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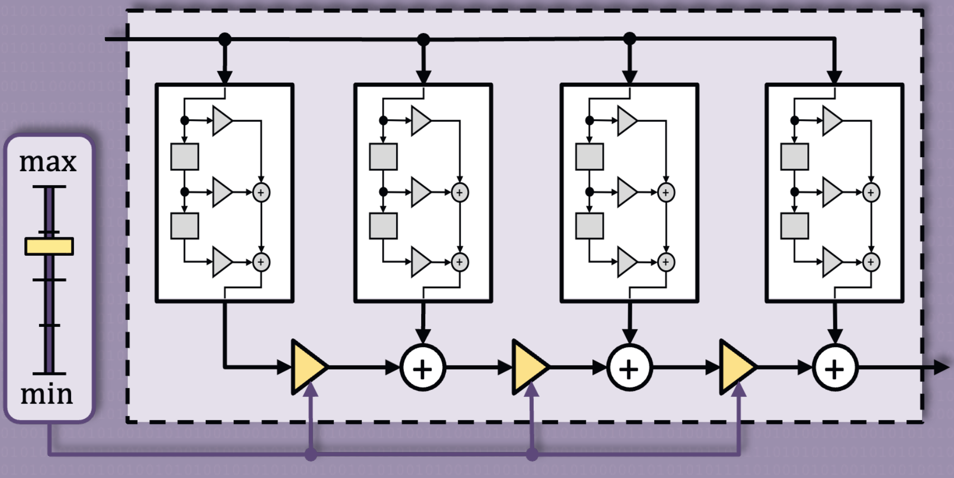
Contributions to Efficient Design and Implementation of Variable Digital Filters

Oksana Moryakova



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Abstract

Complexity reduction is one of the main issues of digital signal processing (DSP) algorithms, especially in communication systems where each new generation brings new requirements towards increasing data rates and improved accuracy positioning, leading to the growth of power consumption and chip area. To meet these requirements and at the same time find a trade-off between high performance and low implementation cost, more sophisticated DSP algorithms need to be developed. Recent communication standards require flexible, adaptive systems capable of real-time frequency-domain tuning. Variable digital filters (VDFs) address these needs by enabling "on-the-fly" frequency response adjustments without the need for online filter design. The key feature of VDFs is that they require only an adjustment of one or a few parameters to change their characteristics, without the need for extensive additional computations. Most VDF coefficients remain fixed after the initial design, allowing for efficient hardware implementation. This makes VDFs essential for modern adaptive communication technologies.

This thesis primarily focuses on the design and low-complexity implementation techniques of VDFs and presents three main contributions. Firstly, it proposes three VDF realizations for simultaneous lowpass filtering and equalization using polynomial channel models, with systematic design procedures based on minimax optimization for all the proposed structures. In addition, a fast design method for the VDFs with several variable parameters, which can substantially decrease the design time, is presented. Secondly, it introduces frequency-domain implementations of VDFs using the overlap-save technique. Based on the assumption that these filters have been designed using a common design approach based on optimizing the impulse response coefficients, the filter DFT coefficients are proposed to be implemented as fixed, hybrid, or variable weights. Lastly, the thesis presents an efficient design approach for a variable-bandwidth digital filter implemented in the frequency domain using the overlap-save method. The proposed approach is based on a hybrid of frequency sampling and optimization, allowing for direct

optimization of the DFT coefficients considering the filter frequency-domain implementation and thereby noticeably reducing the cost of implementation and an online update of the DFT filter coefficients when the bandwidth is varied.

Populärvetenskaplig Sammanfattning

Reduktion av komplexitet är en av huvudfrågorna för digital signalbehandling (DSP) algoritmer, särskilt i kommunikationssystem där varje ny generation ställer nya krav på att öka datahastigheter och förbättrad noggrannhet positionering, vilket leder till en ökning av strömförbrukningen och kretsytan. För att möta dessa krav och samtidigt hitta en avvägning mellan hög prestanda och låg implementeringskostnad behöver mer sofistikerade DSP-algoritmer utvecklas. Senaste kommunikationsstandarder kräver flexibla, adaptiva system som kan frekvensdomäninställning i realtid. Variabla digitala filter (VDF) tillgodoser dessa behov genom att möjliggöra "on-the-fly" frekvenssvarsjusteringar utan behov av onlinefilterdesign. Nyckelegenskapen hos VDF:er är att de bara kräver en justering av en eller ett fåtal parametrar för att ändra deras egenskaper, utan behov av omfattande ytterligare beräkningar. De flesta VDF-koefficienter förblir fixerade efter den ursprungliga designen, vilket möjliggör effektiv hårdvaruimplementering. Detta gör VDF:er väsentliga för modern adaptiv kommunikationsteknik.

Den här avhandlingen fokuserar främst på design och implementeringstekniker med låg komplexitet för VDF:er och presenterar tre huvudsakliga bidrag. För det första föreslår den tre VDF-realiseringar för samtidig lågpassfiltrering och utjämning med användning av polynomkanalmodeller, med systematiska designprocedurer baserade på minimax optimering för alla föreslagna strukturer. Dessutom presenteras en snabb designmetod för VDF:erna med flera variabla parametrar, som avsevärt kan minska designtiden. För det andra introducerar den frekvensdomänimplementationer av VDF:er med överlappningssparateknik. Baserat på antagandet att dessa filter har utformats med användning av en gemensam designmetod baserad på optimering av impulssvarskoefficienterna, föreslås filtrets DFT-koefficienter implementeras som fasta, hybrida eller variabla vikter. Slutligen presenterar avhandlingen en effektiv designansats för ett digitalt filter med variabel bandbredd implementerat i frekvensdomänen med användning av överlappningssparametoden. Det föreslagna tillvägagångssättet är baserat på en hybrid av frekvenssampling och optimering, vilket möjliggör direkt optimering av DFT-koefficienterna med tanke på implementeringen av filterfrekvensdomänen och därigenom märkbart minska kostnaden för implementering och en onlineuppdatering av DFT-filterkoefficienterna när bandbredden är varierande.

Contents

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Many thanks go to Assoc. Prof. Yinan Wang, a co-author of some of the papers included in this thesis, for all the ideas, comments, and suggestions provided during our collaborative work.

I am grateful to all my colleagues at the Division of Communication Systems for providing a wonderful working environment, engaging in technical discussions, and offering kind support. Additionally, I appreciate the opportunities I have been given to attend and present at various conferences, which have certainly served as an additional source of motivation and inspiration.

Finally, and most importantly, I would like to express my heartfelt gratitude to my husband, Aleksandr, and my parents for their unwavering belief in me. Their love, care, and support have sustained me through many stressful days.

Oksana Moryakova
Linköping, September 2024

List of Abbreviations

5G	the fifth generation
6G	the sixth generation
ADC	analog-to-digital converter
AFE	analog-frond-end
ASIC	application specific integrated circuit
DAC	digital-to-analog converter
DFT	discrete Fourier transform
DSP	digital signal processing
FB	filter bank
FC	fast convolution
FCFB	fast-convolution filter bank
FFB	fast filter bank
FFT	fast Fourier transform
FIR	finite impulse response
FPGA	field programmable gate array
HFSO	hybrid of frequency sampling and optimization

IDFT	inverse discrete Fourier transform
IFFT	inverse fast Fourier transform
IIR	infinite-impulse-response
IoT	Internet of Things
LPTV	linear periodically time-varying
LS	least-squares
OLA	overlap-add
OLS	overlap-save
PR	perfect reconstruction
PTVIR	periodically time-varying impulse response
RLPE	reconfigurable lowpass equalizer
VBW	variable bandwidth
VDFs	variable digital filters
VE	variable equalizer
VFD	variable fractional delay
WLS	weighted least squares

Chapter 1

Introduction and Motivation

In the era of digital transformation, our way of life is constantly changing, attracting more digital technologies every day. As a result, demands for constant access to the Internet and various digital devices are growing. The basis of these technologies is digital signal processing (DSP), a fundamental technology that has revolutionized the way signals are processed in various fields, including telecommunications and medicine. DSP involves the sophisticated manipulation of digital signals using mathematical algorithms and computational techniques to extract, analyze, and interpret information embedded within the signals. One of the most significant applications of DSP is in communication systems, where this technology enables not only voice transmission but also multimedia streaming, data exchange and beyond. From wireless to fiber-optic communications, DSP algorithms form the core of seamless, efficient and reliable signal transmission, reception, and processing. To achieve optimal performance in terms of signal quality and security connection, such functionalities as modulation, demodulation, channel equalization, error correction, and signal filtering constitute the base of DSP theory [1–4].

One of the main aspects of DSP algorithms is the implementation complexity, which is crucial, especially in the context of evolving mobile communication systems. With each new generation (e.g., the fifth generation (5G), the sixth generation (6G)), there exists a persistent drive towards achieving higher data rates, lower latency, and improved accuracy positioning that together causes the growth of power consumption and area on a chip. These evolving requirements necessitate the development of more sophisticated DSP algorithms to balance high performance with low implementation complexity of advanced communication system transceivers [5–8]. Along with analog-to-digital and digital-to-analog converters (ADCs and DACs), discrete Fourier transform (DFT) and its inverse (IDFT), synchronization, modulation and demodulation techniques, digital filters, which are the focus of this thesis, play a crucial role in DSP. This is still an active research area, where more advanced solutions for efficient filter design and implementation are proposed every year [9–15].

The evolution of communication standards introduces new requirements and challenges for signal processing including demand for flexible and adaptive systems [7, 8, 16–19]. At first sight, these requirements can be fulfilled by means of regular digital filters having all coefficients general, the values of which can be updated after online filter design based on new specification requirements from time to time. This type of filters can be implemented in general-purpose digital signal processors or field programmable gate arrays (FPGAs). However, the reconfiguration time is relatively long and the filters cannot be reconfigured dynamically, i.e. while processing. Moreover, this approach usually leads to high implementation complexity and thereby high power consumption, which is one of the main limitation, for example in mobile terminals and Internet of Things (IoT) devices [20, 21]. This problem has lead many researchers to variable digital filters (VDFs) and increased interest in their design and implementation techniques [22–72]. The main feature of VDFs is the ability to vary their frequency response "on-the-fly" by adjusting only one or a few parameters without online design. In these filters, most of coefficients are fixed once the filter has been designed, and thus, they can be efficiently implemented in hardware, for example in FPGAs or application specific integrated circuits (ASICs) using various techniques [9–11, 15], while preserving both flexibility and low implementation complexity. This is the main advantage over regular digital filters, which have all coefficients being general (i.e. variable) and require design for every new specification using, for example the least-squares (LS) method or minimax optimization [73]. This means that the overall complexity of regular digital filters consists of implementation of general multipliers and obtaining new values for them, whereas the overall complexity of VDFs contains mainly implementation of constant multipliers, which requires significantly less computational resources than the general ones.

The motivation of this thesis is to make a contribution to the cutting-edge research towards low-complexity DSP algorithms, specifically, design and implementation of VDFs.

1.1 Publications and Contributions

This licentiate thesis comprises research publications that have been produced as a result of research work at Communication Systems Division, Department of Electrical Engineering, Linköping University, Sweden. The papers have been peer-reviewed and published. In these papers, the primary research, numerical experiments, validation, and manuscript writing have been carried out by the thesis author. The co-authors of these papers have significantly contributed to the research by shaping the conceptualization and methodology, engaging in technical discussions, and providing valuable feedback during the manuscript revision process.

The main contributions of the thesis are the following.

- Three low-complexity time-domain implementations of a variable finite impulse response (FIR) filter for simultaneous lowpass filtering and equalization using polynomial channel models are introduced along with systematic design procedures based on minimax optimization of the filter impulse response coefficients. This is presented in Paper A for first-order channel equalization and in Paper B for high-order polynomial channel models with a more detailed analysis.

- A fast design method for VDFs with several variable parameters, that can substantially decrease the design time, is proposed in Paper B.

- Three low-complexity frequency-domain implementations of VDFs using the overlap-save technique are introduced in Paper C. Considering that the filters have been designed using a common design approach based on optimizing the impulse response coefficients, the filter DFT coefficients are proposed to be implemented as fixed, hybrid, or variable weights.

- An efficient design approach for a variable-bandwidth digital filter implemented in the frequency domain using the overlap-save method is presented in Paper D. The proposed approach is based on a hybrid of frequency sampling and optimization, allowing for direct optimization of the DFT coefficients considering the filter frequency-domain implementation and thereby noticeably reducing the cost of implementation and an online update of the DFT filter coefficients when the bandwidth is varied.

Paper A: Reconfigurable FIR Lowpass Equalizers

Authored by: Oksana Moryakova, Yinan Wang, and Håkan Johansson
Published in proceedings of 2022 IEEE Workshop on Signal Processing Systems (SiPS), Rennes, France, Nov. 2-4, 2022, pp. 1-6.

This paper introduces realizations of a reconfigurable FIR filter for simultaneous equalization and lowpass filtering. The proposed structures employ properties of both a variable bandwidth (VBW) filter and a variable equalizer (VE) with an adjustable coefficient using the Farrow structure, therefore they consist of weighted combinations of fixed subfilters. Design procedures using minimax optimization technique are provided. The paper includes a design example and complexity comparisons between the proposed structures of the reconfigurable lowpass equalizer (RLPE) and a common approach of using a regular FIR equalization filter requiring online redesign.

Paper B: Low-Complexity Reconfigurable FIR Lowpass Equalizers for Polynomial Channel Models

Authored by: Oksana Moryakova, Yinan Wang, and Håkan Johansson
Published in Journal Digital Signal Processing, vol. 150, pp. 1-13, Jul. 2024.

This paper introduces realizations of a reconfigurable FIR filter for simultaneous equalization and lowpass filtering, which is the idea from Paper A, extended to high-order polynomial channel models. The main advantage of the proposed solutions is computational complexity reduction compared to existing solutions for a given performance, which leads to reduced hardware complexity. The proposed structures employ properties of both a VBW filter and a VE with variable coefficients. The overall transfer function of the proposed RLPE is a weighted linear combination of fixed subfilters where the weights are directly determined by the bandwidth and one or several parameters of the channel needed to be equalized. The paper provides design procedures based on minimax optimization and introduces a fast design method for the filter with several variable parameters that can substantially decrease the design time. Filter order estimation expressions as well as complexity expressions are presented for all proposed realizations. Design examples include comparison of the RLPE structures and a common approach of using a regular FIR equalization filter requiring online redesign when the bandwidth or channel characteristics are changed. It is shown that the number of general multiplications can be reduced up to 91% using the proposed RLPE.

Paper C: Frequency-Domain Implementations of Variable Digital FIR Filters Using the Overlap-Save Technique

Authored by: Oksana Moryakova and Håkan Johansson
Published in proceedings of 2023 24th International Conference on Digital Signal Processing (DSP), Rhodes, Greece, Jun. 11-13, 2023, pp. 1-5.

The paper introduces frequency-domain implementations of variable digital filters using the overlap-save method. Expressions for implementation and design complexities are derived for real-valued impulse responses. Design examples include implementations of a VBW filter alone as well as a cascade of a VBW filter and a variable fractional delay (VFD) filter. Compared to a time-domain implementation and a filter bank approach, the proposed structures can reduce the implementation complexity significantly and achieve savings up to 95% in the multiplication rate and up to 89% in the addition rate.

Paper D: Efficient Design and Implementation of Fast-Convolution-Based Variable-Bandwidth Filters

Authored by: Oksana Moryakova and Håkan Johansson
Published in proceedings of 2024 32nd European Conference on Signal Processing (EUSIPCO), Lyon, France, Aug. 26-30, 2024, pp. 1-5.

This paper introduces a frequency-domain design approach of a fast-convolution-based VBW filter. The proposed approach is based on a hybrid of frequency sampling and optimization (HFSO), that offers significant computational complexity reduction compared to existing solutions for a given performance. The paper provides a design procedure based on minimax optimization to obtain the minimum complexity of the overall filter. A design example includes comparison of the proposed-design based VBW filter and time-domain designed VBW filters implemented in the time domain and in the frequency domain. It is shown that not only the implementation complexity can be reduced but also the design complexity by excluding any computations when the bandwidth of the filter is adjusted. Moreover, memory requirements are also decreased compared to the existing frequency-domain implementations.

1.2 Thesis Outline

This thesis consists of two main parts. Part I gives an introduction to the area of VDFs, starting from the basics of digital filters in Chapter 2, specifically

FIR filters, which are commonly used as the basic building blocks in VDFs. In Chapter 3, the basic elements of multirate systems and filter banks (FBs) are given, which can help the reader to acquire an overall understanding of FB-based VDFs, which are presented in Chapter 4 along with commonly used Farrow-based VDFs. Chapter 5 concludes Part I of the thesis. Finally, Part II comprises the published papers, the layout of which has been changed and a few typos have been corrected.

Chapter 2

Basics of Digital Filters

In this chapter, relevant basics of digital filters are provided. Generally, FIR filters are used as basic building blocks in VDFs. Therefore, their design and implementation are important to recall here. The main focus is on linear-phase FIR filters and digital differentiators since those two types are used in the author's contribution to this thesis.

2.1 FIR Filters

There are two major classes of digital filters, namely, FIR filters and infinite-impulse-response (IIR) filters. Advantages of the former over the latter are that stability of the filters can be guaranteed and a linear-phase response can be obtained. Therefore, this class of digital filters is commonly used in applications where linear phase response and stability are important, e.g. in channel equalization and anti-aliasing filtering. FIR filters usually have a higher order and, correspondingly, greater group delay compared to IIR filters [73–76]. This is why it is of interest to reduce the complexity of FIR filter implementations [9, 11, 15].

The impulse response $h(n)$ of a causal FIR filter of order N is nonzero only for $0 \leq n \leq N$, while the transfer function and frequency response are defined as

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n} \tag{2.1}$$

and

$$H(e^{j\omega T}) = \sum_{n=0}^{N} h(n) e^{-j\omega T n}, \tag{2.2}$$

respectively, where ωT indicates the discrete frequency[1] for the sampling period T. The frequency response in (2.2) can be represented as

$$H(e^{j\omega T}) = |H(e^{j\omega T})| e^{j\Phi(\omega T)}, \tag{2.3}$$

where $|H(e^{j\omega T})|$ is the magnitude response and $\Phi(\omega T)$ is the phase response.

The output sequence of an FIR filter can be expressed in the time domain as linear convolution of an input sequence $x(n)$ and the filter impulse response as

$$y(n) = \sum_{k=0}^{N} h(k)x(n-k). \tag{2.4}$$

In the frequency domain, it corresponds to the product of the corresponding Fourier transforms, i.e.

$$Y(e^{j\omega T}) = H(e^{j\omega T})X(e^{j\omega T}), \tag{2.5}$$

where $X(e^{j\omega T})$ and $Y(e^{j\omega T})$ are the Fourier transforms of the input and output sequences, respectively.

2.1.1 Linear-Phase FIR Filters

Linear-phase FIR filters are of particular interest in this class of digital filters. The phase response of these filters is linear, i.e. in (2.3), $\Phi(\omega T) = -K\omega T$ for some positive constant K. The impulse response is symmetric or antisymmetric depending on the type of the filter as follows [76]:

Type I (N even):	$h(n) = h(N-n)$,
Type II (N odd):	$h(n) = h(N-n)$,
Type III (N even):	$h(n) = -h(N-n)$,
Type IV (N odd):	$h(n) = -h(N-n)$.

This means that the number of multiplications in filter implementations can be halved. The frequency response of linear-phase FIR filters can be expressed with the help of a real function $H_R(\omega T)$, called *zero-phase frequency response*, as

$$H(e^{j\omega T}) = H_R(\omega T) e^{-j\left(\omega T \frac{N}{2} + c\right)}, \tag{2.6}$$

[1] In some other literature, the discrete frequency is alternatively denoted as ω or Ω.

where $c = 0$ for filters of Types I and II and $c = \pi/2$ for filters of Types III and IV. The zero-frequency response $H_R(\omega T)$ depends on the linear-phase filter type as follows [76]:

$$H_R(\omega T) = \begin{cases} h\left(\frac{N}{2}\right) + \sum_{n=1}^{N/2} 2h\left(\frac{N}{2} - n\right)\cos(\omega T n), & \text{(Type I)} \\ \sum_{n=1}^{\frac{N+1}{2}} 2h\left(\frac{N+1}{2} - n\right)\cos\left(\omega T\left[n - \frac{1}{2}\right]\right), & \text{(Type II)} \\ \sum_{n=1}^{N/2} 2h\left(\frac{N}{2} - n\right)\sin(\omega T n), & \text{(Type III)} \\ \sum_{n=1}^{\frac{N+1}{2}} 2h\left(\frac{N+1}{2} - n\right)\sin\left(\omega T\left[n - \frac{1}{2}\right]\right). & \text{(Type IV)} \end{cases} \tag{2.7}$$

Then, the phase response and group delay of a linear-phase filter $H(z)$ are given, respectively, by

$$\Phi(\omega T) = \begin{cases} -\omega T\frac{N}{2} + c, & H_R(e^{j\omega T}) \geq 0, \\ -\omega T\frac{N}{2} + c \pm \pi, & H_R(e^{j\omega T}) < 0. \end{cases} \tag{2.8}$$

and

$$\tau_g(\omega T) = -\frac{\partial\Phi(\omega T)}{\partial(\omega T)} = \frac{N}{2}. \tag{2.9}$$

To achieve an integer group delay, the filter order must be even, i.e. one should consider Type I or Type III linear-phase FIR filters.

2.2 Design of FIR Filters

For a given specification, filter design generally involves two stages, namely, filter order estimation and obtaining the filter coefficients by approximation of the desired response for an order around the estimated value, where the last step is often iterative until the minimum-order filter is obtained. For an FIR filter, it generally amounts to determine the impulse response values $h(n)$, for which the filter frequency response satisfies the given specification. This type of filter design is common for an FIR filter implemented in the time domain. An alternative way is to obtain the filter DFT coefficients during the second step of the design procedure to meet the desired frequency response specification. This type of filter design can be used for efficient filter implementations in the frequency domain. This section discusses the procedures for both design approaches for frequency-selective filters in detail.

2.2.1 Specification

Among all types of frequency-selective filters, here, we will consider a linear-phase lowpass FIR filter since it is mainly used as the basic element in the author's contributions to this thesis.

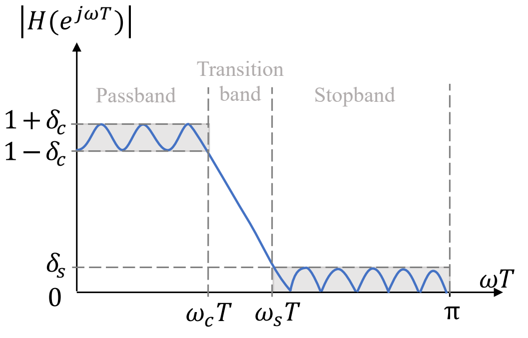


Figure 2.1: Frequency-response specification of a lowpass filter.

The desired frequency response of a causal lowpass FIR filter of order N is given by

$$H_d(e^{j\omega T}) = \begin{cases} e^{-j\omega T \frac{N}{2}}, & \omega T \in [0, \omega_c T], \\ 0, & \omega T \in [\omega_s T, \pi], \end{cases} \tag{2.10}$$

where the frequencies $\omega_c T$ and $\omega_s T$ indicate the passband and stopband edges, respectively. The specification of a lowpass filter $H(e^{j\omega T})$ is shown in Fig. 2.1 and commonly given as

$$\begin{aligned} 1 - \delta_p \leq |H(e^{j\omega T})| \leq 1 + \delta_p, &\quad \omega T \in [0, \omega_c T], \\ |H(e^{j\omega T})| \leq \delta_s, &\quad \omega T \in [\omega_s T, \pi], \end{aligned} \tag{2.11}$$

where δ_p and δ_s denote the passband and stopband ripples. There are no requirements on magnitude response in the transition band, which is generally referred as don't-care band. For a linear-phase FIR filter, the specification in (2.11) can be defined in terms of the zero-frequency response $H_R(\omega T)$ introduced in (2.7) according to

$$\begin{aligned} 1 - \delta_p \leq H_R(\omega T) \leq 1 + \delta_p, &\quad \omega T \in [0, \omega_c T], \\ -\delta_s \leq H_R(\omega T) \leq \delta_s, &\quad \omega T \in [\omega_s T, \pi]. \end{aligned} \tag{2.12}$$

2.2.2 Filter Order Estimation

Compared to IIR filters, there are no analytical expressions for filter order estimation. All available ones have been derived empirically [74, 75, 77, 78]. For a given specification, one can estimate the order of a linear-phase lowpass

FIR filter using, for example, the expression given by [74]

$$\widehat{N} = -\frac{4\pi \log_{10}(10\delta_p\delta_s)}{3(\omega_s T - \omega_c T)}. \tag{2.13}$$

Thus, the narrower the transition band, the higher the filter order required to meet the desired specification.

2.2.3 Design Methods

There are four common methods of FIR filter design, namely, the windowing method, minimax optimization, LS method, and frequency-sampling method.

Design by the Windowing Method

In the windowing method, the impulse response $h(n)$ of an FIR filter is obtained by multiplying the ideal impulse response $h_I(n)$ by a window sequence $w_I(n)$, i.e.

$$h(n) = h_I(n)w_I(n), \tag{2.14}$$

where $h_I(n)$ is the infinite-duration impulse response of an ideal filter obtained using the inverse Fourier transform of the ideal frequency response $H_I(e^{j\omega T})$, i.e. when $\omega_c T = \omega_s T$ in (2.10). In the frequency domain, the response $H(e^{j\omega T})$ is a convolution of the ideal frequency response $H_I(e^{j\omega T})$ with the window's Fourier transform $W_I(e^{j\omega T})$. A variety of windows are available, including rectangular, Hamming, Hanning, Blackman-Harris, and Kaiser windows [73, 79].

Although this method is straightforward and simple in realization, the drawback of it is that the order of designed FIR filters is higher compared to other available methods, that leads to relatively high implementation complexity. Moreover, the windowing approach does not allow en explicit control of the band edges or error weighting.

Minimax Design

Optimization-based design methods allow to obtain an optimal or near-optimal solution in terms of filter parameters and meet the desired specification. One common method is the Parks-McClellan algorithm, also referred as minimax optimization, which is an iterative optimization method that aims to minimize the maximum error between the desired and actual frequency responses. For lowpass FIR filter design, the following problem needs to be solved. For a given filter of order N, find $N+1$ impulse response values $h(n)$ as well as δ to

$$\begin{aligned} &\textbf{minimize } \delta \\ &\textbf{subject to } |E(e^{j\omega T})| \leq \delta, \end{aligned} \tag{2.15}$$

where $E(e^{j\omega T})$ is the *weighted error function*, given by

$$E(e^{j\omega T}) = W(\omega T)\big[H(e^{j\omega T}) - H_d(e^{j\omega T})\big] \tag{2.16}$$

with $H(e^{j\omega T})$ as in (2.2), $H_d(e^{j\omega T})$ as in (2.10), and the *weighting function* $W(\omega T)$ given by

$$W(\omega T) = \begin{cases} 1, & \omega \in [0, \omega_c T], \\ \delta_p/\delta_s, & \omega \in [\omega_s T, \pi]. \end{cases} \tag{2.17}$$

The specification is met if δ after the optimization satisfies $\delta \leq \delta_p$. The resulting filter is optimal in the minimax (Chebyshev) sense, i.e. it is not possible to find another filter of equal or lower order with a smaller value of the error $E(e^{j\omega T})$ in (2.16).

Least-Squares Design

In the applications where requirements on the error energy are given, it is appropriate to design a filter by minimizing the total squared error between the desired filter response $H_d(e^{j\omega T})$ and the actual filter response $H(e^{j\omega T})$ [73]. Thus, the design problem is stated as L_2-norm minimization as follows. For a given filter order N, find $N+1$ impulse values $h(n)$ to

$$\textbf{minimize } ||E||_2^2, \tag{2.18}$$

where

$$||E||_2^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega T}) - H_d(e^{j\omega T})|^2 d(\omega T) \tag{2.19}$$

with $H(e^{j\omega T})$ as in (2.2) and $H_d(e^{j\omega T})$ as in (2.10).

For a linear-phase FIR filter, (2.18) can be expressed as

$$\textbf{minimize } \frac{1}{\pi}\int_{0}^{\pi} |H_R(\omega T) - H_{R,d}(\omega T)|^2 \, d(\omega T), \tag{2.20}$$

where $H_{R,d}(\omega T)$ is the desired zero-frequency response with one in the passband and zero in the stopband. For a Type I linear-phase filter, the response $H_R(\omega T)$ in (2.7) can be written as

$$H_R(\omega T) = \sum_{n=0}^{N/2} b(n) \cos(\omega T n) = \mathbf{b}^{\mathrm{T}} \mathbf{c}(\omega T), \tag{2.21}$$

where

$$\mathbf{b} = [h(N/2), 2h(N/2-1), \ldots, 2h(0)]^{\mathrm{T}}, \tag{2.22}$$

$$\mathbf{c}(\omega T) = [1, \cos(\omega T), \ldots, \cos(\omega T N/2)]^{\mathrm{T}}. \tag{2.23}$$

Then the solution of the problem (2.20) is given by

$$\mathbf{b} = \mathbf{Q}^{-1}\mathbf{p}, \tag{2.24}$$

where $\mathbf{Q}$ is a symmetric positive-definite $(N/2+1) \times (N/2+1)$ matrix with the elements $Q(n,m)$ given by

$$\begin{aligned} Q(n,m) = \frac{1}{\pi} \int_0^{\omega_c T} & \cos(\omega T n) \cos(\omega T m)\, d(\omega T) \\ & + \frac{1}{\pi} \int_{\omega_s T}^{\pi} \cos(\omega T n) \cos(\omega T m)\, d(\omega T). \end{aligned} \tag{2.25}$$

Further, $\mathbf{p}$ represents a vector of length $N/2+1$ with the elements $p(m)$ given by

$$p(m) = \frac{1}{\pi} \int_0^{\omega_c T} \cos(\omega T m)\, d(\omega T). \tag{2.26}$$

If a weighted error function $W(\omega T)$ is desired, the problem in (2.20) is modified as

$$\textbf{minimize}\ \frac{1}{\pi} \int_0^{\pi} W(\omega T) |H_R(\omega T) - H_{Rd}(\omega T)|^2 d(\omega T), \tag{2.27}$$

where $W(\omega T)$ has the gain α for $\omega T \in [0, \omega_c T]$ and $1-\alpha$ for $\omega T \in [\omega_s T, \pi]$. Then, the solution is obtained as in (2.24) with $\mathbf{Q}$ and $\mathbf{p}$ including the corresponding weights in the passband and stopband regions [73, 80].

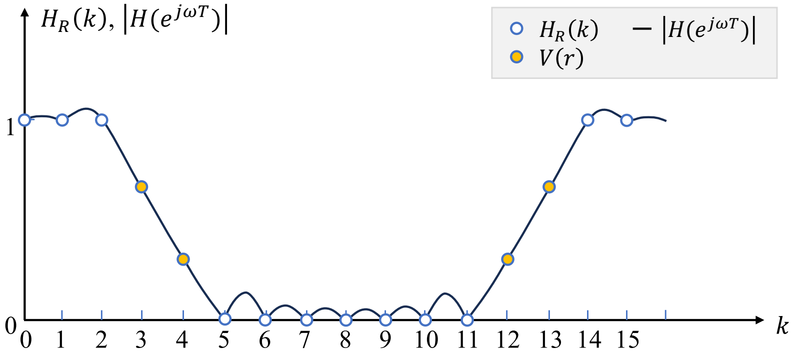


Figure 2.2: Concept of the frequency-sampling-based FIR filter design.

Frequency-Sampling-Based Design

This design method is based on deriving the filter DFT coefficients for the passband and stopband directly by sampling the desired frequency response in (2.10) and optimizing the transition band values, which are generally not specified, to minimize the error between the actual and the desired frequency responses.

Let $H_d(k) = H_d(e^{j\omega_k T})$ be the samples of the desired frequency response for a filter of order $N-1$, uniformly spaced in the interval $[0, 2\pi)$, where $\omega_k T = 2\pi k/N$, $k = 0, \ldots, N-1$. Thus, the coefficients $H(k)$ can be expressed as

$$H(k) = H_R(k) e^{-j\frac{\pi k(N-1)}{N}} \tag{2.28}$$

with the magnitude response samples $H_R(k)$ given by

$$H_R(k) = \begin{cases} 1, & k \in [0, k_1 - 1] \cup [N - k_1 + 1, N - 1], \\ V(k - k_1), & k \in [k_1, k_2] \cup [N - k_2, N - k_1], \\ 0, & k \in [k_2 + 1, N - k_2 - 1], \end{cases} \tag{2.29}$$

where k_1 and k_2 are the first and the last sample indices of the transition band[2]. An example of the magnitude response samples $H_R(k)$ with the transition band samples $V(r)$, $r = k - k_1 = 0, \ldots, k_2 - k_1$, is shown in Fig. 2.2 for $N = 16$.

[2]The expression in (2.29) assumes $k_1 > 1$. For $k_1 = 1$, (2.29) is modified for the passband, which consists of only one sample for $k = 0$.

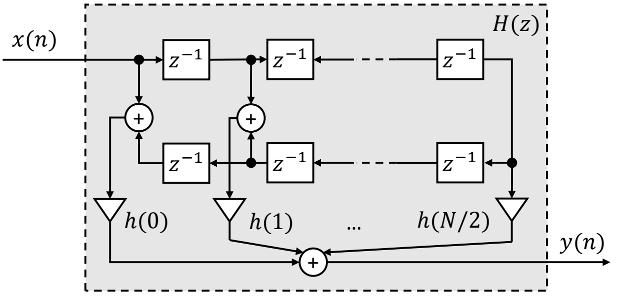


Figure 2.3: Time-domain implementation of a Type I linear-phase FIR filter.

Further, the impulse response coefficients, obtained by the IDFT, are given by [73]

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j2\pi kn/N}. \tag{2.30}$$

Substituting (2.30) in (2.2), the frequency response of the actual filter is obtained as

$$H(e^{j\omega T}) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} H(k) e^{j2\pi kn/N} e^{-j\omega Tn}. \tag{2.31}$$

Therefore, the error between the responses $H_d(e^{j\omega T})$ and $H(e^{j\omega T})$ can be minimized by determining values of $V(r)$, that can be carried out using iterative optimization algorithms [81–83] or the LS method [84].

2.3 Realizations of FIR Filters

2.3.1 Time-Domain Realizations

A time-domain realization of a Type I linear-phase FIR filter is shown in Fig. 2.3. Since the impulse responses of linear-phase filters are either symmetric or antisymmetric, the number of multiplications can be reduced by approximately a factor of two compared to regular FIR filters, however the number of additions and delay element remain the same.

Complexity of Time-Domain Realization

Implementation complexity can be measured by the number of multiplications C_m^{TD}, number of additions C_a^{TD}, and number of delay elements C_d^{TD}. For

Table 2.1: Complexity expressions for linear-phase FIR filters realized in the time domain.

	Type I	Type II	Type III	Type IV
C_m^{TD}	$N/2+1$	$(N+1)/2$	$N/2$	$(N+1)/2$
C_a^{TD}	N	N	$N-1$	N
C_d^{TD}	N	N	N	N

the time-domain filter implementations, these numbers can be also viewed as complexity per output sample. For a linear-phase FIR filter, implementation complexity expressions are given in Table 2.1.

2.3.2 Frequency-Domain Realizations

As mention in Section 2.1, the linear convolution in (2.4) corresponds to the multiplication of DFT coefficients in the frequency domain as in (2.5). It can be efficiently implemented using the fast convolution (FC) with much lower complexity than in the time-domain [11, 85–87]. The most widely-used techniques are the overlap-save (OLS) and overlap-add (OLA) methods which employ the DFT and IDFT [3]. The complexity per sample in terms of multiplications per sample is the same for both methods, however, the former requires slightly less additions per sample than the latter. Thus, the OLS method is more common to implement.

The main idea of the OLS method, shown in Fig. 2.4, is that the input signal is divided into overlapping segments $x_m(n) = x(n+mM)$, $n = 0, 1, \ldots, N-1$, where m is a segment index and the overlapping part is $N-M$. Then, for each segment, the following computations are carried out.

1. The segment x_m is transformed via an N-point DFT.
2. The DFT coefficients $X_m(k)$ are multiplied by the filter DFT coefficients $H(k)$.
3. An N-point inverse discrete Fourier transform (IDFT) is computed.
4. The first $N-M$ samples of the resulting block are discarded, so that the output segments of length M are no longer overlapping.
5. The output sequence is obtained by concatenating the resulting segments as $y(n) = \sum_{m=0}^{\infty} y_m(n-mM)$.

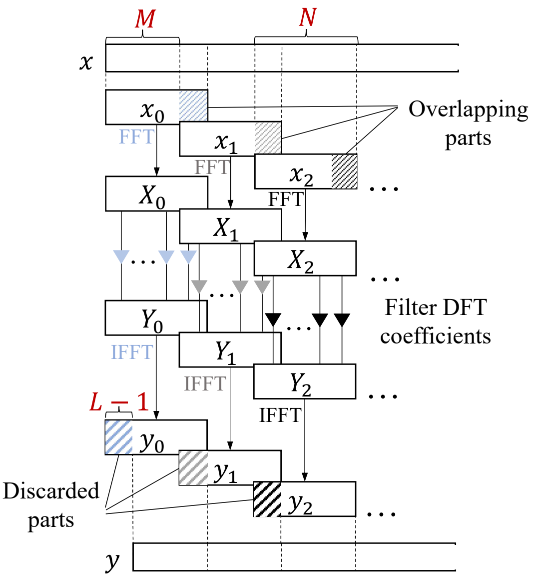


Figure 2.4: Overlap-save method.

In classical OLS filtering, the system is intended to be time-invariant, i.e. the aliasing error is zero. In this case, the DFT coefficients $H(k)$ of an FIR filter correspond to the N-point DFT of the impulse response $h(n)$ of length L, designed by any of the available methods [4]. This means $h(n) = 0$ for $n = L, ..., N-1$, where $L = N - M + 1$.

The FIR filter frequency-domain implementation using the OLS method is shown in Fig. 2.5.

Complexity of Frequency-Domain Realizations

The total implementation complexity of FIR filter frequency-domain implementations is typically expressed as the number of computations per output sample and consists of two parts, namely, DFT/IDFT computing and the multiplications by the filter DFT coefficients. For the OLS technique with M output samples, the complexity is given in terms of multiplication rate R_m^{FD} and addition rate R_a^{FD} as

$$R_m^{\mathrm{FD}} = \frac{2C_m^{\mathrm{DFT}} + C_m^{\mathrm{W}}}{M}, \tag{2.32}$$

$$R_a^{\mathrm{FD}} = \frac{2C_a^{\mathrm{DFT}} + C_a^{\mathrm{W}}}{M}, \tag{2.33}$$

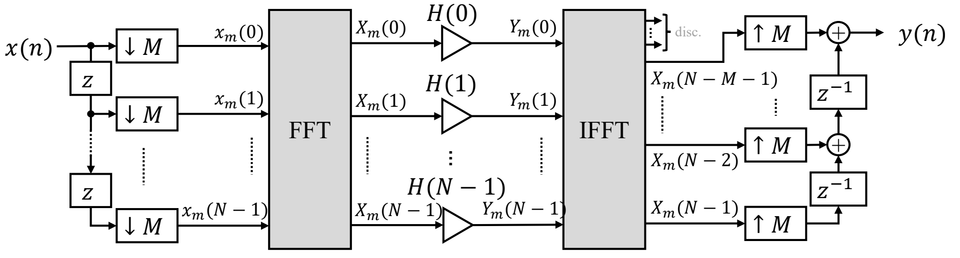


Figure 2.5: FIR filter frequency-domain implementation using the OLS method.

respectively, where C_m^{DFT} and C_a^{DFT} are the numbers of multiplications and additions for the DFT/IDFT, and C_m^{W} and C_a^{W} are the numbers of multiplications and additions for weighting by the DFT filter coefficient $H(k)$, $k = 0, 1, \ldots, N-1$.

The DFT and IDFT can be efficiently implemented using the fast Fourier transform (FFT) and its inverse (IFFT). The complexity of FFT/IFFT depends on two factors, namely, the type of the FFT algorithm and whether the complex multiplications are implemented with three real multiplications and three real additions (referred as the 3/3 algorithm) or with four real multiplications and two real additions (referred as the 4/2 algorithm) [88]. A complex addition requires two real additions. Then, assuming the split-radix FFT algorithm is implemented together with the 3/3 algorithm for a real-valued sequence $x_m(n)$ of length $N = 2^Q$, the complexity of FFT/IFFT is given by

$$C_m^{\mathrm{DFT}} = (1/2)N\log_2 N - (3/2)N + 2, \tag{2.34}$$

$$C_a^{\mathrm{DFT}} = (3/2)N\log_2 N - (5/2)N + 4. \tag{2.35}$$

Further, considering that $h(n)$ is real-valued, the DFT coefficients of $x_m(n)$ and $h(n)$ are conjugate symmetric, and thus only half of the multiplications needs to be carried out. Then, weighting by $H(k)$ require $C_m^{\mathrm{W}} = C_a^{\mathrm{W}} = 3N/2$, and thus the total implementation complexity of an OLS-based FIR filter is

$$R_m^{\mathrm{FD}} = \frac{N\log_2 N - (3/2)N + 4}{M}, \tag{2.36}$$

$$R_a^{\mathrm{FD}} = \frac{3N\log_2 N - (7/2)N + 8}{M}. \tag{2.37}$$

2.4 Digital Differentiators

Digital differentiators find their applications in various domains including communication systems and speech processing. This type of digital filters is generally used to approximate various degrees of signal derivatives that is required, for example, in signal pre-processing and post-equalization in digital communication systems. Digital differentiators can be used as a part of systems for bandwidth extension and signal quality enhancing by compensating channel distortion and mitigating intersymbol interference [76, 89].

The desired frequency response for a causal differentiator of order N_D is given by [90]

$$D_d(e^{j\omega T}) = e^{-j\omega T N_D/2}(j\omega T)^k \tag{2.38}$$

for $|\omega T| \leq \omega_c T \leq \pi$ with k determining the degree of the differentiator.

For design of a digital differentiator, linear-phase FIR filters can be used. Thus, the frequency response of the differentiator can be written as [90]

$$H(e^{j\omega T}) = \begin{cases} e^{-j\omega T N_D/2} H_R(\omega T), & \text{for Type I and II,} \\ j e^{-j\omega T N_D/2} H_R(\omega T), & \text{for Type III and IV,} \end{cases} \tag{2.39}$$

where the function $H_R(\omega T)$ is real-valued. Using (2.38) and (2.39), one can design a digital differentiator by minimizing the error between the desired and actual responses in the minimax sense or LS sense using a similar approach as for a linear-phase FIR filter design mentioned in Section 2.2.3.

Chapter 3

Basics of Multirate Systems and Filter Banks

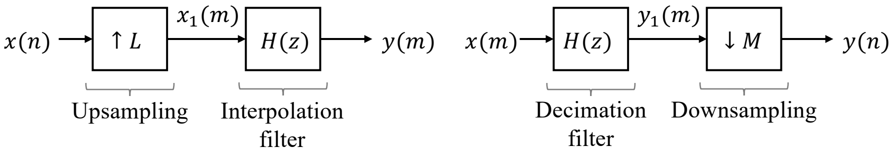
In multirate systems, the sampling rate is changed during the signal processing. Many DSP applications, for example communications and audio processing, utilize multirate techniques because their use can simplify and reduce computational complexity. Variable digital filters can be also efficiently implemented using these techniques. Thus, this chapter gives an introduction to basic elements of multirate systems and filter banks, which can be used to analyze and design time-varying systems as discussed in Paper D.

3.1 Basic Multirate Operations

Generally, different parts of multirate systems operate at different sampling frequencies, and thus an essential part of these systems is sampling rate conversion, which employs the fundamental operations of multirate DSP, namely interpolation and decimation.

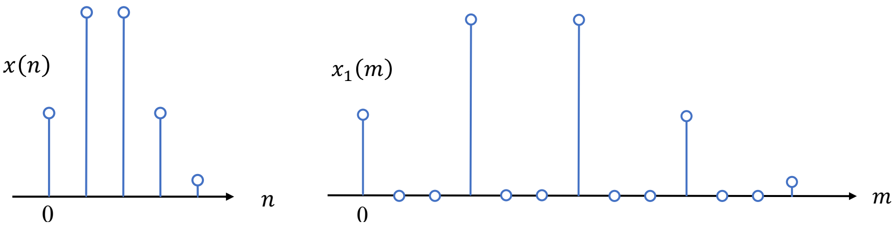
Interpolation

Interpolation generates a new sequence with a higher sampling frequency, while maintaining the original signal information. For interpolation with an integer factor L, this is a two-stage process, which consists of upsampling following by filtering as shown in Fig. 3.1a. The first stage, upsampling by a factor of L, inserts $L-1$ zeros between each input sample pair, i.e. a new signal $x_1(m)$ formed from $x(n)$ is expressed as [80]



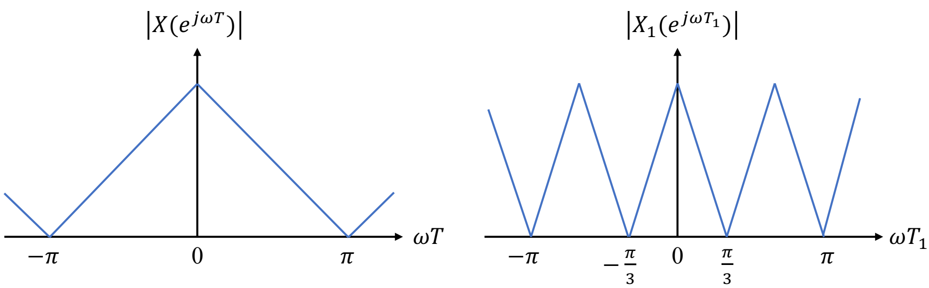
(a) Interpolation by a factor L. (b) Decimation by a factor M.

Figure 3.1: Principle of interpolation and decimation.



(a) Sequence $x(n)$. (b) Sequence $x_1(m)$.

Figure 3.2: Illustration of upsampling by a factor $L = 3$ in the time domain.



(a) Spectrum magnitude of $x(n)$. (b) Spectrum magnitude of $x_1(n)$.

Figure 3.3: Illustration of upsampling by a factor $L = 3$ in the frequency domain.

$$x_1(m) = \begin{cases} x\left(\frac{m}{L}\right), & m = 0, \pm L, \pm 2L, \dots, \\ 0, & \text{otherwise}, \end{cases} \tag{3.1}$$

where $L > 1$. In the frequency domain, the Fourier transform of $x_1(m)$ is given by

$$X_1(e^{j\omega T_1}) = X_1(e^{j\omega T/L}) = X(e^{j\omega T}) = X(e^{j\omega T_1 L}), \tag{3.2}$$

where T and $T_1 = T/L$ are the sampling periods before and after upsampling, respectively. The corresponding z-transform is

$$X_1(z) = X(z^L). \tag{3.3}$$

Illustrations of upsampling by a factor of $L = 3$ in the time domain and in the frequency domain are shown in Figs. 3.2 and 3.3, respectively.

In the second stage, an interpolation filter is used to remove images created due to upsampling. Ideally, it is a lowpass with cutoff frequency π/L. The output signal $y(n)$ is obtained as a convolution of $x_1(n)$ and the filter impulse response $h(n)$.

Decimation

Decimation by an integer factor M, shown in Fig. 3.1b, is also a two-stage process, consisting of filtering followed by downsampling. The filter, which is ideally a lowpass with cutoff frequency π/M, ensures that the signal being decimated is bandlimited to avoid aliasing. Further, downsampling reduces the sampling rate by a factor of M by extracting every Mth value of the signal, i.e. the output signal $y(n)$ is obtained from $y_1(m)$, which is a result of convolution of the input signal $x(n)$ and the filter impulse response $h(n)$, as

$$y(n) = y_1(Mn). \tag{3.4}$$

In the frequency domain, downsampling is expressed as

$$Y(e^{j\omega T}) = \frac{1}{M}\sum_{k=0}^{M-1} Y_1(e^{j(\omega T - 2\pi k)/M}) = \frac{1}{M}\sum_{k=0}^{M-1} Y_1(e^{j(\omega T_1 - 2\pi k/M)}), \tag{3.5}$$

where T_1 and $T = MT_1$ are the sampling periods before and after downsampling, respectively. The corresponding z-transform is

$$Y(z) = \frac{1}{M}\sum_{k=0}^{M-1} Y_1(z^{1/M} W_M^k), \tag{3.6}$$

where $W_M^k = e^{-j2\pi k/M}$ [80]. Thus, the output signal spectrum is the sum of M stretched and shifted versions of $Y_1(e^{j\omega T_1})$.

Sampling rate conversion by a rational factor L/M, where both L and M are integers, can be realized by combining interpolation and decimation [76, 80].

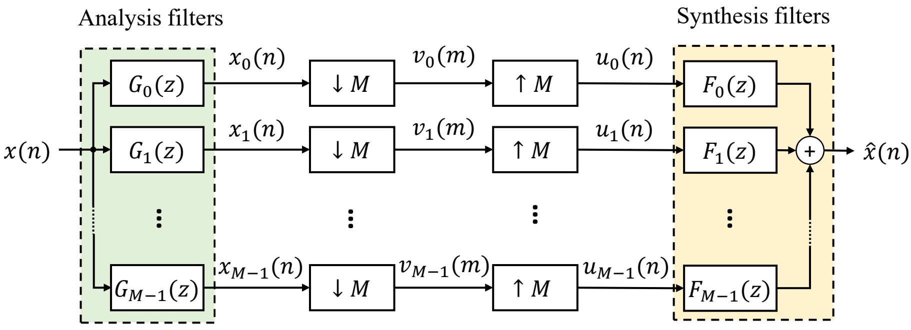


Figure 3.4: M-channel maximally decimated filter bank.

3.2 Filter Banks

A system of digital filters with a common input and/or common output, capable of processing various bands of a signal separately, is referred to as a filter bank (FB). One special type of FBs is an M-channel maximally decimated FB, which is shown in Fig. 3.4. It contains the analysis bank of M filters $G_k(z)$, which splits the input signal $x(n)$ into M subband signals $x_k(n)$, $k = 0, \ldots, M-1$, downsamplers by M, upsamplers by M, and the synthesis bank of M filters $F_k(z)$, which recombine the signals to produce $\hat{x}(n)$. The downsampled signals $v_k(n)$ can be processed individually, which is required in many applications, for instance subband coding in audio processing [80], channelization in wireless communication systems [38, 48, 82, 91], and hearing aid devices [27, 35, 92]. The z-transform of $v_k(n)$ is given by

$$V_k(z) = \frac{1}{M} \sum_{l=0}^{M-1} G_k(z^{1/M} W^l) X(z^{1/M} W^l), \tag{3.7}$$

while the output of the FB is expressed as

$$\widehat{X}(z) = \frac{1}{M} \sum_{l=0}^{M-1} X(zW^l) \sum_{k=0}^{M-1} G_k(zW^l) F_k(z). \tag{3.8}$$

This input-output relation can be expressed as

$$\widehat{X}(z) = \sum_{l=0}^{M-1} A_l(z) X(zW^l), \tag{3.9}$$

where

$$A_l(z) = \frac{1}{M} \sum_{k=0}^{M-1} G_k(zW^l) F_k(z) \tag{3.10}$$

with $A_0(z)$ being the distortion function and $A_l(z)$ being the aliasing functions for $l = 1, \ldots, M-1$. The term $X(zW^l)$ is referred to as the lth aliasing term with the gain $A_l(z)$ [80]. For a perfect reconstruction (PR) FB, i.e. when $\hat{x} = cx(n - n_0)$ for some constant $c \neq 0$ and delay n_0, the distortion and aliasing functions must satisfy

$$A_0(z) = cz^{-n_0}, \quad A_l(z) = 0 \quad \text{for } l = 1, \ldots, M-1. \tag{3.11}$$

For non-maximally decimated FBs, the number of subbands is greater than the downsampling and upsampling factors ($N > M$), and the functions $A_l(z)$ in (3.9) are given by [80]

$$A_l(z) = \frac{1}{M} \sum_{k=0}^{N-1} G_k(zW^l) F_k(z). \tag{3.12}$$

This is commonly used when realizing a so called fast-convolution filter bank (FCFB) [19, 48, 93], as will be discussed in Section 4.3.

Typically, design of FBs starts from design of a prototype filter, for example using the techniques described in Section 2.2.3. Then all the filters involved in the FB can be expressed in terms of the prototype filter to satisfy certain criteria [80].

Linear Periodically Time-Varying Systems

In practice, the filters $G_k(z)$ and $F_k(z)$ are not ideal, i.e. the transition band and stopband gain are non-zero, therefore this can results in aliasing. Unless it is canceled, a general FB with N channels and M-fold downsamplers and upsamplers corresponds to a linear periodically time-varying (LPTV) system with period M [80, 94]. The output of such a system is given by

$$y(n) = \sum_{q=0}^{Q-1} h_n(q) x(n-q), \tag{3.13}$$

where Q is the length of the system M-periodic impulse response $h_n(q) = h_{n+M}(q)$, $n = 0, \ldots, M-1$.

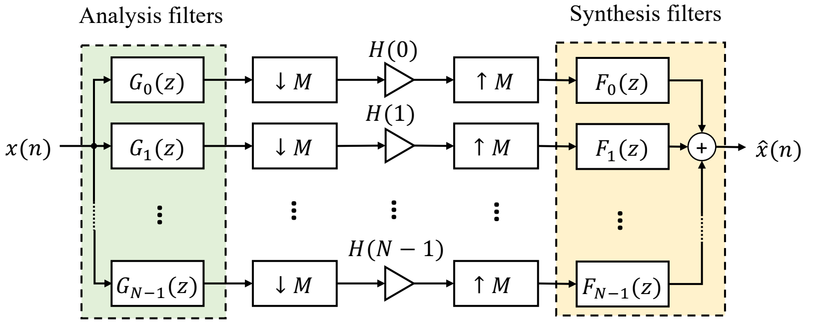


Figure 3.5: Multirate FB representation of the system in Fig. 2.5.

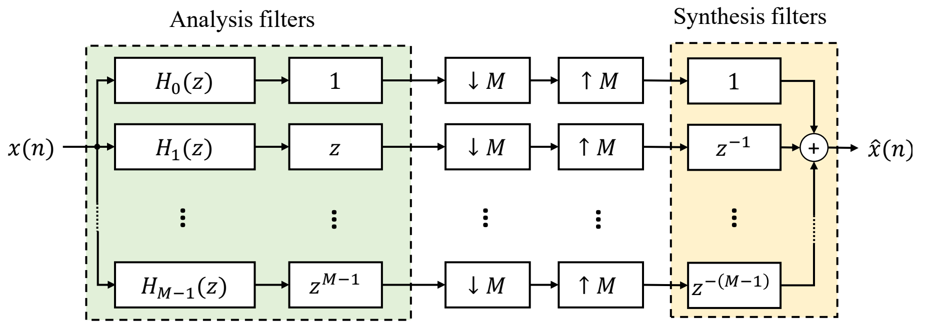


Figure 3.6: PTVIR representation of the system in Fig. 3.5.

Design of LPTV systems has to include measuring a distortion function and aliasing functions, or analysis of periodically time-varying impulse response (PTVIR) representation which corresponds to a set of M time-invariant impulse responses $h_n(q)$ and their corresponding frequency responses $H_n(e^{j\omega T})$ [87, 94–96].

For the first approach, one should ensure that $A_l(z) = 0$ for $l = 1, \dots, M - 1$ in (3.12). To achieve PR, the aliasing and distortion functions must satisfy (3.11). The second approach is commonly used for design of LPTV FIR filters, when aliasing cannot be canceled but can be suppressed to any desired level though a proper design. In this case, the system in Fig. 2.5 can be represented as an equivalent N-channel FB with the multiplication by $H(k)$ between the M-fold downsampler and upsampler in every branch k as shown in Fig. 3.5. Then, the frequency responses of the analysis and synthesis filters are given as [87]

$$G_k(e^{j\omega T}) = \sum_{n=0}^{M-1} e^{j2\pi(n+1)k/N} e^{-j\omega T n} \tag{3.14}$$

and

$$F_k(e^{j\omega T}) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi(n-M)k/N} e^{-j\omega T n}, \tag{3.15}$$

respectively. This N-channel FB with M-fold downsampling and upsampling can be represented as an equivalent PTVIR system, shown in Fig. 3.6, with the M time-invariant filters $H_n(e^{j\omega T})$, given by [87]

$$H_n(z) = z^{-n} \sum_{k=0}^{N-1} H(k) G_k(z) F_{kn}(z^M), \tag{3.16}$$

where $F_{kn}(z)$ is the so called *polyphase component* of $F_k(z) = \sum_{n=0}^{M-1} z^{-n} F_{kn}(z^M)$ [80]. The distortion and aliasing functions in (3.9) can be expressed in terms of $H_n(z)$ as [87]

$$A_l(z) = \frac{1}{M} \sum_{n=0}^{M-1} H_n(zW^l) W_M^{ln}. \tag{3.17}$$

This means that the distortion function $A_0(z)$ is the average of the M filter transfer functions $H_n(z)$ while each aliasing function is the average of frequency-shifted and rotated versions of $H_n(z)$. Thus, to design an LPTV FIR filter (or a corresponding multirate FB representation) by suppressing aliasing to a desired level, each of the responses $H_n(e^{j\omega T})$ should approximate the desired frequency response $H(e^{j\omega T})$ of the overall system.

Chapter 4

Variable Digital Filters

This chapter gives an introduction to VDFs and presents a review of the most common types of VDFs.

4.1 Introduction

Due to the increasing demand for computational complexity reduction while preserving flexibility, reconfigurable systems are getting more attention in the contemporary world of technologies. For instance, recent communication standards require flexible, adaptive systems capable of real-time frequency-domain tuning [97,98]. Such requirements can be fulfilled by means of VDFs. The key feature of VDFs is that they require only an adjustment of one or a few parameters to change their characteristics, without the need for extensive additional computations. Compared to regular digital filters, which require recomputing all coefficients for every new specification, i.e. online filter design using one of the methods mentioned in Section 2.2.3, generally leading to high computational complexity, most coefficients in VDFs are fixed after the initial design. This allows efficient hardware implementation and makes VDFs essential for many modern adaptive technologies, for example in communication systems [28,29,31,37,43,45,46,48,99] and medicine [24,35,42,100].

Research on VDFs emerged in the 1970s [63]. The first methods for designing VDFs were mainly for lowpass digital filters with variable cutoff and center frequencies based on spectral transformations applied to a prototype linear-phase lowpass FIR filter [65, 70, 72]. Further, this approach was developed and extended to bandpass filters [67,69]. For design of variable IIR filters, allpass transformations were utilized to transform a lowpass prototype filter to the desired frequency-selective filters [68, 71, 101]. However, in most cases, transformation-based methods appeared to suffer from some inherent limitations and high computational cost.

Alternative methods, which are more general and based on spectral parameter approximation, were introduced later [61, 62, 64]. The main idea consists of representing the impulse responses or the poles and zeros as polynomials of the variable spectral parameters and a set of constant coefficients. This approach allowed continuous tuning of VDFs directly by the spectral parameters with the reduced implementation complexity compared to the transformation-based methods. Further, it was shown that VDFs can be efficiently implemented based on the Farrow structure [66]. This has led to the growth of research works on FIR VDFs due to their simple design procedures and good filter properties [59, 60, 102], especially digital FIR filters with VFD received considerable attention [25, 26, 29, 30, 36, 39, 51, 52, 59, 102–104]. Later, the Farrow structure was also utilized in digital filters with VBW [28,37,42,44,56] and for simultaneously variable bandwidth and fractional delay [49, 57].

In addition to the transformation-based and spectral parameter approximation based methods, many other approaches have been proposed for some specific applications. Among these, frequency response masking techniques and filter banks were utilized to design VDFs with a sharp transition band or a channelizer with the ability to control different bands separately in real time [27, 31, 35, 37, 38, 43, 47, 48, 50, 54].

In the author's contributions, the proposed FIR VDFs are mainly based on the Farrow structure and/or are compared to existing solutions based on this structure or FB-based structures. Therefore, these two types of VDFs are discussed in more detail below.

4.2 Farrow-Based Variable Digital Filters

The Farrow structure [66] has received great attention in realizing VDFs as it allows to achieve an efficient realization of fine tunability with small computational overhead, especially for filters with VFD [25, 29–31, 36, 39, 49, 51,52,59,102] and VBW [22,28,32,37,41–44,49,56,100,105]. In this technique, the overall transfer function is expressed as a weighted linear combination of $L + 1$ fixed filters, commonly referred to as *subfilters*. A realization of a Farrow-based VDF is shown in Fig. 4.1. Due to fewer multiplications and inherent stability, linear-phase FIR filters are generally the basic elements of

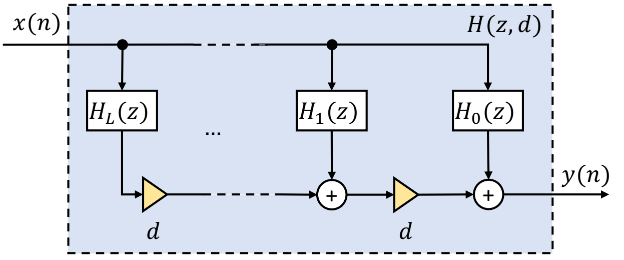


Figure 4.1: Farrow-based realization of a VDF with the variable parameter d.

VDFs. Thus, the overall frequency response is given by

$$H(e^{j\omega T}, d) = \sum_{k=0}^{L} d^k H_k(e^{j\omega T}), \tag{4.1}$$

where $H_k(e^{j\omega T})$ are typically linear-phase FIR filters of order N_{H_k} and d is a variable parameter that can directly determine, for example the fractional delay or bandwidth.

Design of Farrow-based VDFs with the overall frequency response $H(e^{j\omega T}, d)$ can be carried out by approximating the desired frequency response, for example in the minimax sense [22, 56, 102] or in the LS sense [55].

Implementation complexity of Farrow-based VDFs consists of complexity of the fixed part, represented by $L + 1$ subfilters $H_k(z)$ with constant coefficients, and the variable part, i.e. L general multiplications by the variable coefficient d and L additions. Thus, the total complexity can be expressed in terms of fixed multiplications C_{mf}, variable multiplications C_{mf}, and additions C_a given by

$$C_{mf} = \sum_{k=0}^{L} C_{mf,k}, \tag{4.2}$$

$$C_{mv} = L, \tag{4.3}$$

$$C_a = L + \sum_{k=0}^{L} C_{a,k}, \tag{4.4}$$

where $C_{mf,k}$ and $C_{a,k}$ are the number of fixed multiplications and additions

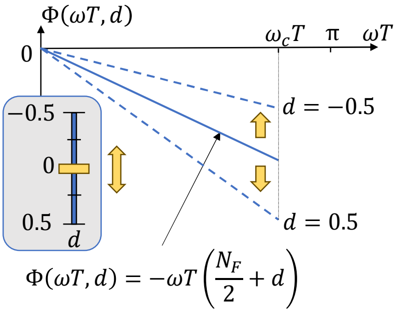


Figure 4.2: Illustration of the VFD filter phase response.

per subfilter k, that depend on the linear-phase FIR filter type as follows

$$C_{mf,k} = \begin{cases} N_k/2 + 1, & \text{Type I,} \\ (N_k + 1)/2, & \text{Type II or IV,} \\ N_k/2, & \text{Type III,} \end{cases} \tag{4.5}$$

$$C_{a,k} = \begin{cases} N_k, & \text{Type I, II, or IV,} \\ N_k - 1, & \text{Type III.} \end{cases} \tag{4.6}$$

Further complexity reduction can be carried out by implementing constant multiplications very efficiently using various techniques for constant multiplications [9–11, 15].

4.2.1 Variable-Fractional-Delay Filters

Digital filters capable of dynamically adjusting a delay that is a fraction of a sample period are referred to as VFD filters. They find applications in telecommunications, audio processing, control systems, and other fields [25, 26, 29–31, 36, 39, 52, 59, 106, 107].

The desired frequency response and phase response of a VFD filter are given by [102]

$$F_d(e^{j\omega T}, d) = e^{-j\omega T(N_F/2+d)}, \quad |\omega T| \leq \omega_c T < \pi \tag{4.7}$$

and

$$\Phi_d(\omega T, d) = -\omega T(N_F/2 + d), \quad |\omega T| \leq \omega_c T < \pi \tag{4.8}$$

respectively, where $\omega_c T$ is the passband edge, N_F is the filter order, and $d \in [-1/2, 1/2]$ is a variable fractional delay. Illustration of the phase response of a VFD filter, controllable by the variable parameter d, is shown in Fig. 4.2.

Farrow-based VFD filters are typically realized as a linear combination of differentiators [36, 49, 52, 59]. This realization, which is shown in Fig. 4.1, has received great attention due to low implementation complexity. The frequency response of a Farrow-structure-based VFD filter can be expressed as [102]

$$F(e^{j\omega T}, d) = \sum_{k=0}^{L} d^k F_k(e^{j\omega T}), \tag{4.9}$$

where each $F_k(e^{j\omega T})$ approximates a differentiator of kth degree and order N_{F_k}, which should preferably be even since this choice results in a lower implementation complexity for the overall VFD filter due to a pure delay in the first branch compared to an odd-order case [102]. Thus, the subfilters $F_k(e^{j\omega T})$ are of Type I or Type III linear-phase filters with symmetric (for even k) or antisymmetric (for odd k) impulse responses $f_k(n)$, i.e. $f_k(n) = f_k(N_{F_k} - n)$ and $f_k(n) = -f_k(N_{F_k} - n)$ for even and odd k, respectively. They can be of different orders, therefore an additional delay $(N_F - N_{F_k})/2$ has to be introduced in a corresponding branch k with lower order to have the same delay $N_F/2$ in every branch, where N_F is the overall filter order given by

$$N_F = \max_k \{N_{F_k}\}. \tag{4.10}$$

The subfilters frequency responses can be written as

$$F_k(e^{j\omega T}) = \begin{cases} e^{-j\omega T N_{F_k}/2} F_{kR}(\omega T), & k \text{ even}, \\ je^{-j\omega T N_{F_k}/2} F_{kR}(\omega T), & k \text{ odd}, \end{cases} \tag{4.11}$$

where $F_{kR}(\omega T)$ should approximate the desired function $F_{kR,d}(\omega T)$, given by

$$F_{kR,d}(\omega T) = \begin{cases} \frac{(-j\omega T)^k}{k!}, & k \text{ even}, \\ -j\frac{(-j\omega T)^k}{k!}, & k \text{ odd}. \end{cases} \tag{4.12}$$

The frequency response $F(e^{j\omega T}, d)$ in (4.9) should approximate the desired response $F_d(e^{j\omega T}, d)$ in (4.7).

Implementation complexity is here computed as in (4.2)-(4.4), considering that the differentiators can be of different orders N_{F_k}. To further reduce the complexity, different modifications of the Farrow structure have been introduced [25, 30, 51, 52]. VFD filters can be designed using the windowing method [108], weighted least squares (WLS) method [30, 36, 52, 55], and minimax optimization algorithm [36, 51, 102].

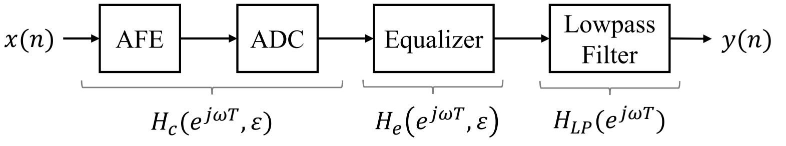
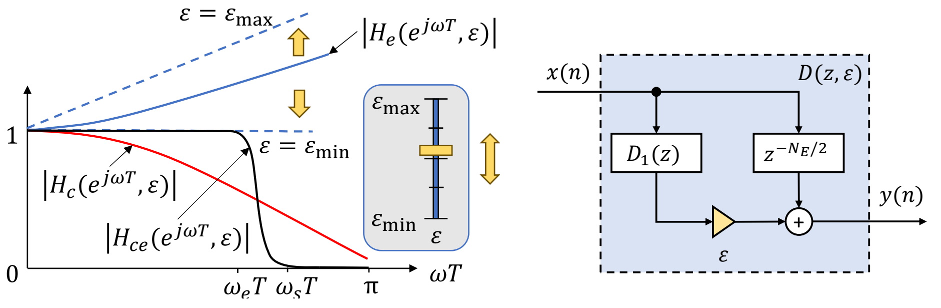


Figure 4.3: Principle of frequency response equalization using a variable equalizer.



(a) Illustration of the VE magnitude response.

(b) VE Farrow-based realization.

Figure 4.4: Illustration of the VE magnitude response and its Farrow-based realization, controllable by the variable parameter ε.

4.2.2 Variable Equalizers

Variable equalizers (VEs) allow to adjust the equalization frequency response by controlling one or a few parameters. This type of digital filters has been utilized mainly in acoustic signal processing for audio equalization with gain as a variable parameter [109, 110]. Although an equalizer is an essential part of communication systems, to the best of the author's knowledge, VEs have not been extensively explored in this context.

The basic principle of the analog-frond-end (AFE) equalization in communication systems is illustrated in Fig. 4.3. The analog input signal is assumed to be bandlimited to $\omega_e < \pi/T$, where T denotes the sampling period. The channel frequency response of the AFE circuits and the following ADC[1] generally distort and attenuate the signal within the band of interest $[0, \omega_e T]$.

[1]Some papers include the ADC in the AFE while some papers do not. Here, the AFE and ADC are considered as separate blocks.

The desired frequency response after channel equalization is given by

$$H_{ce,d}(e^{j\omega T}) = \begin{cases} e^{-j\omega T \frac{N_E}{2}}, & \omega T \in [0, \omega_e T], \\ 0, & \omega T \in [\omega_s T, \pi], \end{cases} \tag{4.13}$$

where N_E is the equalizer order. To recover the original input signal and achieve the desired response in (4.13), it is needed to equalize the channel frequency response $H_c(e^{j\omega T}, \varepsilon)$, which can vary depending on the variable parameter[2] $\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]$. Generally, a lowpass filter is combined with the equalizer in order to remove out-of-band noise and aliasing after the ADC [111,112]. Thus, to design a VE with the frequency response $H_e(e^{j\omega T}, \varepsilon)$, the resulting response $H_{ce}(e^{j\omega T}, \varepsilon)$, given by

$$H_{ce}(e^{j\omega T}, \varepsilon) = H_c(e^{j\omega T}, \varepsilon) H_e(e^{j\omega T}, \varepsilon) H_{LP}(e^{j\omega T}) \tag{4.14}$$

with $H_{LP}(e^{j\omega T})$ being the lowpass filter responses, should approximate $H_{ce,d}(e^{j\omega T})$.

A VE can be realized based on the Farrow structure using digital differentiators as proposed in Papers A and B. Then, the VE frequency response in the simplest case (Paper A) is given by

$$H_e(e^{j\omega T}, \varepsilon) = e^{-j\omega T N_E/2} + \varepsilon D_1(e^{j\omega T}) \tag{4.15}$$

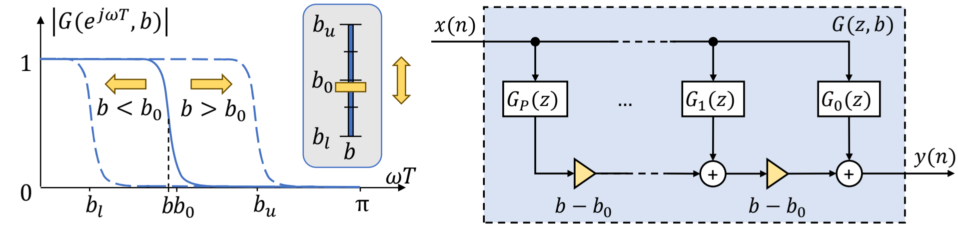
with $D_1(e^{j\omega T})$ being a fixed Type III linear-phase FIR filter with antisymmetric impulse response that should approximate the first-degree differentiator frequency response $D_{1,d}(e^{j\omega T}) = j\omega T e^{-j\omega T N_E/2}$ [32]. The illustration of the variable equalization and the VE Farrow-based realization are shown in Figs. 4.4a and 4.4b, respectively.

VE can be designed in a similar way as VFD filters as their realization also can consist of digital differentiators of different degrees. This is explained in Papers A and B.

4.2.3 Variable-Bandwidth Filters

Digital filters whose bandwidth can be controlled by tuning a single parameter b are referred to as VBW filters. The are required in many DSP applications, for example, spectrum sensing in cognitive radio [37] and adjusting the bandwidth of interest for amplification in hearing aid devices [44]. The

[2] For a higher-order polynomial channel models, several variable parameters for the corresponding VE may be required as proposed in Paper B.



(a) Illustration of the VBW filter magnitude response.

(b) VBW filter Farrow-based realization.

Figure 4.5: Illustration of the VBW lowpass filter magnitude response and its Farrow-based realization, controllable by the variable parameter b.

desired frequency response of a VBW lowpass filter is given by

$$G_d(e^{j\omega T}, b) = \begin{cases} e^{-j\omega T N_G/2}, & \omega T \in [0, b - \Delta/2], \\ 0, & \omega T \in [b + \Delta/2, \pi], \end{cases} \tag{4.16}$$

where the filter order N_G is preferably even[3], $b \in [b_l, b_u]$ is the center of the fixed transition band Δ, whereas $\omega_c T = b - \Delta/2$ and $\omega_s T = b + \Delta/2$ represent variable passband and stopband edges, respectively.

The overall frequency response $G(e^{j\omega T}, b)$ of a Farrow-based VBW filter is given by

$$G(e^{j\omega T}, b) = \sum_{p=0}^{P} (b - b_0)^p G_p(e^{j\omega T}), \tag{4.17}$$

where the subfilters $G_p(e^{j\omega T})$ are generally Type I linear-phase FIR filters[3] of order N_{G_p}. Generally, the subfilters for a VBW filter are chosen to have the same order, i.e. $N_{G_p} = N_G$, since the use of different orders will not bring significant reduction in complexity but complicate the design [58]. For obtaining a more balanced structure with smaller filter coefficients, the selection $b_0 = (b_l + b_u)/2$ tends to make the filter coefficients smaller in magnitude as well as enables one to find a lower bound on the filter order required to meet a given specification [113]. Moreover, for $b = b_0$, $G(e^{j\omega T}, b_0) = G_0(e^{j\omega T})$ is a regular lowpass filter with passband and stopband edges at $b_0 - \Delta/2$ and $b_0 + \Delta/2$, respectively. The frequency response $G(e^{j\omega T}, b)$ in (4.17) should approximate the desired response $G_d(e^{j\omega T}, b)$ in (4.16). An

[3]The assumption on an even filter order gives an integer group delay, as outlined in Section 2.1.1.

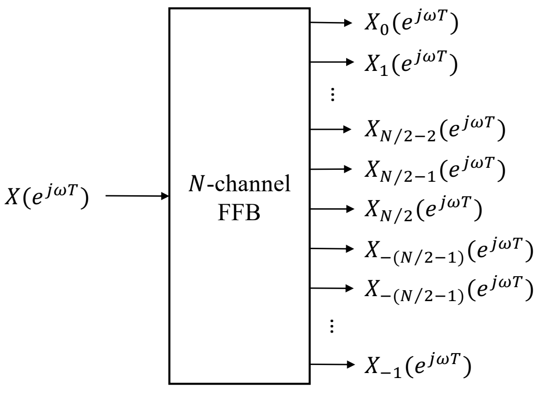


Figure 4.6: N-channel FFB.

illustration of a VBW lowpass filter magnitude response and its Farrow-based realization are shown in Figs. 4.5a and 4.5b, respectively.

VBW filters are typically designed using the minimax optimization [113] or using the WLS method [61]. The implementation complexity of a Farrow-based VBW filter is computed as in (4.2)-(4.4), considering that the subfilters $G_p(z)$ are of the same order N_G.

4.3 Filter-Bank-Based Variable Digital Filters

For some applications, e.g. when a sharp transition band is required and Farrow-based VDFs may cause high computational complexity per sample or for flexible channelization of wireless communication systems, VDFs can be realized based on a FB. There are two common FB-based approaches that have received great attention: based on a fast filter bank (FFB) and based on a FCFB.

4.3.1 Fast Filter Banks

FFBs are referred to as an efficient realization of filter banks based on the sliding FFT [114], which itself is a recursive realization of the DFT [77]. FFBs are typically organized in a tree structure FB, i.e. a multistage FB, where the filters in each stage split the signal further into 2 or 3 channels [54, 114].

In FFB-based VDFs, an input signal is split into subbands by an analysis FFB as shown in Fig. 4.6, and each band can be processed independently. For example, a discretely tunable bandwidth can be realized in a simple way by combining different bands or discarding the bands out of the desired

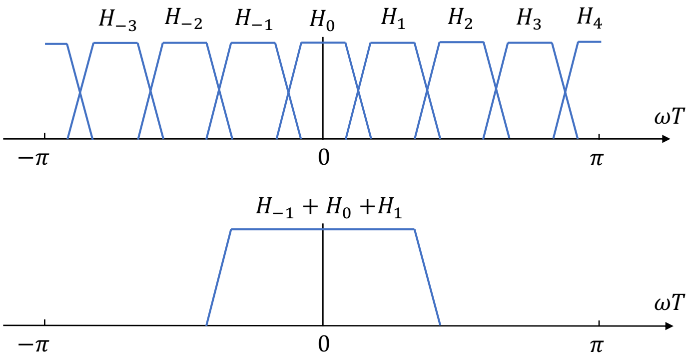


Figure 4.7: Magnitude response of a discretely tunable FFB-based VBW filter.

passband as shown in Fig. 4.7 for $N = 8$ channels [54]. Additionally, to make continuous variability of the bandwidth, shaping bandpass filters need to be introduced in the realizations [47, 50, 54, 91]. Thus, the overall computational complexity of an FFB-based VBW filter consists of the FFB implementation, shaping filter implementation, and implementation of band shifting [54]. A variable fractional delay can be realized as a variable phase shift in each subband, resulting in the desired phase shift [50].

In this technique, one can implement simultaneously many filters with different center frequencies, resulting in reduced computational complexity per subband. This approach significantly reduces computational costs, particularly when compared to Farrow-based structures for very sharp transition bands [54, 91] and for two variable parameters [47, 50]. FFB-based VFDs are generally designed based on a prototype filter for every stage and a shaping filter [47, 50, 54, 91].

Applications of the FFB-based VDFs also include spectrum sensing and channelization of wireless communication systems, where compatibility of channelizers with different standards is required [91].

This type of VDFs is used in Paper C for comparison with the proposed frequency-domain implementations.

4.3.2 Fast-Convolution-Based Filter Banks

Another approach for efficient implementation of VDFs is based on FCFB [48], which is widely applied in realizing multiplexers for spectrum sensing and dynamic spectrum allocation in wireless communication systems [38, 115, 116]. The main idea of this method is based on combination of the frequency-domain implementation of FIR filters based on the OLS technique, described

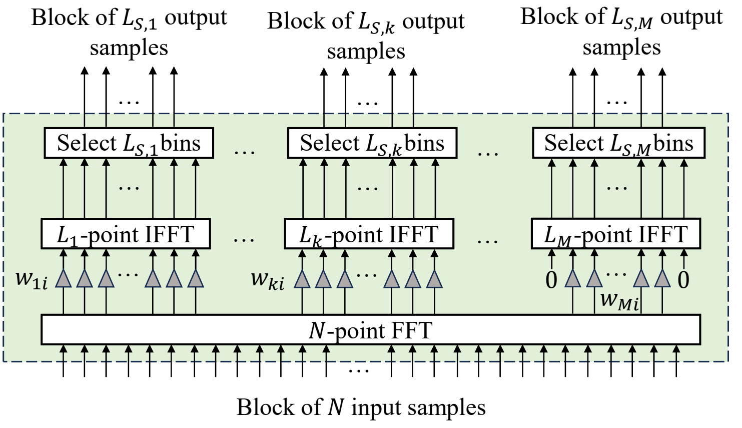


Figure 4.8: Fast-convolution based flexible analysis FB [48].

in Section 2.3.2, and individual subband processing using FBs, mentioned in Section 3.2, that together form a multirate version of a fast convolution. In this structure, after taking the FFT of an input signal of length N with overlapped block-processing, the signal frequency bins are split into blocks of length L_k, $k = 1, \ldots, M$, and each of them is multiplied by the corresponding band weights (filter DFT coefficients). Each subband can be easily configured for unequal bandwidths, different center frequencies, and adjustable sampling rates $R_k = N/L_k$, determined by changing the IFFT length L_k and the corresponding subband filter bandwidth. The latter can be easily performed without additional computations, because the fact that these filters are designed based on the frequency sampling method, when the passband and stopband values are set as ones and zeros, respectively, and only the transition band values need to be optimized during the FB design, allows to vary the passband and stopband width by inserting or removing ones or zeros, respectively, while keeping the transition band fixed [48].

In Paper D, an approach for VBW filter design based on a hybrid of frequency sampling and optimization is proposed, that is based on the idea from FCFB-based VDFs.

Chapter 5

Concluding Remarks

In this licentiate thesis, we consider design and implementation of FIR filters with one or a few variable parameters, capable of changing the filter frequency responses "on-the-fly" without online design.

In Paper A, we propose three realizations of a reconfigurable FIR filter with simultaneous equalization and lowpass filtering for a first-order channel model, which typically can model ADC attenuation. This idea is extended in Paper B to higher-order polynomial channel models, also considering AFE distortions. In this research work, more detailed design procedures of the proposed structures are presented along with a fast design method for VDFs with several variable parameters. Compared to regular equalizers, the proposed solutions show substantial implementation complexity reduction.

In Paper C, we propose three low-complexity frequency-domain implementations of VDFs designed by optimizing the impulse response coefficients and implemented using the fast convolution. Comparison to existing solutions shows that the proposed structures offer significant savings in terms of multiplications and additions per output sample.

In Paper D, we introduce a new efficient design approach of the fast-convolution-based VDFs, which is based on a hybrid of frequency sampling and optimization. Compared to Paper C and other existing solutions, this approach allows to optimize the filter DFT coefficients directly considering the filter frequency-domain implementation and offers significant computational complexity reduction.

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