Assignment 3 (ML for TS) - MVA

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December 16, 2025

1 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \ldots, T-1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1:
- Sequence 2:

2 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

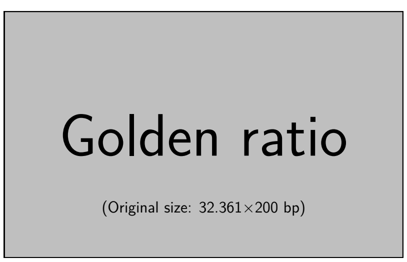
$$\hat{g}^U(\lambda) := \frac{1}{2} \left[1 + \cos\left(2\pi \left(\frac{\lambda}{aR} + \frac{1}{2} \right) \right) \right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

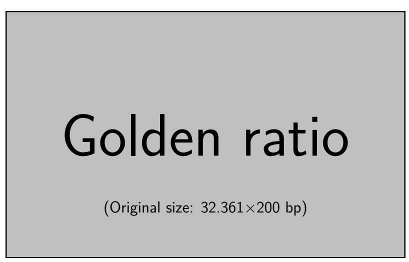
Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R?

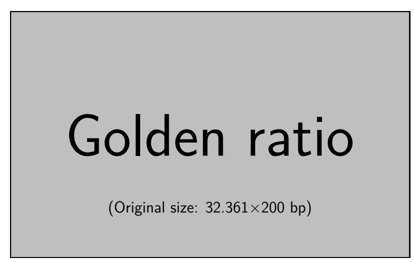
Answer 3



(a) $R = 1$



(b) $R = 3$



(c) $R = 5$

Figure 1: The SAGW kernels functions

3 Molene temperature graph

We study the Molene dataset using temperature signals.

Question 4

Construct the graph using distance-based exponential smoothing.

- Remove stations with missing values.
- Choose the minimum threshold ensuring connectivity and average degree ≥ 3.
- Identify the least smooth and smoothest timestamps.

Answer 4

The stations with missing values are ...

The threshold is ...

The least smooth signal occurs at ...

The smoothest signal occurs at ...

4 Node frequency classification

For each node v, we use

$$[W_f(1,v),\ldots,W_f(M,v)]$$

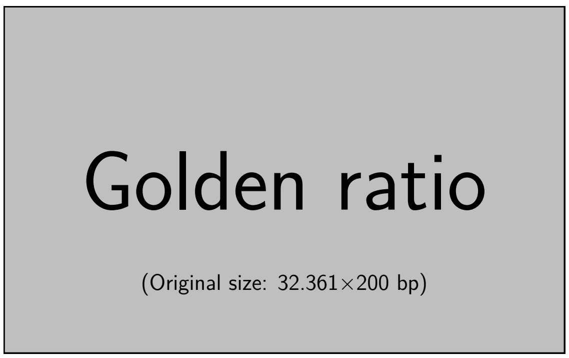
as features.

Nodes are classified as: low (scales 1–3), medium (4–6), high (7–9) frequency.

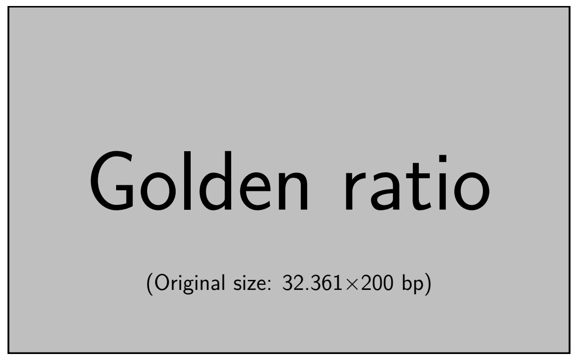
Question 5

Apply this classification to: least smooth signal, smoothest signal, first timestamp. Display results on the map.

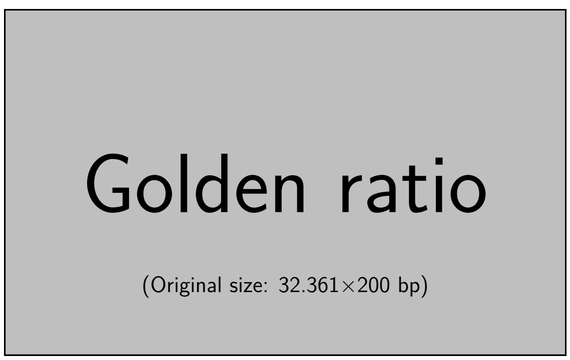
Answer 5



(a) Least smooth



(b) Smoothest



(c) First timestamp

Figure 2: Node frequency classification

Question 6

Display average temperature with marker colours equal to the majority class.

Answer 6

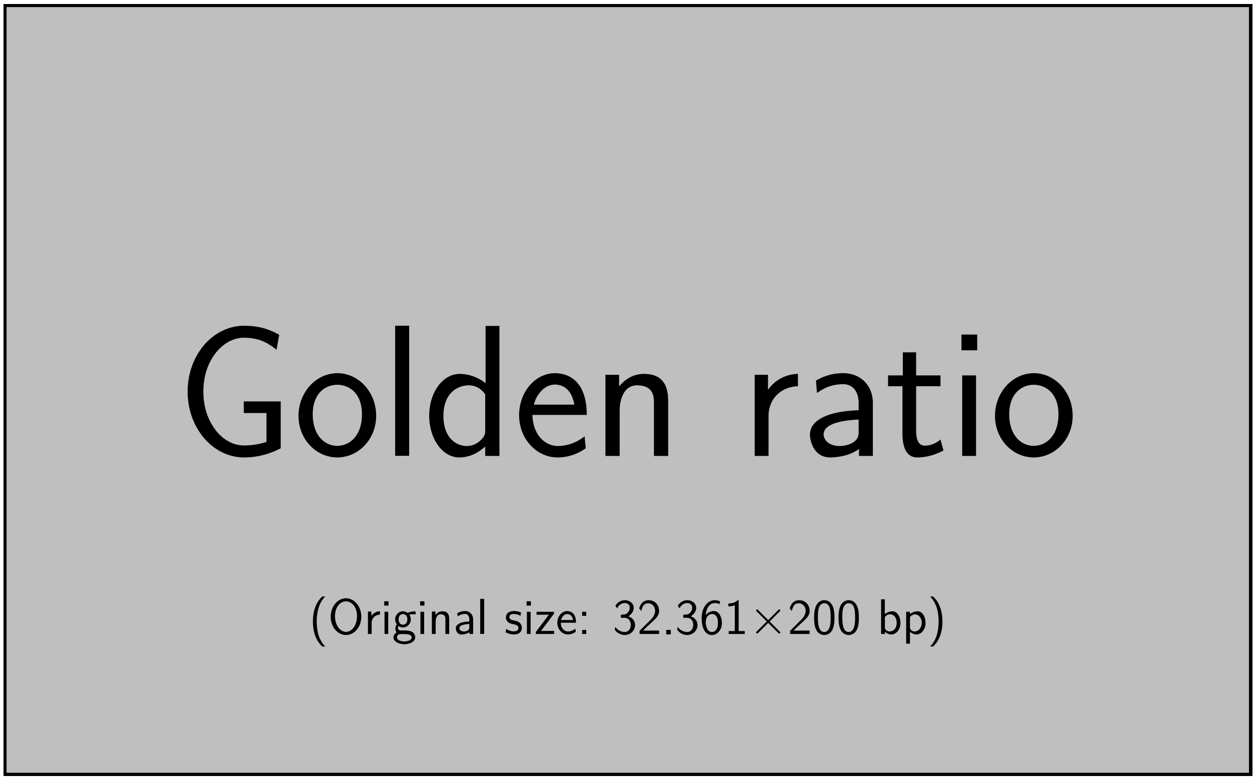


Figure 3: Average temperature with majority frequency class

5 Spatio-temporal graph

We construct a graph $H = G \,\square\, G'$, where G' is a temporal line graph.

Question 7

- Express L_H using Kronecker products.
- Derive eigenpairs of L_H.
- Compute wavelet transform.
- Classify nodes and display results.

Answer 7

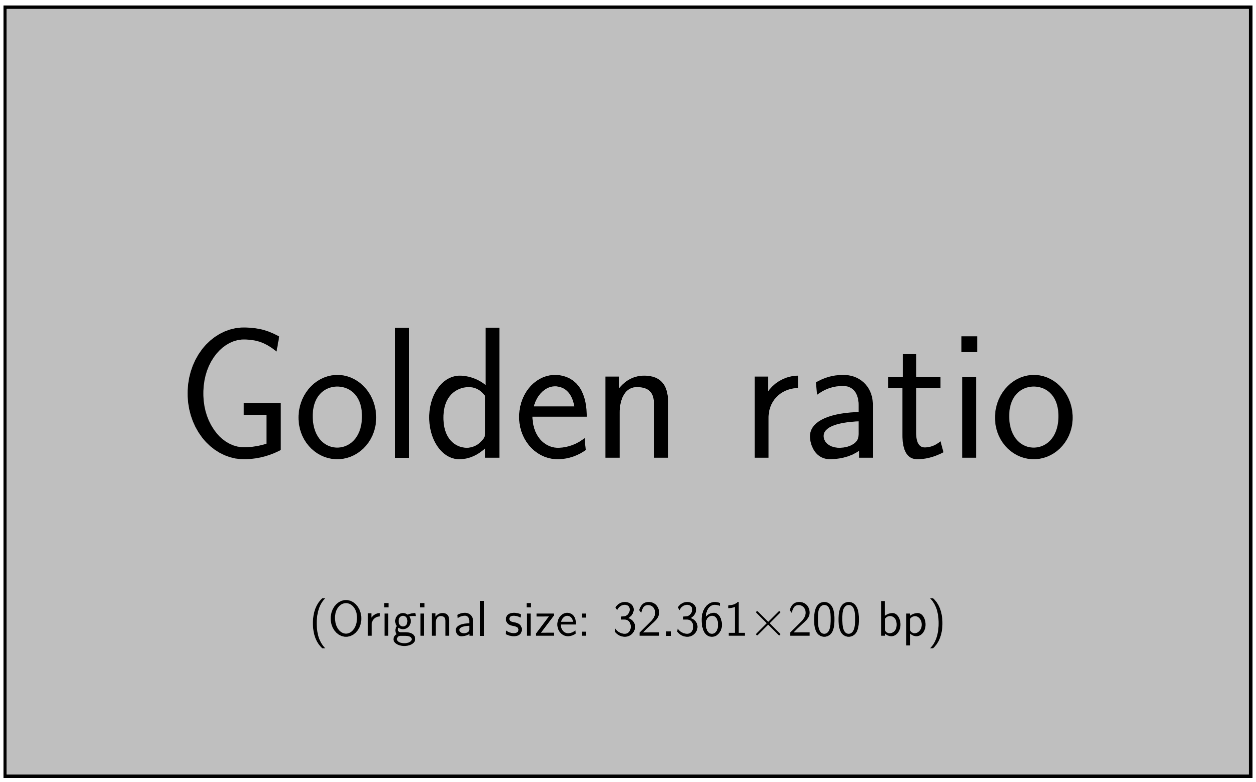


Figure 4: Spatio-temporal frequency classification