Midterm Exam (Differential Calculus)

October 16-18, 2024

I. Multiple Choice. Choose the correct answer and write it on the test booklet. (3 points each).

1. The derivative of $y = e^x \cos x^2$ is
   a. $-e^x \sin x^2$ c. $e^x (\cos x^2 - 2x \sin x^2)$
   b. $e^x (\cos x^2 - 2x \sin x^2$ d. $-2xe^x \, sinx$

2. The derivative of $y = (x+1)^3 - x^3$ is
   a. $3x + 6$ c. $3x - 3$
   b. $6x - 3$ d. $6x + 3$

3. The derivative of $y = \sec (x^2 + 2)$ is
   a. $2x \, cosx(x^2+2)$ c. $-cosx(x^2+2)cot(x^2+2)$
   b. $2xsec(x^2+2)tan(x^2+2)$ d. $cos(x^2+2)$

4. The derivative of $y = \ln (1 - x^2)$ is
   a. $\frac{1}{1-x^2}$ b. $\frac{1}{1+x}$ c. $\frac{1}{1-x}$ d. $\frac{1}{1+x^2}$

5. The derivative of $x^2 \, tanx$ is
   a. $2xsec^2x$ c. $2xtanx + x^2cotx$
   b. $2xtanx + (xsecx)^2$ d. none of these

II. Find the derivative of the following using the general formula. (5 points each)

6. $f(x) = x^2 - x$
7. $f(x) = \frac{1}{\sqrt{x}}$

III. Find the derivative of the following using the rules of differentiation. (5 points each)

8. $f(x) = (4x^2 - x)(x^3 - 8x^2 + 12)$

9. $f(x) = \frac{\sqrt{x}+2x}{7x-4x^2}$

10. $f(x) = 2e^x - \cos x^2$

IV. Find the derivative of the following using implicit differentiation. (5 points each)

11. $3x^2 - 2y^3 = 0$
12. $\ln(x+y) = x + y$

V. Find the higher derivatives of the following as stated. (5 points each)

13. $2y + 3x^3 = 1, find \; its \; second \; derivatives.$
14. $f(x) = \sqrt{x} - 3x^2, find \; its \; third \; derivatives.$

VI. Find the partial derivatives of the following. (10 points each)

15. $F = f(x,y,z) = xy^3z^2 + 3x^3yz, find \; \frac{\partial F}{\partial x} \; and \; \frac{\partial F}{\partial z}$

16. $F = f(x,y) = e^x cosxy, find \; \frac{\partial F}{\partial x} \; and \; \frac{\partial F}{\partial y}$