Ejercicio 1

a) $L = \{ w \in L \mid |w|_{ab} > 0 \}$

b) $L = \{ w \in L^* \mid \forall x_1, x_2 \;\; w = x_1 ab x_2 \Rightarrow |x_2|_{ba} > 0 \}$

c) $L = \{ w \in L^* \mid |w|_{ab} > 0 \wedge |w|_{ba} > 0 \}$

d) $L = \{ w \in L^* \mid \forall_{x_1, x_2, x_3} \;\; w = x_1 b x_2 b x_3 \Rightarrow |x_2|_a > 0 \}$

e) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \;\; w = x_1 b x_2 \Rightarrow |x_1| \notin \dot{2} \}$

f) $L = \{ w \in L^* \mid \exists_{x_1, x_2} \;\; w = x_1 x_2 \wedge |x_1|_b \geq |x_1|_a \}$

g) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \;\; w = x_1 x_2 \Rightarrow |x_1|_b \geq |x_1|_a \}$

h) $L = \{ w \in L^* \mid \exists_{x_1, x_2} \;\; w = x_1 x_2 \wedge |x_1|_b \geq |x_1|_a \wedge |x_1| \in \dot{2} \}$

i) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \;\; (w = x_1 x_2 \wedge |x_1| \in \dot{2}) \Rightarrow |x_1|_b \geq |x_1|_a \}$

j) $L = \{ w \in L^* \mid \exists_{x_1, x_2, y_1, y_2} \;\; w = x_1 x_2 = y_1 y_2 \wedge x_1 = y_2 \wedge 0 < |x_1| < |w| \}$

Ejercicio 2

a) Fals → contraexemple $x = a$, $y = b$ $\quad xy = ab$, $yx = ba$ $\quad ab \neq ba$

b) Red. Absurdo

$(xy = xz \Rightarrow y \neq z)$ por contrarecíproco $y = z \Rightarrow xy \neq xz \Rightarrow$

$\overset{(y=z)}{\Rightarrow}$ $\boxed{xy \neq xy}$ contradicción $\Rightarrow$ cierto

c) $AB = \{xy \mid x \in A \wedge y \in B\} = \{k \mid k \in AB\} \Rightarrow \boxed{xy = k}$ (1)

$BC = \{yz \mid y \in B \wedge z \in C\} = \{j \mid j \in BC\} \Rightarrow \boxed{yz = j}$ (2)

$$\left.\begin{aligned} A(BC) &= \{xj \mid x \in A \wedge j \in BC\} = \{xj \in A(BC)\} \overset{(2)}{\Rightarrow} \{xyz \in A(BC)\} \\ (AB)C &= \{kz \mid k \in (AB) \wedge z \in C\} = \{kz \in (AB)C\} \overset{(1)}{\Rightarrow} \{xyz \in (AB)C\} \end{aligned}\right\} A(BC) = (AB)C$$

d)

Ejercicio 3

b) $L_1^* L_2^* \subseteq (L_1 L_2)^*$ fals

$L_1 = \{a\}$ $L_1 L_2 = \{ab, ba, \lambda\}$ ab

$L_2 = \{b\}$

c) $L_1^* L_2^* \supseteq (L_1 L_2)^*$ fals

$L_1 = \{a\}$ $L_1 L_2 = \{ab, ba, \lambda\}$ ab

$L_2 = \{b\}$

d) $L_1^* \subseteq L_2^* \rightarrow L_1 \subseteq L_2$ fals

$L_2 \{a\}$ $L_2 = \{\lambda, a, aa, aaa, \ldots\}$

$L_1 \{aa\}$ $L_1 = \{\lambda, aa, aaa, \ldots\}$

$L_1 \notin L_2$

$aa \notin a$

r) $L^2 \subseteq L \iff L = L^+$

Ejercicio 4

a) $(xy)^R = y^R x^R$ $\quad$ α $\left[(aw)^R = w^R a \mid a \in \Sigma \wedge w \in \Sigma^*\right]$

$\forall_{i \in (1,|x|)}\ x_i \in x \Rightarrow x = x_1 x_2 x_3 x_4 \cdots x_m \quad m = |x|$

$\forall_{j \in (1,|y|)}\ y_j \in y \Rightarrow y = y_1 y_2 y_3 y_4 \cdots y_m \quad m = |y|$

$$xy = x_1 x_2 x_3 x_4 \cdots x_m \cdot y_1 y_2 y_3 y_4 \cdots y_m$$

$$(xy)^R = (x_1 \underbrace{x_2 x_3 x_4 \cdots x_m \cdot y_1 y_2 y_3 y_4 \cdots y_m}_{= w_1})^R = (x_1 \cdot w_1)^R = w_1^R \cdot x_1$$

$$w_1^R \cdot x_1 = (\underbrace{x_2 x_3 x_4 \cdots x_m \cdot y_1 y_2 y_3 y_4 \cdots y_m}_{= w_2})^R \cdot x_1 = (x_2 \cdot w_2)^R \cdot x_1 = w_2^R \cdot x_2 \cdot x_1$$

$$\vdots$$

$$w_m^R \cdot x_{m-1} \cdot \cdots \cdot x_2 \cdot x_1 = (x_m \underbrace{y_1 \cdot y_2 \cdot y_3 \cdot y_4 \cdots y_m}_{w_{m+1}})^R \cdot x_{m-1} \cdot \cdots \cdot x_2 \cdot x_1 =$$

$$= (x_m \cdot w_{m+1})^R \cdot x_{m-1} \cdot \cdots \cdot x_2 \cdot x_1 = (w_{m+1})^R \cdot x_m \cdot x_{m-1} \cdot \cdots \cdot x_2 \cdot x_1 = (w_{m+1})^R \cdot x^R =$$

$$= (y_1 \cdot y_2 \cdot y_3 \cdot y_4 \cdots y_m)^R \cdot x^R = \boxed{y^R \cdot x^R}$$

f) $(L^*)^R = (L^R)^*$ $\quad$ α $\left[w^R \in L \Leftrightarrow w \in L^R\right]$ $\quad$ β $\left[(L^m)^R = (L^R)^m\right]$

$$w \in (L^*)^R \overset{\alpha}{\Longleftrightarrow} w^R \in L^*$$
$$\Longleftrightarrow w^R \in L^0 \cup L^1 \cup L^2 \cup \cdots$$
$$\Longleftrightarrow w^R \in L^0 \vee w^R \in L^1 \vee \cdots$$
$$\overset{\beta}{\Longleftrightarrow} w \in (L^0)^R \vee w \in (L^1)^R \vee \cdots$$
$$\Longleftrightarrow w \in (L^R)^0 \vee w \in (L^R)^1 \vee \cdots$$
$$\Longleftrightarrow w \in (L^R)^*$$

g) $(L_1 L_2)^R = L_1^R L_2^R \Rightarrow L_1 = L_2$ $\quad$ Fals

$L_1 = \{a\}$ $\qquad a^R = a^R \cdot \lambda^R = a^R \not\Rightarrow \{a\} = \{\lambda\}$

$L_2 = \{\lambda\}$

Ejercicio 5

a)

Ejercicio 6

$$\sigma: \Sigma^* \to \Gamma^*$$

$$\forall_{x,y} \in \Sigma^* \quad \sigma(xy) = \sigma(x)\cdot\sigma(y)$$

a) $\sigma(L_1L_2) = \sigma(L_1)\sigma(L_2)$

$y \in \sigma(L_1L_2) \iff \exists_x \in L_1L_2 \mid y \in \sigma(x)$

$\iff \exists_{x_1} \in L_1 \; \exists_{x_2} \in L_2 \mid x = x_1x_2 \wedge y \in \sigma(x_1x_2)$

$\iff \exists_{x_1} \in L_1 \; \exists_{x_2} \in L_2 \mid y \in \sigma(x_1)\cdot\sigma(x_2)$

$\iff \exists_{x_1} \in L_1 \; \exists_{x_2} \in L_2 \; \exists_{y_1} \in \sigma(x_1) \; \exists_{y_2} \in \sigma(x_2) \mid y = y_1y_2$

$\iff \exists_{y_1} \in \sigma(L_1) \; \exists_{y_2} \in \sigma(L_2) \mid y = y_1y_2$

$\iff y \in \sigma(L_1)\cdot\sigma(L_2)$

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b) $\sigma(L^m) = \sigma(L)^m$

$\alpha \; [\sigma(L_1L_2) = \sigma(L_1)\cdot\sigma(L_2)]$

$$\sigma(L^m) = \sigma(L\cdot L^{m-1}) \overset{\alpha}{=} \sigma(L)\cdot\sigma(L^{m-1}) = \overbrace{\sigma(L)\cdot\sigma(L)\cdot\ldots\sigma(L)}^{m \text{ veces}} = \sigma(L)^m$$

$x \in L^m \iff \exists_x \in L^m \mid y \in \sigma(x)$

$\iff \exists_{x_1} \in L \; \exists_{x_2} \in L^{m-1} \mid x = x_1x_2 \wedge y \in \sigma(x_1x_2)$

$\iff \exists_{x_1} \in L \; \exists_{x_2} \in L^{m-1} \mid x = x_1x_2 \wedge y \in \sigma(x_1)\cdot\sigma(x_2)$

$\iff \exists_{x_1} \in L \; \exists_{x_2} \in L^{m-1} \; \exists_{y_1} \in \sigma(x_1) \; \exists_{y_2} \in \sigma(x_2) \mid y = y_1y_2$

$\iff y \in \sigma(L)\cdot\sigma(L^{m-1})$

este proceso m veces : obtienes $\overbrace{\sigma(L)\cdot\sigma(L)\cdot\ldots\cdot\sigma(L)}^{m \text{ veces.}} = \sigma(L)^m$

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c) $\sigma(L_1 \cup L_2) = \sigma(L_1) \cup \sigma(L_2)$

$x \in \sigma(L_1 \cup L_2) \Rightarrow x \in \sigma(L_1) \vee x \in \sigma(L_2) \Rightarrow x \in \sigma(L_1) \cup \sigma(L_2)$

d) $\sigma(L^*) = \sigma(L)^*$