REVERBERATION TIME AND MEAN ABSORPTION IN CONCERT HALLS

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1 INTRODUCTION

Are Concert Halls random number generators? This question may sound provocative, but many properties of the impulse responses of a concert hall are statistical in nature. This was indeed the major contribution of Manfred Schroeder[1] to room acoustics. Following his pioneer work, it was indeed possible to derive a stochastic model for impulse responses that allows accurate predictions of some correlation properties of impulse responses[2]. Later, the model was developed into a simulation algorithm[3] that shed further light on some statistical properties of concert hall measurements. Its main result was a confirmation of Barron's revised theory[4].

The present papers attempt to go one step further. On the basis of the measurements carried out by John Bradley, Anders Christian Gade and Gary W. Siebein in 1992, the simulation was applied to 9 US Concert Halls and compared with the measurement data from Anders Christian Gade[5]. These results have never been presented before.

The paper first explains how the simulation algorithm is constructed, then compares the results form the simulation to actual measurement data. An unexpected result is the relative constancy of the mean absorption coefficient for all the halls analysed; its generality is therefore checked on hand other cases.

2 THE STOCHASTIC MODEL

The stochastic model for impulse responses[2] describes the response of a room as the random superposition of modes decaying exponentially, as was assumed by Manfred Schroeder[1]. He proved that the frequency response of a room is purely statistical with a Gaussian probability distribution for its amplitude, and that its correlation properties can be predicted from only one parameter: the reverberation time.

Schroeder's model has, however, a counterpart in the time domain, which is obtained by simply using the Fourier transform. Since the Fourier transformation of a randomly distributed function with Gaussian distribution is another randomly distributed function with Gaussian distribution. Thus, the impulse response can be viewed as the superposition of random pulses whose distribution must be compatible with Schroeder's theory. Simple analysis shows that the distribution of the arrival times of the pulses must follow a Poisson distribution with a mean increasing with time; and the amplitude of the pulses must decrease exponentially with time. This is the basic idea behind the simulation algorithm used in this paper[3].

Each of the pulses reaching the receiver can be considered as originating from an image of the source. Its time of arrival is given by the length of the ray that links the image source to the receiver. Thus, the distribution of the arrivals can be deduced from the distribution of the image sources. Besides, each ray emitted by an image source hits several times the walls of the room where it is reflected. The number of hits varies from ray to ray, but the mean number of hits is simply given by the length of the ray divided by the mean free path. Together with the reflection coefficient on the walls, the distribution of hits each ray is the second input to the algorithm, and makes it possible to fit the decay of the impulse response to the reverberation time.

More precisely, the algorithm is constituted of two uncorrelated random number generators. The first random generator simulates the arrival times of the reflections as a Poisson process. The only parameter of this random generator is the mean number of reflections arriving at any time. Assuming rectangular rooms, this mean number is easily calculated: it is proportional to the square of the time, and inversely proportional to the volume of the room, as is well known from more conventional computer simulations of rooms. The second random generator computes the number of hits of each ray on the walls. This is simply done by assuming a Poisson process again, with a parameter equal to the mean number of reflections, that is, proportional to the time elapsed since the ray left the source divided by the means free path.

In order to compute the mean free path, the geometry of the hall is needed. Since we are interested in general trends only, this geometry is simplified by assuming rectangular halls of the same volume. The three dimensions are then taken from drawings of the halls, so that they roughly correspond to the overall shape of the hall while keeping the volume to its actual size. For theatres with a circular plan, this procedure certainly over evaluates the surface area of the walls, and this should be remembered when looking at the results later on.

The geometry of the hall is also needed for computing the reflection coefficients on the walls. We use the same equivalent rectangular hall, further assuming a uniform reflection coefficient. Thus, the mean absorption coefficient is computed from the measured reverberation time with the help of Sabine formula, using the surface area of the equivalent rectangular hall.

In summary, the parameters needed for the simulation are: the volume of the hall, the reverberation time at mid-frequency, and the dimensions of the equivalent rectangular hall. These parameters are either measured or evaluated from the drawings of the hall. Then the mean free path and the mean absorption coefficient are calculated. No further parameters are required to run the simulation.

3 US CONCERT HALLS

The data from the 9 concert halls of this study were kindly provided by Anders Christian Gade. Table 1 gives the list of these halls, together with their general shape characteristics. More details on most of these halls can be found in Beranek[6]. In fact, the dimensions of the equivalent rectangular halls were taken from the drawings available in Beranek. They are given in Table 2 together with the other data used for the simulation.

City	**Hall name**	**Abbreviation**	**Shape features**
Akron	E.J. Thomas Hall, concert mode	AKR	wide fan, two balcony levels
Baltimore	Joseph Meyerhoff Symphony Hall	BAL	oval, two balcony levels
Boston	Symphony Hall	BOS	rectangular, two balcony levels
Buffalo	Kleinhans Music Hall	BUF	wide fan, one balcony level
Cleveland	Severance Hall	CLE	horse shoe, box + large rear balcony
Detroit	Orchestra Hall	DET	horse shoe, box + large rear balcony
Philadelphia	Academy of Music	PHI	horse shoe, three balcony levels
Troy	Saving Bank Music Hall	TRO	rectangular, two balcony levels
Washington	Kennedy Center Concert Hall	WAS	rectangular, two balcony levels

Table 1: List of the 9 US halls; names, general shape characteristics and abbreviation used for identification in the graphs (from Gade[5])

Two points are worth mentioning concerning Table 2. Firstly, in most cases, the mean free path is of the same order of magnitude as the shortest dimension of the hall, in all cases its height. This first result is general for rectangular rooms where one dimension is notably smaller than the others, and can be used as a check of the validity of the data. For example, in the case of Philadelphia, it confirms that the dimensions do not correspond to the volume of the hall: they over estimate it.

The second point worth mentioning concerning Table 2 is the relative constancy of the mean absorption coefficient. With the exception of Philadelphia, Washington, and to a lesser extent Cleveland, the absorption coefficient remains equal to 0.3 ± 10%. We shall come back on this point in Section 5 below.

Hall	Volume (m^3)	RT (s)	length (m)	width (m)	height (m)	surface (m^2)	mean free path (m)	mean absorption
AKR	18406	1.79	46.8	31.3	15.6	5366.4	13.7	0.31
BAL	21420	2.27	50.0	28.1	16.6	5402.9	15.9	0.28
BOS	18740	2.46	43.8	22.0	17.2	4190.7	17.9	0.29
BUF	18220	1.81	40.6	36.0	14.0	5068.0	14.4	0.32
CLE	15700	1.75	37.5	25.0	17.0	4000.0	15.7	0.36
DET	16360	1.94	40.0	25.0	15.6	4028.0	16.2	0.33
PHI	15700	1.29	37.5	25.0	20.3	4412.5	14.2	0.44
TRO	11320	2.58	28.1	18.7	17.2	2660.9	17.0	0.26
WAS	21620	1.85	46.9	26.6	16.6	4935.3	17.5	0.38

Table 2: Parameters used for the simulation

As a first check of the quality of the simulation, Figure 1 presents the mean reverberation times measured in the halls and calculated from the simulated impulse responses. There is a very good agreement between the measured data (left bars – in blue) and the simulated data (right bars – in red). This was, indeed, to be expected since the simulations are based on the measured reverberation times. However, it confirms that the simulation introduces no distortion in the reverberation.

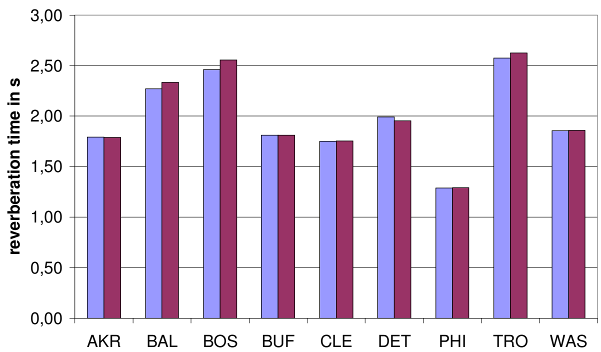

Figure 1: Mean reverberation times of the nine halls; for each hall, measured value is on the right, and simulated value on the left.

A further check for the quality of the simulation can be obtained from the comparison of other indices. Since the study was inspired by Gade[5], we chose to compare his empirical prediction models for Clarity (C80) and Strength (G). Figure 2 presents the data for Clarity. Gade's empirical model for Clarity gives: $C = -0.4 + C_{exp}$ with $C_{exp} = 10*log(exp(1.104/RT)-1)$ dB. And Figure 3 presents the data for Strength, for which Gade's empirical model gives: $G = -1.6 + 0.9*G_{exp}$ with $G_{exp} = 10*log(RT/V) + 45$ dB. In the two Figures, measured (empty circles) and simulated (filled squares) data are very close to each other, but for a few exceptions, confirming that the simulation algorithm retains the basic geometrical properties of concert halls. There is, however, a trend for simulated data for Clarity and Strength to lie above the measured values. Notice also that simulated data better follow Gade's model, thus confirming its statistical validity.

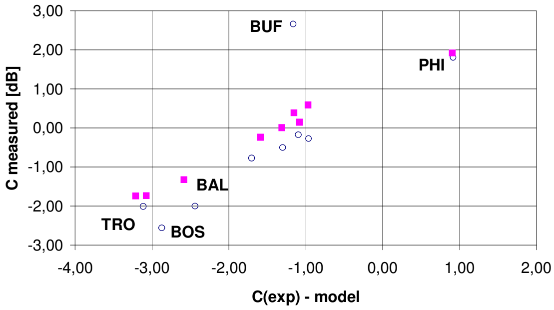

Figure 2: Clarity vs. Gade' s model; empty blue circles represent the measured data and filled pink squares the simulated data.

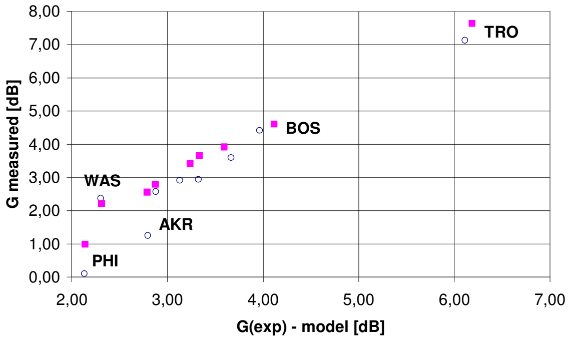

Figure 3: Strength vs. Gade' s model; empty blue circles represent the measured data and filled pink squares the simulated data.

A closer look at Figure 2 also reveals that Gade's model underestimates Clarity in the order of 1 dB. This is both the case for measured and simulated data. No such systematic deviation is observed in Figure 3, but low values of the Strength are overestimated while high values are underestimated.

Finally, two halls in Figure 2 and 3 do not behave according to Gade's model: Buffalo for Clarity, and Akron for Strength. Gade[5] has given some explanation for this abnormal behaviour, which is not reproduced by the simulation. This proves that impulse responses in these halls systematically differ from stochastic processes.

4 WITHIN HALL DIFFERENCES

Figure 4 presents plots of Clarity versus source-receiver distance for the 9 halls on its left column, and plots of Strength versus source-receiver distance on its right column. For most halls, this distance varies between 5m and 45m, except for Troy which is much smaller than the other halls, as can be seen on Table 2. Philadelphia is also a smaller hall, but the minimum distance is larger than in all other halls except Akron. When looking at general trends, Clarity tends to stabilize at a constant value roughly equal to its mean value at distances larger than 15m, whereas Strength regularly decreases with distance from the source. The latter is in agreement with earlier simulations[2] and with Barron' s revised theory[4], but not the former finding: earlier simulations show regular decrease of Clarity with distance, in agreement with Barron' s theory. It should be however noted that the earlier simulations were carried out on a much smaller room. In fact, the largest source-receiver distance in the earlier simulations did not exceed 13m, that is, did not reach the distance above which Clarity remains constant in Figure 4.

When comparing measurements and simulation, there is in general better agreement between the measured and simulated values for Strength than for Clarity, confirming that Strength is a statistically well-defined index whereas Clarity is not[7]. For Clarity, exceptions are Buffalo, Cleveland, Washington and to a lesser extend Baltimore and Troy. In all these halls, measured Clarity tends to increase with increasing source-receiver distances instead of remaining constant for large distances; measurements do not give higher values of Clarity for short source-receiver distances either - except for Buffalo. These are indeed significant differences with the statistical case obtained by simulation, but these differences are not correlated with the shape of the halls (Table 1).

For Strength, halls were simulation data do not agree with measurements are Akron, Cleveland, Detroit, and to a lesser extend Philadelphia and Troy. In the last two halls, which are the smallest ones, measured Strength values are lower than the simulated values at medium distances (15-25m for Troy, 20-30m for Philadelphia); in both cases, but most notably in Troy, measured Strength values increase at the longest source-receiver distances, a situation reminiscent of corridors[8]. Indeed, as can be seen in Table 2, these two halls have similar width and height, much smaller than the length, as is found in corridors; besides, they are the only halls presenting this particularity. We refer to Picaud and al.[8] for an explanation of this behaviour.

As for Akron, Cleveland and Detroit, they present different characteristics. In Akron, the measured Strength values are systematically lower than the simulated, revealing a deficit of reflections that is to be expected in a wide fan-shaped hall; however, Buffalo, which is even wider, does not present the same systematic difference. In Cleveland, measured Strength values are low at short distances, probably due to a lack of reflecting surfaces for the floor area. And in Detroit, a group of measured Strength values (red circle on Figure 4) are much lower than the other measured values and the simulated values: they correspond to two specific areas of the hall.

Last but not least, we would like to single out the case of Boston Symphony Hall, because of its outstanding acoustic reputation. For this hall, measurements and simulation agree particularly well for Strength, both locally when the source-receiver distance varies as in Figure 4, and globally as in Figure 3. As for Clarity, the apparent agreement between measured and simulated value in Figure 4 is not confirmed at the global level (Figure 2). Thus, Boston Symphony Hall indicates that the simulation can indeed be used as a reference in order to assess to which extent a hall conforms to its expected behaviour, but only for specific indices such as Strength.

5 PIANO PRACTICE ROOMS AND OTHER ROOMS

We would now like to come back on the earlier finding concerning the relative constancy of the mean absorption coefficient across all halls. From Table 2, it can be seen that the mean absorption coefficient of most US halls is about 0.30. Can this mean absorption coefficient be generalized to other size of rooms? Two publications confirm it.

The first one concerns the acoustical characteristics of piano practice rooms[9]. Fujita and Yamaguchi carried out extensive experiments with professional musicians in three rooms of different floor areas (27.8, 11.5, and 7.4 m^2) in order to find out what is the adequate "liveness" of a practice room. They found out that preference is well correlated with the mean absorption coefficient of the rooms: the preferred practice conditions corresponds to a mean absorption coefficient of 0.3 ± 20%.

The other publication is the IEC recommendation for listening tests for loudspeakers[10]. It recommends a listening room of 80m^3 with dimensions 6.7x4.2x2.8m and a mean reverberation time of 0.4 ± 0.05 s. Translated in terms of mean absorption coefficient, this corresponds to a value of 0.27.

Therefore, we conclude that the preferred value for absorption is a mean coefficient of 0.3, irrespective of the size of the room.

6 CONCLUSION

Are Concert Halls random number generators? At the end of the present study, we can conclude that measurements in concert halls present systematic deviations from simulation obtained by generating reflections randomly. Therefore, concert halls cannot be considered as pure random number generators. However, the random number generator can be used to simulate a reference case, a procedure usually known as Monte Carlo method, in order to assess the actual measurements of a given hall, as illustrated here in the case of Boston Symphony Hall.

A side result of the present study is the finding that a mean absorption coefficient of 0.30 is a target value for all rooms, irrespective of their size. This probably is the major result of the present study and can be used as a rule for room design.

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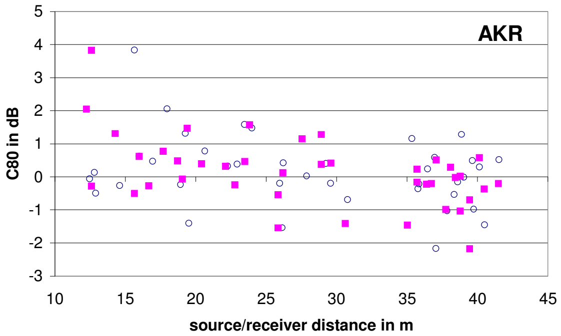

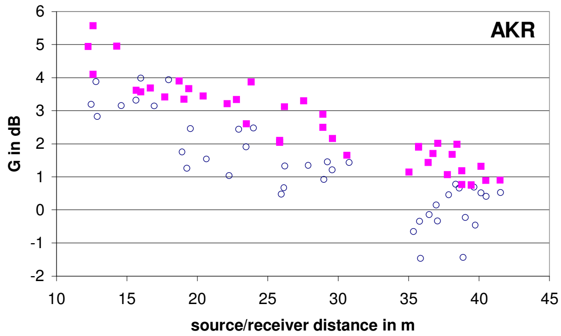

Figure 4: Clarity (left) and Strength (right) as function of source-receiver distance; empty blue circles are measured data, filled pink square simulated data; each row correspond to a single hall, as indicated by label.

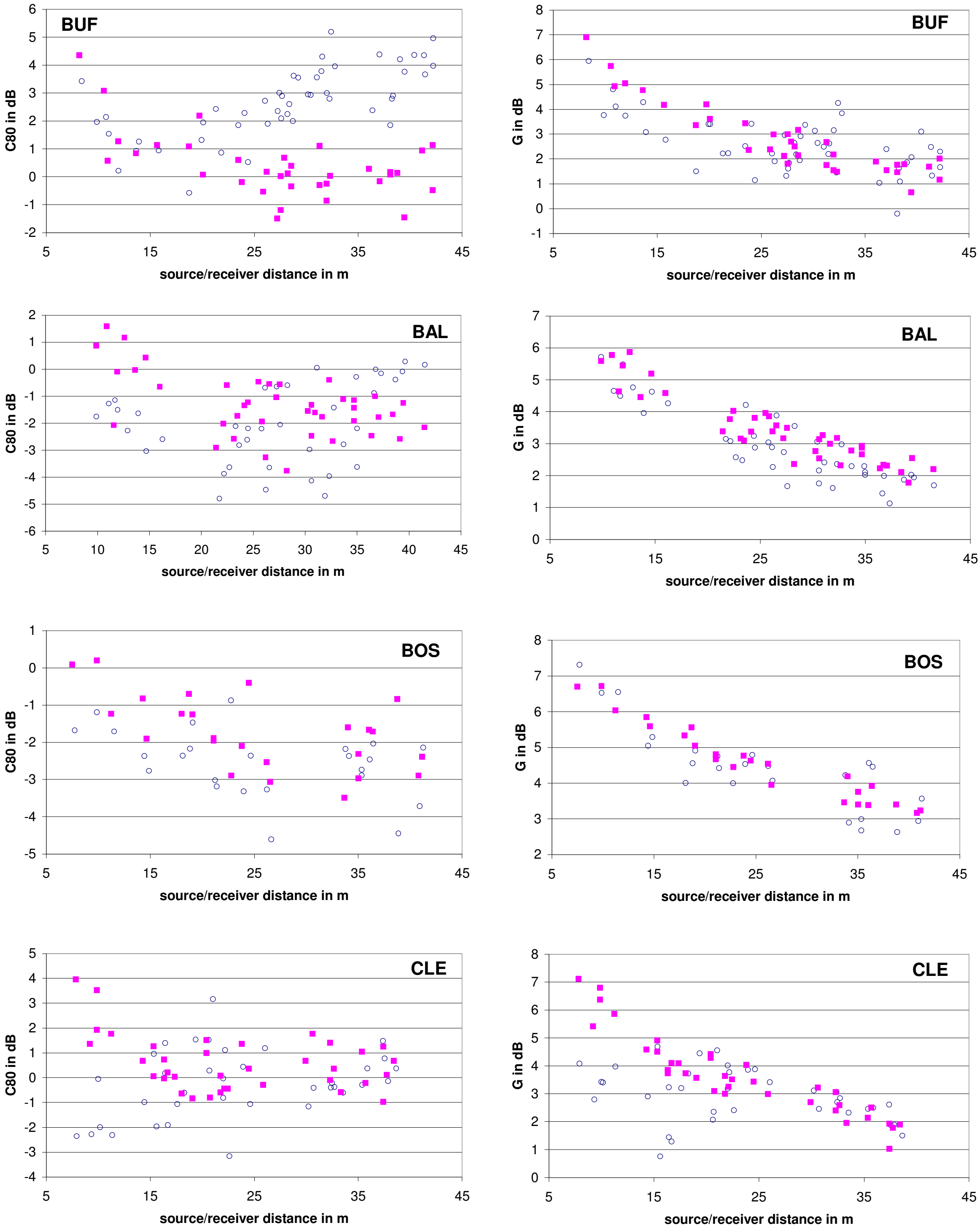

Figure 4 (continued)**:** Clarity (left) and Strength (right) as function of source-receiver distance; empty blue circles are measured data, filled pink square simulated data; each row correspond to a single hall, as indicated by label.

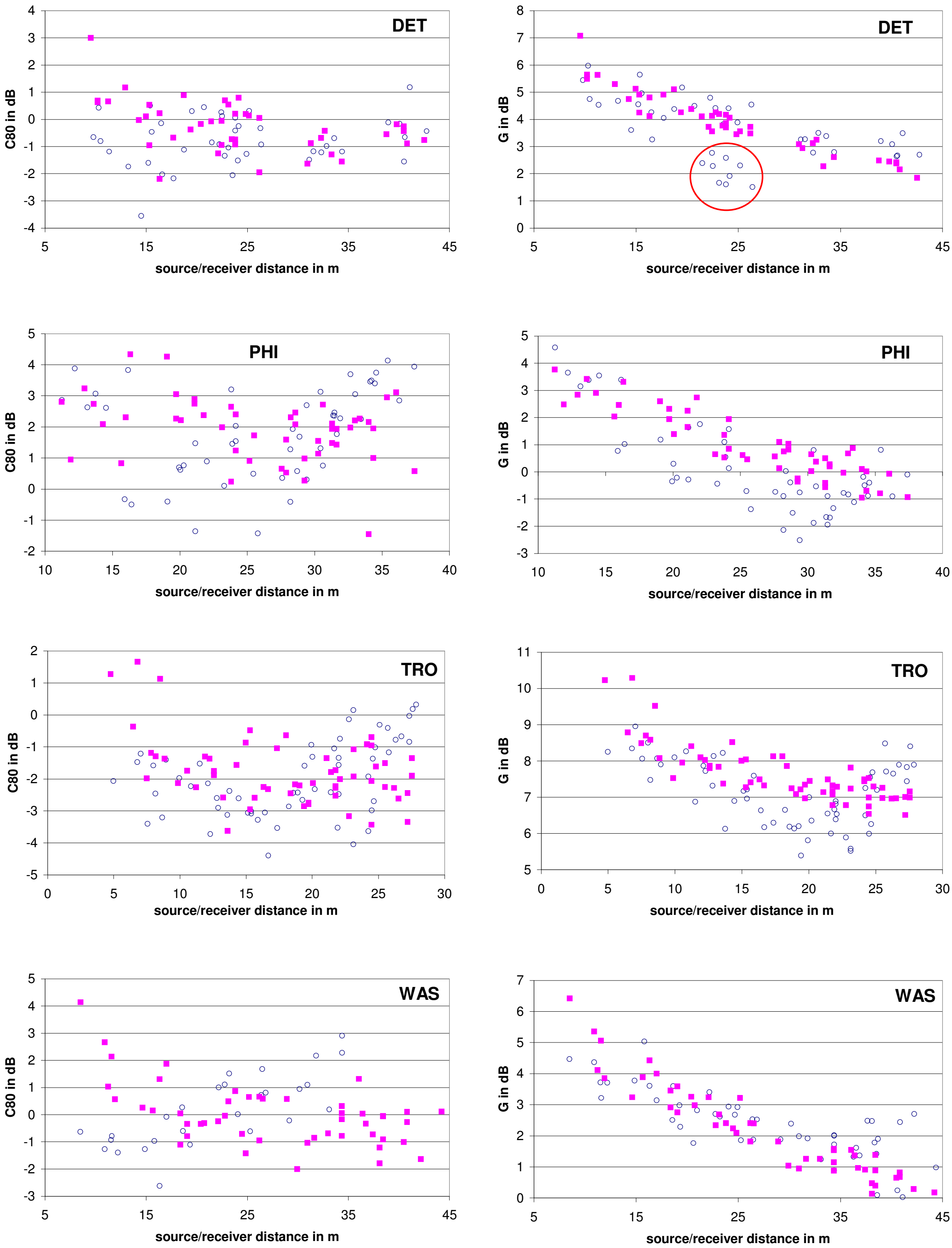

Figure 4 (finished): Clarity (left) and Strength (right) as function of source-receiver distance; empty blue circles are measured data, filled pink square simulated data; each row correspond to a single hall, as indicated by label.