MA2215 Linear Programming **Tutorial 9**

Reading Chapter 6.

1. Consider the following balanced transportation problem.

		Destination 1	2	3	Supply
	1	5	1	2	25
Source	2	7	2	4	10
	3	3	3	5	15
Demand		10	20	20	

Find a starting basic feasible solution by the (i) north west corner method and (ii) least cost method.Compare the total transportation costs at both starting basic feasible solutions.

[Solution] (i) NW-method

		1	2	3	Supply
	1	5 (10)	1 (15)	2	25
Source	2	7	2 (5)	4 (5)	10
	3	3	3	5 (15)	15
Demand		10	20	20	

Cost $= 10(5) + 15(1) + 5(2) + 5(4) + 15(5) = 170.$

(ii) Least-cost method

		1	2	3	Supply
	1	5	1 (20)	2 (5)	25
Source	2	7	2	4 (10)	10
	3	3 (10)	3	5 (5)	15
Demand		10	20	20	

Cost $= 20(1) + 5(2) + 10(4) + 10(3) + 5(5) = 125.$

2. Consider the balanced transportation problem.

		Destination 1	2	3	Supply
Source	1	5	1	2	25
	2	7	2	4	10
	3	3	3	5	15
Demand		10	20	20	

(a) Verify that the basic feasible solution given by

$$x_{11} = 10, x_{13} = 15, x_{22} = 10, x_{32} = 10, x_{33} = 5$$

is not optimal. Find the entering variable and leaving variable, and obtain the next basic feasible solution.

(b) Proceed to find an optimal solution.

[Solution] (a)

		1	2	3	Supply	u
	1	5 (10)	1 1	2 (15)	25	0
Source	2	7 0	2 (10)	4 0	10	2
	3	3 −5	3 (10)	5 (5)	15	3
Demand		10	20	20		
v		5	0	2		

Because $\bar{c}_{31} < 0$, the solution is not optimal.

(b) Loop: $x_{31}(+) \to x_{11}(-) \to x_{13}(+) \to x_{33}(-) \to x_{31}$. $\theta = 5$. New solution

		1	2	3	u
	1	5 (5)	1 −4	2 (20)	0
Source	2	7 5	2 (10)	4 5	−3
	3	3 (5)	3 (10)	5 5	−2
v		5	5	2	

Loop: $x_{12}(+) \rightarrow x_{32}(-) \rightarrow x_{31}(+) \rightarrow x_{11}(-) \rightarrow x_{12}$. $\theta = 5$. New solution

		1	2	3	u
	1	5 4	1 (5)	2 (20)	0
Source	2	7 5	2 (10)	4 1	1
	3	3 (10)	3 (5)	5 1	2
v		1	1	2	

All $\bar{c}_{ij} \geq 0$. Thus, Optimal solution:

$$x_{12} = 5, x_{13} = 20, x_{22} = 10, x_{31} = 10, x_{32} = 5, \text{ others } = 0.$$

Cost $= 5(1) + 20(2) + 10(2) + 10(3) + 5(3) = 110$.

3. Consider the transportation problem in Table A, the total demand exceeds total supply. Suppose that the penalty costs per unit of unsatisfied demand are 5, 3 and 2 for destinations 1,2 and 3.

   (a) Find a starting basic feasible solution by the Vogel's Approximation method, and proceed to find an optimal solution.

   (b) Suppose the unit transportation cost from Source 1 to Destination 3 is reduced to 2. Determine whether the optimal solution is optimal. If not proceed to find an optimal solution.

Table A

		Destination 1	2	3	Supply
	1	5	1	7	10
Source	2	6	4	6	40
	3	3	2	5	15
Demand		25	20	50	

[Solution] (a)

$$TD = 25 + 20 + 50 = 95$$

$$TS = 10 + 40 + 15 = 65$$

Dummy Supply $= TD - TS = 30$.

Find a starting solution by Vogel's method:

	1	2	3	Supply	Penalty
1	5	1 (10)	7	10	4
2	6 (10)	4 (10)	6 (20)	40	2
3	3 (15)	2	5	15	1
Dummy S	5	3	2 (30)	30	1
Demand	25	20	50		
Penalty	2	1	3		
	3	2	1		

Proceed to find the optimal solution:

	1	2	3	u
1	5, 2	1, (10)	7, 4	0
2	6, (10)	4, (10)	6, (20)	3
3	3, (15)	2, 1	5, 2	0
Dummy S	5, 3	3, 3	2, (30)	-1
v	3	1	3	

All $\bar{c}_{ij} \geq 0$. Thus, Optimal solution:

$x_{12} = 10, x_{21} = 10, x_{22} = 10, x_{23} = 20, x_{31} = 15, x_{43} = 30(Dummy),$ others $= 0.$

Cost $= 10(1) + 10(6) + 10(4) + 20(6) + 15(3) + 30(2) = 335.$

(b) We shall re-calculate the reduced costs

	1	2	3	u
1	5 2	1 (10)	2 -1	0
2	6 (10)	4 (10)	6 (20)	3
3	3 (15)	2 1	5 2	0
Dummy S	5 3	3 3	2 (30)	-1
v	3	1	3	

This is not optimal, ($\bar{c}_{13} < 0$). Proceed to find the optimum.

Loop: $x_{13}(+) \to x_{23}(-) \to x_{22}(+) \to x_{12}(-) \to x_{13}$. $\theta = 10$. New solution:

	1	2	3	u
1	5 3	1 1	2 (10)	0
2	6 (10)	4 (20)	6 (10)	4
3	3 (15)	2 1	5 2	1
Dummy S	5 3	3 3	2 (30)	0
v	2	0	2	

Cost $= 20 + 60 + 80 + 60 + 45 + 60 = 325$.

4. Consider the unbalanced transportation problem in Table B. Suppose the demand at Destination 1 must be shipped from Source 4.

   Use the Vogel's Approximation method to find a starting basic feasible solution, and proceed to determine an optimal solution.

Table B

		Destinations 1	2	3	Supply
	1	5	1	0	20
Sources	2	3	2	4	10
	3	7	5	2	15
	4	9	6	0	25
Demand		5	10	15	

[Solution]

Let $x_{41} = 5$ and $x_{k1} = 0$ for $k = 1, 2, 3$. Do not consider destination 1 in the further computation.

$TS = 20 + 10 + 15 + 20 = 65$

$TD = 10 + 15 = 25$

Dummy demand $= TS - TD = 40$. Costs for transporting to Dummy Demand are 0.

Find starting BFS by Vogel's approximation method:

	2	3	Dummy D	Supply	Penalty
1	1 (10)	0 (10)	0	20	0
2	2	4	0 (10)	10	2
3	5	2	0 (15)	15	2
4	6	0 (5)	0 (15)	20	0
Demand	10	15	40		
Penalty	1	0	0		

Compute the reduced costs:

	2	3	Dummy D	u
1	1 (10)	0 (10)	0 0	0
2	2 1	4 4	0 (10)	0
3	5 4	2 2	0 (15)	0
4	6 5	0 (5)	0 (15)	0
v	1	0	0	

All reduced costs are nonnegative. Thus, solution is optimal.

Optimal solution: $x_{12} = 10, x_{13} = 10, x_{41} = 5, x_{43} = 10$, other $x_{ij} = 0$.

Cost $= 10(1) + 10(0) + 5(9) + 10(0) = 55$

5. Three electric power plants with capacities of 25, 40, and 30 million kilowatt hour (kWh) supply electricity to three cities whose maximum demands are estimated at 30, 35, and 25 million kWh. The costs in dollars of selling power to the different cities per million kWh are given in the table.

		City 1	City 2	City 3
Plant	1	600	700	400
	2	320	300	350
	3	500	480	450

Formulate the problem as a transportation problem (in the form of transportation tableau) for the purpose of establishing the most economical distribution plan from the standpoint of the power company and find the optimal solution.

[Solution]

Formulate problem (add a dummy destination) and find starting BFS by least-cost method:

	1	2	3	D.D.	Supply
1	600	700	400 (20)	0 (5)	25
2	320 (5)	300 (35)	350	0	40
3	500 (25)	480	450 (5)	0	30
Demand	30	35	25	5	

First iteration:

	1	2	3	D.D.	u
1	600 150	700 370	400 (20)	0 (5)	0
2	320 (5)	300 (35)	350 80	0 130	-130
3	500 (25)	480 0	450 (5)	0 -50	50
v	450	430	400	0	

Entering variable: x_{34}.

Loop: $x_{34}(+) \to x_{33}(-) \to x_{13}(+) \to x_{14}(-) \to x_{34}$. $\theta = 5$.

Second iteration:

	1	2	3	D.D.	u
1	600 100	700 220	400 (25)	0 (0)	0
2	320 (5)	300 (35)	350 130	0 180	-180
3	500 (25)	480 0	450 50	0 (5)	0
v	500	480	400	0	

Optimal cost $= 10000 + 0 + 1600 + 10500 + 12500 + 0 = 34600$.

6. A wholesale company supplies goods to three retailers who each requires 30 units. The company has 2 warehouses. Warehouse 1 has 40 units available and Warehouse 2 has 30 units available. The costs of shipping 1 unit of goods from warehouses to retailers are shown in the table:

	To		
From	Retailer 1	Retailer 2	Retailer 3
Warehouse 1	\$15	\$35	\$25
Warehouse 2	\$10	\$50	\$40

Suppose that the penalty costs are, respectively, \$ 90, \$ 80 and \$ 110 for each unit of Retailer 1's , Retailer 2's and Retailer 3's unmet demands.

(i) Formulate the problem as a transportation model to minimize the sum of shortage and shipping costs.

(ii) Find an optimal solution of the problem.

(<u>Answer</u> (ii) $W1 \xrightarrow{10} R2$, $W1 \xrightarrow{30} R3$, $W2 \xrightarrow{30} R1$ cost = 3000.)

[Solution] (i) and (ii)

Formulate problem (adding dummy source) and find starting BFS by least-cost method:

	1	2	3	Supply
1	15 (0)	35 (10)	25 (30)	40
2	10 (30)	50	40	30
D.S.	90	80 (20)	110	20
Demand	30	30	30	

Compute reduced costs:

	1	2	3	u
1	15 (0)	35 (10)	25 (30)	0
2	10 (30)	50 20	40 20	-5
D.S.	90 30	80 (20)	110 40	45
Demand	15	35	25	

All reduced costs are ≥ 0.

Optimal solution: $x_{11} = 0, x_{12} = 10, x_{13} = 30, x_{21} = 30, x_{32} = 20$ (Dummy), others = 0.

Cost $= 0(15) + 10(35) + 30(25) + 30(10) + 20(80) = 3000$.

7. (*Assignment as a transportation model.*)
Joshop wants to assign four different categories of machines to five types of tasks. The number of machines available in the four categories are 25, 30, 20 and 30. The number of jobs in the five tasks are 20, 20, 30, 10 and 25. Machine category 4 cannot be assigned to task type 4. The following table provides the unit cost (in dollars) of assigning a machine category to a task type. The objective of the problem is to determine the optimum number of machines in each category to be assigned to each task type. Set up the problem as a transportation problem (in transportation tableau form).

		Task Type				
		1	2	3	4	5
	1	10	2	3	15	9
Machine	2	5	10	15	2	4
Category	3	15	5	14	7	15
	4	20	15	13	–	8

[Solution] Numbers of machines and jobs are considered as supplies and demands. The problem is balanced.

	1	2	3	4	5	No. machines
1	10	2	3	15	9	25
2	5	10	15	2	4	30
3	15	5	14	7	15	20
4	20	15	13	M	8	30
No. jobs	20	20	30	10	25	

You are not required to find the optimal solution.