Modern Wage Dynamics ODD

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0.1 Model Purpose

The Modern Wage Dynamics Model is a generative model of coupled economic production and allocation systems. Each simulation describes a series of interactions between a single aggregate firm and a set of households through both labour and goods markets. The firm produces a representative consumption good using labour provided by the households, who in turn purchase these goods as desired using wages earned, thus the coupling. The firm has incomplete information with regards to the aggregate households' demand, and attempts to learn this relationship over time.

Each model iteration the firm, in its responsibility to produce for the consumption needs and desires of the households, decides wage, price and labour hours to match sugar supply to sugar demand as well as hour supply and demand. Given price and wage, households decide both effort and hours worked based on a utility function for leisure and consumption. A labour market construct chooses the minimum of hours required and aggregate hours supplied, and aggregates the hours provided. The firm then uses these inputs to produce goods. Given the hours actually worked, the households decide actual consumption and a market chooses the minimum of goods supplied and aggregate demand. The firm uses information gained through observing market transactions about effort and consumption demand to refine their conceptions of the population's sugar demand.

The purpose of this model is to explore the general behaviour of an economy with coupled production and allocation systems, directly addressing the opposing dynamics of higher wages suppressing labour hours but increasing demand for goods produced by that labour.

0.2 Model Overview

The model consists of a single aggregate firm representing all economic production and multiple households. The firm produces a single representative good, called *sugar*. The households provide wage labour to the firm and consume sugar. The model begins with the firm choosing the wage, ω, price of sugar, p, and hours requested, H_D. Given ω and p, each household decides the hours to supply to the production process according to its preference for leisure and consumption. A labour market determines H_M as the lesser of H_D and H_S. Given H_M, the firm supplies a quantity of sugar to the households, S_S, based on its production function. Each household then decides how much sugar it desires according to its budget, S_i. The market then aggregates the household demand and determines the quantity of sugar sold as the lesser of S_S and S_D. Finally, the firm updates its expectation of household sugar demand, and makes modifications to price and wage as appropriate. Figure 1 shows a schematic overview of the model process. Households are assumed to have a necessary base level of consumption, which implies a minimum level of

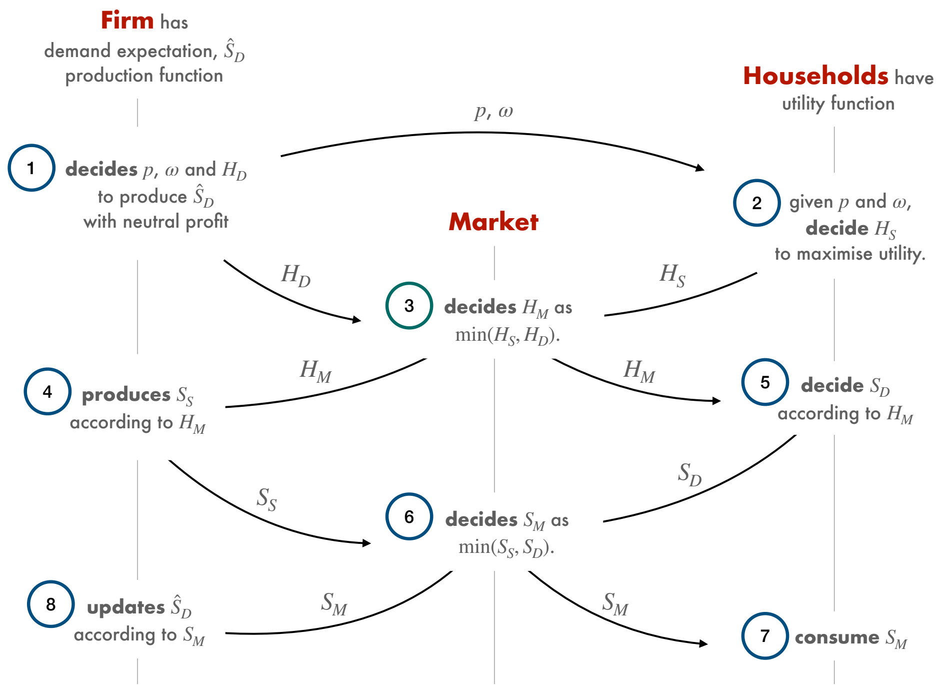

Figure 1: Modern Wage Dynamics Logic Flow

The firm starts with an initial price and wage value, as well as an expectation of sugar demand from the current model state. Firm production is modeled as a variant of the Solow production function,

$$S = AH^{\gamma} \tag{1}$$

where A combines the total factor productivity coefficient as well as the capital contribution. Thus, Y is solely a function of labour.

The firm has an expectation of aggregate household demand, and in its role as representing all economic production will attempt to meet that demand. Thus, the hours requested by the firm are

$$H_D = \left(\frac{S_D}{A}\right)^{\frac{1}{\gamma}}. \tag{2}$$

The parameter value for A is derived from making it possible that there exists the technology such that households can at least self-produce the required consumption, S_N, with maximum household hours at constant returns to scale, or

$$A \geq \frac{S_N}{H_{max}}. \tag{3}$$

For communal production, γ will represent increasing returns to scale, as the division of labour claims higher labour productivity than each household producing for itself. The model value for

γ is derived to be large enough that multiple households can produce nS_N with each household working the minimum hours required to obtain wages sufficient to consume that necessary quantity (H_N).

$$\gamma \geq \frac{\ln\left(\frac{nS_N}{A}\right)}{\ln(nH_N)}. \tag{4}$$

Each household decides the quantity of work hours it will provide to the firm. These hours are determined based on a Cobb-Douglas utility function for consumption and leisure. Since sugar is the aggregate consumption good in the model, the quantity of household consumption is not completely arbitrary as the household requires a necessary base level of consumption to survive, S_N. Therefore, each household will work a number of hours regardless of the utility from those hours, H_N.[1]

Given and maximum number of available hours per household, H_{max}, leisure can be described as $(H_{max} - H_N - H_O)$ were H_O are discretionary hours, and the household utility function is

$$U(H_O, S_O) = (H_{max} - H_N - H_O)^{\alpha}(S_O)^{\beta} \tag{5}$$

where $\alpha + \beta = 1$ and S_O represents discretionary sugar consumption. The total household budget, B, is the wage income, $\omega(H_N + H_O)$ plus any positive monetary holdings, $\min(0, m)$.[2]

Tribute hours, H_N, are calculated as

$$H_N = \max\left(0,\ \frac{pS_N - m}{\omega}\right) \tag{6}$$

and we end up with two different cases, one where tribute hours are 0 and one where they are greater than 0. If $H_N > 0$ the household doesn't have enough non-wage holdings to cover S_N, but anything they do have goes toward S_N thus reducing H_N. Therefore discretionary consumption is based on the wage earned from discretionary hours alone. If $H_N = 0$, then the household has enough non-wage funds to cover necessary consumption, and can apply the remainder toward discretionary consumption. In the first case,

$$U(H_O) = (H_{max} - H_N - H_O)^{\alpha}\left(\frac{\omega H_O}{p}\right)^{\beta} \tag{7}$$

and maximising utility yields

$$H_O = \beta(H_{max} - H_N). \tag{8}$$

[1]These hours can be described as *tribute hours*, per the late David Graeber, since they are not available for the free use of the worker, but owed to someone else in order to obtain material goods for survival.

[2]Given that households require a base level of consumption, S_N, households may encounter negative holdings as they incur 'debt' to meet basic consumption needs.

Total hours supplied by the household will be $H_O + H_N$, or

$$H = \beta H_{max} + \alpha H_N \tag{9}$$

Applying Equation 6, we find

$$H = \beta H_{max} + \frac{\alpha}{\omega}\left(pS_N - m\right) \text{ for } H_N > 0. \tag{10}$$

Note that since $pS_N - m$ is positive, as ω increases H decreases, and as ω decreases H increases.

In the second case, $H_N = 0$, $m \geq pS_N$ and there may be a remainder of the non-wage funds that can be applied toward discretionary consumption, such that

$$U(H_O) = (H_{max} - H_O)^{\alpha}\left(\frac{\omega H_O}{p} + \frac{m}{p} - S_N\right)^{\beta}. \tag{11}$$

Utility maximising yields

$$H_O = \beta H_{max} - \frac{\alpha}{\omega}\left(m - pS_N\right). \tag{12}$$

Since $H_N = 0$,

$$H = \beta H_{max} - \frac{\alpha}{\omega}\left(m - pS_N\right) \text{ for } H_N = 0. \tag{13}$$

Note that since $m - pS_N$ is positive, H increases and decreases with ω, which differs from the $H_N > 0$ case. Thus we have two different hour response regimes to wage, but Equations 10 and 13 are identical.

Equations 10 and 13 give hours worked per household, H_i, specific to that household's unique values of α, β and m. The market aggregates the hours offered by the households into H_S, as

$$H_S = \sum_{i}^{n} H_i$$

and completes the labor market exchange such that H_M is the lesser of H_D and H_S. This market also determines hours worked per household, H_W.

$$H_W = \begin{cases} H & \text{for } H_S \leq H_D, \\ \dfrac{H_D}{H_S} H & \text{for } H_S > H_D \end{cases} \tag{14}$$

such that in the case of labour supply greater than labour demand, each household provides a proportional amount of labour to meet demand.

The quantity of sugar produced, S_S, is given by the production function described by Equation 1.

The households will buy the sugar they can afford in addition to the necessary quantity[3], S_N,

[3] According to Keynes's marginal propensity to consume.

so

$$S_i = \max\left(S_N,\ \frac{1}{p}(\omega H_i + m_i)\right) \tag{15}$$

and the aggregate demand is

$$S_D = \sum_i^n S_i. \tag{16}$$

The market chooses the lesser of S_S and S_D as the quantity of sugar exchanged. In the case of $S_S < S_D$, the sugar supplied by the firm is divided proportionately between the households and notated as S_C, sugar consumed.

$$S_C = \begin{cases} S & \text{for } S_D \leq S_S, \\ \dfrac{S_S}{S_D} S & \text{for } S_D > S_S. \end{cases} \tag{17}$$

Any unspent wages are saved for future consumption and added to m_i. Sugar, on the other hand, is not considered storable and any unsold sugar is removed from the system. Households will incur debt if necessary in order to purchase S_N, which is represented by negative m values, though this debt does not affect future consumption decisions, unlike positive holding values.

The sugar demand at the offered price and wage is observed as S_D. The firm remembers the last μ observed values of S_D and takes the weighted mean of those values, then updates the values of p or ω in order to match supply to this demand. The heuristics used by the firm to modify p and ω values are based on sugar supply and demand as well as the hours supply and demand relationships. Setting $H_S = H_D$,

$$\beta H_{max} + \frac{\alpha}{\omega}\left(pS_N - m\right) = H_D \tag{18}$$

which can be solved for both p and ω.

$$p = \frac{1}{S_N}\left(\frac{\omega}{\alpha}\left(H_D - \beta H_{max}\right) + m\right) \tag{19}$$

and

$$\omega = \frac{\alpha(pS_N - m)}{H_D - \beta H_{max}} \tag{20}$$

for $m \geq 0$. If $H_S > H_D$, then adjusting toward equality would lower p or raise ω. In the case of $H_S < H_D$, adjusting to equality would raise p or lower ω.

0.3 Model Initialisation

At the start of the simulations, initial values for wage and price are specified as input parameters. The fixed unique household parameters α and β are assigned by random draws of the uniform distribution at the start of the model run. Sugar demand is initalised by randomly selecting μ households and calculating their demand with initial wage and price, finding the average, then

	$S_S > S_D$	$S_S < S_D$
$H_S > H_D$	$p = \frac{1}{S_N}\left(\frac{\omega}{\alpha}\left(H_D - \beta H_{max}\right) + m\right)$ (decrease p)	$\omega = \frac{\alpha(pS_N - m)}{H_D - \beta H_{max}}$ (increase ω)
$H_S \leq H_D$	$\omega = \frac{\alpha(pS_N - m)}{H_D - \beta H_{max}}$ (decrease ω)	$p = \frac{1}{S_N}\left(\frac{\omega}{\alpha}\left(H_D - \beta H_{max}\right) + m\right)$ (increase p)

Table 1: Explicit and qualitative changes to p and ω given relative sugar and hours conditions.

multiplying this value by the number of households.

0.4 Entities, State Variables and Scale

The model entities are a single firm, n households, and a market through which the firm and the households interact. All household values are stored as vectors indexed by household number, which allows for numerous opportunites to employ vectorised operations instead of iterating through all households. Table 2 gives the models parameters that are set at the start of the simulation and remain constant. Table 3 gives the model state values that update at varying times over the course of the simulation.

0.5 Process Overview and Scheduling

Each model step represents a week time duration, and iterative operations are conducted in consecutive household order. The market interactions are based on aggregate household outcomes, and a single household outcome does not impact another household during any given operation, though aggregate outcomes do affect all households over time.

We run the model by calling `main.py`, without any arguments. All parameters including output directory and filenames are specified in the `series_params.py` file. The `main.py` file also calls

Parameter	Description	Values
n	number of households	100
ω_0	initial value for wage	10
p_0	initial value for price	1
μ	firm's memory length for observed demand values	5
A	production function coefficient	3
γ	firm's production function exponent	1.2
H_{max}	maximum available household hours per step	400
S_N	amount of consumption necessary for household survival	300
α_i	leisure term exponent in utility function for household i	$\mathcal{U}[0,1]$
β_i	consumption term exponent in utility function for household i	$1-\alpha_i$

Table 2: Modern Wage Dynamics Parameter Values.

Variable	Description
ω	wage set by the firm
p	price of sugar set by the firm
H_D	quantity of hours demanded by the firm
π	firm profit
I	quantity of sugar in the firm's inventory
H_i	desired number of hours supplied by household i
H_S	aggregate number of hours supplied by the households
H_M	number of work hours exchanged in the labour market
$H_{W,i}$	number of hours worked by the household i
N	aggregate effective labour provided by the households
S_S	quantity of sugar the firm produces according to available effective labour
ι_i	wage income for household i
S_i	quantity of sugar demanded by household i
S_M	quantity of sugar exchanged in the sugar market
$S_{C,i}$	quantity of sugar consumed by household i
m_i	monetary holdings for household i
U_i	utility for household i

Table 3: Modern Wage Dynamics State Variables.

`initialisation_functions.py`, `firm_functions.py`, `household_functions.py` and `market_function.py`.

The process overview is:

```
main.py reads parameter sets from series_params.py
for each distinct parameter set:
    for each repetition of parameter set:
        assign parameter values
        initialise households
        initialise S_D
        save initial values
        for each step:
            firm decides hours demanded
            households decide effort and hours supplied
            market conducts labour exchange
            firm produces sugar with market labour
            households determine sugar demand
            market conducts sugar exchange
            firm calculates profit
            households calculate utility
            households update ledgers
            firm updates inventory
            firm adjusts wage or price values
            save step values
write data frames to files
```

0.6 Considerations and Explorations

The model can currently be run with multiple parameter values and assumptions.

1. The model can be run without necessary consumption, in which case $S_N = 0$.
2. The model can be used as a base to implement *efficiency wage theory*, whereby a worker's effort per hour, e, is a function of wage, and production depends on eH instead of simply H.
3. The model can be modified to run numerous policy experiments, such as
    (a) minimum wage, ω_{min},
    (b) unemployment benefits, υ, which results in households having a reserve wage, ω_r,
    (c) earned income tax credits (EITC), τ,
    (d) and universal income supplement, σ.

4. The model is very sensitive to production function parameters, A and γ. How does the model behaviour change with A, γ and n? What meaning could these changes have? Is it possible to quantify these relationships formulaically?

5. How does the model behavoiur change if the firm uses existing inventory to meet demand, thus decreasing production and hours demanded if $I > 0$?

0.7 Inputs and Outputs

The model reads simulation parameter values from the `series_params.py` and sends simulation results to a designated `.csv` file. Input parameters are described in Table 4, and some parameters accept multiple values to allow for numerous parameter sets within the same series.

Parameter	Description	value
`directory`	name of output file directory	single
`series_name`	filename for series results	single
`seed`	rng seed for replicable results	single
`t_max`	the number of time steps the model will run	single
`omega_max`	firm's maximum wage for initialising η	single
`H_max`	maximum available household hours per step	single
`mu`	firm's memory length for updating η	single
`n`	number of households	multi
`omega_0`	initial wage value	multi
`A`	value of production function coefficient	multi
`gamma`	value of production function exponent	multi
`S_N`	required household consumption in units of sugar	multi
`p_0`	initial price value	multi

Table 4: Input parameters specified in the `series_params.py` file.

For each time step the model saves the following simulation parameter and variable values to a dataframe: `set`, `run`, `step`, `t_max`, `n`, `omega_max`, `H_max`, `A`, `gamma`, `mu`, `S_N`, `omega_0`, `p_0`, `I`, `pi`, `total_pi`, `omega`, `p`, `H_D`, `S_S`, `S_P`, `perceived_effort`, `household`, `alpha`, `beta` ,`H_N`, `H`, `H_W`, `m`, `S`, `S_C`, `income`, `U`, `expenditure`, `N`, `H_S`, `H_M`, `S_D`, `S_M`. The variable *set* is the specific combination of parameter values, and *run* is the iteration under those values.