Assignment 3 (ML for TS) - MVA

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1 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \dots, T-1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

The decoding procedure utilizes a time-frequency representation via a Short-Time Fourier Transform (STFT) with a 4096-point FFT to achieve high frequency resolution at $f_s = 22.05$ kHz. Segmentation is performed by applying the Pruned Exact Linear Time (PELT) algorithm with an L_2 cost function to the normalized, median-filtered frame energy. Valid segments are isolated from noise using an energy threshold, and dominant frequencies in the low $(650 - 1000$ Hz) and high $(1150 - 1700$ Hz) bands are identified via spectral peak detection and mapped to the DTMF grid. Hyperparameters, including the segmentation penalty, energy threshold, and window length, were calibrated through an exhaustive grid search. This optimization process targeted the minimization of the mean Levenshtein distance between predicted and ground-truth symbol sequences. The final calibrated parameters were determined to be a window of 512 samples, a penalty of 0.1, and an energy threshold of 0.7, resulting in an average distance of 3.06.

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1: 51C9
- Sequence 2: #17#126#1

2 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure **??**. What is the influence of R?

Answer 3

The parameter R directly controls the spectral support of each kernel, which is

$$\mathrm{supp}(\hat{g}_m) = [a_m - Ra,\, a_m).$$

As R increases, the kernels become wider in the spectral domain, leading to stronger overlap between adjacent scales and smoother frequency coverage. Conversely, smaller values of R produce narrower kernels with reduced overlap, resulting in a more localized and selective spectral tiling.

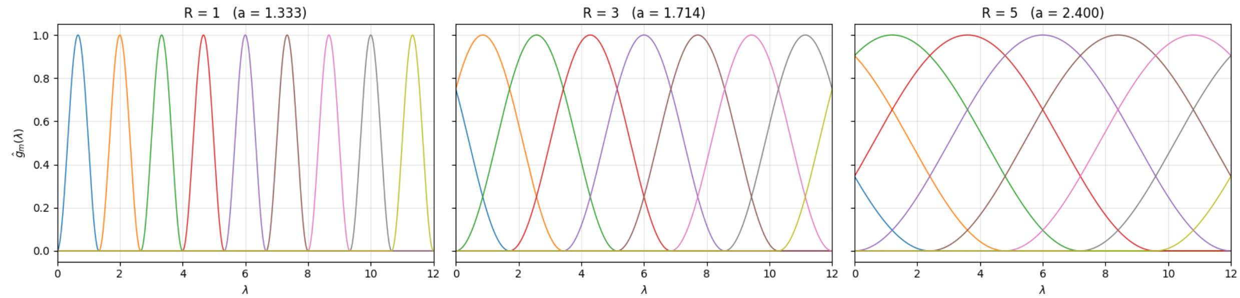


Figure 1: Spectrum Adapted Graph Wavelet kernels $\hat{g}_m(\lambda)$ for $M = 9$, $\lambda_n = 12$, and different values of the overlap parameter R. Each curve corresponds to one scale m.

3 Molene temperature graph

We study the Molene dataset using temperature signals.

Question 4

Construct the graph using distance-based exponential smoothing.

- Remove stations with missing values.
- Choose the minimum threshold ensuring connectivity and average degree ≥ 3.
- Identify the least smooth and smoothest timestamps.

Answer 4

The stations with missing values are: ARZAL, BATZ, BEG_MEIL, BREST-GUIPAVAS, BRIGNOGAN, CAMARET, LANDIVISIAU, LANNAERO, LANVEOC, OUESSANT-STIFF, PLOUAY-SA, PLOUDALMEZEAU, PLOUGONVELIN, QUIMPER, RIEC-SUR-BELON, SIZUN, ST-NAZAIRE-MONTOIR, and VANNES-MEUCON.

The threshold is 0.77 (using Lambert II étendu)

The least smooth signal occurs at 2014-01-21 06:00:00

The smoothest signal occurs at 2014-01-24 19:00:00

4 Node frequency classification

For each node v, we use

$$[W_f(1, v), \ldots, W_f(M, v)]$$

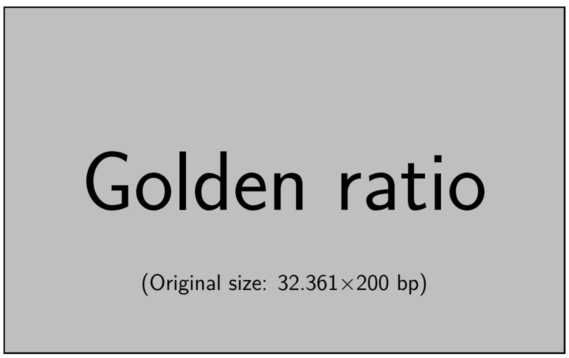
as features.

Nodes are classified as: low (scales 1–3), medium (4–6), high (7–9) frequency.

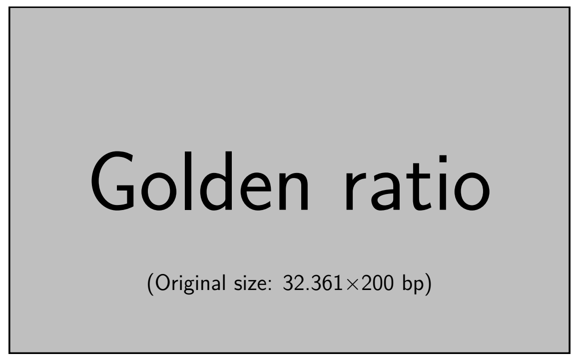
Question 5

Apply this classification to: least smooth signal, smoothest signal, first timestamp. Display results on the map.

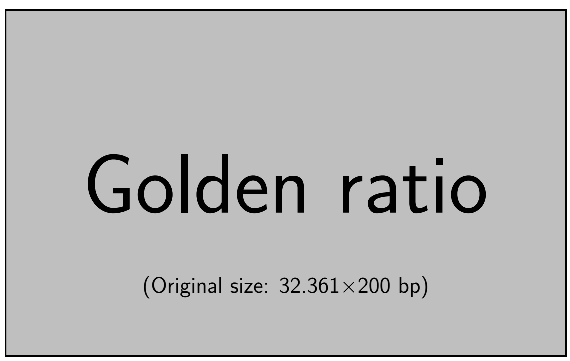
Answer 5



(a) Least smooth



(b) Smoothest



(c) First timestamp

Figure 2: Node frequency classification

Question 6

Display average temperature with marker colours equal to the majority class.

Answer 6

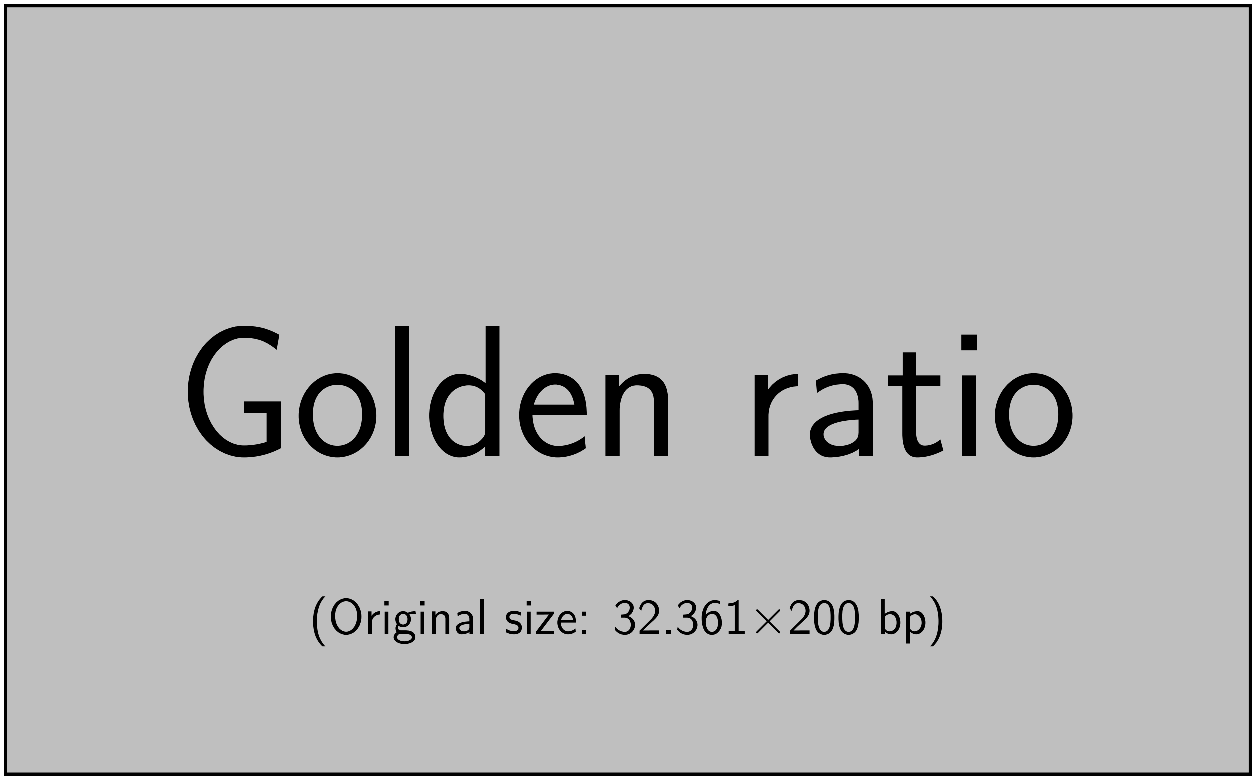


Figure 3: Average temperature with majority frequency class

5 Spatio-temporal graph

We construct a graph $H = G \,\square\, G'$, where G' is a temporal line graph.

Question 7

- Express L_H using Kronecker products.
- Derive eigenpairs of L_H.
- Compute wavelet transform.
- Classify nodes and display results.

Answer 7

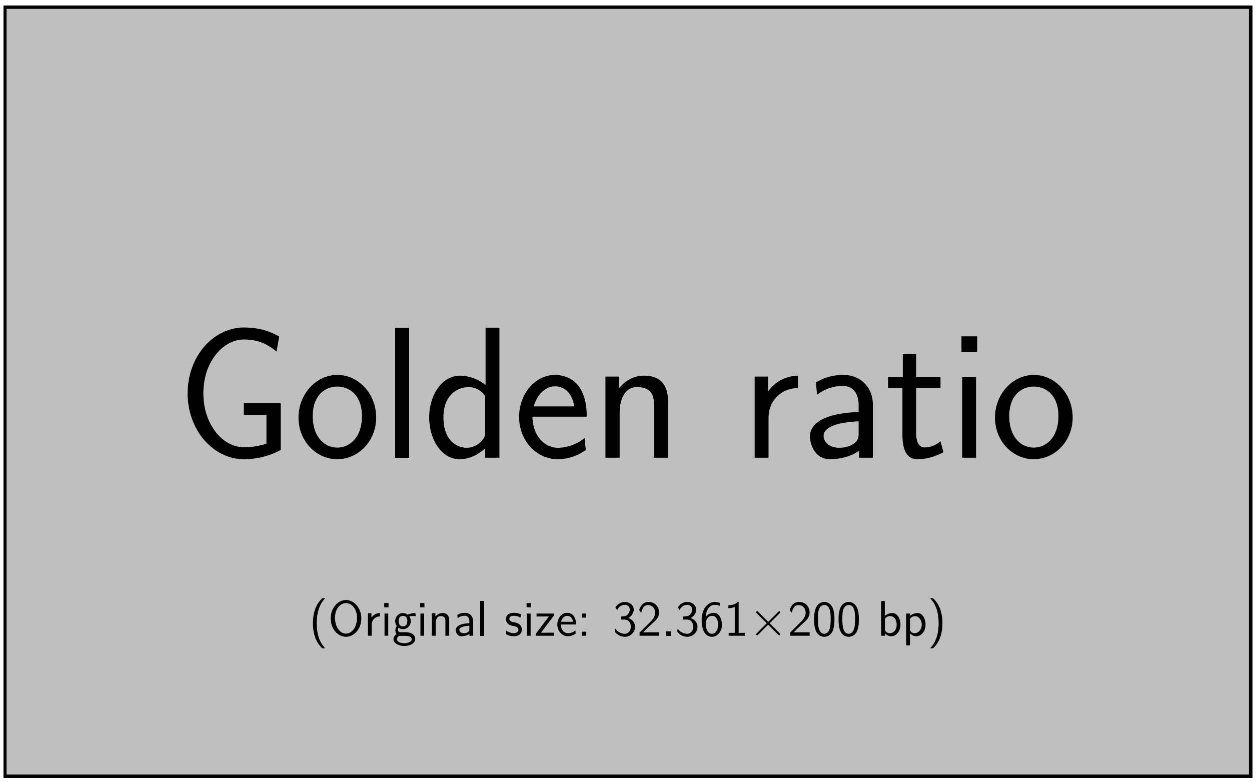


Figure 4: Spatio-temporal frequency classification