Ejercicio 1

a) $L = \{ w \in L \mid |w|_{ab} > 0 \}$

b) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \ w = x_1 ab x_2 \Rightarrow |x_2|_{ba} > 0 \}$

c) $L = \{ w \in L^* \mid |w|_{ab} > 0 \land |w|_{ba} > 0 \}$

d) $L = \{ w \in L^* \mid \forall_{x_1, x_2, x_3} \ w = x_1 b x_2 b x_3 \Rightarrow |x_2|_a > 0 \}$

e) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \ w = x_1 b x_2 \Rightarrow |x_1| \not\in \dot{2} \}$

f) $L = \{ w \in L^* \mid \exists_{x_1, x_2} \ w = x_1 x_2 \land |x_1|_b \geqslant |x_1|_a \}$

g) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \ w = x_1 x_2 \Rightarrow |x_1|_b \geqslant |x_1|_a \}$

h) $L = \{ w \in L^* \mid \exists_{x_1, x_2} \ w = x_1 x_2 \land |x_1|_b \geqslant |x_1|_a \land |x_1| \in \dot{2} \}$

i) $L = \{ w \in L^* \mid \forall_{x_1, x_2} \ (w = x_1 x_2 \land |x_1| \in \dot{2}) \Rightarrow |x_1|_b \geqslant |x_1|_a \}$

j) $L = \{ w \in L^* \mid \exists_{x_1 x_2, y_1, y_2} \ w = x_1 x_2 = y_1 y_2 \land x_1 = y_2 \land 0 < |x_1| < |w| \}$

Ejercicio 2

a) fals → contraexemple $x = a$ $xy = ab$ $ab \neq ba$
                            $y = b$ $yx = ba$

b) Red. Absurdo

$(xy = xz \Rightarrow y \neq z)$ por contrarecíproco $y = z \Rightarrow xy \neq xz \Rightarrow$

$(y = z)$

$\Rightarrow \boxed{xy \neq xy}$ contradicción $\Rightarrow$ <u>cierto</u>

c) $AB = \{xy \mid x \in A \wedge y \in B\} = \{K \mid K \in AB\} \Rightarrow \boxed{\overline{xy = K}}$ ①

$BC = \{yz \mid y \in B \wedge z \in C\} = \{j \mid j \in BC\} \Rightarrow \boxed{yz = j}$ ②

$A(BC) = \{xj \mid x \in A \wedge j \in BC\} = \{xj \in A(BC)\} \overset{②}{\Rightarrow} \{xyz \in A(BC)\}$

$(AB)C = \{Kz \mid K \in (AB) \wedge z \in C\} = \{Kz \in (AB)C\} \overset{①}{\Rightarrow} \{xyz \in (AB)C\}$

$\left. \right\} A(BC) = (AB)C$

d)

Ejercicio 3

b) $L_1^* L_2^* \subseteq (L_1 L_2)^*$ <u>fals</u>

$L_1 = \{a\}$ $L_1 L_2 = \{ab, ba, \lambda\}$ ab
$L_2 = \{b\}$

c) $L_1^* L_2^* \supseteq (L_1 L_2)^*$ <u>fals</u>

$L_1 = \{a\}$ $L_1 L_2 = \{ab, ba, \lambda\}$ ab
$L_2 = \{b\}$

d) $L_1^* \subseteq L_2^* \rightarrow L_1 \subseteq L_2$ fals

    $L_2 \{a\}$ $L_2 = \{\lambda, a, aa, aaa, \ldots\}$
    $L_1 \{aa\}$ $L_1 = \{\lambda, aa, aaa, \ldots\}$

$L_1 \nsubseteq L_2$
$aa \nsubseteq a$

r) $L^2 \subseteq L \iff L = L^+$

Ejercicio 4

a) $(xy)^R = y^R x^R$ $\textcircled{\alpha}\left[(aw)^R = w^R a \mid a \in \Sigma \wedge w \in \Sigma^*\right]$

$\forall_{i \in (1, |x|)} \; x_i \in X \implies X = x_1 x_2 x_3 x_4 \cdots x_m \quad m = |x|$

$\forall_{j \in (1, |y|)} \; y_j \in Y \implies y = y_1 y_2 y_3 y_4 \cdots y_m \quad m = |y|$

$xy = x_1 x_2 x_3 x_4 \cdots x_m \cdot y_1 y_2 y_3 y_4 \cdots y_m$

$(xy)^R = \left(x_1 \underline{x_2 x_3 x_4 \cdots x_m y_1 y_2 y_3 y_4 \cdots y_m}\right)^R = (x_1 \cdot w_1)^R = w_1^R \cdot x_1$
$\qquad \qquad \qquad \qquad \overset{||}{w_1}$

$w_1^R \cdot x_1 = \left(\underline{x_2 x_3 x_4 \cdots x_m y_1 y_2 y_3 y_4 \cdots y_m}\right)^R \cdot x_1 = (x_2 \cdot w_2)^R \cdot x_1 = w_2^R \cdot x_2 \cdot x_1$
$\qquad \qquad \qquad \overset{||}{w_2} \qquad \vdots$

$w_m^R \cdot x_{m-1} \cdots \cdot x_2 \cdot x_1 = \left(x_m \underline{y_1 \cdot y_2 \cdot y_3 \cdot y_4 \cdots y_m}\right)^R \cdot x_{m-1} \cdots \cdot x_2 \cdot x_1 =$
$\qquad \qquad \qquad \qquad \qquad \qquad \quad w_{m+1}$

$= (x_m \cdot w_{m+1})^R \cdot x_{m-1} \cdots \cdot x_2 \cdot x_1 = (w_{m+1})^R \cdot x_m \cdot x_{m-1} \cdots x_2 \cdot x_1 = (w_{m+1})^R \cdot x^R =$

$= (y_1 \cdot y_2 \cdot y_3 \cdot y_4 \cdots y_m)^R \cdot x^R = \boxed{y^R \cdot x^R}$

f) $(L^*)^R = (L^R)^*$ $\textcircled{\alpha}\left[w^R \in L \Longleftrightarrow w \in L^R\right]$ $\textcircled{\beta}\left[(L^m)^R = (L^R)^m\right]$

$w \in (L^*)^R \overset{\textcircled{\alpha}}{\Longleftrightarrow} w^R \in L^*$

$\qquad \Longleftrightarrow w^R \in L^0 \cup L^1 \cup L^2 \cup \cdots$

$\qquad \Longleftrightarrow w^R \in L^0 \vee w^R \in L^1 \vee \cdots$

$\qquad \overset{\textcircled{\beta}}{\Longleftrightarrow} w \in (L^0)^R \vee w \in (L^1)^R \vee \cdots$

$\qquad \Longleftrightarrow w \in (L^R)^0 \vee w \in (L^R)^1 \vee \cdots$

$\qquad \Longleftrightarrow w \in (L^R)^*$

g) $(L_1 L_2)^R = L_1^R L_2^R \implies L_1 = L_2$ fals

$L_1 = \{a\}$ $a^R = a^R \cdot \lambda^R = a^R \; \not\Longrightarrow \{a\} = \{\lambda\}$
$L_2 = \{\lambda\}$

Ejercicio 5

a)

Ejercicio 6

$$\sigma: \Sigma^* \to \Gamma^*$$

$$\forall_{x,y} \in \Sigma^* \quad \sigma(xy) = \sigma(x) \cdot \sigma(y)$$

a) $\sigma(L_1 L_2) = \sigma(L_1) \sigma(L_2)$

$y \in \sigma(L_1 L_2) \iff \exists x \in L_1 L_2 \mid y \in \sigma(x)$

$\iff \exists x_1 \in L_1 \; \exists x_2 \in L_2 \mid x = x_1 x_2 \land y \in \sigma(x_1 x_2)$

$\iff \exists x_1 \in L_1 \; \exists x_2 \in L_2 \mid y \in \sigma(x_1) \cdot \sigma(x_2)$

$\iff \exists x_1 \in L_1 \; \exists x_2 \in L_2 \; \exists y_1 \in \sigma(x_1) \; \exists y_2 \in \sigma(x_2) \mid y = y_1 y_2$

$\iff \exists y_1 \in \sigma(L_1) \; \exists y_2 \in \sigma(L_2) \mid y = y_1 y_2$

$\iff y \in \sigma(L_1) \cdot \sigma(L_2)$

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b) $\sigma(L^m) = \sigma(L)^m$ $\boxed{\text{ⓐ} \; \sigma(L_1 L_2) = \sigma(L_1) \cdot \sigma(L_2)}$

$\sigma(L^m) = \sigma(L \cdot L^{m-1}) \overset{\text{ⓐ}}{=} \sigma(L) \cdot \sigma(L^{m-1}) = \overbrace{\sigma(L) \cdot \sigma(L) \cdot \ \cdots \ \sigma(L)}^{m \text{ veces}} = \sigma(L)^m$

$x \in L^m \iff \exists x \in L^m \mid y \in \sigma(x)$

$\iff \exists x_1 \in L \; \exists x_2 \in L^{m-1} \mid x = x_1 x_2 \land y \in \sigma(x_1 x_2)$

$\iff \exists x_1 \in L \; \exists x_2 \in L^{m-1} \mid x = x_1 x_2 \land y \in \sigma(x_1) \cdot \sigma(x_2)$

$\iff \exists x_1 \in L \; \exists x_2 \in L^{m-1} \; \exists y_1 \in \sigma(x_1) \; \exists y_2 \in \sigma(x_2) \mid y = y_1 y_2$

$\iff y \in \sigma(L) \cdot \sigma(L^{m-1})$

Este proceso m veces: obtienes $\overbrace{\sigma(L) \cdot \sigma(L) \cdot \ \cdots \ \sigma(L)}^{m \text{ veces.}} = \sigma(L)^m$

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c) $\sigma(L_1 \cup L_2) = \sigma(L_1) \cup \sigma(L_2)$

$x \in \sigma(L_1 \cup L_2) \implies x \in \sigma(L_1) \lor x \in \sigma(L_2) \implies x \in \sigma(L_1) \cup \sigma(L_2)$

d) $\nabla(L^*) = \nabla(L)^*$