Estimating the Hubble Constant via Gravitational Waves

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Introduction

Measuring the Hubble constant (H_0) is central to understanding the universe's expansion, age, structure, and fate. Traditionally, two methods are used: the **cosmic distance ladder**—which relies on **Cepheid variables** and **Type Ia supernovae** as **standard candles**—and the analysis of cosmic microwave background fluctuations using early-universe physics [1]. These methods, however, yield conflicting values, a discrepancy known as the "Hubble tension," which may point to new physics. An alternative approach involves using "standard sirens"—the gravitational waves produced by binary black hole mergers—and identifying their host galaxies to correlate the gravitational-wave-inferred distance with redshift, thereby offering an independent route to measuring H_0. The Hubble's law is defined as [2]:

$$\boxed{v_H = H_0 d}$$

Where v_H is the Hubble flow velocity of an object, and d is its distance.

Galaxy Dataset

We used a dataset of approximately 320,000 galaxies—each a potential host for a binary black hole merger—sampled uniformly to create a robust cosmic distribution. For each galaxy, a merger "particle" was assigned a position based on the ratio of its true electromagnetic distance to its GW distance (incorporating measurement uncertainties) and by randomly sampling azimuthal and polar angles from a Fisher distribution. This method accurately captures both the spatial distribution and observational uncertainties essential for analysing the Hubble constant via gravitational wave signals.

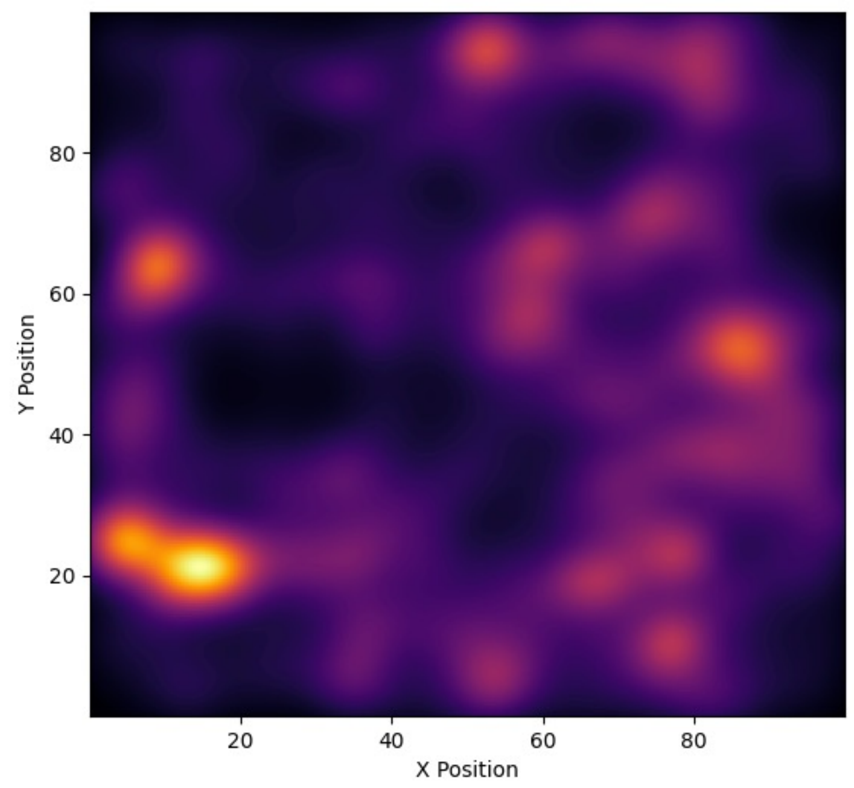

Fig 1. 2D sliced contour plot of galaxies and binary black hole mergers

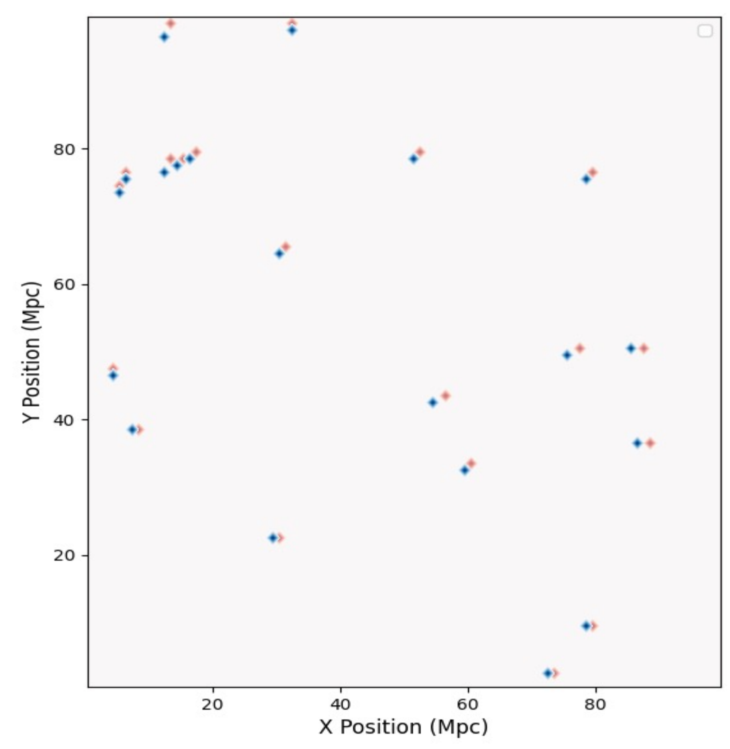

Fig 2. 2D sliced plot showing positions of galaxies (orange) and mergers (blue)

Statistical Theory

Firstly, a random sample of 1000 binary black hole merger events was generated from a catalogue of host galaxies. Recognizing that distance estimates from gravitational wave observatories carry an uncertainty of roughly 30%—meaning the true distance may be significantly higher or lower than the measured value—this uncertainty is explicitly incorporated into our simulation framework. To achieve this, the host galaxy positions are perturbed to reflect the typical measurement errors, thereby generating merger events that mimic real observational challenges. Furthermore, the likelihood of any given galaxy being the true host of a merger is assessed by comparing the merger's inferred location from the GW data with the galaxy's position. This probabilistic approach enables us to effectively account for spatial uncertainties and is essential for calibrating the distance–redshift relation used in estimating the Hubble constant.

$$\boxed{\chi = \sum_{\substack{all\\ Galaxies}} \sum_{\substack{all\\ Mergers}} \mathcal{L}\left(\frac{d_{EM}}{d_{GW}}\right) \mathcal{L}(\Theta)}$$

Simulation and Results

The simulation involved running a likelihood algorithm over all galaxies and their associated merger events. Recession velocities were first computed using an established Hubble constant value. Then, by generating an array of candidate Hubble constant values (in this case 30-120), new merger distances were recalculated based on the recession velocities. The likelihood was evaluated for each candidate, and the value corresponding to the maximum likelihood was adopted as the best-fit H_0.

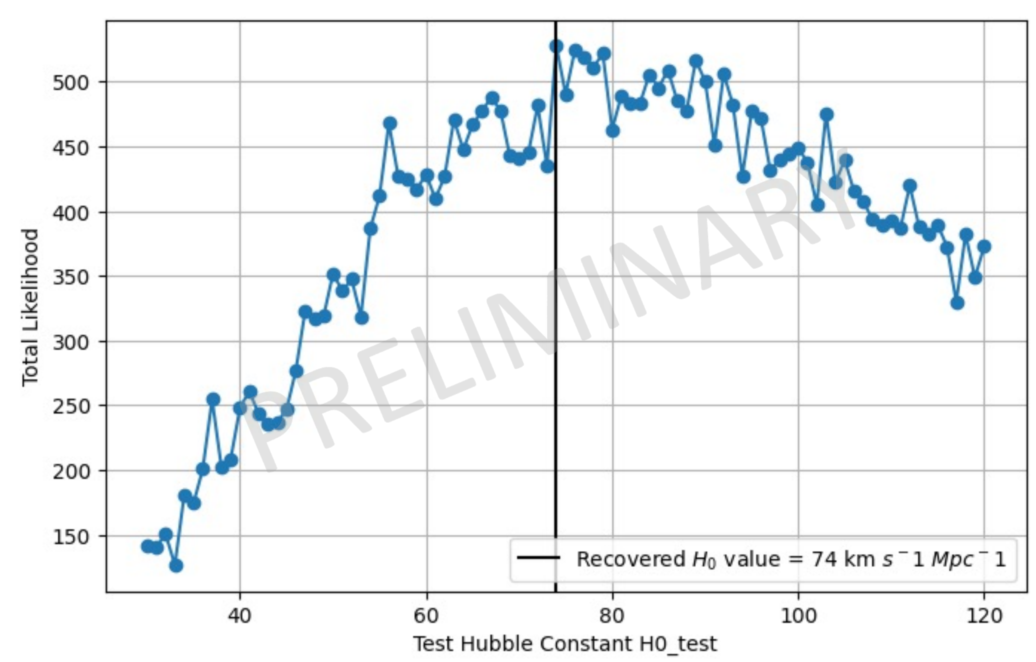

Fig 3. Simulation showing likelihood estimation for test values of Hubble constant.

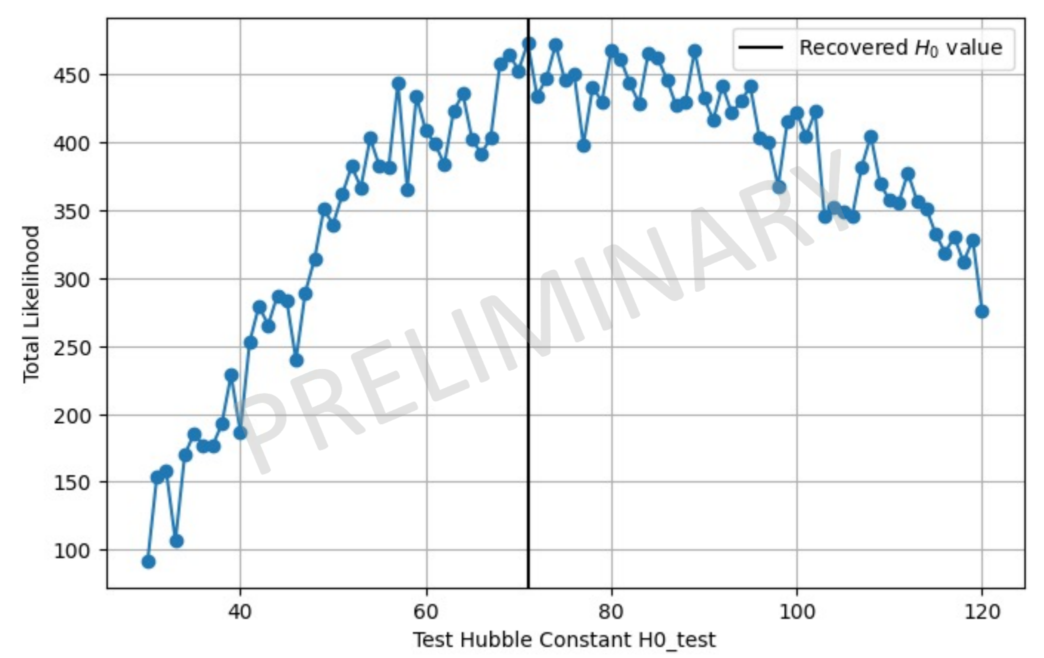

Fig 4. Simulation showing likelihood estimation for test values of Hubble constant.

Two tests were performed: one with 1000 galaxies and mergers yielded 74 km/s/Mpc—likely inflated due to limited data—and one using the full galaxy set with 100 mergers produced 69 km/s/Mpc, closely matching the Eagle simulation value of 67.77 km/s/Mpc [3]; discrepancies likely arise from additional uncertainties in GW distance and position measurements.

Future Directions in H_0 Estimation:

We plan to expand the simulation beyond the original 100 Mpc cube by replicating the host galaxy data, allowing us to include many more galaxies and mergers. This larger-scale approach will improve our H_0 estimates, and we will apply Bayesian inference to derive robust posterior distributions for H_0, fully accounting for observational uncertainties.

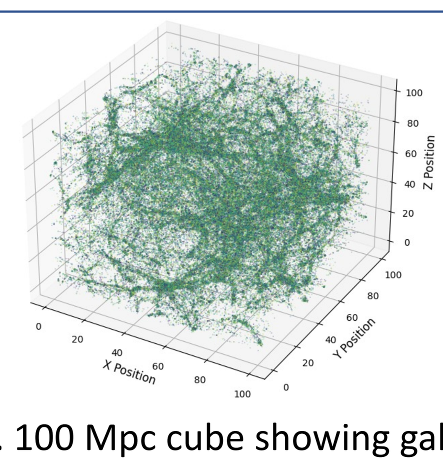

Fig 5. 100 Mpc cube showing galaxy cluster

References:

[1] The University of Western Australia (2011) *Measuring the Universe 4: Explanation of the cosmic distance ladder (background sheet)*, *Department of Education WA*, pp. 1–3. https://www.uwa.edu.au/science/-/media/Faculties/Science/Docs/Explanation-of-the-cosmic-distance-ladder.pdf.

[2] "Hubble's Law | Astronomy 801: Planets, Stars, Galaxies, and the Universe," (2023)

[3] Schaye, Joop, et al. "The EAGLE project: simulating the evolution and assembly of galaxies and their environments." Monthly Notices of the Royal Astronomical Society 446.1 (2015): 521-554.