INSURANCE FRAMEWORK IN ACTUARIAL AND EPIDEMIOLOGY MODELING

Jingyuan Cao*

Department of Mathematical Sciences, Purdue University Fort Wayne
Supervisor: Alessandro Maria Selvitella*

Introduction

Since 2019, COVID-19 has been a huge pandemic affecting the world. It is highly viral and difficult to cure. It also caused the death of many people and the world suffered a great wound. The world faces a huge challenge in how to address and analyze the COVID-19 situation and future trends.

This is a huge epidemiological challenge and difficulty for the world. In this essay, we talk about several models from epidemiology and apply them into an insurance framework. We use models to predict changes and future trends in mortality and morbidity from a pandemic and biological point of view. In actuarial science and medical research epidemiology, many models are used to solve data and simulate trends. I believe that through this research some model analysis and risk model analysis can be simply applied to actuarial.

In the following sections, I will divide two models into epidemiology and analyze their application to the insurance framework. I will use compartmental models, which include deterministic version and stochastic version.

Model

1. SIR Model

For the population of size N(t) indexed by t(time). I use the SIR model, which is one of the simplest compartmental models. This model consists of three compartments: S(susceptible individuals), I(infectious individuals), R(removed individuals). N is the sum of population which was no births and no immigration. For the β is an infection rate and is a recovery rate. The following system of differential equation:

$$\begin{cases} S'(t) = -\beta S(t)I(t)/N \\ I'(t) = \beta S(t)I(t)/N - \alpha I(t) \\ R'(t) = \alpha I(t) \end{cases} \quad (1)$$

which implies $N = S(t) + I(t) + R(t)$. S(t) is susceptible population, I(t) is infected population, R(t) is removed population, and N is the sum of these three.

2. Insurance Application

For using the basic SIR model. From the Runge-Kutta method, I need use some differential equations to show that reserve at time t. Let P(t) is the accumulated value of premiums collected up to t and B(t) is the accumulated value of benefits paid up to t. V(t) is the accumulated benefit reserve at t.

The following differential equation:

$$\begin{cases} P'(t) = \pi S(t) + \gamma P(t), t > 0 \\ B'(t) = I(t) + \gamma B(t), t > 0 \\ V(t) = P(t) - B(t) = \pi \int_0^t S(x)dx - \int_0^t I(x)dx \end{cases} \quad (2)$$

In the insurance application section, four possible shapes of graph as time t and it was discovered these shapes are linked to the general reproduction number R. Form the following table:

Shape of $V(\pi, t)$	**Interval for values of π**
Increasing concave	At least $1/R_t - 1$
Increasing concave-then-convex	Between $1/R_{tm} - 1$ and $1/R_t - 1$
Non-monotonic concave-then-convex	Between $1/R_0 - 1$ and $1/R_{tm} - 1$
Non-monotonic convex	Between 0 and $1/R_0 - 1$

Results for SIR model and V(t)

In the SIR model section, I discuss this project results. From the equation (1), when $\beta S(t) - \alpha > 0$,the disease spread that's meaning I'(t) is positive. If $\beta S(t) - \alpha < 0$,the disease die out(I'(t) is negative). The motivation of the relevance of a quantity called general reproductive number, $R_t = \beta S(t)/\alpha$ which is the average number of secondary infections depend on a single infections individual at time t. when t=0, R_0 called as the basic reproduction number which is common use in measure to determine a disease spread out during the early phase of the outbreak. From the former, I discussed about beta and alpha that easy to consider that if $R_0 > 1$, the disease begin to spread. If $R_0 < 1$, the disease will die out.

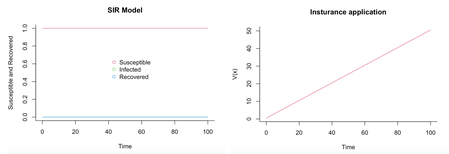

Fig. 1: $\beta = 0.001$ *and* $\alpha = 0.1$ *or* $\beta = 1$ *and* $\alpha = 100$.

For $\beta = 0.001$ and $\alpha = 0.1$, or $\beta = 1$ and $\alpha = 100$, there is a two horizontal straight lines for y=0(Recovered) and y=1(Susceptible). Since $\beta S(t) - \alpha < 0$, so the disease absolutely die out. And from the insurance application graph, this is a linear equation of one dimension through the origin (0,0). Because the disease is all die out and no infected.

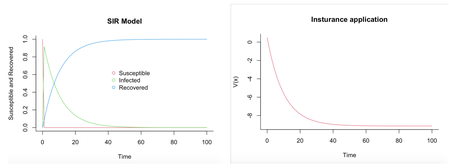

Fig. 2: $\beta = 100$ *and* $\alpha = 0.1$

For $\beta = 100$ and $\alpha = 0.1$, $\beta S(t) - \alpha > 0$ the disease extremely spread out. Susceptible to the dominate curve drops dramatically. It's almost straight down to zero at first, and then Infected slightly to zero. Then the Recovered curve rises at the same rate as the Infected one. Since $0 < \pi < (1/R_0) - 1$, insurance application's curve is Non-monotonic convex .

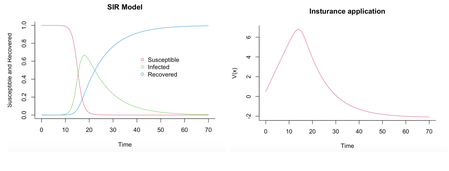

Fig. 3: $\beta = 1$ *and* $\alpha = 0.1$

For $\beta = 1$ and $\alpha = 0.1$, there is three curves which two curves decrease to 0 and one curve increase to 1. As Susceptible line equal to Infected line, insurance application to achieve vertex. Since $\beta S(t) - \alpha > 0$ the disease spread out with time t, V(t) will decrease to state of equilibrium. V(t) is increasing concave-then-convex which depend on $(1/R_{tm}) - 1 < \pi < (1/R_t) - 1$.

Discussion

In this two figures, I choose same $\beta = 1$ and $\alpha = 0.1$ for different π. In the former, I discuss the $\pi = 0.5$ that Figure 6 V(t) is increasing concave-then-convex. Same $\beta and \alpha$ that's meaning R0 and Rt is same. Thus, we only change π to see the graphs' change. For figure 9 ($\pi = 0.01$), at the same vertex(Infected=Susceptible) and then decrease to equilibrium. Since $(1/R_0) - 1 < \pi < (1/R_{tm}) - 1$, V(t) is Non-monotonic concave-then-convex. For figure 10($\pi = 100$), this is straight line to increase at the same vertex(Infected=Susceotible) and then into the balance(horizontal line). Since $(1/R_t) - 1 < \pi$, V(t) is increasing concave.

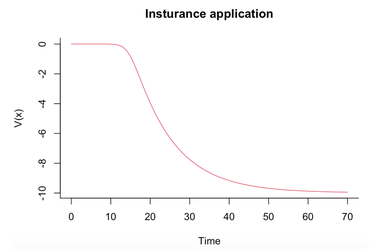

Fig. 4: $\beta = 1 and \alpha = 0.1 and \pi = 0.01$

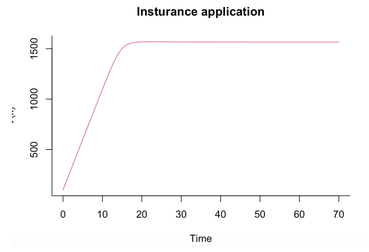

Fig. 5: $\beta = 1 and \alpha = 0.1 and \pi = 100$

Conclusion

In this paper, I discuss insurance framework in acturial and epidemiology, and provide SIR model of results. Also, Using insurance application V(t) which can influenced by disease spread out or die out. From the graphs, we can see it is very difficult to control an epidemic once it has started, so the hope is that this risk can be avoided at an early stage.

References

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