Size and Shape Analysis: Brain Disorders

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1. Introduction

Schizophrenia is characterized as a serious mental illness. Symptoms range from delusions and hallucinations to social and economic deficits. Regardless of the experienced symptoms, cognitive abilities are always at risk of impairment with a schizophrenia diagnosis. As schizophrenia is a mental disorder, the brain remains the primary focus of study.

The aim of this project is to identify any notable differences between the brain shape of individuals without a mental disorder diagnosis and of schizophrenia patients. There is no desire to attain results applicable to the diagnosis and treatment of this illness.

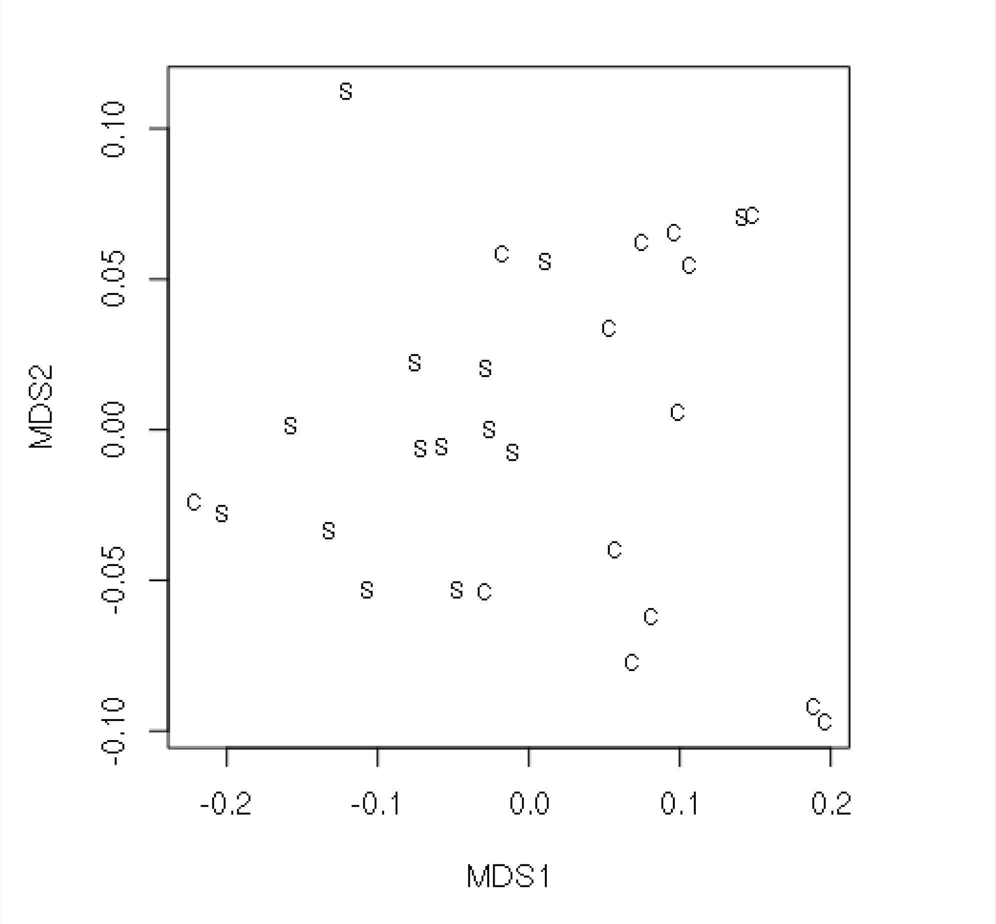
4. Analysis

The data consisted of brain scans from 14 control volunteers and 14 schizophrenia patients. The thirteen landmarks within the brain studied are: splenium, genu, top of corpus callosum, top of head, tentorium of cerebellum, top of cerebellum, tip of fourth ventricle, bottom of cerebellum, top of pons, bottom of pons, optic chiasm, frontal pole, and superior colliculus. Here the term landmark refers to a point of correspondence on each object that matches between and within populations.

As mentioned in the methods, multidimensional scaling was used to plot the brain shape of schizophrenia patients and the brain shape of the control group. With the patient data given in matrix form, the Riemannian distance in size-and-shape space was computed. The calculated distance values were then placed in a multidimensional scaling plot, showing the brain reconstructions of the two groups. From the Riemannian distance, the centroid size of each landmark was calculated and then placed in a box plot, allowing direct comparison of size.

- The Riemannian size-and-shape distance for each landmark was computed in R using the function *ssriemdist*.
- The multidimensional scaling was plotted using the R *cmdscale* function.
- The centroid sizes of the control group and the schizophrenia patients were illustrated in box plots by means of the R function *centroid.size*.

Figure 1. Resulting Multidimensional Scaling Plot



2. Dataset and Software

- The analysis was performed using the R software and the R package *shapes*.
- The data is publicly available under the title *schizophrenia* in the R database.
- The code utilized for the analysis can be found in the text *Statistical Shape Analysis: With Applications in R*, authored by Ian L. Dryden and Kanti V. Mardia.

3. Multidimensional Scaling

Multidimensional scaling is a form of geometric shape analysis that results in a reconstructed configuration of an organism. This form of geometric analysis uses the actual coordinates of landmarks directly in distance calculations. In this case, multidimensional scaling is used to reconstruct the brain shape of schizophrenia patients in comparison to the brain shape of the control group.

The multidimensional scaling here is performed using Riemannian distance in size-and-shape space. Note that the size-and-shape of a configuration matrix **X** is all the geometrical information about **X** that is invariant under location and rotation. Also, here the size-and-shape space is the space of all size-and-shapes.

Riemannian Distance

The Riemannian distance $\rho(X_1, X_2)$ is the closest great circle distance between Z_1 and Z_2 on the pre-shape sphere, where $Z_j = HX_j/\|HX_j\|, j = 1, 2$. The minimization is carried out over rotations.

The Riemannian distance ρ is:

$$\rho(X_1, X_2) = \arccos(\sum_{i=1}^{m} \lambda_i),$$

where $\lambda_i, i = 1, ..., m$ are eigenvalues.

Riemannian Distance in Size-and-Shape Space

Consider two k-point configurations in m dimensions, $X_1^o, X_2^o \in R^{km}$ ($k \times m$) matrices.

The Riemannian distance in size-and-shape space is given by:

$$d_S(X_1^o, X_2^o) = \sqrt{S_1^2 + S_2^2 - 2S_1S_2\cos\rho(X_1^o, X_2^o)},$$

where S_1,S_2 are the centroid sizes of X_1^o,X_2^o and ρ is the Riemannian shape distance.

Centroid Size

With Riemannian distance, the centroid sizes of the landmarks can be derived and compared using the inputted multidimensional data. Centroid size is derived from the square root of the sum of squared distances of all the landmarks of an object from their center of gravity. This centroid size results in the discrepancy between corresponding landmarks and is an ideal measure of shape difference.

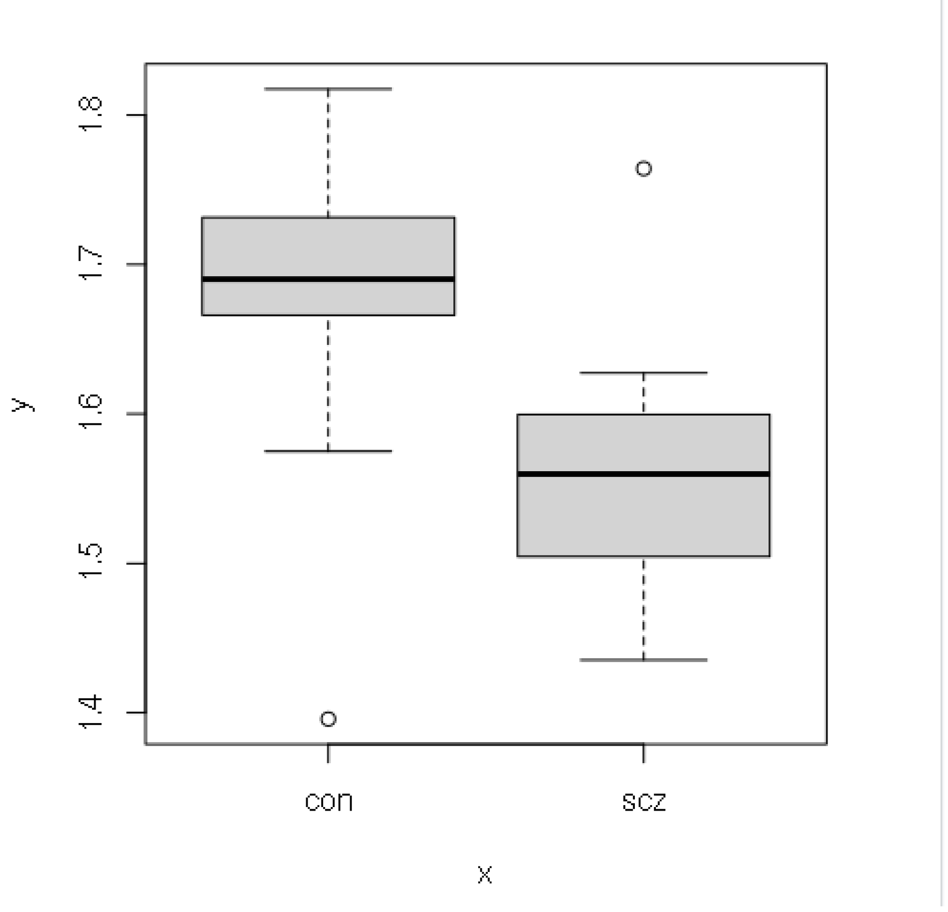
5. Results and Discussion

The Riemannian size-and-shape distance for each landmark was computed in R using the function *ssriemdist*. These values were then assigned to the matrix **distmat** within the software and used in the R distance structure *cmdscale*.

Along with the factor values assigned to the schizophrenia and control categories, the **distmat** matrix was plotted using multidimensional scaling, the *cmdscale* function. The resulting plot illustrated that the landmark distances and overall brain reconstruction of the schizophrenia patients were different than those of the control group.

To help illustrate these differences, box plots were used to show the centroid sizes of the control group and the schizophrenia patients by means of the R function *centroid.size*. The resulting box plots revealed that those within the control group were larger in centroid size.

Figure 2. Box Plots of Centroid Size



Altogether, both the multidimensional scaling plot using Riemannian distance and the box plots of the centroid size display a clear difference between the two groups. The control group is presented as being larger in centroid size in comparison to its schizophrenic counterpart. This suggests that there is a significant difference in the size and shape of the brains for those diagnosed with schizophrenia from the brain size and shape of those without the disorder.

References

Dryden, I. L. and Mardia, K. V. (2016). *Statistical Shape Analysis: With Applications in R*. John Wiley and Sons, Incorporated.

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