Algebraic properties of limits

---

1 Finite limits

If $\lim a_n = a$ and $\lim b_n = b$ with $a, b \in \mathbb{R}$, then

$$\begin{aligned}
&\lim(a_n \pm b_n) = a \pm b \\
&\lim(a_n b_n) = ab \\
&\lim \frac{a_n}{b_n} = \frac{a}{b} \quad \text{for} \quad b \neq 0 \quad \text{and} \quad b_n \neq 0 \quad \text{for all} \quad n \in \mathbb{N} \\
&\lim (b_n)^{a_n} = b^a \quad \text{for} \quad b > 0 \quad \text{and} \quad b_n > 0 \quad \text{for all} \quad n \in \mathbb{N} \\
&\lim \ln(a_n) = \ln a \quad \text{for} \quad a > 0 \quad \text{and} \quad a_n > 0 \quad \text{for all} \quad n \in \mathbb{N}
\end{aligned}$$

2 Infinite limits

If $\lim a_n = a$ with $a \in \mathbb{R}$, $\lim b_n = \lim c_n = +\infty$ and $\lim d_n = 0$, then

$$\begin{aligned}
&\lim(a_n \pm b_n) = \pm\infty & &\lim(\pm b_n \pm c_n) = \pm\infty \\
&\lim(a_n b_n) = \pm\infty \quad \text{for} \quad a \neq 0 & &\lim(b_n c_n) = +\infty \\
&\lim \frac{a_n}{b_n} = 0 & &\lim \frac{a_n}{d_n} = \pm\infty \\
&\lim \frac{b_n}{a_n} = \pm\infty & &\lim \frac{b_n}{d_n} = \pm\infty \\
&\lim \frac{d_n}{a_n} = 0 & &\lim \frac{d_n}{b_n} = 0 \\
&\lim a_n^{b_n} = +\infty \quad \text{for} \quad a > 1 & &\lim a_n^{b_n} = 0 \quad \text{for} \quad 0 < a < 1 \\
&\lim b_n^{a_n} = +\infty \quad \text{for} \quad a > 0 & &\lim b_n^{a_n} = 0 \quad \text{for} \quad a < 0 \\
&\lim c_n^{b_n} = +\infty & &\lim c_n^{-b_n} = 0
\end{aligned}$$

3 Indeterminate forms

$$\infty - \infty\,, \quad 0 \cdot \infty\,, \quad \frac{\infty}{\infty}\,, \quad \frac{0}{0}\,, \quad 1^\infty\,, \quad 0^0\,, \quad \infty^0$$