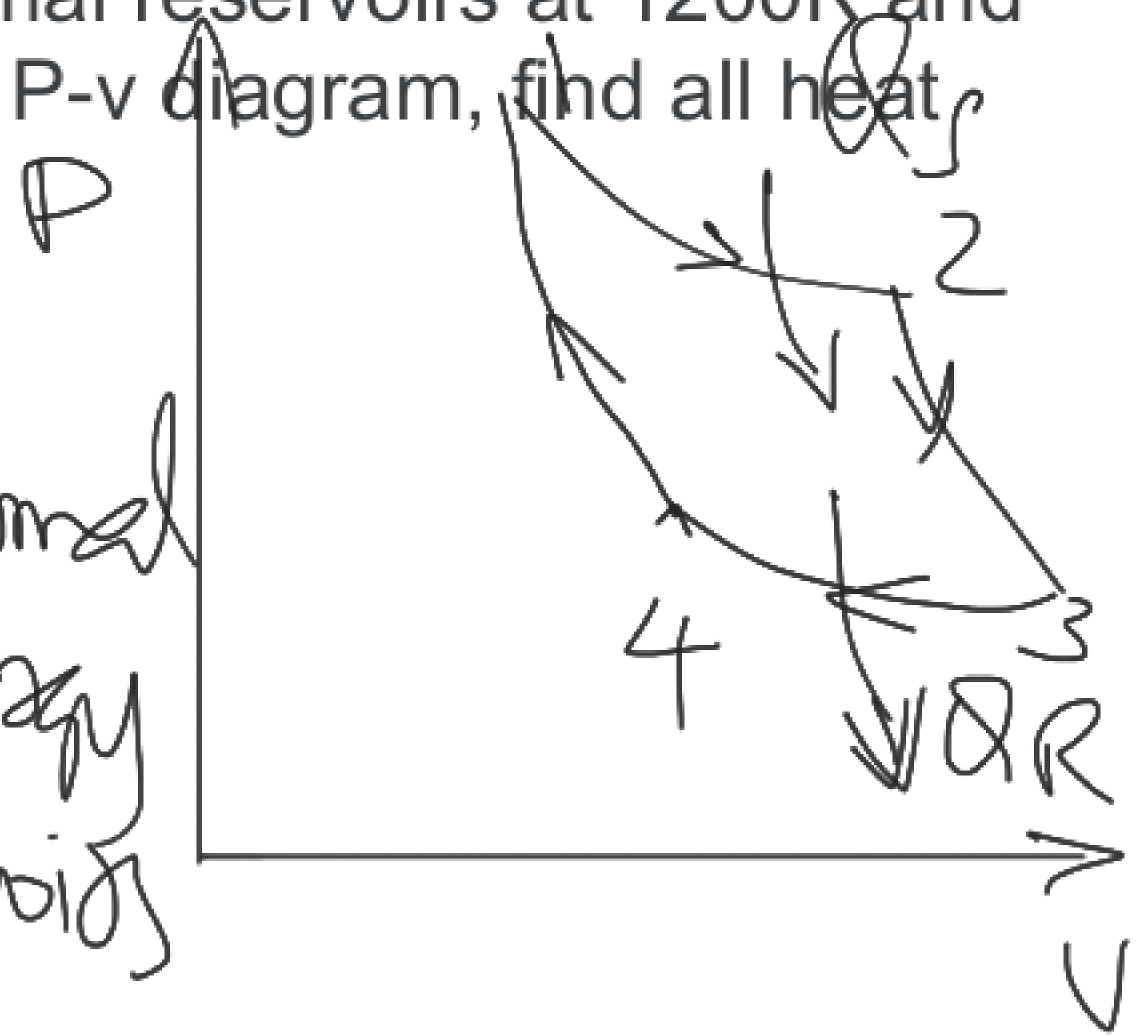
Air undergoes a Carnot cycle inside a piston-cylinder between thermal reservoirs at 1200K and 400K. During heat addition, the volume triples. Draw the cycle on a P-v diagram, find all heat transfers in kJ/kg, and the overall cycle efficiency.

Solution

$T_1 = T_2 = 1200K$ (Source)

$T_3 = T_4 = 400K$ (Sink)

thermal energy reservoirs



Heat transfers

$$Q_S = P_1 V_1 \ln(V_2/V_1) = mRT_1 \ln(V_2/V_1)$$

$$R_{air} = 287 J/kgK$$

$V_2 = 3V_1$ (during heat addition)

$$\frac{V_2}{V_1} = 3$$

$\therefore Q_S = 1 \times 287 \times 1200 \times \ln(3) =$ J/kg or kJ/kg

$$Q_R = mRT_3 \ln\left(\frac{V_3}{V_4}\right) = mRT_3 \times \ln\left(\frac{V_2}{V_1}\right)$$

$$= 1 \times 287 \times 400 \times \ln(3)$$

$=$ J/kg or kJ/kg

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{400}{1200} = 1 - \frac{1}{3} = \underline{\underline{2/3}}$$

$$\text{or } \underline{\underline{66.667\%}}$$