Grau en Enginyeria Informàtica
Facultat d'Informàtica de Barcelona
Universitat Politècnica de Catalunya

MATEMÀTIQUES 1

Part I: Teoria de grafs

Respostes a alguns exercicis

Setembre 2016

Aquest document conté les respostes a alguns dels problemes de la segona part de l'assignatura Matemàtiques 1. Aprofitem per fer constar i agrair la tasca del becari docent Gabriel Bernardino en la redacció de les solucions.

Us ho agraïrem si ens comuniqueu qualsevol errada que detecteu.

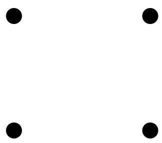
Anna de Mier
Montserrat Maureso
Dept. Matemàtica Aplicada 2

Respostes

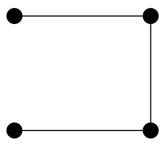
Conceptes bàsics de grafs

1.1

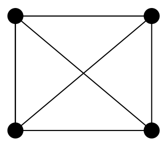
1) N_4



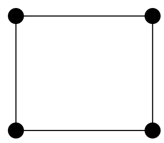
T_4



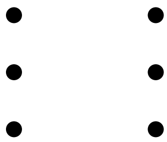
K_4



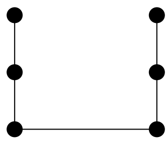
C_4



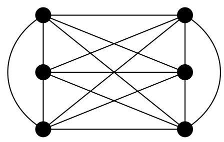
N_6



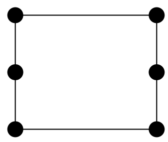
T_6



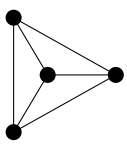
K_6



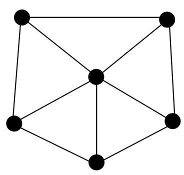
C_6



W_4



W_6



2)

$$M_A(N_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad M_A(K_5) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

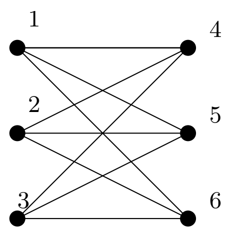
$$M_A(T_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad M_A(C_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$M_A(W_5) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

3) Per a $n \geq 3$ ($n \geq 4$ en el cas del graf W_n):
$N_n = (V, E): \ |V| = n, \ |E| = 0, \delta(N_n) = 0, \ \Delta(N_n) = 0$
$K_n = (V, E): \ |V| = n, \ |E| = \binom{n}{2}, \ \delta(K_n) = n - 1, \ \Delta(K_n) = n - 1$
$T_n = (V, E): \ |V| = n, \ |E| = n - 1, \ \delta(T_n) = 1, \ \Delta(T_n) = 2$
$C_n = (V, E): \ |V| = n, \ |E| = n, \ \delta(C_n) = 2, \ \Delta(C_n) = 2$
$W_n = (V, E): \ |V| = n, \ |E| = 2 \cdot n - 2, \ \delta(W_n) = 3, \ \Delta(W_n) = n - 1$

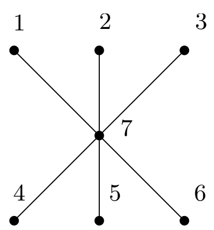
1.2

- Solució d' 1. i 2.



1	2	3	4	5	6
4	4	4	1	1	1
5	5	5	2	2	2
6	6	6	3	3	3

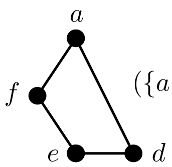
- Solució de 3. i 4.



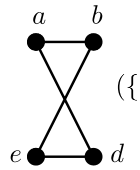
1	2	3	4	5	6	7
7	7	7	7	7	7	1
						2
						3
						4
						5
						6

1.4 1) $\frac{r \cdot n}{2}$; 2) $r \cdot s$;

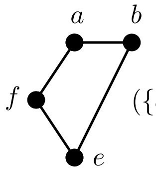
1.5



$(\{a, d, e, f\}, \{ad, de, ef, fa\})$

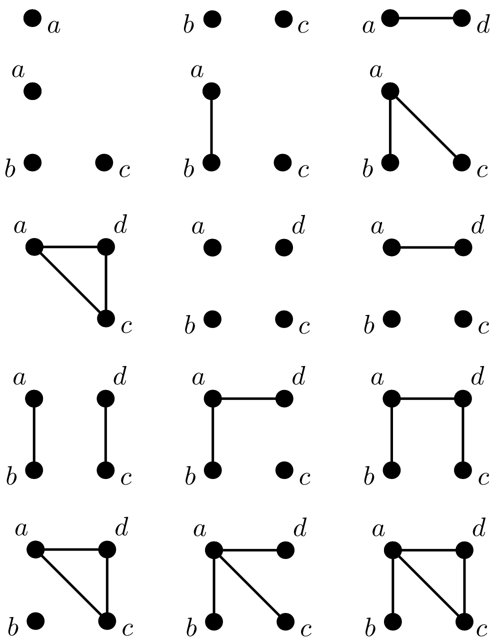


$(\{a, b, e, d\}, \{ab, be, ed, da\})$



$(\{a, b, e, f\}, \{ab, be, ef, fa\})$

1.6



1.7

1) 5; 4.

2) 4; 2.

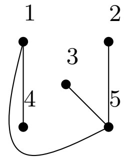
3) 5; 5.

4) 9; 8.

5) 9; 12.

1.8

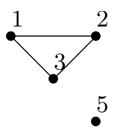
1) G^c



$A = \{14, 15, 25, 35\}$

$$M_A(G^c) = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad M_I(G^c) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

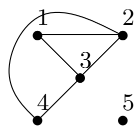
2) $G - 4$



$A = \{12, 13, 23\}$

$$M_A(G-4) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad M_I(G-4) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

3) $G - 45$

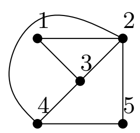


$A = \{12, 13, 23, 24, 34\}$

$$M_A(G{-}45) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_I(G{-}45) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4) $G + 25$

$A = \{12, 13, 23, 24, 25, 34, 45\}$



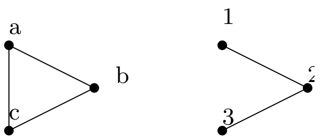
$$M_A(G{+}25) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \quad M_I(G{+}25) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

1.9

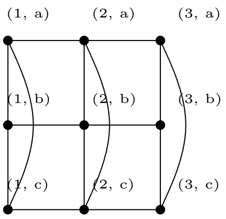
- $G^c = (V, A)$: $|V| = n$; $|A| = \binom{n}{2} - m$.
- $G - v = (V, A)$: $|V| = n - 1$; $|A| = m - g(u)$.
- $G - a = (V, A)$: $|V| = n$; $|A| = m - 1$.

1.10

1) $K_3 \cup T_3$ $A = \{ab, ac, bc, 12, 23\}$



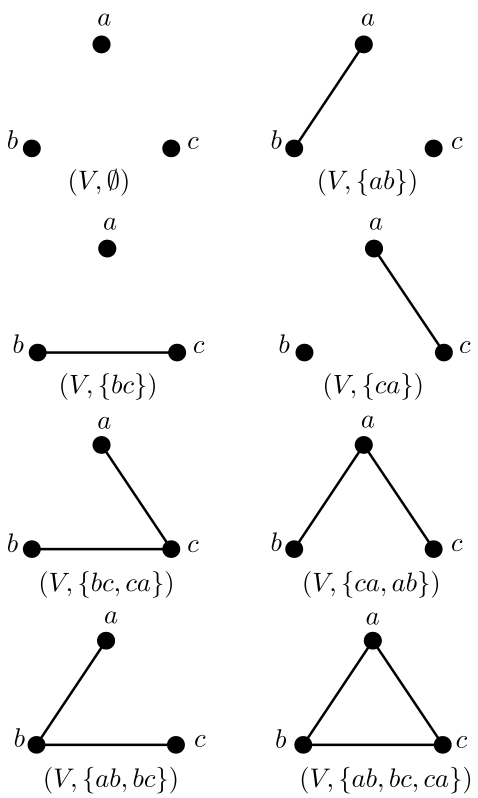
2) $T_3 \times K_3$



$A = \{$(1,a)(1,b); (1,a)(1,c); (1,a)(2,a); (1,b)(1,c); (1, b)(2,b); (1,c)(2,c); (2,a)(2,b); (2,a)(2,c); (2,a)(3,a); (3,a)(3,b); (3,a)(3,c); (3,b)(3,c)$\}$

1.11 Ordre $|V_1||V_2|$, $g_{G_1 \times G_2}(u_1, u_2) = g_{G_1}(u_1) + g_{G_2}(u_2)$ i mida $|V_1||A_2| + |V_2||A_1|$.

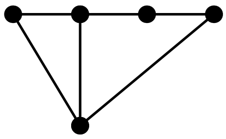
1.13



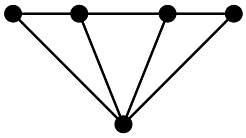
1.14 20349.

1.15 $2^{n(n-1)/2}$.

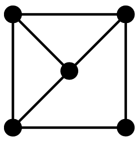
1.17

1) 

2) No existeix.

3) 

4) No existeix.

5) 

6) No existeix.

1.23 4 la Maria i 4 la parella.

1.24 

1.25

- $G_1 \cong G_2$
- $G_3 \cong G_4$
- $G_5 \cong G_6$
- G_7
- $G_8 \cong G_9 \cong G_{10}$
- G_{11}
- G_{12}

- G_{13}

1.27 2.

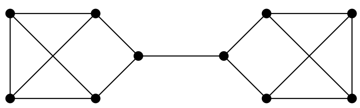
Recorreguts, connexió i distància

2.1 G_1: Camí de longitud 9: 1 2 3 4 5 10 7 9 6 8. No hi ha camins de longitud 11 ja que té ordre 10. Cicles: 1 2 3 4 5 1; 1 2 3 8 10 5 1; 1 6 8 10 7 9 4 5 1; 1 2 3 4 9 7 10 8 6 1.
G_2: 1 2 3 4 5 10 6 7 8 9. No hi ha camins de longitud 11 ja que té ordre 11. Cicles: 1 2 3 4 5 1; 5 10 6 11 9 4 5; 2 3 4 5 10 9 8 7 2; 5 1 2 3 4 9 11 6 10 5.

2.2 K_6: 4; 12; 24; 24; 0. $K_{3,3}$: Si els vèrtexs pertanyen a la mateixa part estable: 3; 0; 6; 0; 0. Altrament: 0; 4; 0; 4; 0.

2.5 1) $\langle\{a,b,d,e,f,g,i,j\}\rangle \bigcup \langle\{c,h\}\rangle$. 2) $\langle\{a,b,d,e,g,h,j,m\}\rangle \bigcup \langle\{c,f,i,k\}\rangle \bigcup \langle\{l\}\rangle$.

2.14 $n = 10$



2.17

- G_1 no és 2-connex, ja que 4 és un vèrtex de tall.
- G_2 no és 2-connex, ja que 3 i 6 són vèrtexs de tall.
- G_3 és 2-connex.

2.18 1)

v	a	b	d	e	f	g	i	j
d(a,v)	0	2	1	1	1	3	3	3
d(b,v)	2	0	1	2	2	1	1	1

2)

v	a	b	d	e	g	h	j	m
d(a,v)	0	1	2	2	2	2	1	3
d(b,v)	1	0	1	1	1	1	1	2

2.19 1) 1. 2) $D(G_1) = 2$, $D(G_2) = 4$. 3) 2. 4) $\lfloor n/2 \rfloor$. 5) 2. 6) $n-1$.

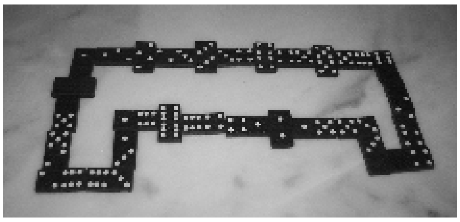
2.20 1) $G = W_6$ i u un vèrtex de grau 3. 2) $G = W_7$, u el vèrtex de grau 6. 3) $G = ([4], \{12, 13, 14, 23\})$, $u = 4$.

2.21 1) a) G_1: $e(v) = 2$, $1 \leq v \leq 10$; $r(G) = 2$; tots els vèrtexs són centrals. G_2: $e(1) = e(11) = 4$, $e(v) = 3$, $2 \leq v \leq 10$; $r(G) = 3$; v vèrtex central si $2 \leq v \leq 10$. b) G: $e(4) = 2$, $e(v) = 3$, $v \neq 4$; $r(G) = 2$; l'únic vèrtex central és el 4. 2) C_6. 3) T_5.

Grafs eulerians i hamiltonians

3.1 Només és eulerià el graf G_4

3.3



3.4 r i s parells.

3.5 2) 2^n; $n2^{n-1}$; Q_n és n-regular. 3) n parell.

3.6 Si els dos components són complets, 4; altrament, 3.

3.7 Tots, llevat del primer dibuix.

3.8 1) 7; 2) 4.

3.10 Només són hamiltonians els grafs G_1 i G_2.

3.11 $n!$

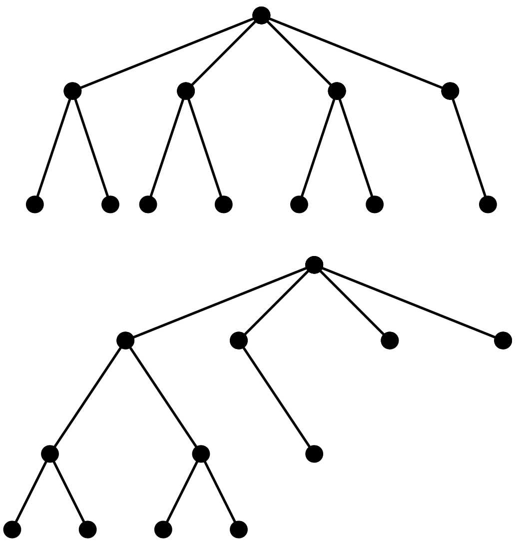
3.18 Dues.

Arbres

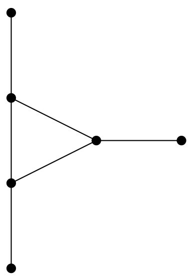
4.5 $n = 18$; ordre de T_2: 36; mida de T_2: 35.

4.6 $n(n-1)$.

4.7 4,3,3,3,2,1,1,1,1,1,1,1.

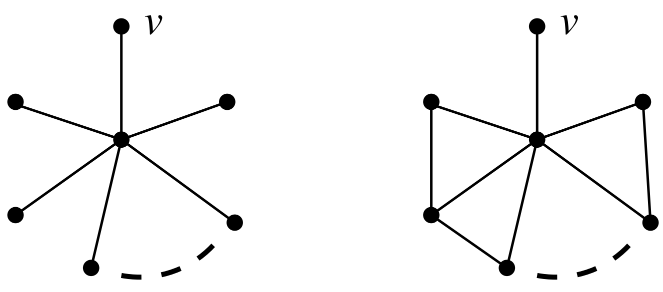


4.8



4.16 n i $r2^{r-1}$.

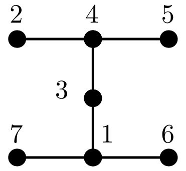
4.17



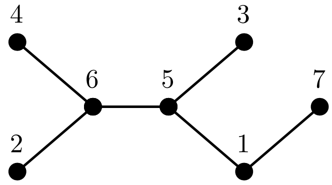
4.18 Un.

4.20 $(1,1,1,5)$; $(1,1,2,2,2,1)$; $(3,3,1,2,4,4,2,5,5)$.

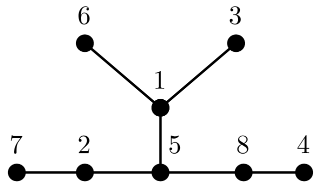
4.21 1)



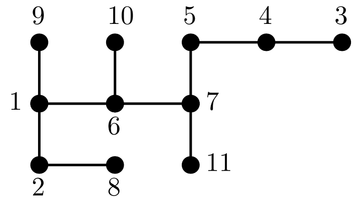
2)



3)



4)



4.22 Els trajectes d'ordre 3.

4.23 Els grafs estrella.

Excercicis de repàs i consolidació

A.1

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

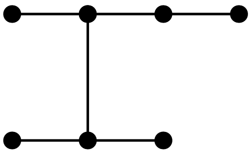
A.9 4 components connexos. 7,14,2,4,6,8,10,12,3,9,15,5.

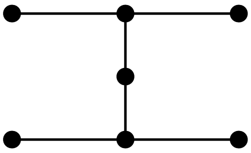
A.19 Sí; no.

A.22 K_1 i T_4.

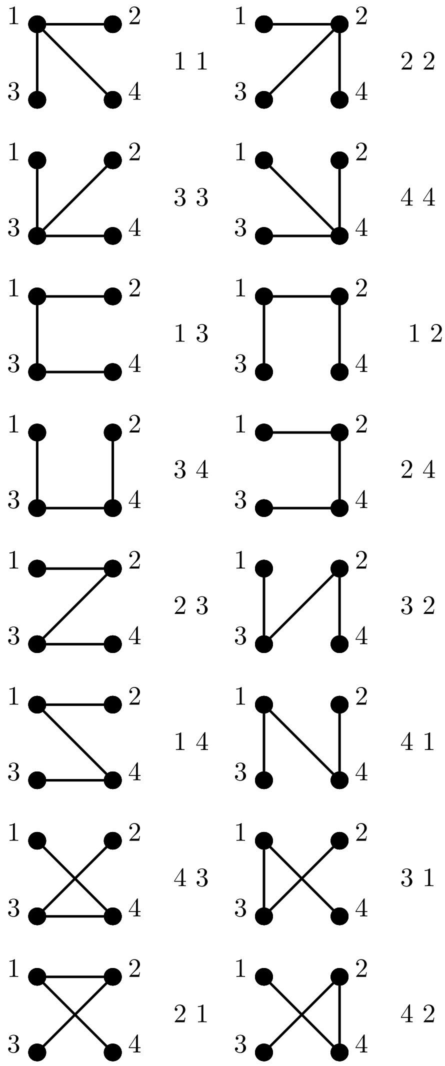
A.23 $k-1$.

A.24 3,3,2,1,1,1,1.





A.29



A.30 Els trajectes d'ordre $n \geq 4$.