Fuzzy fractional differential equation involving the fuzzy conformable derivative and the α-semigroups

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Abstract

In this study, the initial problem of conformable fuzzy fractional orders is explored using generalized Hukuhara differentiability [1] . Fuzzy conformable derivatives based on this differentiability are introduced, revealing many relevant properties [3]. The fuzzy fractional semigroup method is then used to determine the analytical solution of the conformable fuzzy fractional differential equation [2]. Krasnoselskii's fixed point theorem is employed to ensure the existence and uniqueness of the solution[4]. Finally, an application of abstract Cauchy problems is resolved to demonstrate the effectiveness of these methods. The results indicate that the fuzzy fractional-order semigroup method is a practical tool for solving these equations, offering prospects for future research in this area.

Main results

The system studied in this article is an abstract Cauchy problem formulated using a fuzzy fractional differential equation involving the fuzzy conformable derivative and α-semigroups. The equation under consideration can be represented as follows :

$$\begin{cases} \Psi^{(\alpha)}(\tau) = A\Psi(\tau) \oplus \Phi(\tau, \Psi(\tau)), \quad \tau > 0, \\ \Psi(0) = \Psi_0, \end{cases} \tag{1}$$

where $\Psi^{(\alpha)}(\tau)$ denotes the fuzzy conformable derivative of order α of Ψ, A is a linear operator defined on a subset $D(A)$ of a fuzzy space X, $\Phi(\tau, \Psi(\tau))$ is a continuous and α-conformably differentiable function, and $\oplus$ represents addition in the fuzzy number space.

The studied system is motivated by the generalization of the classical Cauchy problem to fuzzy spaces using fractional derivatives. This approach is particularly suitable for modeling phenomena where uncertainty and variability are present, such as nonlinear oscillations, fluid flow in porous media, and other complex dynamics in physics and engineering.

By leveraging the properties of fuzzy α-semigroups, the paper investigates the conditions for the existence and uniqueness of the solution using Krasnoselski's fixed-point theorem. This method ensures that the obtained solution satisfies the constraints imposed by the initial system and the boundary conditions.

Références

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