COMMENTARIES

Commentaries are informative essays dealing with viewpoints of statistical practice, statistical education, and other topics considered to be of general interest to the broad readership of *The American Statistician*. Commentaries are similar in spirit to Letters to the Editor, but they involve longer discussions of background, issues, and perspectives. All commentaries will be refereed for their merit and compatibility with these criteria.

Use of the Correlation Coefficient With Normal Probability Plots

STEPHEN W. LOONEY and THOMAS R. GULLEDGE, JR.*

The use of the correlation coefficient is suggested as a technique for summarizing and objectively evaluating the information contained in probability plots. Goodness-of-fit tests are constructed using this technique for several commonly used plotting positions for the normal distribution. Empirical sampling methods are used to construct the null distribution for these tests, which are then compared on the basis of power against certain nonnormal alternatives. Commonly used regression tests of fit are also included in the comparisons. The results indicate that use of the plotting position $p_i = (i - .375)/(n + .25)$ yields a competitive regression test of fit for normality.

KEY WORDS: Shapiro–Wilk test; Shapiro–Francia test; Filliben test; Plotting position; Empirical power comparison; Regression tests of fit.

1. INTRODUCTION

Probability plots are frequently recommended for assessing the goodness of fit of a hypothesized distribution (e.g., see Nelson 1982, chap. 3; Johnson and Wichern 1982, pp. 152–156; and Snedecor and Cochran 1980, pp. 59–63). Shapiro and Brain (1981) went so far as to urge that any formal test for goodness of fit be augmented by the construction of a probability plot, since "the plot can better describe the data than a single test statistic" (p. 22). We suggest that a preferable approach for testing goodness of fit is to examine the probability plot first and then augment it with a formal hypothesis test that is based on the plot. Besides providing an objective criterion for interpreting the plot, such a test also provides a useful summary of the information contained therein. Of course, other formal hypothesis tests could also be performed for corroboration.

Several authors have proposed formal goodness-of-fit tests that they claim are based on the information contained in probability plots. These tests are usually referred to as *regression tests of fit* (Shapiro and Brain 1981, p. 2). For the normal distribution, these include the tests proposed by Shapiro and Wilk (1965), Shapiro and Francia (1972), Filliben (1975), and LaBrecque (1977). What these authors call probability plots, however, are not the plots that are usually constructed in practice. Instead, they define probability plots in terms of plotting positions that are not used by practitioners. The history of this problem is discussed in Section 2.

In this article, we examine the use of the Pearson product–moment correlation coefficient as a technique for constructing a test statistic from a realistic version of a normal probability plot. We construct such a test for several commonly used plotting positions for the normal distribution and generate the null distribution for each one using empirical sampling methods. These tests are then compared with other regression tests of fit in terms of power against various alternative distributions. Recommendations concerning these tests are given in Section 3. Section 4 contains a description of the computational methods used in this study, and Section 5 contains a summary and some discussion of the results.

2. PROBABILITY PLOTS AND PLOTTING POSITIONS

Let $\{y_{(1)}, y_{(2)}, \ldots, y_{(n)}\}$ denote the order statistics of a sample from a distribution whose cdf is hypothesized to be of the form $F[(y - \mu)/\sigma]$, where μ and σ are location and scale parameters, respectively. (Note that the $y_{(i)}$ could be the order statistics of the transformed sample values after an apparently appropriate transformation has been made.) To construct a probability plot, the sample order statistic $y_{(i)}$ is plotted (usually on the vertical axis) against $x_i = F^{-1}(p_i)$ (usually on the horizontal axis), where p_i is an estimate of $F[(y_{(i)} - \mu)/\sigma]$. This estimate is called the plotting position. The two most commonly used plotting positions are

$$p_i = (i - .5)/n \tag{2.1}$$

(Hazen 1914) and

$$p_i = i/(n + 1) \tag{2.2}$$

(Weibull 1939). Many other plotting positions have been

*Stephen W. Looney and Thomas R. Gulledge, Jr., are Assistant Professors, Department of Quantitative Business Analysis, Louisiana State University, Baton Rouge, LA 70803.

proposed in the literature, including several that are based on the mean or median order statistics of the reduced variate $(Y - \mu)/\sigma$.

Let $W_{(i)} = (Y_{(i)} - \mu)/\sigma$ denote the ith order statistic of the reduced variate, and let $E(W_{(i)})$ and $M(W_{(i)})$ denote its mean and median, respectively. Shapiro and Wilk (1965), Shapiro and Francia (1972), and LaBrecque (1977) defined probability plots in terms of the plotting position

$$p_i = F[E(W_{(i)})], \tag{2.3}$$

whereas Filliben (1975) defined them in terms of

$$p_i = F[M(W_{(i)})]. \tag{2.4}$$

Although several authors have offered justification for the use of (2.3) (Kimball 1960, Barnett 1975, and Cunnane 1978) or (2.4) (Benard and Bos-Levenbach 1953 and Filliben 1975) as a plotting position, in practice one almost never uses either. Typically, either (2.1) (Nelson 1979, p. 8) or (2.2) (Chilko 1978, p. 4) is used; most authors seem to prefer (2.1).

Some authors, however, have proposed other plotting positions that are based on approximations of (2.3) or (2.4). For example, Filliben (1975) suggested using

$$\begin{aligned} p_i &= 1 - m_n, \qquad i = 1, \\ &= (i - .3175)/(n + .365), \\ &\qquad i = 2, 3, 4, \ldots, n - 1, \\ &= (.5)^{(1/n)}, \qquad i = n, \end{aligned} \tag{2.5}$$

as an approximation to (2.4). Several authors, including Kimball (1960) and Cunnane (1978), have recommended that an approximation of (2.3) developed by Blom (1958, p. 71) be used as a plotting position for the normal distribution:

$$p_i = (i - .375)/(n + .25). \tag{2.6}$$

This plotting position has seen increasing acceptance among practitioners in recent years; for example, the normal probability plot produced by the UNIVARIATE procedure of the Statistical Analysis System (SAS) is based on this plotting position (SAS Institute Inc. 1982, p. 580). Applied textbooks that recommend the use of (2.6) for the normal distribution include King (1981, p. 6).

We will restrict our attention in this article to the plotting positions (2.1), (2.2), and (2.6), since these seem to be the most commonly used. Many other plotting positions, however, have been proposed for use with the normal distribution, including those of Benard and Bos-Levenbach (1953), Tukey (1962), Cunnane (1978), and Dixon (1981, p. 126).

Once the plotting position has been selected and the probability plot constructed, an objective criterion for interpreting the plot is needed. If the hypothesized distribution is the correct one, then the plot of $y_{(i)}$ against $x_i = F^{-1}(p_i)$ will be approximately linear. Since the Pearson product–moment correlation coefficient between $y_{(i)}$ and x_i is equal to one if the plot is exactly linear, Filliben (1975) suggested that one use it to measure the linearity of a probability plot. Following his lead, we propose the use of the correlation coefficient in constructing a goodness-of-fit test statistic from a plot based on any particular plotting position. Since the $Y_{(i)}$ are highly correlated and heteroscedastic, the usual distributional results for the correlation coefficient do not apply. Instead, empirical sampling methods must be used to determine the null distribution of the test statistic. Two of the test statistics already mentioned can be constructed in this way: the Filliben (1975) test statistic r is the correlation coefficient statistic based on plotting position (2.5), and the Shapiro–Francia test statistic W' (Shapiro and Francia 1972) is a squared version of the correlation coefficient statistic based on plotting position (2.3). In addition, the test for normality presented in Johnson and Wichern (1982, pp. 155–156) is a correlation coefficient test based on position (2.1).

Another procedure for constructing a test statistic from a probability plot was proposed by Shapiro and Wilk (1965). They considered the ratio of two estimators of σ^2: the square of the generalized least squares estimate of the slope of the regression line fitted to the probability plot and the sample variance calculated from the $y_{(i)}$'s. If the hypothesized normal distribution is the appropriate model and the straight line assumption is correct, then these estimates will be close to each other, and their ratio will be close to one. Departures from normality are indicated by values significantly different from one. Similarly, LaBrecque (1977) based his tests for normality on ratios of estimates of σ^2 that are appropriate when certain types of nonlinearity are present in the plot. We do not consider test statistics based on such ratios in this article because they do not directly assess the linear nature of the plot; rather, they are more concerned with the quality of an estimate of σ^2 obtained from the plot. Since probability plots are more commonly used today for testing goodness of fit than for estimation, the class of test statistics based on the correlation coefficient seems more appropriate for our purposes. The Shapiro–Wilk (1965) test, however, is included in our discussion for comparison, since it has been shown to be one of the most powerful tests for normality (Pearson et al. 1977).

Various authors have proposed other techniques for objectively evaluating probability plots, including Mage (1982), Stirling (1982), and Michael (1983). Each suggested ways of using either the Kolmogorov–Smirnov test statistic or a modification of it, such as the Lilliefors (1967) statistic, as an aid in the interpretation of probability plots. Although these techniques may prove to be useful in a given application, they suffer from the same disadvantage as the Shapiro–Wilk test: none of them assesses the linearity of the probability plot as directly as the correlation coefficient does. For this reason, we do not consider any of these techniques in this article.

3. RESULTS AND CONCLUSIONS

To compare the powers of the correlation coefficient tests corresponding to positions (2.1), (2.2), and (2.6) with those of the Shapiro–Wilk (1965), Shapiro–Francia (1972), and Filliben (1975) tests for normality, an empirical sampling study was conducted in which samples of sizes $n = 20$, 50, and 100 were generated from each of the 25 alternative distributions used for $n = 50$ in Filliben's study. The results of the study are summarized in Table 1, and computational details are given in Section 4.

If we restrict ourselves to those tests based on the correlation coefficient, that is, all except the Shapiro–Wilk test, an examination of Table 1 indicates that for symmetric, shorter-tailed alternatives, the test based on Hazen's (1914) plotting position $p_i = (i - .5)/n$ is the most powerful. It is slightly inferior, however, to the other correlation coefficient tests in terms of power against symmetric longer-tailed alternatives and skewed alternatives. The test based on the Blom (1958) position $p_i = (i - .375)/(n + .25)$ has power that is comparable to that of Hazen's for shorter-tailed alternatives and is generally the most powerful against longer-tailed and skewed alternatives. In view of this and the fact that the Blom position is preferable to the Hazen position whenever the plot is to be used for estimation purposes (Kimball 1960 and Cunnane 1978), we recommend that the Blom position be used in constructing normal probability plots. Table 2 contains percentage points for the corresponding goodness-of-fit test for $n = 3(1)50(5)100$.

The Weibull (1939) plotting position $p_i = i/(n + 1)$ yields a test that is generally inferior to the Blom test, especially against shorter-tailed alternatives, and we do not recommend its use for normal probability plots. The test based on Blom's position dominates both the Filliben and Shapiro–Francia tests, although the dominance is slight in most cases. Although the Shapiro–Wilk test is certainly the most powerful of these tests against shorter-tailed alternatives, it has less power than the test based on Blom's position for some longer-tailed and skewed alternatives. In general, these results indicate that the correlation coefficient test based on the position $p_i = (i - .375)/(n + .25)$ is a competitive regression test of fit for normality.

4. COMPUTATIONAL METHODS

For each of the plotting positions (2.1), (2.2), and (2.6), a test statistic for the composite hypothesis of normality was constructed, using the Pearson product–moment correlation coefficient between the sample order statistics $y_{(i)}$ and $x_i = F^{-1}(p_i)$, where F denotes the standard normal distribution function. The inversion of F was accomplished by using the algorithm given by Abramowitz and Stegun (1964, p. 933, formula 26.2.23). The empirical percentage points of the null distribution of each of these test statistics were determined for $n = 20$, 50, and 100 by generating $N = 10{,}000$ normal random samples on the Louisiana State University IBM 370/3081 computer using the GRAND generator (Brent 1974). $N = 10{,}000$ was used because longer run lengths affected the percentage points only in the fourth decimal place. These percentage points were smoothed by replicating each experiment 12 times and taking the average; therefore, 120,000 samples were generated for each sample size.

Since only $N = 1{,}000$ normal random samples were used in generating the published percentage points for the Shapiro–Francia test for $n = 50$ (Shapiro and Francia 1972, p. 215), and since percentage points are not given in their paper for $n = 20$ and $n = 100$, the procedure just described was also used to determine the percentage points for $n = 20$, 50, and 100 for the Shapiro–Francia test. Calculation of the order statistic means was achieved by using the al-

Table 1. Empirical 5% Level Power (in %) of Correlation Coefficient Tests Based on Plotting Positions

	Power[a] for n = 20						Power for n = 50						Power for n = 100					
Distribution	W	F	W'	B	H	S	W	F	W'	B	H	S	W	F	W'	B	H	S
Symmetric alternatives shorter-tailed than normal																		
Arcsine	30	43	47	51	56	73	96	99	99	100	100	100	100	100	100	100	100	100
Johnson bounded, JSB(0, .5)	12	20	23	26	30	45	68	86	88	90	92	99	100	100	100	100	100	100
Tukey λ(1.5)	5	9	10	12	15	25	34	57	60	65	70	93	94	98	99	99	100	100
Uniform	4	7	8	9	11	20	24	44	48	53	58	88	85	95	97	98	99	100
Tukey λ(.75)	2	4	5	6	7	13	11	25	28	32	36	72	58	79	85	89	91	100
Anglit	1	2	2	3	3	5	1	4	4	6	6	21	5	12	17	20	22	66
Triangular	1	2	2	2	2	3	1	1	2	2	2	9	1	3	4	5	6	32
Symmetric alternatives longer-tailed than normal																		
Logistic	15	14	14	15	14	11	26	26	24	27	23	14	40	37	36	41	35	13
LaPlace	33	32	32	33	30	26	63	62	60	62	58	42	86	84	84	86	82	54
Johnson unbounded, JSU(0, 1)	50	48	48	49	47	42	83	82	81	83	80	69	97	97	97	97	96	86
Tukey $\lambda(-.5)$	68	67	67	67	66	62	95	95	95	95	94	90	100	100	100	100	100	99
Cauchy	89	89	89	89	88	86	100	100	100	100	100	99	100	100	100	100	100	100
Tukey $\lambda(-1.5)$	98	98	98	98	97	97	100	100	100	100	100	100	100	100	100	100	100	100
Skewed alternatives																		
Weibull (10)	14	15	15	16	15	15	32	35	34	37	35	34	57	60	62	66	63	59
Weibull (3)	3	4	4	4	4	4	3	4	4	5	4	4	4	5	6	7	7	14
Skewed, λ(1.5, .5)	15	19	21	23	25	33	56	72	73	77	79	93	98	99	100	100	100	100
Extreme Value Type 1	30	31	31	33	32	32	65	68	68	70	68	68	91	93	93	94	94	92
Half-normal	34	37	38	40	40	44	82	88	89	90	90	95	100	100	100	100	100	100
Power lognormal, PLN(.5)	48	50	51	52	52	52	89	91	91	92	92	93	100	100	100	100	100	100
Extreme Value Type 2(5)	62	64	64	66	65	65	96	97	96	97	97	97	100	100	100	100	100	100
Exponential	76	79	80	81	82	84	100	100	100	100	100	100	100	100	100	100	100	100
Pareto (10)	84	86	86	88	88	90	100	100	100	100	100	100	100	100	100	100	100	100
Lognormal	89	91	91	92	92	93	100	100	100	100	100	100	100	100	100	100	100	100
Chi-Squared (1)	96	97	97	98	98	98	100	100	100	100	100	100	100	100	100	100	100	100
Extreme Value Type 2(1)	99	99	99	99	99	99	100	100	100	100	100	100	100	100	100	100	100	100

[a] W, Weibull Position; F, Filliben test; W', Shapiro–Francia test; B, Blom position; H, Hazen position; S, Shapiro–Wilk test.

Table 2. Empirical Percentage Points for Correlation Coefficient Test Based on Blom's Plotting Position

	Level													
n	*.000*	*.005*	*.010*	*.025*	*.050*	*.100*	*.250*	*.500*	*.750*	*.900*	*.950*	*.975*	*.990*	*.995*
3	.866	.867	.869	.872	.879	.891	.924	.966	.992	.999	.9997	.9999	1.000	1.000
4	.785	.813	.824	.846	.868	.894	.931	.958	.979	.992	.996	.998	.999	1.000
5	.729	.807	.826	.856	.880	.903	.934	.960	.977	.988	.992	.995	.997	.998
6	.686	.820	.838	.866	.888	.910	.939	.962	.977	.986	.990	.993	.996	.997
7	.651	.828	.850	.877	.898	.918	.944	.964	.978	.986	.990	.992	.995	.996
8	.623	.840	.861	.887	.906	.924	.948	.966	.978	.986	.990	.992	.994	.995
9	.599	.854	.871	.894	.912	.930	.952	.968	.980	.986	.990	.992	.994	.995
10	.578	.862	.879	.901	.918	.934	.954	.970	.980	.987	.990	.992	.994	.995
11	.560	.870	.886	.907	.923	.938	.957	.972	.981	.987	.990	.992	.994	.995
12	.544	.876	.892	.912	.928	.942	.960	.973	.982	.988	.990	.992	.994	.995
13	.529	.885	.899	.918	.932	.945	.962	.974	.983	.988	.991	.992	.994	.995
14	.516	.890	.905	.923	.935	.948	.964	.976	.984	.989	.991	.992	.994	.995
15	.504	.896	.910	.927	.939	.951	.965	.977	.984	.989	.991	.993	.994	.995
16	.493	.899	.913	.929	.941	.953	.967	.978	.985	.989	.991	.993	.994	.995
17	.483	.905	.917	.932	.944	.954	.968	.979	.986	.990	.992	.993	.994	.995
18	.473	.908	.920	.935	.946	.957	.970	.979	.986	.990	.992	.993	.9945	.9952
19	.465	.914	.924	.938	.949	.958	.971	.980	.987	.990	.992	.993	.9946	.9953
20	.457	.916	.926	.940	.951	.960	.972	.981	.987	.991	.992	.994	.9947	.9954
21	.449	.918	.930	.943	.952	.961	.973	.982	.987	.991	.993	.994	.995	.996
22	.442	.923	.933	.945	.954	.963	.974	.982	.988	.991	.993	.994	.995	.996
23	.435	.925	.935	.947	.956	.964	.975	.983	.988	.991	.993	.994	.995	.996
24	.429	.927	.937	.949	.957	.965	.976	.983	.988	.992	.993	.994	.995	.996
25	.422	.929	.939	.951	.959	.966	.976	.984	.989	.992	.993	.994	.995	.996
26	.417	.932	.941	.952	.960	.967	.977	.984	.989	.992	.993	.994	.995	.996
27	.411	.934	.943	.953	.961	.968	.978	.985	.989	.992	.994	.995	.9955	.9960
28	.406	.936	.944	.955	.962	.969	.978	.985	.990	.992	.994	.995	.9955	.9960
29	.401	.939	.946	.956	.963	.970	.979	.985	.990	.993	.994	.995	.9956	.9961
30	.396	.939	.947	.957	.964	.971	.979	.986	.990	.993	.994	.995	.9957	.9962
31	.392	.942	.950	.958	.965	.972	.980	.986	.990	.993	.994	.995	.9957	.9962
32	.387	.943	.950	.959	.966	.972	.980	.987	.991	.993	.994	.995	.9958	.9963
33	.383	.944	.951	.961	.967	.973	.981	.987	.991	.993	.994	.995	.9959	.9963
34	.379	.946	.953	.962	.968	.974	.981	.987	.991	.993	.994	.995	.996	.997
35	.375	.947	.954	.962	.969	.974	.982	.987	.991	.994	.9945	.9953	.996	.997
36	.371	.948	.955	.963	.969	.975	.982	.988	.991	.994	.9946	.9954	.996	.997
37	.368	.950	.956	.964	.970	.976	.983	.988	.991	.994	.995	.9955	.9962	.997
38	.364	.951	.957	.965	.971	.976	.983	.988	.992	.994	.995	.9956	.9963	.997
39	.361	.951	.958	.966	.971	.977	.983	.988	.992	.994	.995	.9957	.9963	.997
40	.358	.953	.959	.966	.972	.977	.984	.989	.992	.994	.995	.9957	.9964	.997
41	.354	.953	.960	.967	.973	.977	.984	.989	.992	.994	.995	.996	.9965	.9968
42	.351	.954	.961	.968	.973	.978	.984	.989	.992	.994	.995	.996	.9965	.9969
43	.348	.956	.961	.968	.974	.978	.984	.989	.992	.994	.995	.996	.9966	.9969
44	.346	.957	.962	.969	.974	.979	.985	.989	.993	.9945	.9953	.996	.9966	.9970
45	.343	.957	.963	.969	.974	.979	.985	.990	.993	.9945	.9954	.996	.9966	.9970
46	.340	.958	.963	.970	.975	.980	.985	.990	.993	.995	.9955	.9961	.9968	.9971
47	.337	.959	.965	.971	.976	.980	.986	.990	.993	.995	.9956	.9962	.9968	.9972
48	.335	.959	.965	.971	.976	.980	.986	.990	.993	.995	.9956	.9962	.9968	.9972
49	.332	.961	.966	.972	.976	.981	.986	.990	.993	.995	.9957	.9963	.9968	.9972
50	.330	.961	.966	.972	.977	.981	.986	.990	.993	.995	.9957	.9963	.9969	.9972
55	.319	.965	.969	.974	.979	.982	.987	.991	.994	.995	.996	.9966	.9971	.9974
60	.309	.967	.971	.976	.980	.984	.988	.992	.994	.9956	.9963	.9968	.9973	.9975
65	.300	.969	.973	.978	.981	.985	.989	.992	.994	.996	.9965	.9969	.9974	.9977
70	.292	.971	.975	.979	.983	.986	.990	.993	.995	.996	.9966	.9971	.9975	.9978
75	.284	.973	.976	.981	.984	.987	.990	.993	.995	.996	.9968	.9972	.9976	.9979
80	.277	.975	.978	.982	.985	.987	.991	.993	.995	.996	.9970	.9974	.9978	.9980
85	.271	.976	.979	.983	.985	.988	.991	.994	.996	.9966	.9971	.9975	.9979	.9981
90	.266	.977	.980	.984	.986	.988	.992	.994	.996	.9967	.9972	.9976	.9979	.9981
95	.260	.979	.981	.984	.987	.989	.992	.994	.996	.9969	.9973	.9977	.9980	.9982
100	.255	.979	.982	.985	.987	.989	.992	.995	.996	.9970	.9974	.9978	.9981	.9983

gorithm provided by Royston (1982a). Published percentage points provided by Filliben were used for the Filliben test (Filliben 1975, p. 113). Percentage points for the Shapiro–Wilk test were not required; instead, p values were calculated by using an algorithm provided by Royston (1982b).

The empirical power comparisons for these tests were conducted by generating $N = 1{,}000$ samples of sizes $n = 20$, 50, and 100 from each of the 25 alternative distributions used for $n = 50$ in Filliben's study. The required uniform random numbers were generated by using the algorithm presented by Wichmann and Hill (1982) and the normal random numbers were generated by using the GRAND generator (Brent 1974). The power calculations were smoothed by replicating the experiment 12 times and taking the average.

5. DISCUSSION

In this article, we have examined the use of the correlation coefficient as a technique for summarizing the information contained in probability plots. We constructed a test statistic in this manner for the three most commonly used probability plotting positions for the normal distribution: $p_i = (i - .5)/n$ (Hazen 1914), $p_i = i/(n + 1)$ (Weibull 1939), and $p_i = (i - .375)/(n + .25)$ (Blom 1958). The null distribution of each of the resulting test statistics was determined by using empirical sampling methods, and the tests were compared in terms of power against various alternative distributions. Also included in the comparison were the Shapiro–Francia (1972) and Filliben (1975) tests for normality, both of which are based on correlation coefficients that are calculated from rather unrealistic versions of probability plots. To determine how well the power of the correlation coefficient tests compares with that of the other tests for normality, the Shapiro–Wilk (1965) test was also included in this study.

Our results indicate that the Blom plotting position $p_i = (i - .375)/(n + .25)$ yields a correlation coefficient test that is more powerful overall than either the Shapiro–Francia or Filliben tests. This test also compares favorably with those based on the Hazen position $p_i = (i - .5)/n$ and the Weibull position $p_i = i/(n + 1)$. Furthermore, comparisons with the Shapiro–Wilk test indicate that the correlation coefficient test based on Blom's position compares favorably with goodness-of-fit tests for normality that are not calculated by using the correlation coefficient.

[*Received March 1984. Revised July 1984.*]

REFERENCES

Abramowitz, M., and Stegun, I. A. (eds.) (1964), *National Bureau of Standards Applied Mathematics Series, 55: Handbook of Mathematical Functions*, Washington, DC: U.S. Government Printing Office.

Barnett, V. (1975), "Probability Plotting Methods and Order Statistics," *Applied Statistics*, 24, 95–108.

Benard, A., and Bos-Levenbach, E. C. (1953), "The Plotting of Observations on Probability Paper," *Statistica*, 7, 163–173.

Blom, G. (1958), *Statistical Estimates and Transformed Beta Variables*, New York: John Wiley.

Brent, R. P. (1974), "A Gaussian Pseudo-Random Number Generator (G5)," *Communications of the ACM*, 17, 704–706.

Chilko, D. M. (1978), "Probability Plotting," SAS Technical Report No. A-106, SAS Institute Inc., Cary, NC.

Cunnane, C. (1978), "Unbiased Plotting Positions—A Review," *Journal of Hydrology*, 37, 205–222.

Dixon, W. J. (ed.) (1981), *BMDP Statistical Software 1981*, Berkeley: University of California Press.

Filliben, J. J. (1975), "The Probability Plot Correlation Coefficient Test for Normality," *Technometrics*, 17, 111–117.

Hazen, A. (1914), "Storage to Be Provided in the Impounding Reservoirs for Municipal Water Supply," *Transactions of the American Society of Civil Engineers*, 77, 1547–1550.

Johnson, R. A., and Wichern, D. W. (1982), *Applied Multivariate Statistical Analysis*, Englewood Cliffs, NJ: Prentice-Hall.

Kimball, B. F. (1960), "On the Choice of Plotting Positions on Probability Paper," *Journal of the American Statistical Association*, 55, 546–560.

King, J. R. (1981), *Probability Charts for Decision Making* (rev. ed.), Tamworth, NH: Technical and Engineering Aids for Management.

LaBrecque, J. (1977), "Goodness-of-Fit Tests Based on Nonlinearity in Probability Plots," *Technometrics*, 19, 293–306.

Lilliefors, H. W. (1967), "On the Kolmogorov–Smirnov Test for Normality With Mean and Variance Unknown," *Journal of the American Statistical Association*, 62, 399–404.

Mage, D. T. (1982), "An Objective Graphical Method for Testing Normal Distributional Assumptions Using Probability Plots," *The American Statistician*, 36, 116–120.

Michael, J. R. (1983), "The Stabilized Probability Plot," *Biometrika*, 70, 11–17.

Nelson, W. (1979), *How to Analyze Data with Simple Plots*, Milwaukee: American Society for Quality Control.

——— (1982), *Applied Life Data Analysis*, New York: John Wiley.

Pearson, E. S., D'Agostino, R. B., and Bowman, K. O. (1977), "Tests for Departures From Normality: Comparison of Powers," *Biometrika*, 64, 231–246.

Royston, J. P. (1982a), "Algorithm AS 177: Expected Normal Order Statistics (Exact and Approximate)," *Applied Statistics*, 31, 161–165.

——— (1982b), "Algorithm AS 181: The *W* Test for Normality," *Applied Statistics*, 31, 176–180.

SAS Institute Inc. (1982), *SAS User's Guide: Basics, 1982 Edition*, Cary, NC: Author.

Shapiro, S. S., and Brain, C. W. (1981), "A Review of Distributional Testing Procedures and Development of a Censored Sample Distributional Test," in *Statistical Distributions in Scientific Work*, eds. C. Taillie, G. P. Patil, and B. A. Baldessari, Dordrecht, Holland: D. Reidel, pp. 1–24.

Shapiro, S. S., and Francia, R. S. (1972), "An Approximate Analysis of Variance Test for Normality," *Journal of the American Statistical Association*, 67, 215–216.

Shapiro, S. S., and Wilk, M. B. (1965), "An Analysis of Variance Test for Normality (Complete Samples)," *Biometrika*, 52, 591–611.

Snedecor, G. W., and Cochran, W. G. (1980), *Statistical Methods* (7th ed.), Ames: Iowa State University Press.

Stirling, W. D. (1982), "Enhancements to Aid Interpretation of Probability Plots," *The Statistician*, 31, 211–220.

Tukey, J. W. (1962), "The Future of Data Analysis," *Annals of Mathematical Statistics*, 33, 1–67.

Weibull, W. (1939), "The Phenomenon of Rupture in Solids," *Ingeniors Vetenskaps Akademien Handlingar* (153), 17.

Wichmann, B. A., and Hill, I. D. (1982), "Algorithm AS 183: An Efficient and Portable Pseudo-Random Number Generator," *Applied Statistics*, 31, 188–190.