# Assignment 3 (ML for TS) - MVA

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## 1 Introduction

**Objective.** The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

## 2 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \ldots, T - 1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of $(f_1, f_2)$ represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

### Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

### Answer 1

The decoding procedure utilizes a time-frequency representation via a Short-Time Fourier Transform (STFT) with a 4096-point FFT to achieve high frequency resolution at $f_s = 22.05$ kHz. Segmentation is performed by applying the Pruned Exact Linear Time (PELT) algorithm with an $L_2$ cost function to the normalized, median-filtered frame energy. Valid segments are isolated from noise using an energy threshold, and dominant frequencies in the low $(650 - 1000$ Hz) and high $(1150 - 1700$ Hz) bands are identified via spectral peak detection and mapped to the

DTMF grid. Hyperparameters, including the segmentation penalty, energy threshold, and window length, were calibrated through an exhaustive grid search. This optimization process targeted the minimization of the mean Levenshtein distance between predicted and ground-truth symbol sequences. The final calibrated parameters were determined to be a window of 512 samples, a penalty of 0.1, and an energy threshold of 0.7, resulting in an average distance of 3.06.

## Question 2

What are the two symbolic sequences encoded in the test set?

## Answer 2

- Sequence 1: 51C9
- Sequence 2: #17#126#1

# 3 Wavelet transform for graph signals

Let $G$ be a graph defined a set of $n$ nodes $V$ and a set of edges $E$. A specific node is denoted by $v$ and a specific edge, by $e$. The eigenvalues and eigenvectors of the graph Laplacian $L$ are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of $f$ is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of $f$ and $\hat{g}_m$ are $M$ kernel functions. The number $M$ of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n/(M + 1 - R)$,

$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right]\mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

## Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of $R$?

## Answer 3

The parameter $R$ directly controls the spectral support of each kernel, which is

$$\text{supp}(\hat{g}_m) = [a_m - Ra, a_m).$$

As $R$ increases, the kernels become wider in the spectral domain, leading to stronger overlap between adjacent scales and smoother frequency coverage. Conversely, smaller values of $R$ produce narrower kernels with reduced overlap, resulting in a more localized and selective spectral tiling.
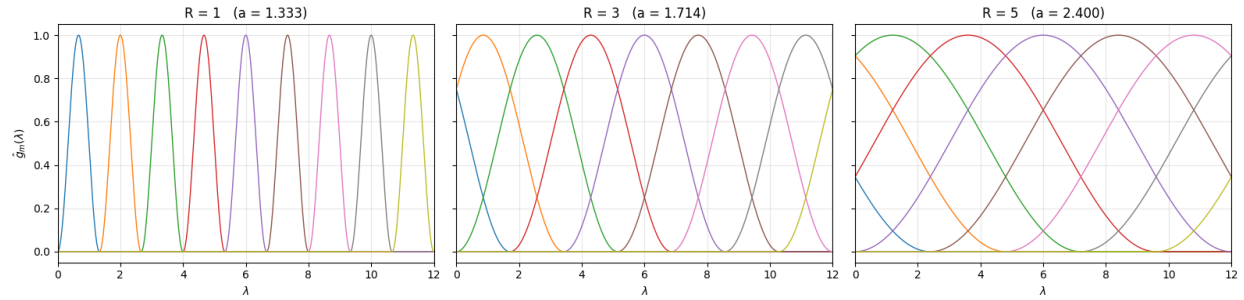
Figure 1: Spectrum Adapted Graph Wavelet kernels $\hat{g}_m(\lambda)$ for $M = 9$, $\lambda_n = 12$, and different values of the overlap parameter $R$. Each curve corresponds to one scale $m$.

## 4 Molene temperature graph

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

### Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

### Answer 4

The stations with missing values are: ARZAL, BATZ, BEG_MEIL, BREST-GUIPAVAS, BRIGNO-GAN, CAMARET, LANDIVISIAU, LANNAERO, LANVEOC, OUESSANT-STIFF, PLOUAY-SA, PLOUDALMEZEAU, PLOUGONVELIN, QUIMPER, RIEC-SUR-BELON, SIZUN, ST-NAZAIRE-MONTOIR, and VANNES-MEUCON.

The threshold is equal to 0.83 (using Lambert II étendu)

The least smooth signal occurs at 2014-01-10 09:00:00

The smoothest signal occurs at 2014-01-24 19:00:00

# 5 Node frequency classification

**Question 5**

(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node $v$, the vector $[W_f(1, v), W_f(2, v), \ldots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low/medium/high frequency:

- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,

- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,

- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

# Answer 5



(a) Least smooth signal
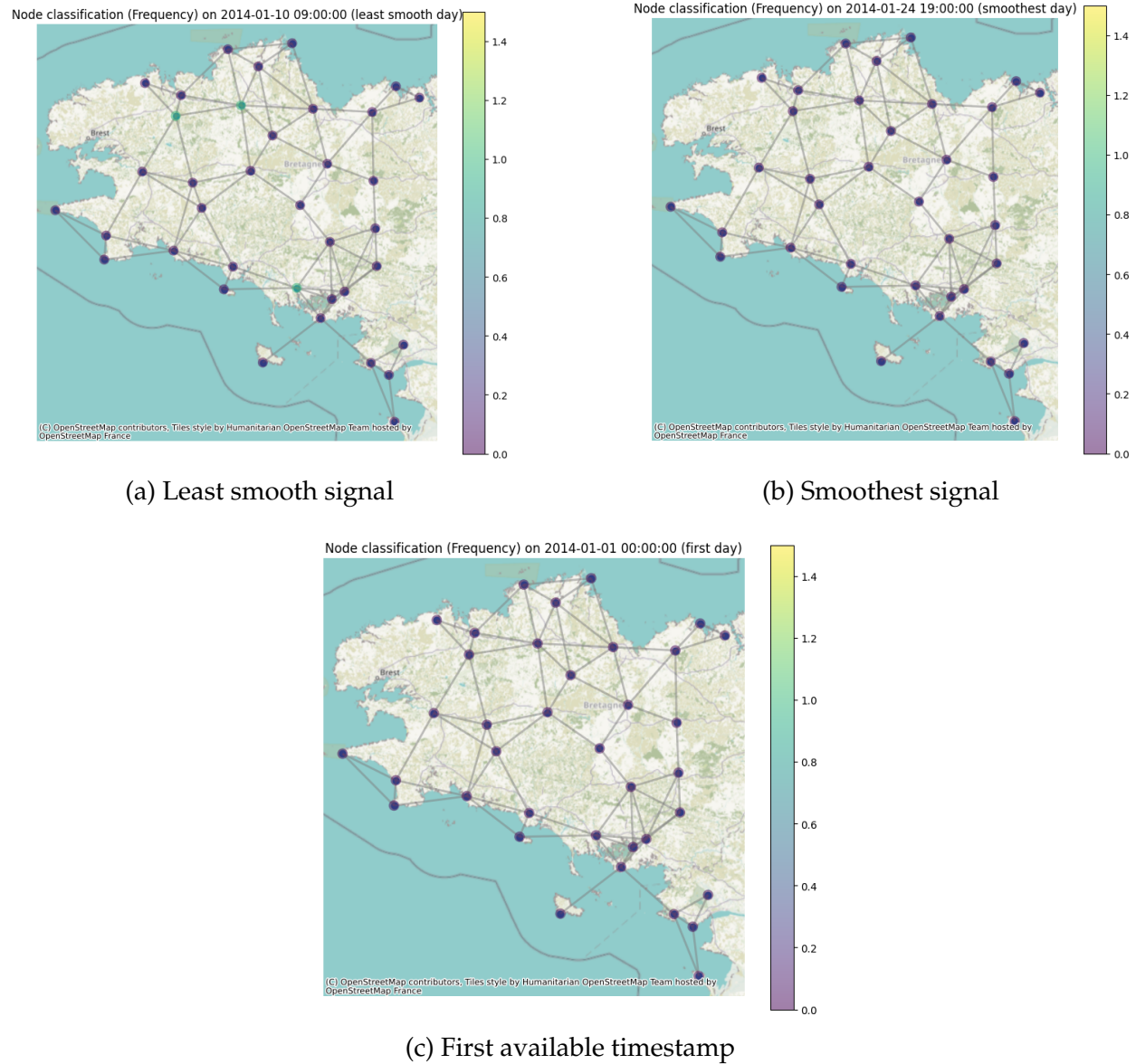


(b) Smoothest signal



(c) First available timestamp

Figure 2: Classification of nodes into low/medium/high frequency

As we can see in Figure 2, all nodes are considered low frequency for the first available timestamp (c) and for the smoothest signal (b). The least smooth signal (a) however presents some medium and high frequency nodes.

# Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph.
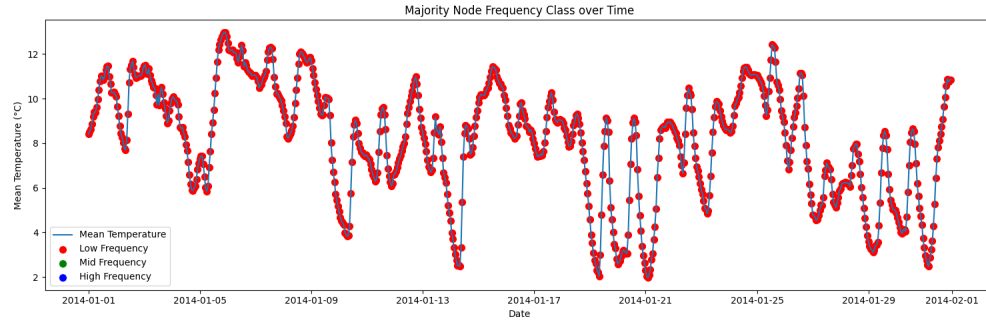
**Answer 6**



Figure 3: Average temperature. Markers' colours depend on the majority class.

Figure 3 shows only low frequency markers, indicating that the average temperature signal is very smooth across the graph.

# 6 Spatio-temporal graph

## Question 7

The previous graph $G$ only uses spatial information. To take into account the temporal dynamic, we construct a larger graph $H$ as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to $G$) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph $H$ is the Cartesian product of the spatial graph $G$ and the temporal graph $G'$ (which is simply a line graph, without loop).

- Express the Laplacian of $H$ using the Laplacian of $G$ and $G'$ (use Kronecker products).

- Express the eigenvalues and eigenvectors of the Laplacian of $H$ using the eigenvalues and eigenvectors of the Laplacian of $G$ and $G'$.

- Compute the wavelet transform of the temperature signal.

- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

## Answer 7

- Let $L_H, L_G, L_{G'}$ be the Laplacians of graphs $H, G, G'$ respectively. We have: $H = G \times G'$. Denoting by $I_n$ the identity matrix of size $n$, and using Kronecker products, we have:

$$L_H = L_G \otimes I_T + I_N \otimes L_{G'}$$

where $N$ is the number of stations and $T$ the number of timestamps.

- The Laplacian $L_H$ has $N + T$ eigenvectors. Let $u, v$ be eigenvectors of $L_G, L_{G'}$ with eigenvalues $\lambda, \mu$ respectively. Then,

$$\begin{aligned} L_H(u \otimes v) &= L_G u \otimes I_T v + I_N u \otimes L_{G'} v \\ &= \lambda(u \otimes v) + \mu(u \otimes v) \\ &= (\lambda + \mu)(u \otimes v) \end{aligned}$$

Thus, the eigenvalues of $L_H$ are all the sums $\lambda + \mu$ where $\lambda, \mu$ are eigenvalues of $L_G, L_{G'}$ respectively, and the corresponding eigenvectors are $u \otimes v$.
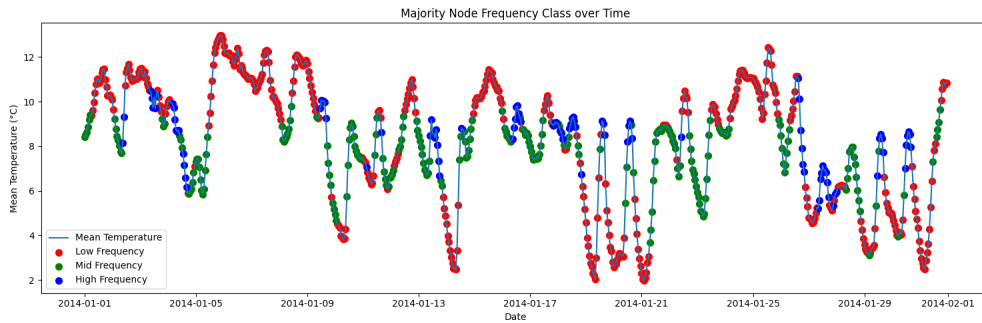


Figure 4: Average temperature. Markers' colours depend on the majority class.

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As we can see in Figure 4, the classification based on the wavelet transform finds mid-frequency as well as high-frequency nodes. The computed graphs in the notebook also show that the low-frequency nodes appear mostly near the sea, the mid-frequency nodes are usually more inland, and the high-frequency nodes are even more inland.