



# **MODELING COUPLED PRODUCTION AND ALLOCATIONS SYSTEMS: THE MODERN WAGE DYNAMICS MODEL**

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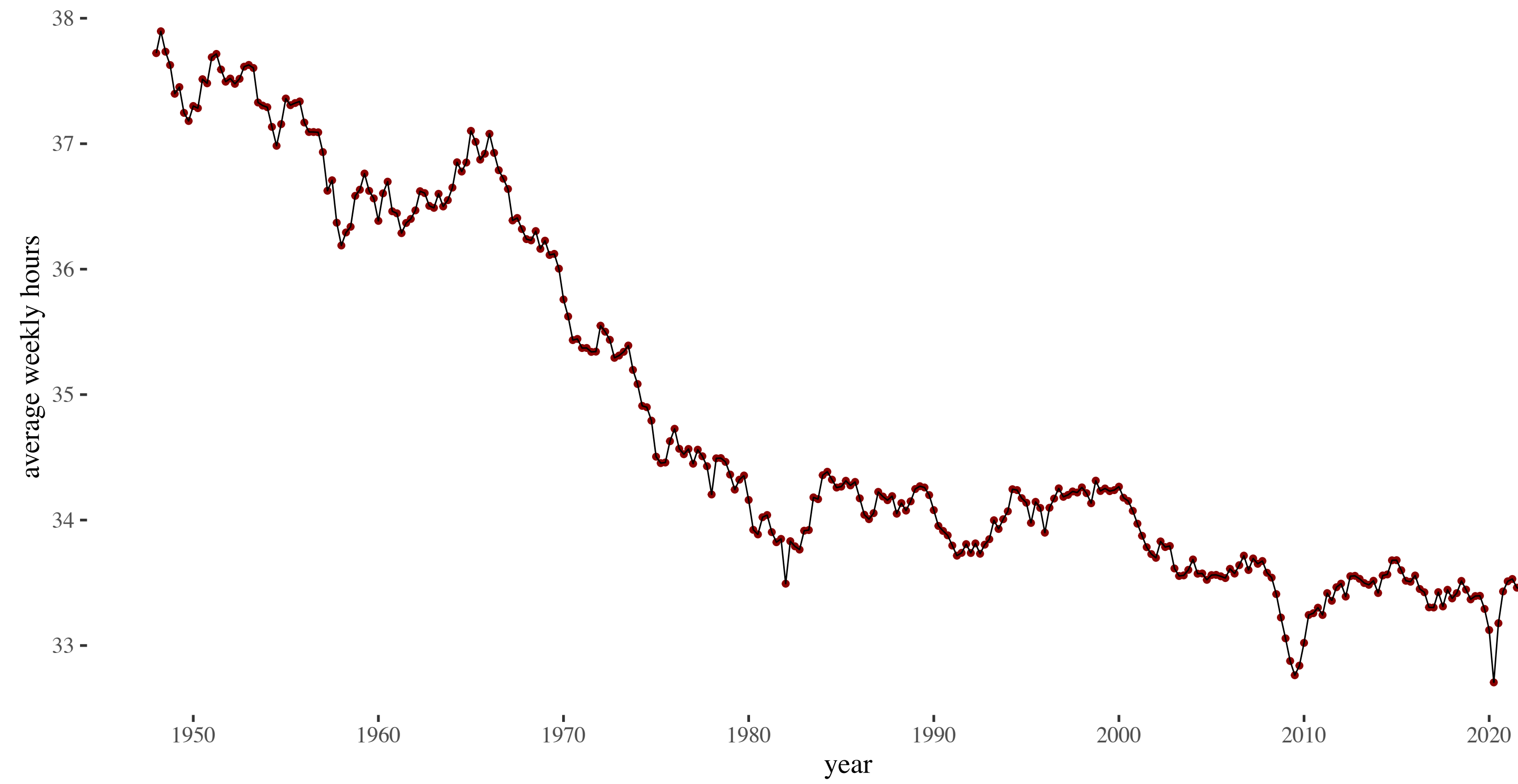
Keynes famously predicted that within his lifetime we would 'be able to perform all the operations of agriculture, mining, and manufacture with a quarter of the human effort to which we have been accustomed', and that the resulting technological unemployment would cause 'only a temporary phase of maladjustment' until people became accustomed to a purposeful life of abundance and leisure.

— *Economic Possibilities for Our Grandchildren* (1930)

More recently, Arthur posits that we are already at the point where we don't need people to work because we can produce enough for everyone's needs, and that we have moved away from questions of production to questions of distribution.

— *Where is Technology Taking the Economy* (2017)





# COUPLED PRODUCTION - ALLOCATION SYSTEM

Most modern economies have coupled production and allocation systems, where production, employment, price and wage are all interconnected. Households produce intermediate and final services and goods through an organizing structure, the firm, and those services and goods are ultimately consumed by the households. Wages are used to remunerate households for that production, which they then use to consume what they produced.

‘. . . this is what social life is actually about, the production of people (of which the production of things is simply a subordinate moment).’ — Graeber, *Consumption* (2011)

# ARRAY OF SOLUTIONS, OUTCOMES ALL CONTENTIOUS

1. minimum wage; ambiguous as to whether increases or decreases employment, but intensity changes are apparently difficult to tease out econometrically.
2. unemployment benefits; ambiguous as to whether this has a deleterious effect on employment, though sometimes intended to do just that.
3. earned income tax credits (EITC); ambiguous effect on employment, except for evidence that single mothers will enter the workforce with EITC (not a leisure choice). More interesting is the undetermined effect of the threat of benefit loss with higher wages on reduced hours.
4. universal basic income (UBI); will people continue to work what is required for high levels of consumption?

These studies are nearly all theoretically based on labour demand.

# THE MODEL FUNDAMENTALS

We assume numerous heterogeneous households, an aggregate firm representing all intermediate and final production, producing a single representative good called *sugar*.

Firms produce with modified Solow production function:

$$Y = AK^\eta L^\gamma = A' L^\gamma$$

Households decide hours to provide based on Cobb-Douglas preferences for leisure and consumption:

$$U(H_o, S_o) = (H_{max} - H_N - H_o)^\alpha (S_o)^\beta .$$

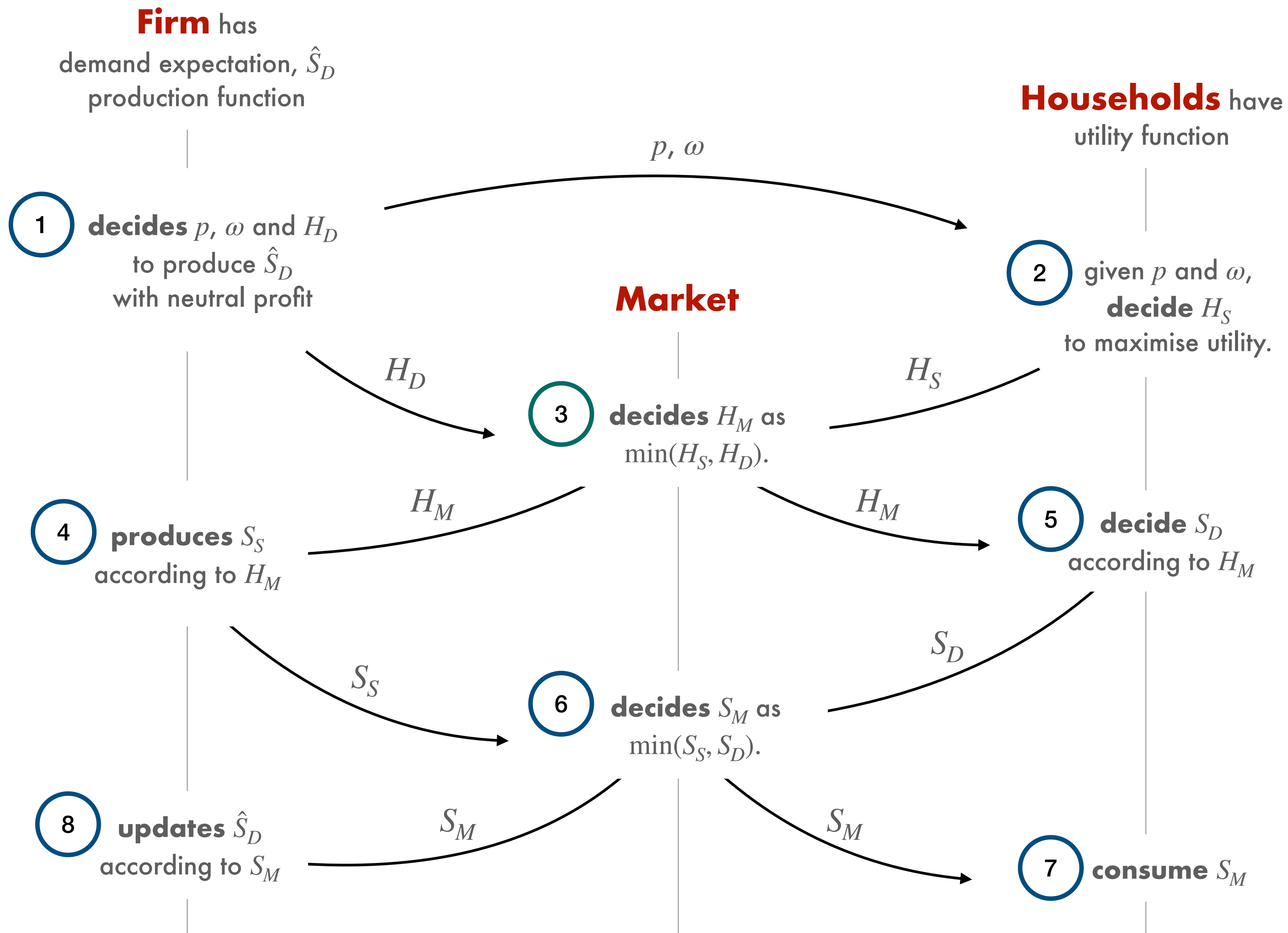
# NECESSARY CONSUMPTION

Each household requires a certain amount of consumption for survival,  $S_N$ , from which it does not obtain utility.

Each household therefore requires a certain amount of labour to obtain  $S_N$ , which we call tribute hours,  $H_N$ .

The number of tribute hours,  $H_N$ , required depends on wage, price and how much money the household has retained from previous earnings:

$$H_N = \max \left( 0, \frac{pS_N - m}{\omega} \right)$$





# THE LABOUR MARKET

- 1 Given  $S_{D,0}$ ,  $p_0$  and  $\omega_0$  as initial conditions, the firm will request the hours to meet that demand given its production function:

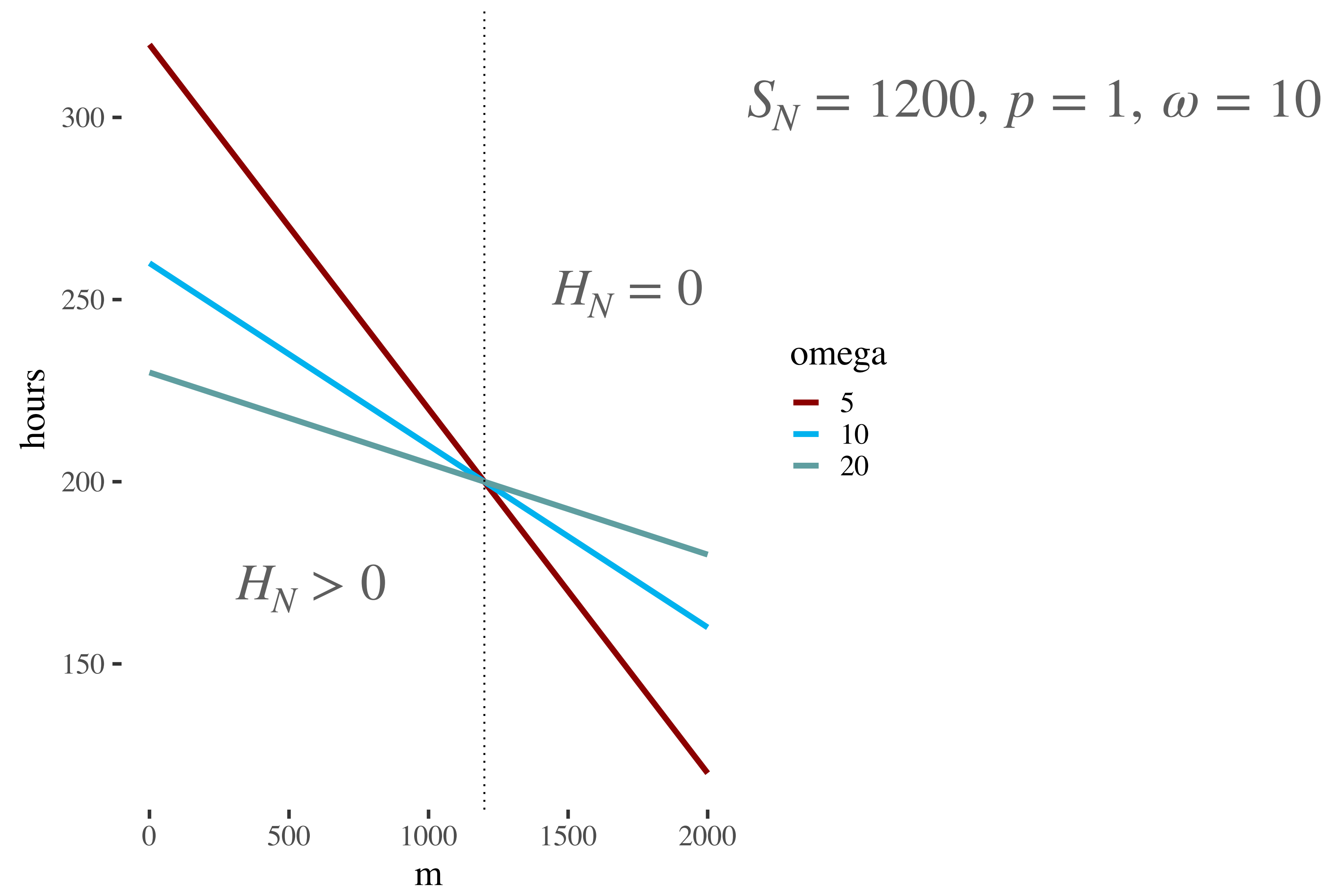
$$H_D = \left( \frac{S_D}{A} \right)^{\frac{1}{\gamma}}$$

- 2 Households determine utility maximising hours:

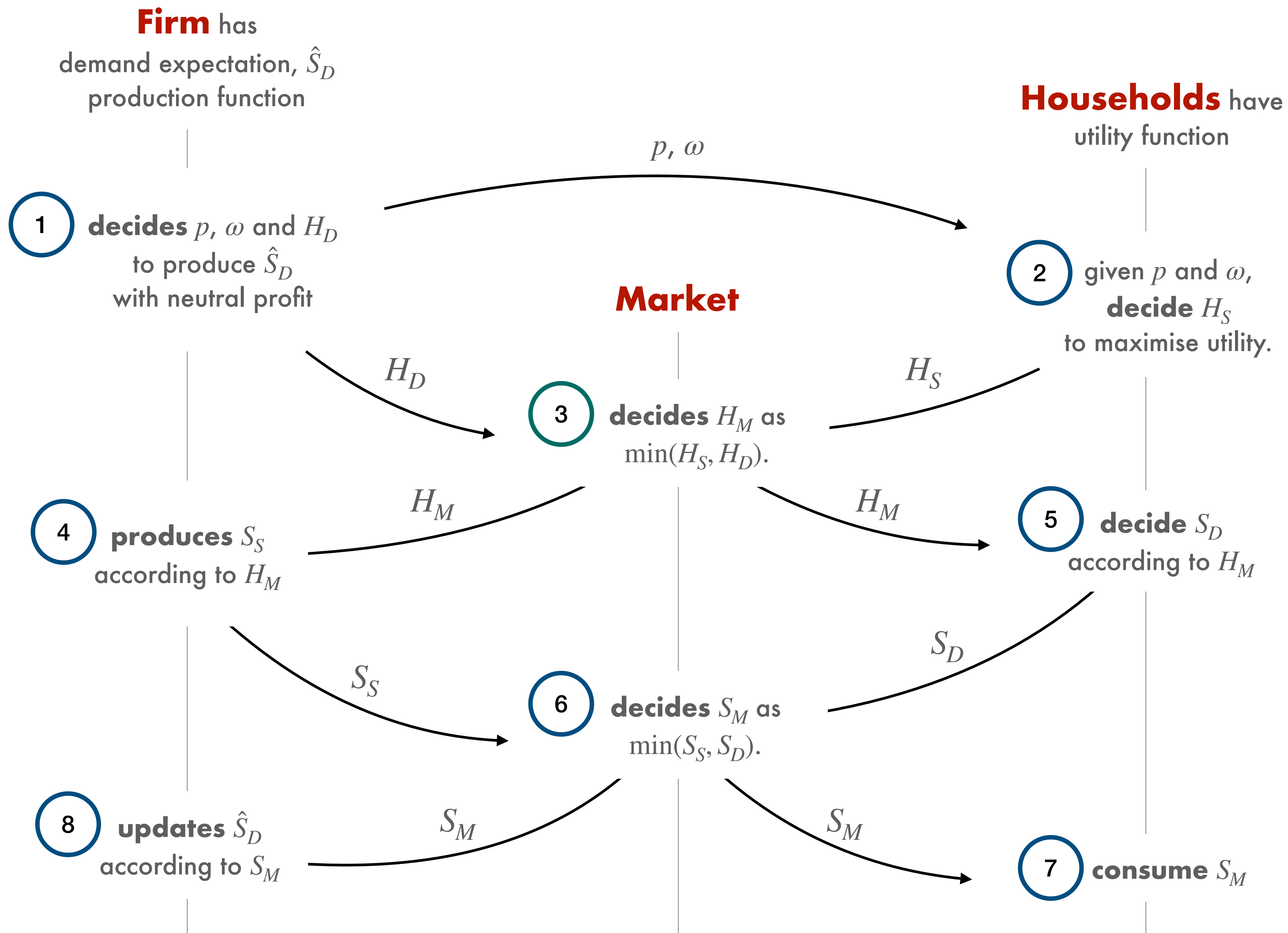
$$H = \beta H_{max} + \frac{\alpha}{\omega} (pS_N - \max(0, m))$$

- 3 Market determines labor exchanged, where  $H_S = \sum_i^n H_i$   
and  $H_M = \min(H_S, H_D)$

# DISTINCT WAGE AND HOUR REGIMES







# THE SUGAR MARKET

④ Firm produces with  $H_M$  according to its production function:  $S_S = AH_M^\gamma$

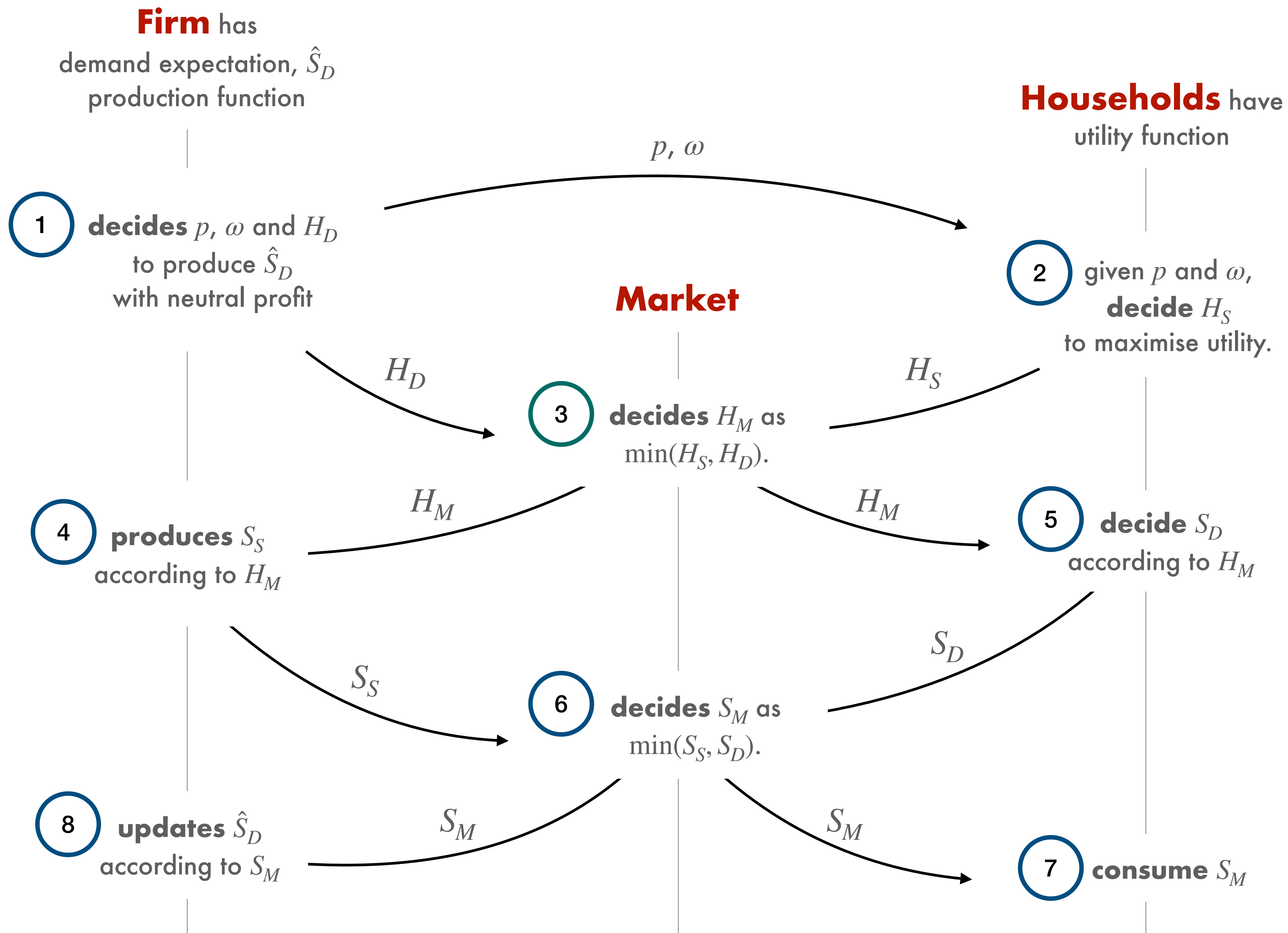
⑤ Households buy the sugar they can afford:

$$S_i = \max \left( S_N, \frac{1}{p}(\omega H_i + \min(0, m_i)) \right)$$

⑥ Market determines sugar sold, where

$$S_D = \sum_i^n S_i \text{ and } S_M = \min(S_S, S_D).$$





# CONSUMPTION AND EXPECTATION UPDATES

- 7 Households consume sugar they can afford, and less if  $S_S < S_D$ , in which case they keep unspent earnings.
- 8 Firm updates expectation of  $S_D$  according to specified algorithm.



# PRODUCTION PARAMETER CONSTRAINTS

To parameterise the production function,  $A$  and  $\gamma$ , we make a couple of assumptions:

1. Each household could self-produce what it needs with maximum hours and maximum effort, and constant returns to scale. Thus

$$S_N = AH_{max}^1 \text{ or } A = \frac{S_N}{H_{max}}.$$

2. Division of labour implies increasing returns to labour compared to self-production. All households working together at  $H_N$  should produce  $nS_N$ .

$$nS_N = A(nH_N)^\gamma \text{ or } \gamma \geq \ln \left( \frac{nS_N}{A} \right) \frac{1}{\ln(nH_N)}.$$

# FIRM PRICE AND WAGE DECISIONS

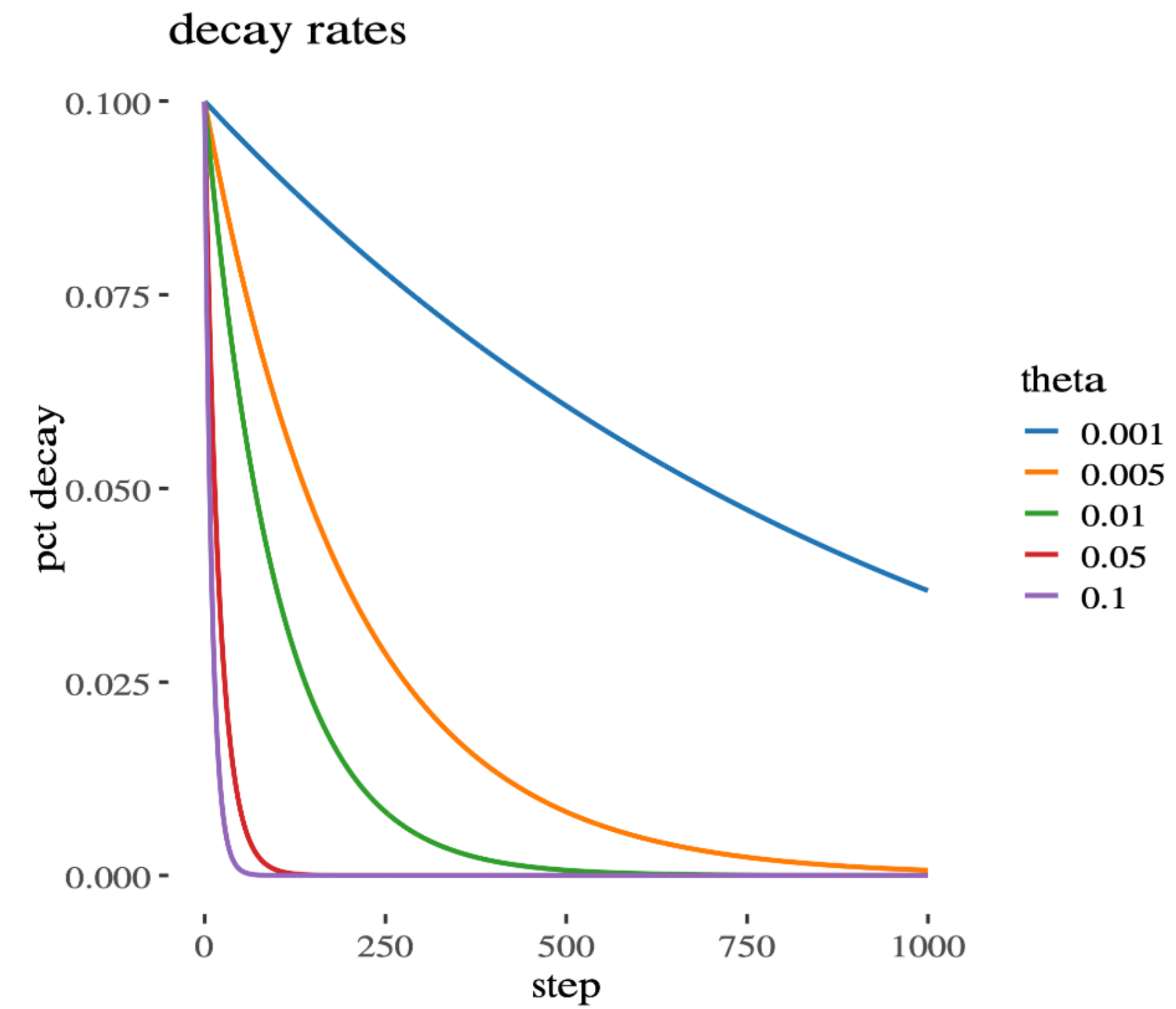
In addition to the desire to match sugar supply with demand, the firm strives to clear the labour market such that  $H_S = H_D$ .

	$S_S > S_D$	$S_S < S_D$
$H_S > H_D$	<div><math display="block">p = \frac{1}{S_N} \left( \frac{\omega}{\alpha} (H_D - \beta H_{max}) + m \right)</math><div>(decrease <math>p</math>)</div></div>	<div><math display="block">\omega = \frac{\alpha(pS_N - m)}{H_D - \beta H_{max}}</math><div>(increase <math>\omega</math>)</div></div>
$H_S \leq H_D$	<div><math display="block">\omega = \frac{\alpha(pS_N - m)}{H_D - \beta H_{max}}</math><div>(decrease <math>\omega</math>)</div></div>	<div><math display="block">p = \frac{1}{S_N} \left( \frac{\omega}{\alpha} (H_D - \beta H_{max}) + m \right)</math><div>(increase <math>p</math>)</div></div>



# SIMULATED ANNEALING

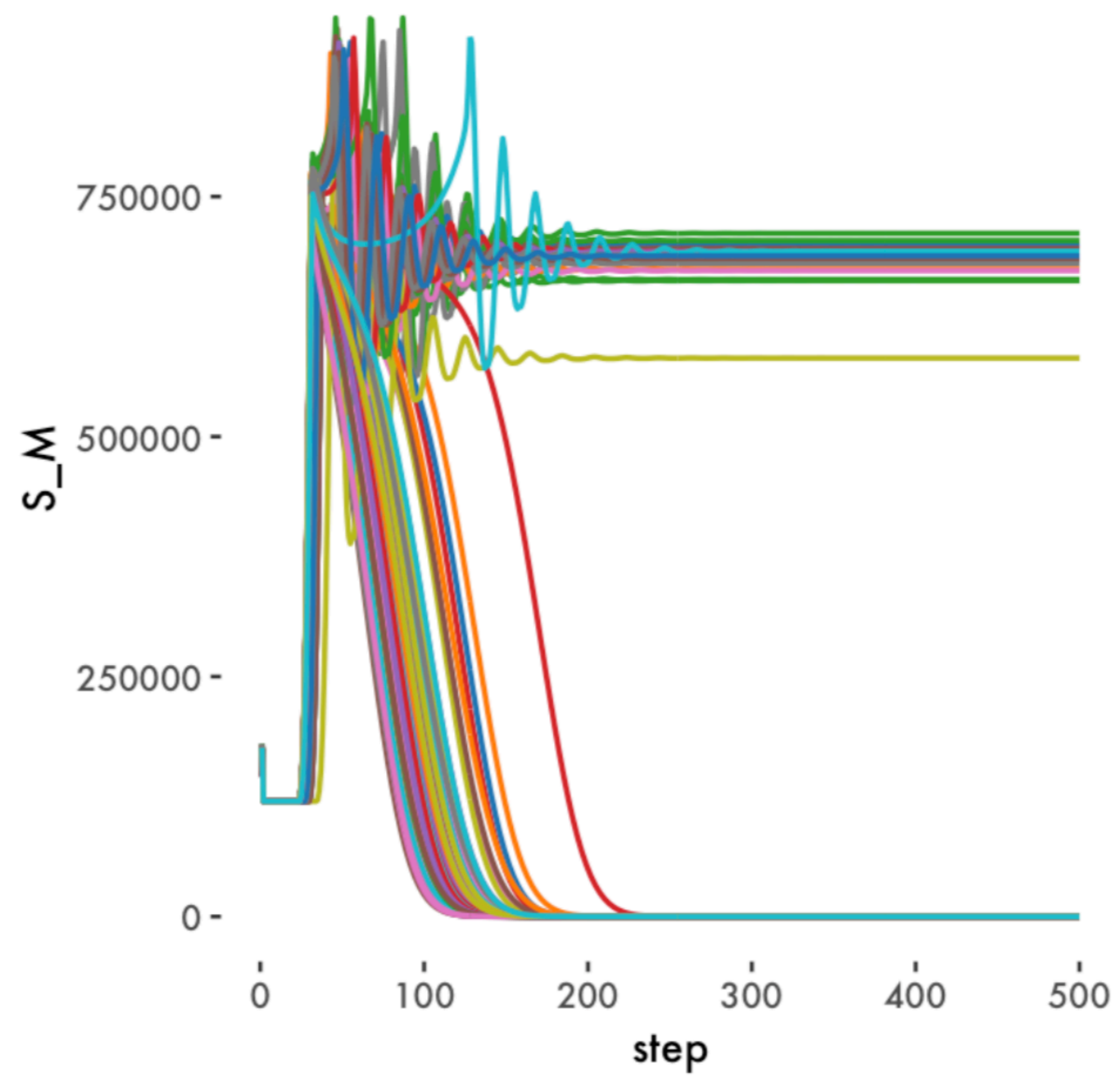
$$\delta_0 e^{-\theta x}$$



# RESTATEMENT OF MODEL PURPOSE

To be clear, we are not intending to suggest that economies evolve in the manner indicated, but rather we understand this model as a tool to discover the 'optimal'  $\omega : p$  ratio for a given system. Ultimately, our purpose is to explore how wage policies affect that ratio, as well as production, hours worked and debt. Therefore, we do see that a significant first step is to construct a model of a coupled system that does indeed consistently produce a notable  $\omega : p$  ratio for a specific institutional scenario.

# SAMPLE RESULTS



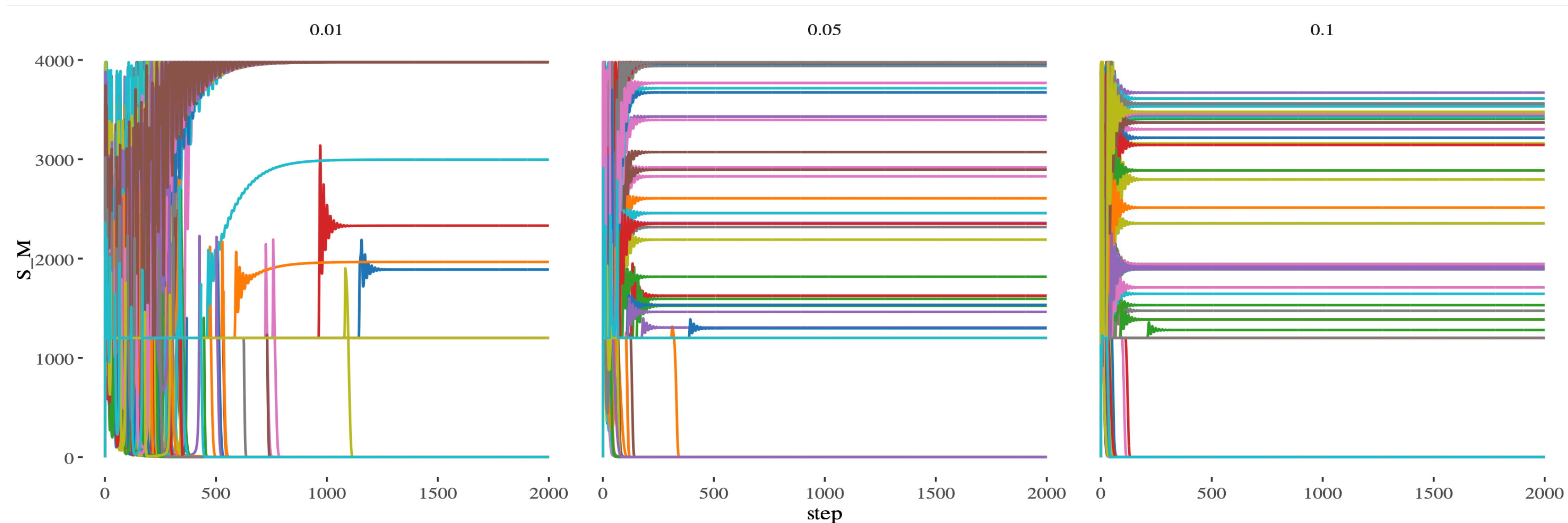
restrict beta!



**WHAT YOU COULD DO WITH THE MODEL**

# EXPLORE MODEL BEHAVIOUR

1. Simulated annealing speed or other convergence mechanism.



2. Number of households.

3. Changes in parameter values such as  $\mu$  and  $\beta$ ?

4. Convergence mechanism with money = 0?

# EFFICIENCY WAGES

## Labor Discipline and Aggregate Demand: A Macroeconomic Model

By SAMUEL BOWLES AND ROBERT BOYER\*

The neoclassical theory of employment and output may be characterized by its two most basic abstractions: the acceptance of Say's law and the representation of labor as a commodity like any other input. In practice, Say's law is nothing more than the assertion that product market clearing will be achieved through some combination of price level and interest rate effects, and that the process by which these effects work is sufficiently rapid and regular to justify ab-

ployers, the level of government redistributive expenditure will influence both the distribution of private incomes and, independently of this, the level of aggregate demand; it will be modeled explicitly and generated endogenously. As we will see, taking account of the effect of the v aggregate demand and the en termination of output per la level of employment may r positively or negatively to ch ing rise to wh profit-led employ

Lawrence F. Katz  
UNIVERSITY OF CALIFORNIA, BERKELEY, AND NBER

## Efficiency Wage Theories: A Partial Evaluation

### 1. Introduction

The question of why unemployed workers are unable to bid down the wages of seemingly comparable employed workers and gain jobs has long perplexed economists. A burgeoning literature on efficiency wage theories suggests that the answer may lie in the negative incentive effects of low wages. The basic efficiency wage hypothesis states that workers' productivities depend positively on their wages. If this is the case, firms

## CHAPTER 16

## The Fair Wage-Effort Hypothesis and Unemployment

GEORGE A. AKERLOF AND JANET L. YELLEN

### 1. INTRODUCTION

This chapter explores the consequences of a hypothesis concerning worker behavior, which we shall call the fair wage-effort hypothesis.<sup>1</sup> According to this hypothesis, the fair wage is proportional to effort. The fair wage is the fair

## Relative Wages, Efficiency Wages, and Keynesian Unemployment

Lawrence H. Summers

WORKING PAPER 2590      DOI 10.3386/w2590      ISSUE DATE May 1988

While modern economic theorists have produced a variety of explanations for the failure of wages to fall in the face of unemployment, Keynes emphasis on relative wages has not been reflected in most contemporary discussions. This short paper suggests that relative wage theories in which workers' productivity depends primarily on their relative wage provide the best available apparatus for understanding actual unemployment and its fluctuations. Such theories are very closely related to the efficiency wage theories that have received widespread attention in recent years.

# EXPLORE CONTENTIOUS WAGE AUGMENTATION STRATEGIES

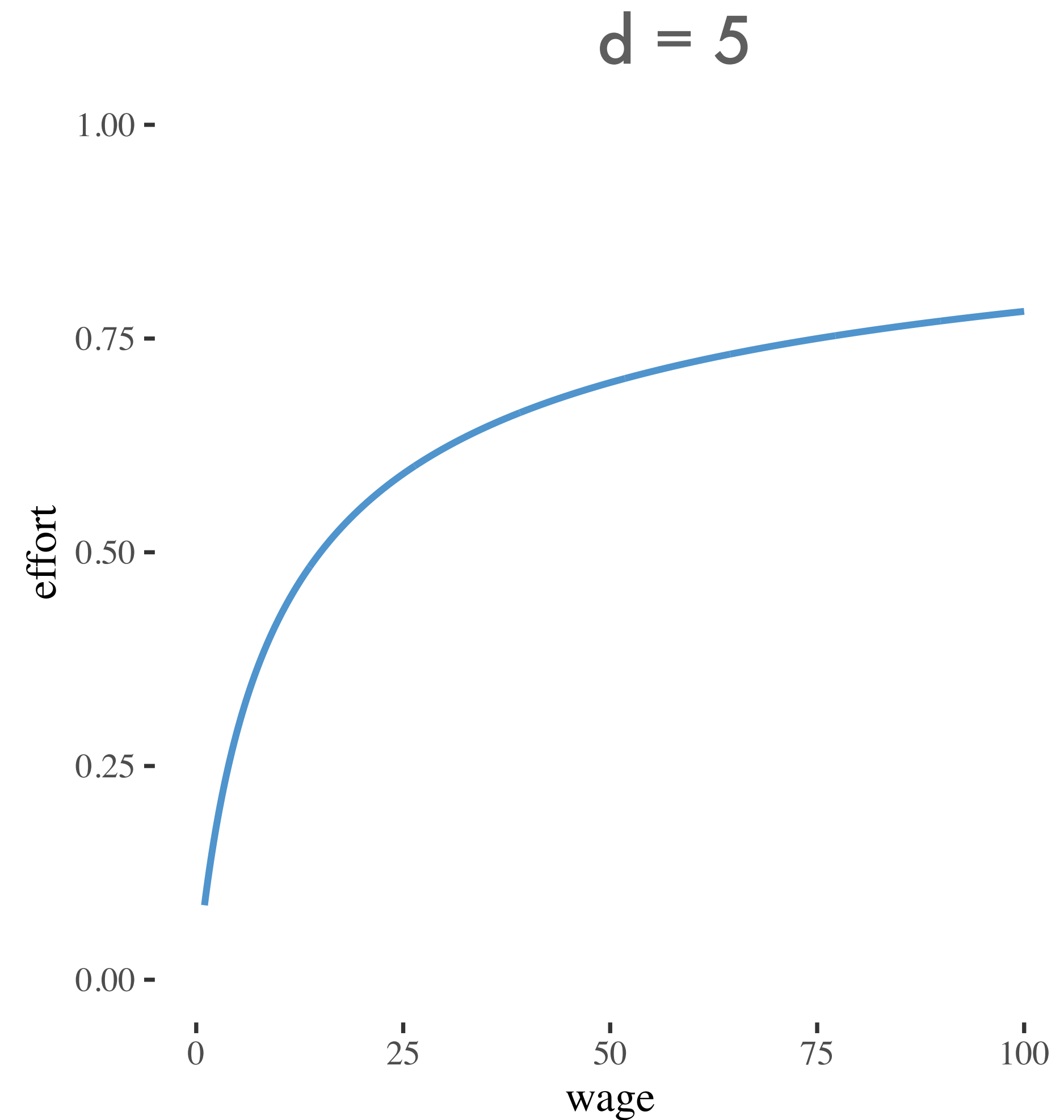
1. minimum wage:  $\omega_{min}$  affects firm wage decision
2. unemployment benefits:  $v$ , results in reserve wage,  $\omega_r$ ,
3. earned income tax credits (EITC):  $\tau$  based on income,  $l$ ,
4. universal income supplement:  $\sigma$ .



# EFFORT RESPONSE CURVE

$$e = 1 - \left( \frac{d}{\omega + d} \right)^{\frac{1}{2}}$$

$d$  represents a household's  
disutility of effort,  
measured in wage units

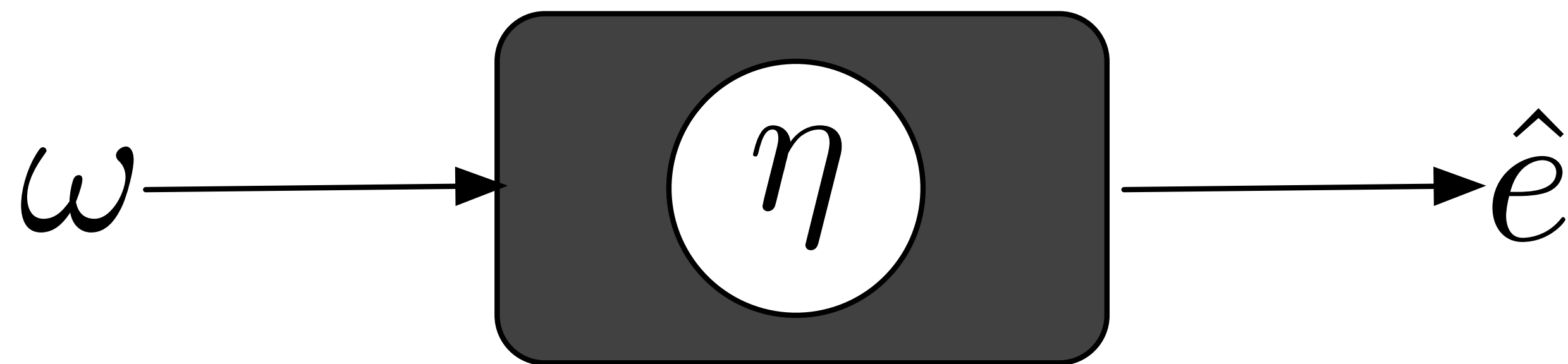


# EFFORT WITH POLICY OPTIONS

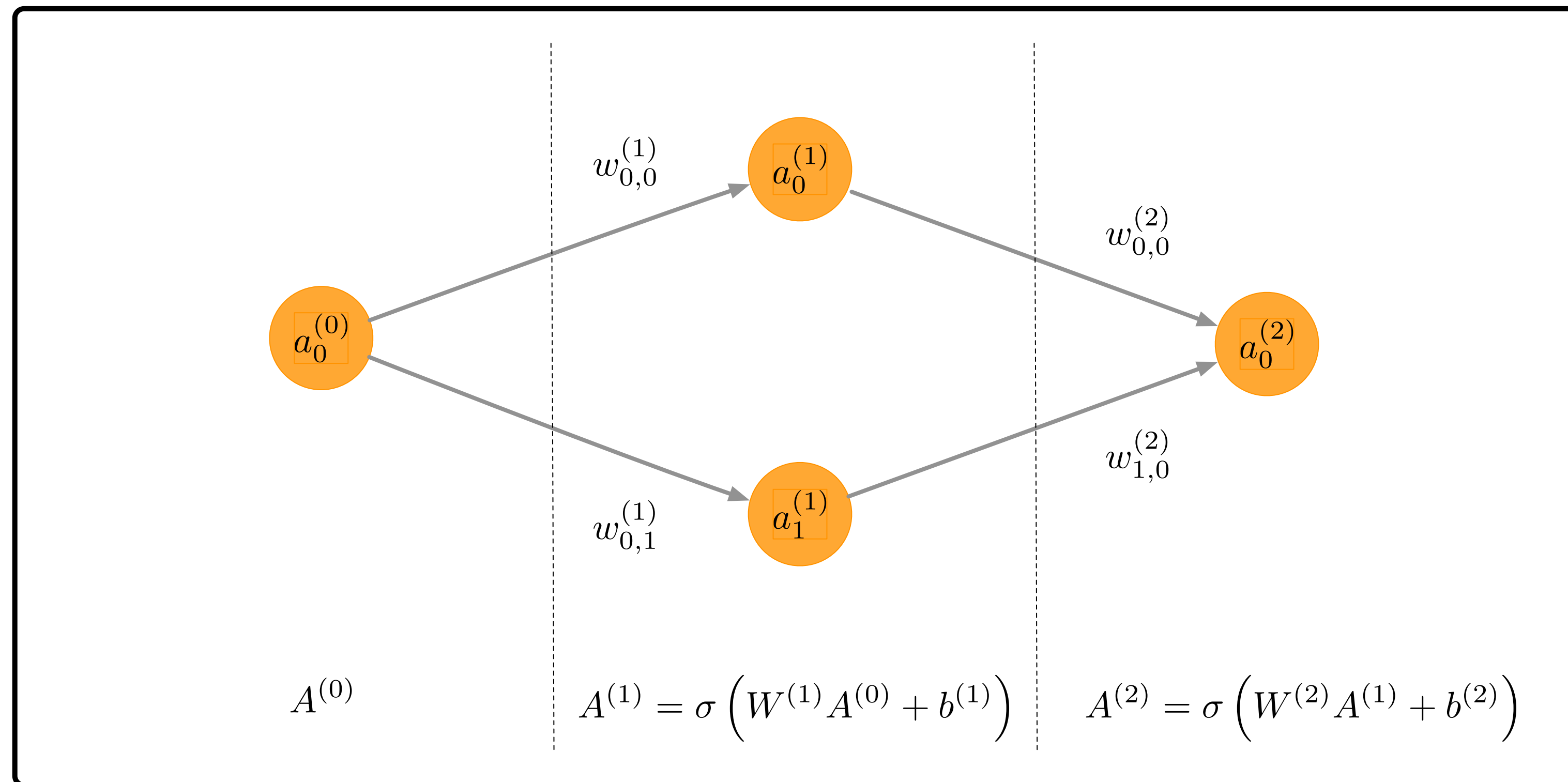
$$e_i = \begin{cases} 1 - \left( \frac{d_i}{\omega - v + \tau_i + d_i} \right)^{\frac{1}{2}} & \text{for } \omega > \omega_{r,i} \\ 0 & \text{for } \omega \leq \omega_{r,i} \end{cases}$$

$$\text{where } \omega_r = \frac{2v}{H_{max}}$$

# FIRM DOESN'T KNOW HOUSEHOLD EFFORT RESPONSE



# NEURAL NETWORK TO MODEL EFFORT EXPECTATION



$$\hat{e} = \eta(\omega) = \sigma(w_{0,0}^{(2)} \sigma(w_{0,0}^{(1)} \omega + b^{(1)}) + w_{1,0}^{(2)} \sigma(w_{0,1}^{(1)} \omega + b^{(1)}) + b^{(2)})$$

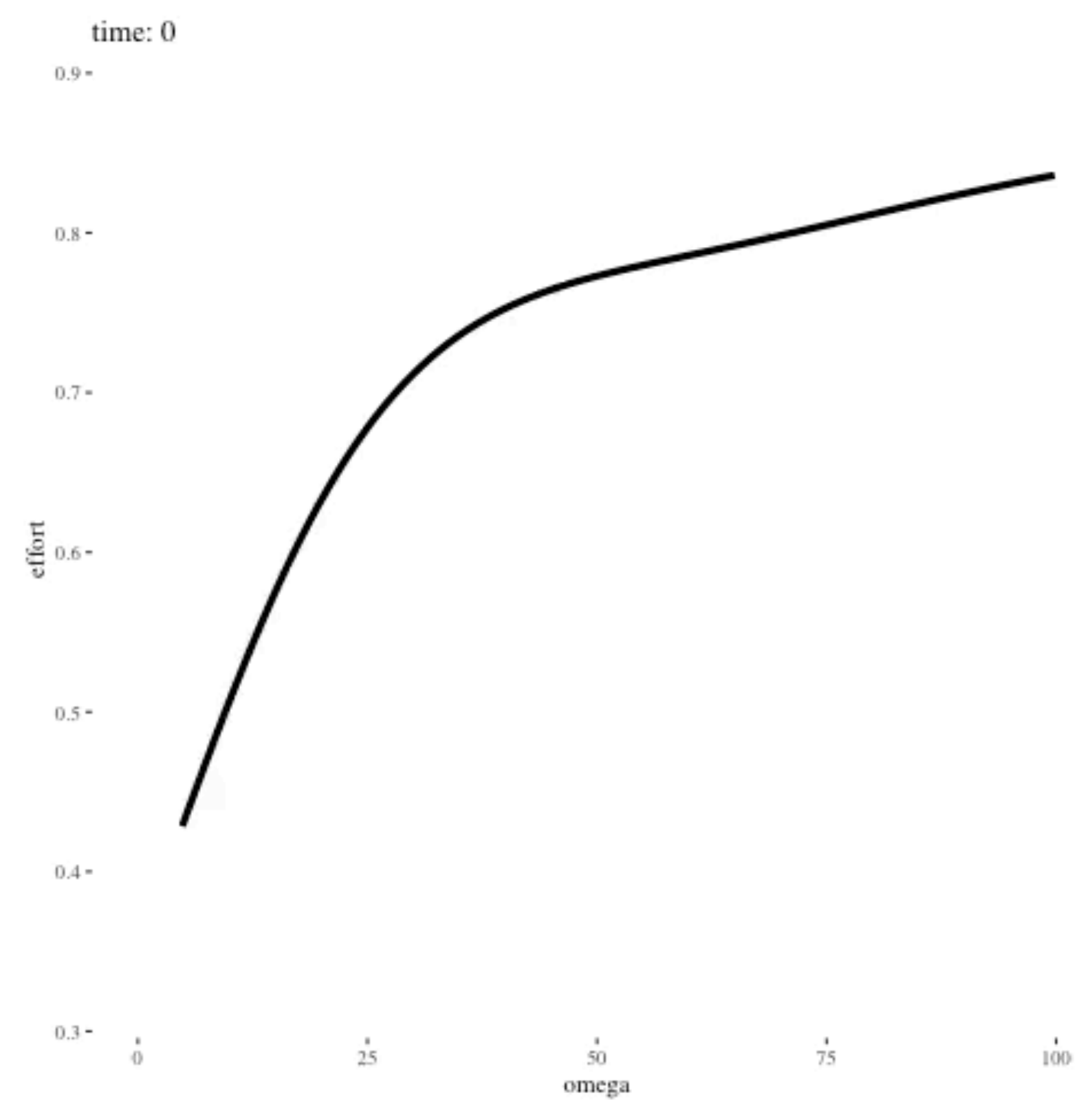


# OBSERVED EFFORT

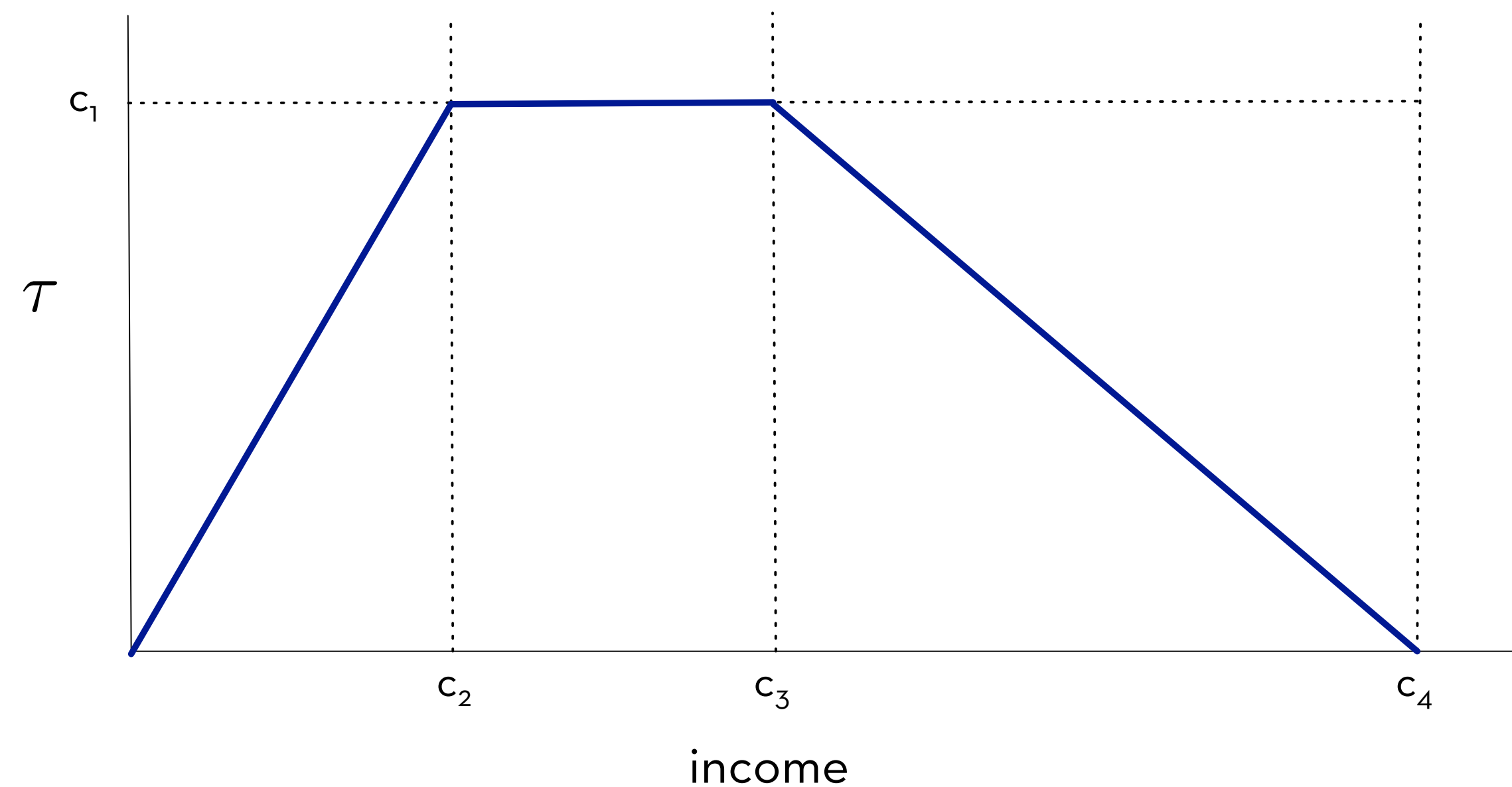
$$e_{obs} = \frac{1}{H_M} \left( \frac{S_S}{A} \right)^{\frac{1}{\gamma}}$$

Firm uses  $\hat{e}$  along with memory of other effort-wage combinations to update  $\eta$ .

# NEURAL NET RESOLVES IN ABOUT TEN STEPS

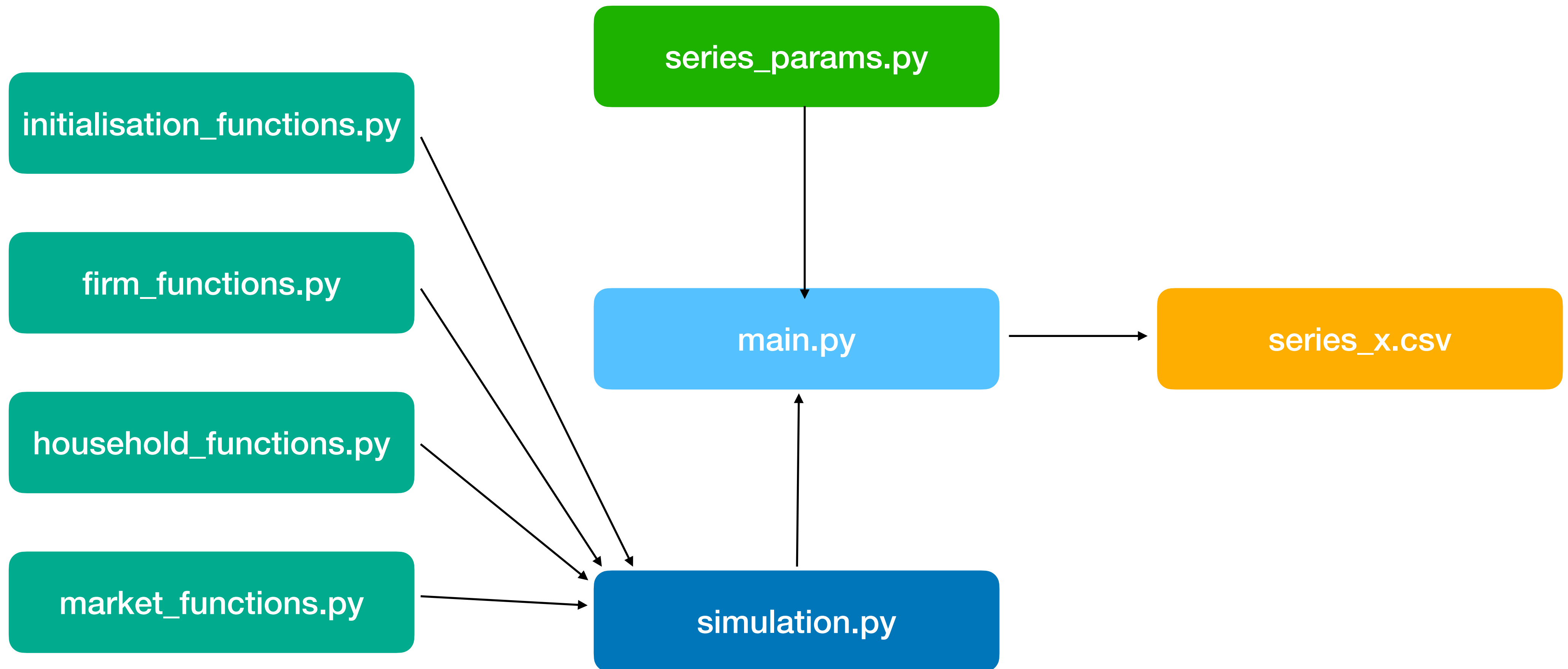


# EARNED INCOME TAX CREDITS (EITC)



$$\tau_i = \begin{cases} \frac{c_1}{c_2} l_i & \text{for } 0 < l_i < c_2, \\ c_1 & \text{for } c_2 \leq l_i \leq c_3, \\ -\frac{c_1}{c_4 - c_3} l_i + \frac{c_1 c_4}{c_4 - c_3} & \text{for } c_3 < l_i < c_4, \\ 0 & \text{for } l_i \geq c_4. \end{cases}$$

# MODEL FILES





**[HTTPS://GITHUB.COM/JMAPPLEGATE/ABMSS2024](https://github.com/JMAPPLEGATE/ABMSS2024)**