Make FunBlocks alive

Marvin FOURASTIE

Master project

Motivations

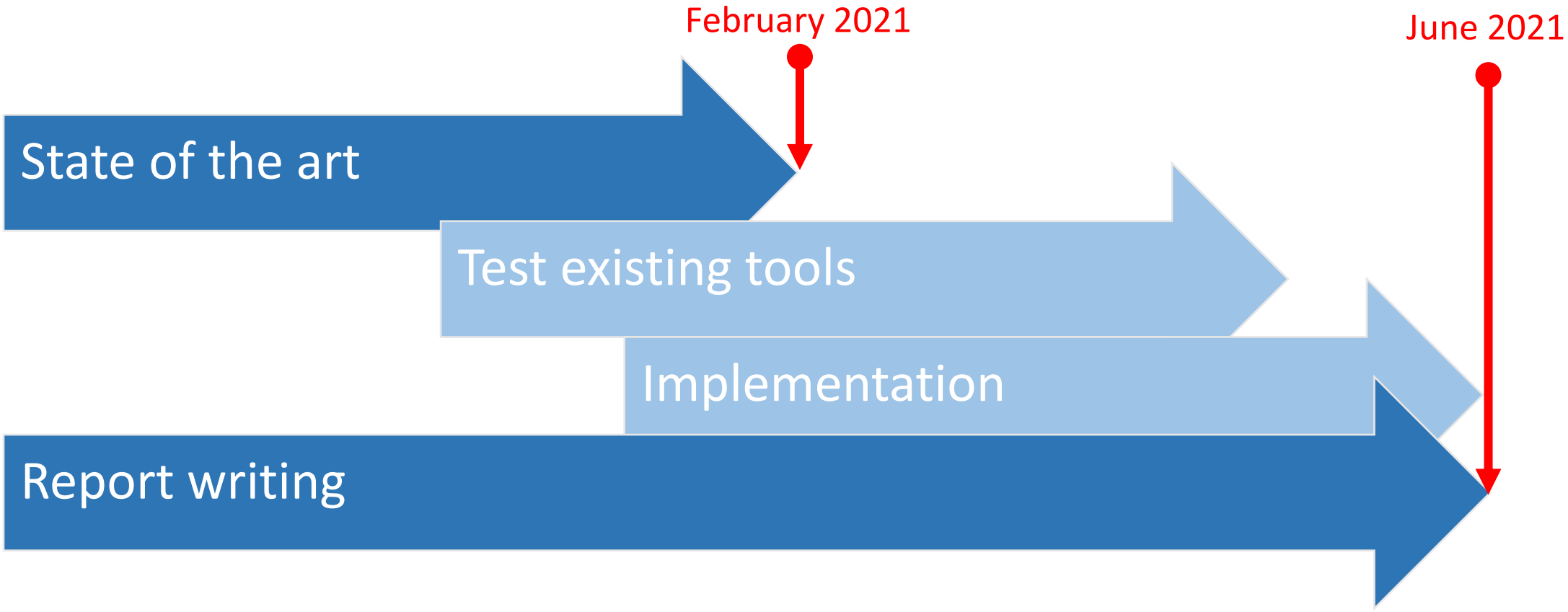
Educational purpose

Imperative paradigm alternative

Based on rewrite systems

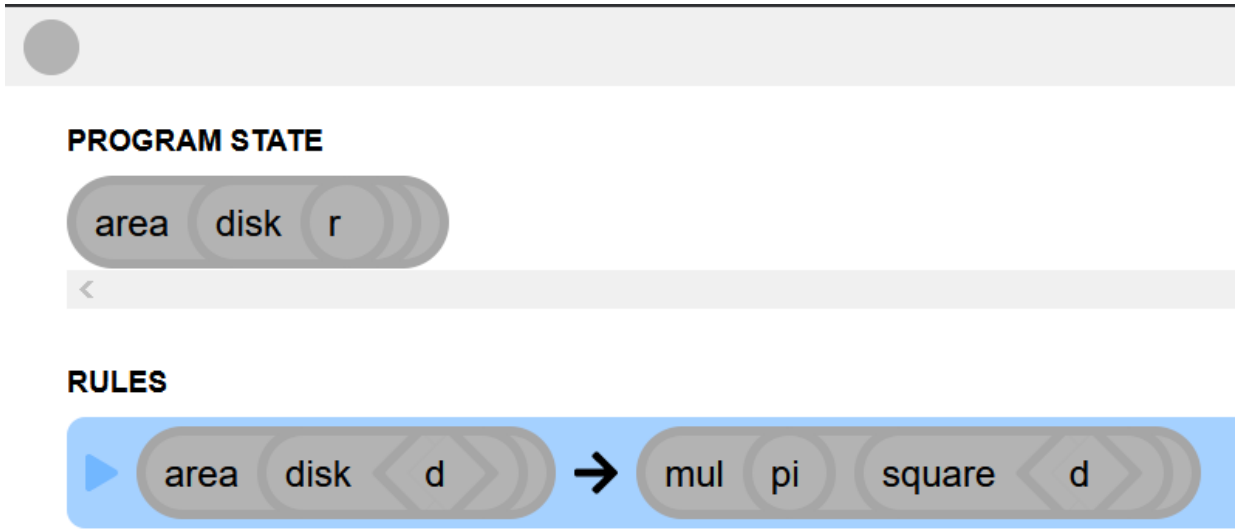
FunBlocks

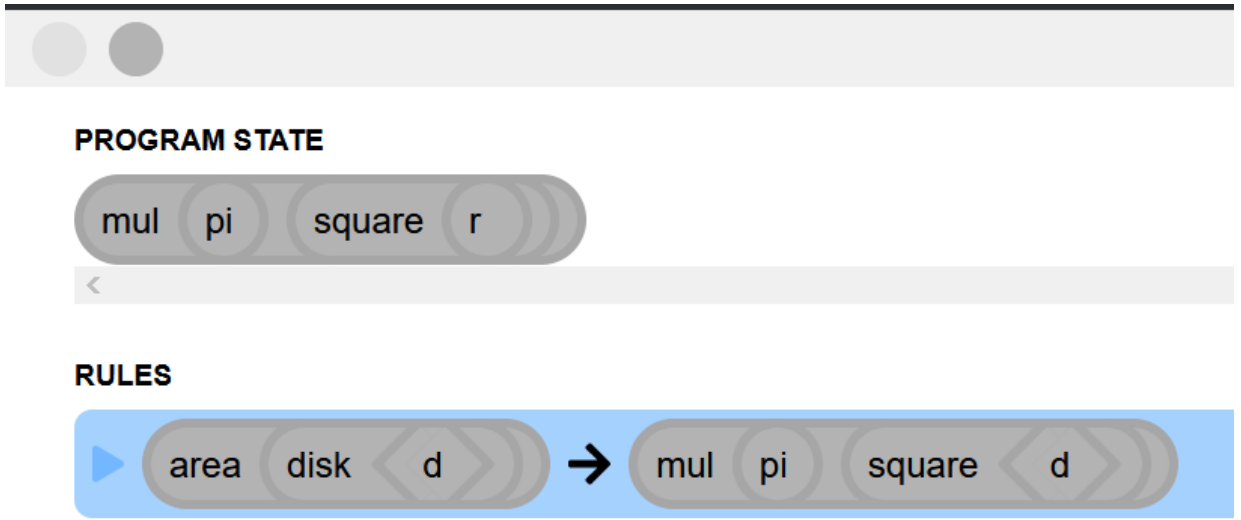
Road Map


February 2021
June 2021
State of the art
Test existing tools
Implementation
Report writing

FunBlocks

```
1  init area(disk(r))
2
3  case area(disk($d)) => mul(pi,square($d))
```





FunBlocks

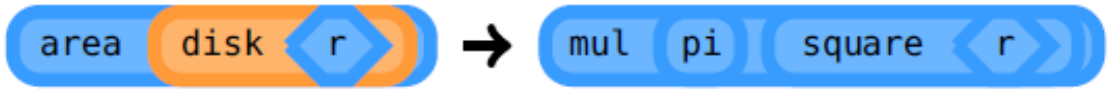
Declarative paradigm

```
area(disk(r))
```

Based on rewrite systems

```
case area(disk($d)) => mul(pi,square($d))
```

Static typing




```
type Tree $t :: empty | leaf $t | node (Tree $t) (Tree $t)
```

Goals

Provide users with valuable insights about their program

Verification of rewrite systems

Rewrite systems

Stack operators

Zero = {0}

Nat = Zero $\cup$ succ(Nat)

Empty = Λ

Stack = Empty $\cup$ push(Nat, Stack)

top : Stack $\rightarrow$ Nat

pop : Stack $\rightarrow$ Stack

alternate : Stack $\times$ Stack $\rightarrow$ Stack

Rewrite systems

Canonical rewrite system

top(push(x, y)) = x

pop(push(x, y)) = y

alternate(Λ, z) = z

alternate(push(x, y), z) = push(x, alternate(z, y))

top(push(x, y)) → x

pop(push(x, y)) → y

alternate(Λ , z) → z

alternate(push(x, y), z) → push(x, alternate(z, y))

Rewrite systems

Termination

Confluence

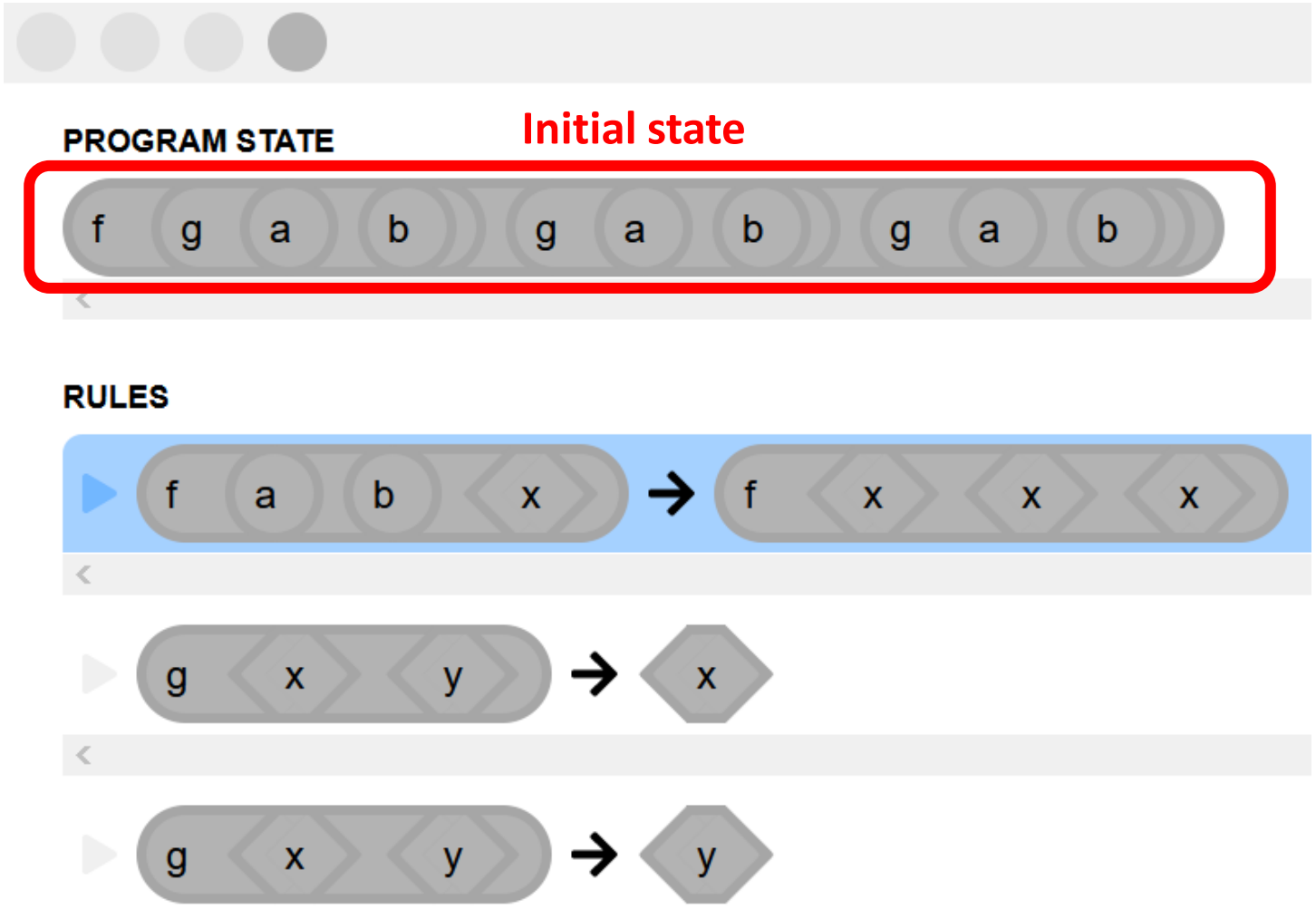
Soundness

Completeness

Correctness

Undecidable in general

Termination



Reduction order

Monotone → $s_i > t \rightarrow f(s_1, \ldots, s_i, \ldots, s_n) > f(s_1, \ldots, t, \ldots s_n)$
For f of arity n

Close under substitution → $s > t \rightarrow \sigma s > \sigma t$, for all substitution σ

Well-founded → no infinite descending chain

$(\mathbb{N}, <)$ is well-founded
$(\mathbb{Z}, <)$ is not well-founded

Termination

A term rewriting system is terminating

if and only if

it admits a compatible reduction order $<$
(if $l > r$ for every rewrite rule $l \to r$)

Verification of termination

Polynomial interpretation

$$f(a, x) \to x$$

$$f(g(x), y) \to g(f(x, y))$$

weight

$$w(a) = 1$$

$$w\big(g(t)\big) = 1 + w(t)$$

$$w\big(f(t_1, t_2)\big) = 2w(t_1) + w(t_2)$$

Polynomial interpretation

$$f(a,x) \to x$$

$$w\big(f(a,x)\big) = \mathbf{2} + \boldsymbol{w(x)}$$

$$w(x) = \boldsymbol{w(x)}$$

$$w\big(f(a,x)\big) > w(x)$$

$$f(g(x),y) \to g(f(x,y))$$

$$w(f(g(x),y)) = \mathbf{2} + \mathbf{2}\boldsymbol{w(x)} + \boldsymbol{w(y)}$$

$$w(g(f(x,y)) = \mathbf{1} + \mathbf{2}\boldsymbol{w(x)} + \boldsymbol{w(y)}$$

$$w(f(g(x),y)) > w(g(f(x,y))$$

Reduction order → Termination

Algorithms

Recursive Path Ordering	→	Order based on the mutisets
Knuth-Bendix Ordering	→	Based on weights assigned to operators
Dependency pairs	→	Prove innermost termination

Termination

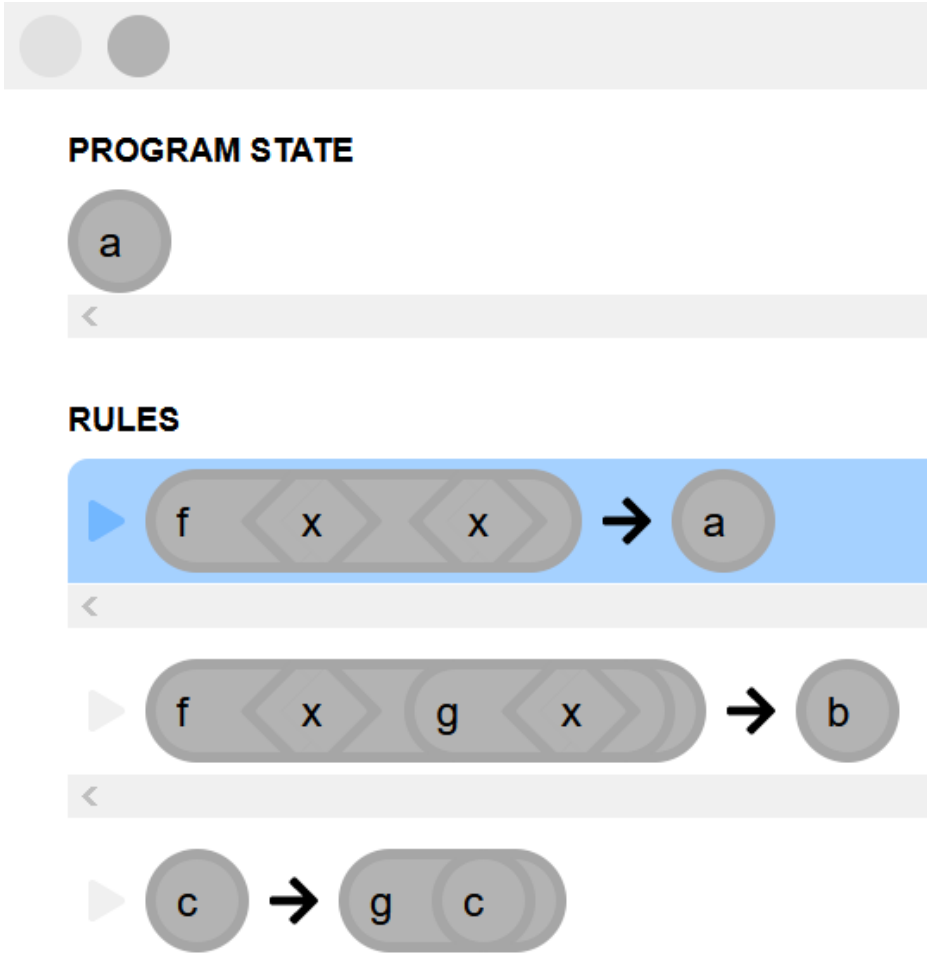
=

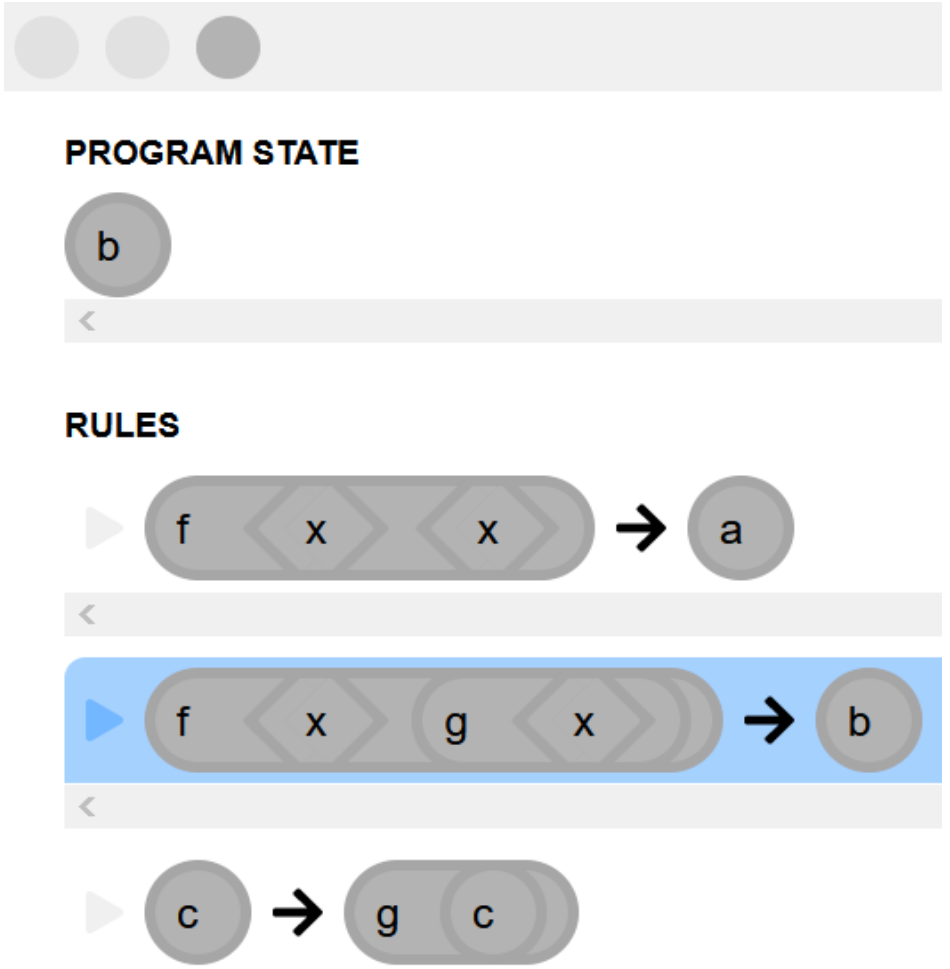
Direct proof
(polynomial, LBO, KBO,…)

+

Dependency pairs and
size-change principle

Confluence

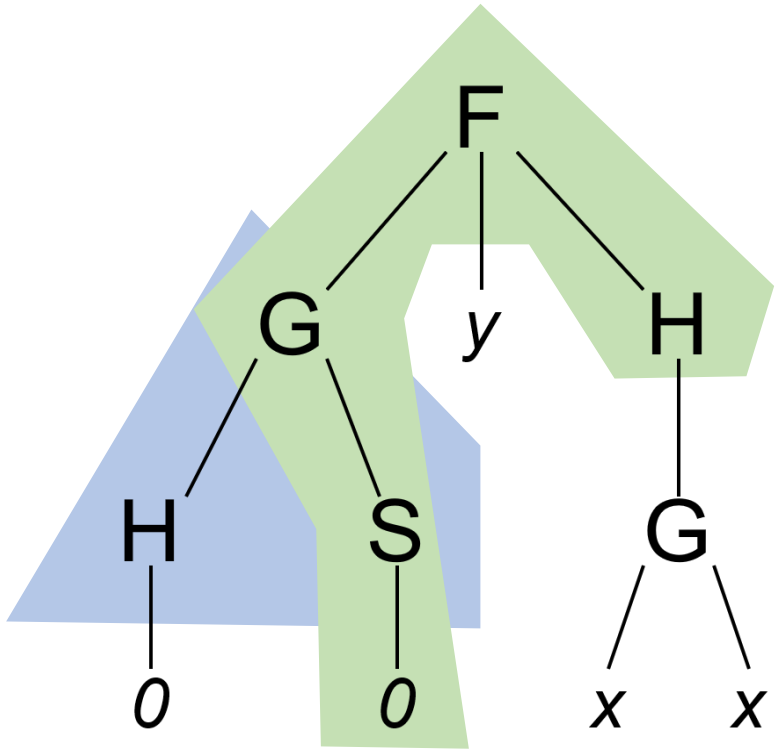

PROGRAM STATE
a
RULES
f x x → a
f x g x → b
c → g c


PROGRAM STATE
b
RULES
f x x → a
f x g x → b
c → g c

Overlap and critical pairs

$\rho_1 : F\left(G\left(x, S(0)\right), y, H(z)\right) \to x$

$\rho_2 : G(H(x), S(y)) \to y$



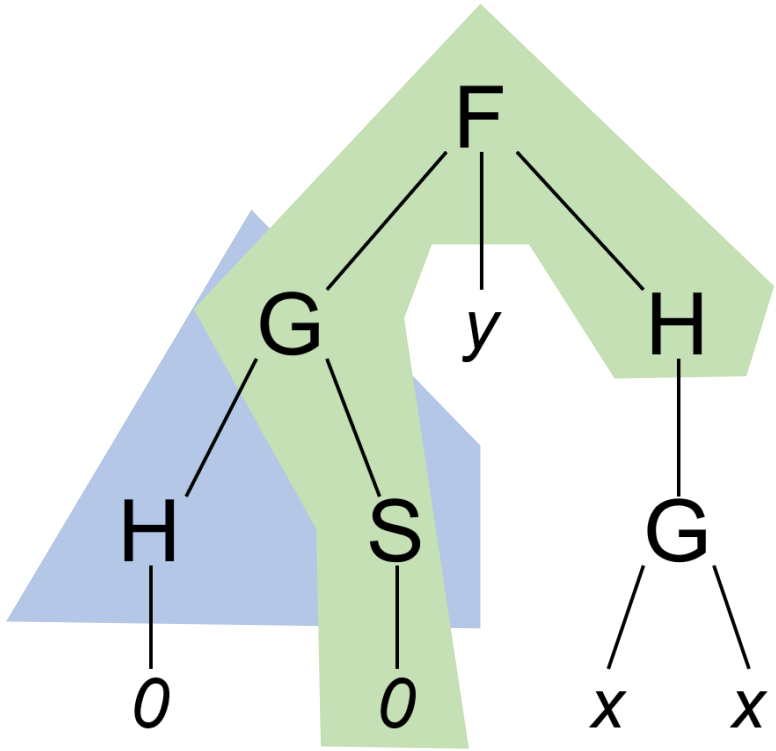
Overlap and critical pairs

Overlapping:

Term: $F\left(G\left(H(0), S(0)\right), y, H(z)\right)$

$F\left(G\left(\square, S(0)\right), \square, H(\square)\right)$

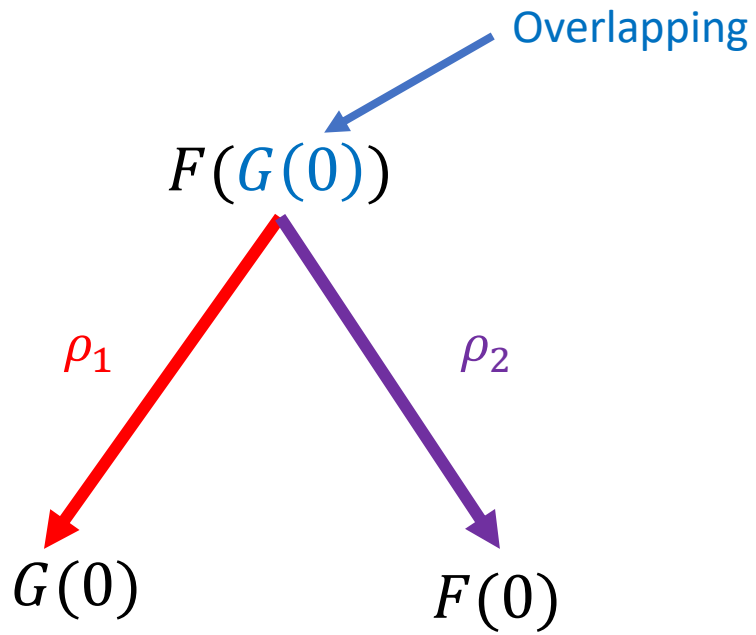
$G(H(\square), S(\square))$



Overlap and critical pairs

$\rho_1 : F(x) \rightarrow G(0)$

$\rho_2 : G(x) \rightarrow 0$



$< G(x), F(x) >$ is called critical pair

Critical Pair Lemma

A terminating rewriting system is confluent

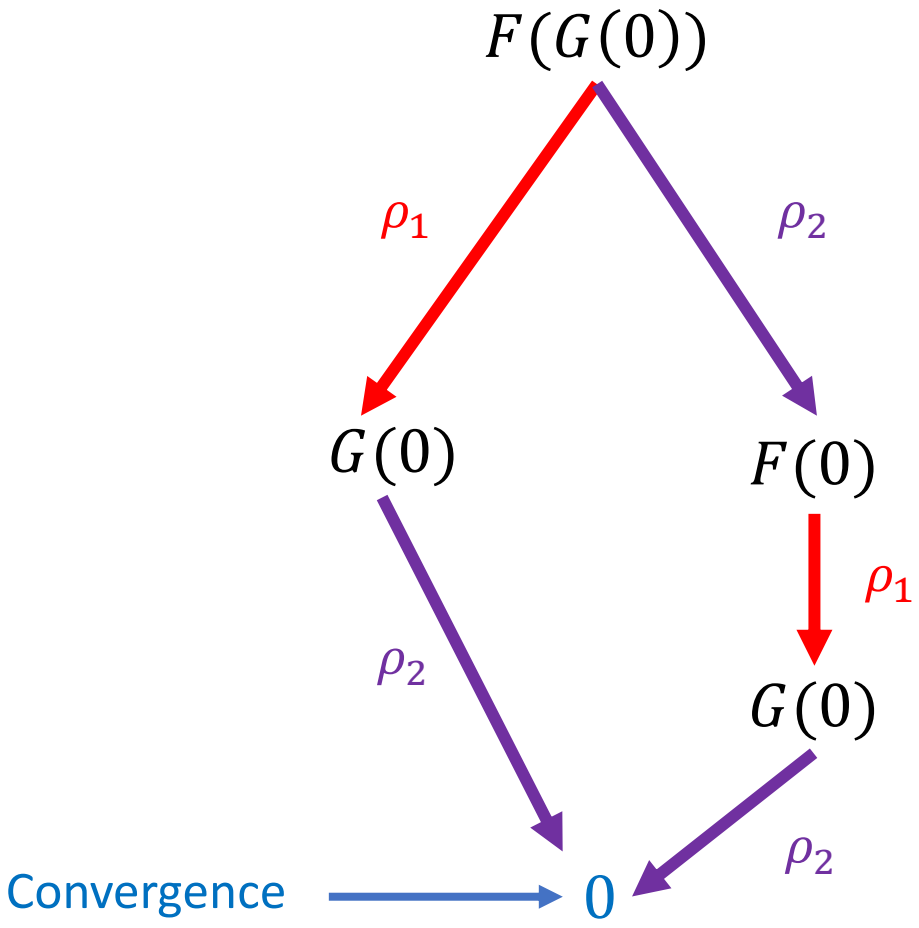
if and only if

all critical pairs are convergent

Critical Pair Lemma

$\rho_1 : F(x) \to G(0)$

$\rho_2 : G(x) \to 0$



Knuth-Bendix completion

Input:

A set of equation

A reduction ordering <

$$
\begin{aligned}
1 \cdot x &= x \\
x^{-1} \cdot x &= 1 \\
(x \cdot y) \cdot z &= x \cdot (y \cdot z)
\end{aligned}
$$

Non-confluent

Knuth-Bendix completion

Output:

Terminate successfully	→	Terminating and confluent rewrite system
Loop indefinitely	→	Non-terminating rewrite system
Fail	→	Rule which cannot be ordered (i.e. commutative operator)

Knuth-Bendix completion

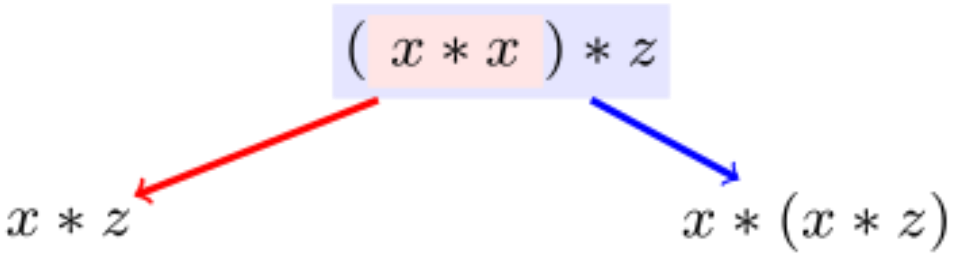
Basic rules:

Orienting	→	Transform $s = t$ to $s \to t$
Adding	→	Add $s = t$ in the set of equation
Simplifying	→	Simplify $s = t$ in $s' = t'$
Deleting	→	Delete trivial rules $s = s$

Knuth-Bendix completion

Adding  Add $s = t$ in the set of equation

$$(x * y) * z \quad \rightarrow x * (y * z)$$
$$x * x \quad \rightarrow x$$



No convergent

$$(x * y) * z \quad \rightarrow x * (y * z)$$
$$x * x \quad \rightarrow x$$
$$x * (x * z) \quad \rightarrow x * z$$

New rule added

Knuth-Bendix completion

Completion process:

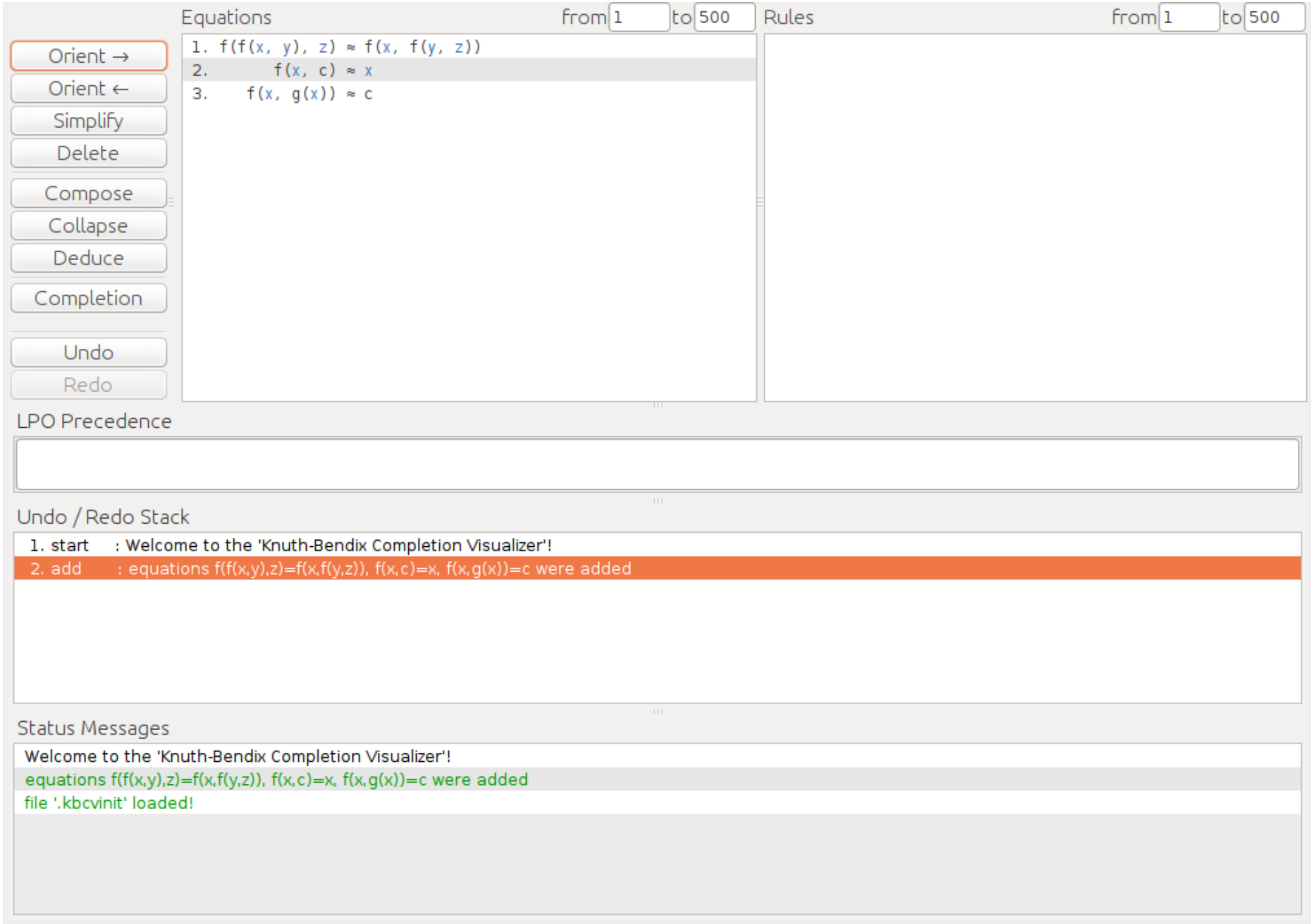
1. For each equation $s = t$ reduce s and t to normal form s' and t'
2. Fill the set of rules using basic operators and reduction ordering
3. If the algorithm terminate successfully: terminating and confluent rewrite system

Knuth-Bendix completion

Completion for axioms of groups:

$$
\begin{aligned}
1 \cdot x &= x \\
x^{-1} \cdot x &= 1 \\
(x \cdot y) \cdot z &= x \cdot (y \cdot z)
\end{aligned}
$$

$$
\begin{aligned}
1 \cdot x &\to x \\
x^{-1} \cdot x &\to 1 \\
(x \cdot y) \cdot z &\to x \cdot (y \cdot z) \\
x^{-1} \cdot (x \cdot y) &\to y \\
1^{-1} &\to 1 \\
x \cdot 1 &\to x \\
(x^{-1})^{-1} &\to x \\
x \cdot x^{-1} &\to 1 \\
x \cdot (x^{-1} \cdot y) &\to y \\
(x \cdot y)^{-1} &\to y^{-1} x^{-1}
\end{aligned}
$$


Equations
from 1 to 500
Rules
from 1 to 500
1. f(f(x, y), z) ≈ f(x, f(y, z))
2. f(x, c) ≈ x
3. f(x, g(x)) ≈ c
Orient →
Orient ←
Simplify
Delete
Compose
Collapse
Deduce
Completion
Undo
Redo
LPO Precedence
Undo / Redo Stack
1. start : Welcome to the 'Knuth-Bendix Completion Visualizer'!
2. add : equations f(f(x,y),z)=f(x,f(y,z)), f(x,c)=x, f(x,g(x))=c were added
Status Messages
Welcome to the 'Knuth-Bendix Completion Visualizer'!
equations f(f(x,y),z)=f(x,f(y,z)), f(x,c)=x, f(x,g(x))=c were added
file '.kbcvinit' loaded!

Existing tools

MU-TERM

CoLoR

HOPS

Maude

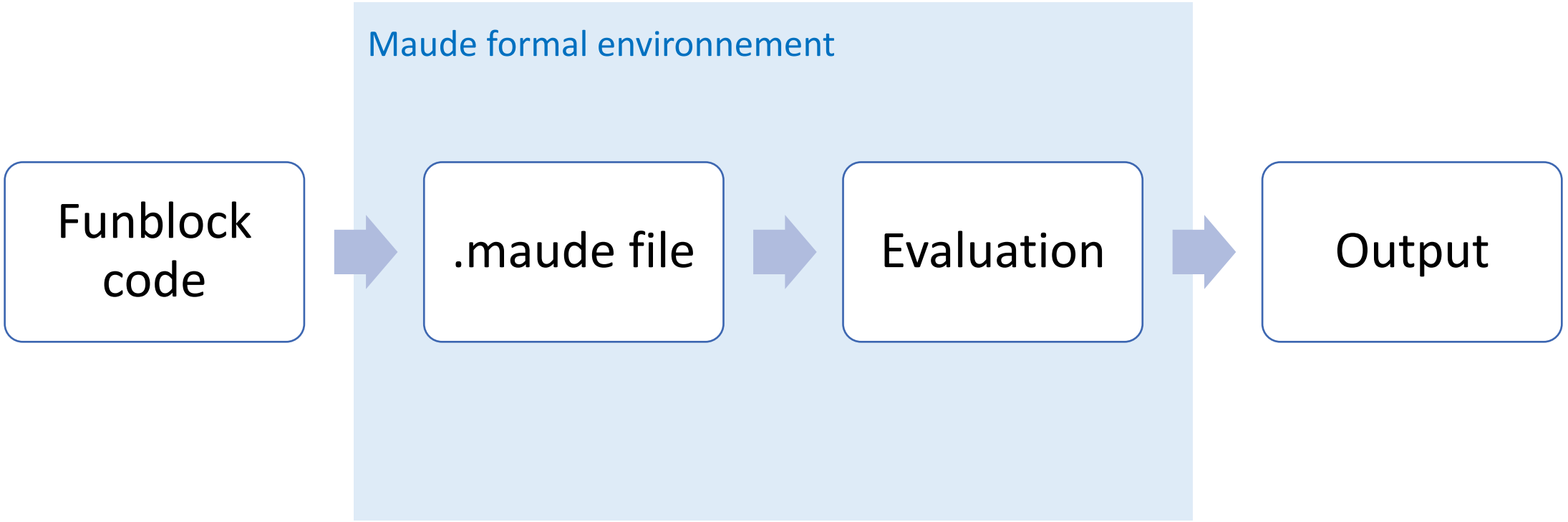
MoudE∃

Simplicity

Expressiveness

Performance

Procedure


Maude formal environnement
Funblock code
.maude file
Evaluation
Output

Maude

```
fmod BASIC-NAT is
        sort Nat .

        op 0 : -> Nat .
        op s : Nat -> Nat .
        op _+_ : Nat Nat -> Nat .

        vars N M : Nat .

        eq 0 + N = N .
        eq s(M) + N = s(M + N) .
endfm
```

Maude

```
5       fmod FACTORIAL is
6         protecting NAT .
7         op _! : Nat -> NzNat .
8         var N : Nat .
9         eq 0 ! = 1 .
10        eq (s N) ! = (s N) * N ! .
11      endfm
```

```
> load factorial.maude
> red 100 ! .
Reduce in FACTORIAL : 100 ! .
rewrites: 201 in 0ms cpu (0ms real) (~ rewrites/second)
result NzNAT:
933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168640000000000000000000000
```

Maude

```
 7        mod VENDING-MACHINE is
 8          including VENDING-MACHINE-SIGNATURE .
 9          var M : Marking .
10          rl [add-q] : M => M q .
11          rl [add-$] : M => M $ .
12          rl [buy-c] : $ => c .
13          rl [buy-a] : $ => a q .
14          rl [change] : q q q q => $ .
15        endm
```

Maude

Inductive Theorem Prover (ITP)

Sufficient Completeness Checker (SCC)

Church-Rosser Checker (CRC)

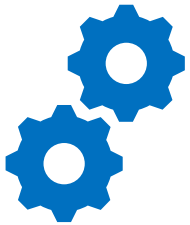
Coherence Checker (ChC)

Maude Termination Tool (MTT)

} Maude Formal Environment (MFE)

CiME

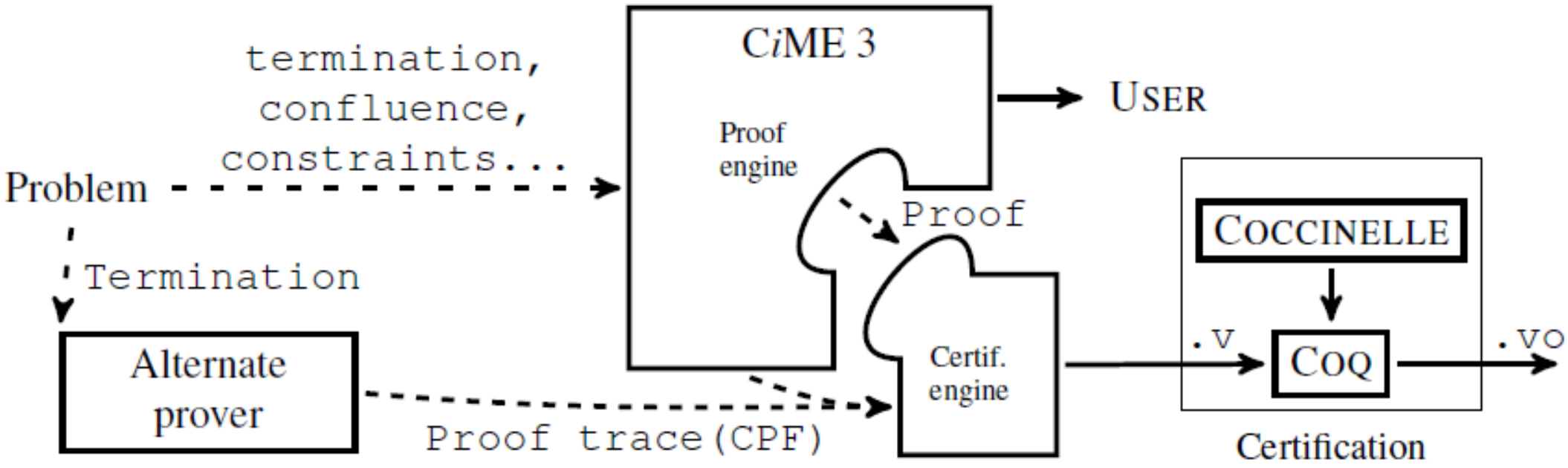
Rewriting toolkit



Proof engine

Proof certification

CiME


termination,
confluence,
constraints...
Problem
Termination
Alternate prover
CiME 3
Proof engine
Proof
Certif. engine
Proof trace(CPF)
USER
COCCINELLE
.v
COQ
.vo
Certification

CiME

Definition of signatures

```
CiME> let F_peano = signature "
      0 : constant; s : unary; +,* : infix binary;
      ";

F_peano : signature = signature "* : 2; s : 1; + : 2; 0 : 0"
```

```
CiME>let X = variables "x,y,z";

X : variable_set = variables "z,x,y"
```

CiME

Definition of algebra and terms

```
CiME> let A_peano = algebra F_peano ;

A_peano : F_peano algebra = algebra F_peano
```

```
CiME> let t = term A_peano "s(s(s(0)))*(s(0)+s(s(0)))";

t : F_peano term = s(s(s(0))) *(s(0)+s(s(0)))
```

CiME

Term rewriting system

```
CiME> let R_peano = trs A_peano "
      x+0 -> x;
      x+s(y) -> s(x+y);
      x*0 -> 0;
      x*s(y) -> (x*y)+x;
      ";

 R_peano : F_peano trs = trs A_peano "
           x+0 -> x;
           x+s(y) -> s(x+y);
           x *0 -> 0;
           x *s(y) -> (x *y)+x "
```

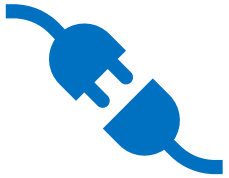
```
CiME> termination R_peano;

CiME> coq_certify_proof R_peano;

CiME> convergence R_peano ;

…
```

What's next?



Integration of the tools

Test the existing rewrite system checker

References

1. Didier Buchs, modelisation verification course material, 2018
2. Dimitri Racordon, Emmanouela Stachtiri, Damien Morard, Didier Buchs, Functional Block Programming and Debugging, 2020
3. Nachum Dershowitz, Jean-Pierre Jouannaud, Rewrite Systems, 1990
4. Nachum Dershowitz, Computing with Rewrite Systems, 1985
5. Terese, Term Rewriting Systems, 2003
6. Thomas Sternagel and Harald Zankl, KBCV-Knuth-Bendix Completion Visualizer, 2012
7. Thomas Artsa, Jürgen Giesl, Termination of term rewriting using dependency pairs, 2000

References

8. Jürgen Giesl, René Thiemann, Peter Schneider-Kamp, Stephan Falke, Automated Termination Proofs with AProVE, 2004
9. D. Kapur, P. Narendran, Path ordering for proving termination of term rewriting systems, 1985
10. Jeremy Dick, John Kalmus and Ursula Martin, Automating the Knuth Bendix ordering, 1990
11. E. Contejean, P. Courtieu, J. Forest, O. Pons, X. Urbain, Automated Certified Proofs with CiME3, 2011

References (links)

Database of Rewriting Systems

- http://rewriting.loria.fr/systems.html

Knuth-Bendix Completion Visualizer

- http://cl-informatik.uibk.ac.at/software/kbcv/

Knuth-Bendix Completion subject based thesis

- https://homepage.divms.uiowa.edu/~astump/papers/thesis-wehrman.pdf

References (links)

Prolog implementation of the Knuth-Bendix completion procedure

- https://www.metalevel.at/trs/

Maude tools

- http://maude.lcc.uma.es/CRChC/
- http://www.lcc.uma.es/%7Eduran/MTT/
- http://maude.sip.ucm.es/debugging/

Wikipedia

- https://fr.wikipedia.org/wiki/Compl%C3%A9tion_de_Knuth-Bendix
- https://fr.wikipedia.org/wiki/Paire_critique

Make FunBlocks alive

Marvin FOURASTIE

Master project