3.

K) $\overline{L^*} \supseteq \overline{L} \supseteq (\overline{L})^*$ $\qquad L^* \supseteq L \qquad \overline{L} \supseteq \overline{L^*}$

↳ conjunto de todas las palabras posibles a generar con lenguaje L.

$A \subseteq B$

$\overline{B} \subseteq \overline{A}$

$$L^* = \{ w_1 \cdot w_2 \cdot [\ldots] \cdot w_m \mid w_1, w_2, \ldots, w_m \in L \}$$

$$= L^0 \cup L^1 \cup L^2 \cup L^m$$

L contiene símbolos $\{a, b, c\}$ $\qquad \overline{L}$ = cualquier símbolo $\neq \{a, b, c\}$ =

$= \{d, e, f, \ldots\}$

L^* λ, $\underline{a, b, c}$, ab, ac, ...

$\overline{L^*}$ =

$L = \{a, b\}$

$\overline{L} = \{c, d, e, f, \ldots\}$ diferente de "a" y de "b" $\rightarrow L^* - (L^0 \cup L^1)$

cc dd

$L^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, acb, aba, \ldots\} = L^0 \cup L^1 \cup L^2 \cup L^3$

$\overline{L^*}$ = todo menos que no esté solo formado por as y bs.

$\{c, d, e, f, \ldots, ac, ad, abc, abd, \ldots, bd, bdf, ff, \ldots\}$

$(\overline{L})^* = \{c, d, e, f, cc, dd, ee, ce, cde, \ldots\}$

1)

$\exists \rightarrow \wedge$
$\forall \rightarrow \Rightarrow$

a)

b) $\{x \in \Sigma^* \mid \forall_{x_1 x_2}\ x = x_1 ab\, x_2 \Rightarrow |x_2|_{ba} > 0\}$

c) $\{w \in \Sigma^* \mid |w|_{ab} > 0 \wedge |w|_{ba} > 0\}$

d) $\{w \in \{a, b, c\}^* \mid \forall_{x,y,z}\ w = xbybz \Rightarrow |y|_a > 0\}$

e)

f) $\{w \in \Sigma^* \mid \exists_{x,y}\ w = xy \wedge |x|_b \geqslant |x|_a\}$

g)

h) $\{x \in \Sigma^* \mid \exists_{y,z,K}\ |y| = 2K \wedge |y|_b \geqslant |y|_a \wedge yz = x\}$

i)

j) $\{w \in \Sigma^* \mid \exists_{x_1, x_2, y_1, y_2}\ w = x_1 y_1 = x_2 y_2 \wedge x_1 = x_2 \wedge |x_1| \neq 0 \wedge |x_1| \neq |w|\}$

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3. K) $\boxed{A \subseteq B \iff \overline{B} \subseteq \overline{A}}$ REGLA

$a \rightarrow b \equiv \neg b \rightarrow \neg a$

2.

a) $xy = yx$

Fals $\Sigma = \{a, b\}$ $\begin{matrix} x = a \\ y = b \end{matrix}$

$xy = yx$

$ab \neq ba$

b)

c) $A \cdot B = \{xy \mid x \in A \wedge y \in B\}$

d) $AB = BA$ No xd

e) $A \neq \emptyset \wedge AB = AC \Rightarrow B = C$

contraejemplo $\longrightarrow$

$A = \{a^{2k} \mid k \geq 0\}$

$B = \{\lambda, aa\}$

$C = \{\lambda, aaaa\}$

$A \subseteq AB \subseteq A$

$A \subseteq AC \subseteq A$

$\underline{\underline{B \neq C}}$

h) $(A \cap B)C \subseteq AC \cap BC$

$w = w_1 w_2$ $w \in (A \cap B)C \Rightarrow w_1 \in (A \cap B) \wedge w_2 \in C \Rightarrow$

$\Rightarrow w_1 \in A \wedge w_1 \in B \wedge w_2 \in C \Rightarrow$

$\Rightarrow w_1 w_2 \in AC \wedge w_1 w_2 \in BC \Rightarrow$

$\Rightarrow w \in AC \wedge w \in BC \Rightarrow$

$\Rightarrow w \in AC \cap BC$

3

a) $L^* = L^0 \cup \underset{\substack{\uparrow \\ a,b}}{L^1} \cup L^2 \cup \cdots$ $\quad L^* = \{a,b\}^* = \{a,b\} \subseteq L$

g) $(L_1 \cap L_2)^* \subseteq (L_1^* \cap L_2^*)$

$\underbrace{L_1 \cap L_2}_{L_3}$, $L_3 \subseteq L_1 \wedge L_3 \subseteq L_2$

$\Rightarrow L_3^* \subseteq L_1^* \wedge L_3^* \subseteq L_2^* \Rightarrow L_3^* \subseteq (L_1^* \cap L_2^*)$

j)
k) Tomeu & Me

o)

p) $(\lambda \in L) \wedge (L^2 \subseteq L) \Longleftrightarrow L = L^*$

① $L = L^* = \underset{\substack{\downarrow \\ \lambda}}{L^0} \cup L^1 \cup \underset{\substack{\downarrow \\ L^2}}{L^2}$ cert

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② $L^2 = \{w \cdot v \mid w, u \in L\}$ $\quad L^2 = L \cdot L \subseteq L \Rightarrow wu \in L$

$L^K \subseteq L$, $L^{K+1} = L^K \cdot L \subseteq L \cdot L \subseteq L$

$L = L^*$

$L^* \subseteq L \iff \forall k \ L^k \subseteq L$

$L^k \subseteq L$ per inducció sobre k

$k = 0: \quad L^0 = \{\lambda\} \subseteq L$

$k > 0: \quad L^k = L \cdot L^{k-1} \subseteq L \cdot L = L^2 \subseteq L$