Oscillator Assignment

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Introdutron

In this assignment, a code is provided for the parameterised oscillator scheme, as given in Eq. (58) in the notes. The scheme depends on an angular frequency $w0$, as well as a free paramater alpha. You should derive a scheme of the form of: $u^{n+1} = a \times u^n - u^{n-1}$, for some constant a, which you will determine below.

There are three questions Q1 through Q3 below. Please edit this Matlab code and return it with the questions answered!

Eq.58 : $\delta_{tt}u^n + \omega_0^2(\alpha + (1-\alpha)\mu_{to})u^n$

Eq.22 : $\delta_{tt} = \delta_{t^+}\delta_{t^-} = \frac{1}{k^2}(e_{t^+} - 2 + e_{t^-})$

Eq.23 : $\mu_{to} = \mu_{t^+}\mu_{t^-} = \frac{1}{4}(e_{t^+} + 2 + e_{t^-})$

Initial Values

In [2]:
```
Fs = 44100;         % Sampling rate
f0 = 1842;          % Frequency of the oscillator
Tf = 1;             % Duration of the generated signal
u0 = 1;             % Initial value of u (movement)
v0 = 0;             % Initial value of v (speed)
alpha = 0.8;        % Parameter of the oscillator
```

Derived quantities

In [6]:
```
k = 1/Fs;           % Temporal Step
w0 = 2*pi*f0;       % angular frequency
Nf = floor(Tf*Fs);% number of dot to generate (integer)
```

Initialize

In [7]:
```
u2 = u0;
u1 = u0+k*v0;
H = zeros(Nf,1);
out = zeros(Nf,1);
```

Question N°1 : Value of a

Insert proper value of the parameter a to be used in the update in the main loop (see comments at top of assignment).

to access to a value we have to write equation 58 with 22 and 23 expressions :

Eq.58 : $\delta_{tt}u^n + \omega_0^2(\alpha + (1-\alpha)\mu_{to})u^n = 0$

$$\delta_{tt}u^n + \omega_0^2\alpha u^n + (1-\alpha)w_0^2\mu_{to}u^n = 0$$

$$\frac{1}{k^2}(u^{n+1} - 2u^n + u^{n-1}) + \omega_0^2\alpha u^n + \frac{(1-\alpha)w_0^2}{4}(u^{n+1} + 2u^n + u^{n-1}) = 0$$

$$\left[\frac{1}{k^2} + \frac{(1-\alpha)\omega_0^2}{4}\right]u^{n+1} + \left[-\frac{2}{k^2} + \omega_0^2\alpha + \frac{(1-\alpha)\omega_0 2}{4}\right]u^n + \left[\frac{1}{k^2} + \frac{(1-\alpha)\omega_0^2}{4}\right]u^{n-1} = 0$$

$$Au^{n+1} + Bu^n + Cu^{n-1} = 0$$

$$A = C = \frac{1}{k^2} + \frac{(1-\alpha)\omega_0^2}{4} = \frac{(1-\alpha)\omega_0^2k^2+4}{4k^2}$$

$$B = \left[-\frac{2}{k^2} + \omega_0^2\alpha + \frac{(1-\alpha)\omega_0 2}{4}\right] = \frac{(1+\alpha)w_0^2k^2-4}{2k^2}$$

We put the equation into the $u^{n+1} = au^n - u^{n-1}$ format

$$Au^{n+1} + Bu^n + Cu^{n-1} = 0$$

$$u^{n+1} = -\frac{B}{A}u^n - \frac{C}{A}u^{n-1} = 0$$

$A = C$ so :

$$u^{n+1} = -\frac{B}{A}u^n - u^{n-1} = 0$$

In [21]:
```
a = (2/(k^2) - (w0^2)*(alpha + 1)/2)/(1/(k^2) + (w0^2)*(1-alpha)/4)
```

```
a =  1.9314
```

Question N°2 : Stability condition

Check stability condition, which gives bound on k, given alpha and w0 (see the notes). Here, perform a check to make sure that the stability condition is

- satisfied, for any values of the parameters. If it is not, the code
- should exit with an error message.

The stability condition is the following:

$$\frac{\alpha w_0^2 k^2}{4} \leq 1$$

The parametric equation (58) is allways stable if $\alpha \leq 0$, but if $\alpha > 0$ then :

$$k \leq \frac{2}{\omega_0 \sqrt{\alpha}}$$

In [22]:
```
if k <= 2/w0*sqrt(alpha)
    fprintf('INFO : Stability condition : satisfied!\n');
else
    error('WARNING! : Stability condition : NOT satisfied!\n');
    return;
end
```

```
INFO : Stability condition : satisfied!
```

Question N°3 : Conserved energy expression

Derive an expression for conserved energy and insert here below: Using the identities from the notes at Expr. (38) and Expr. (53) and using the parametrized scheme obtained in Q1, we get:

$$\delta_{tt}u^n + \omega_0^2\alpha u^n + (1-\alpha)w_0^2\mu_{to}u^n = 0$$

$$\delta_{t0}u^n\delta_{tt}u^n + \omega_0^2\alpha\delta_{t0}u^n u^n + (1-\alpha)w_0^2\delta_{t0}u^n\mu_{t0}u^n = 0$$

But :

$$\delta_{t0}u^n\delta_{tt}u^n = \delta_{t+}\left[\frac{1}{2}(\delta_{t-}u^n)^2\right]$$

$$\delta_{t0}u^n\mu_{t0}u^n = \delta_{t+}\left[\frac{1}{2}(\mu_{t-}u^n)^2\right]$$

$$\delta_{t0}u^n = \delta_{t+}\mu_{t-}u^n$$

So :

$$\delta_{t+}\left[\frac{1}{2}(\delta_{t-}u^n)^2\right] + \omega_0^2\alpha\delta_{t+}\mu_{t-}u^n u^n + (1-\alpha)w_0^2\delta_{t+}\left[\frac{1}{2}(\mu_{t-}u^n)^2\right] = 0$$

$$\delta_{t+}\left[\frac{1}{2}(\delta_{t-}u^n)^2 + \omega_0^2\alpha\mu_{t-}u^n u^n + (1-\alpha)w_0^2\left[\frac{1}{2}(\mu_{t-}u^n)^2\right]\right] = 0$$

$$\delta_{t+}[H] = 0$$

$$H(n) = \frac{1}{2k^2}(u^n - u^{n-1})^2 + \frac{\omega_0^2}{2}(\alpha u^n u^{n-1} + \frac{(1-\alpha)}{2}(u^n + u^{n-1})^2)$$

In [25]:
```
tic

for n=1:Nf
    u = a*u1-u2;
    out(n) = u2;

    % Q3: derive an expression for conserved energy and insert here below:
    % Using the identities from the notes at Expr. (38) and Expr. (53) and
    % using the parametrized scheme obtained in Q1, we get:

    H(n) = 0.5*(u1-u2)^2/k^2 + 0.5*w0^2*(alpha*u1*u2 + (1 - alpha)*(0.5*(u1+u2)^2));

    u2 = u1;
    u1 = u;
end

toc

% plot

tax = [0:Nf-1]'*k;

Herr = (H-H(1))/H(1);
%plot(tax,Herr,'k.');
plot(tax,Herr,'k.');
xlabel('t');
ylabel('H');
title('Energy variation');

% play sound

soundsc(out,Fs)
```

```
Elapsed time is 0.793613 seconds.
```