SOME DETAILS ABOUT REWIRING SYSTEMS

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Strongly based on a présentation of Roel de Vrijer and Alexis Marechal

RECALL

- Signature $\Sigma = \langle S, \; F \rangle$
- From Σ we can derive the set T_Σ, *i.e.*, the set of all terms generated by Σ.
- Give a set X of S indexed variables, we can derive the set $T_{\Sigma,X}$ of terms with variables.
- We can give a semantics to all of that by adding equations (called axioms), the whole pack Σ + X + Axioms = Adt

$\Sigma = \langle S,\ F \rangle \qquad S = \{nat, bool\} \qquad X_s = \{x_{nat}, b_{bool}\}$

$F = \{true_{\varepsilon,bool}, false_{\varepsilon,bool}, zero_{\varepsilon,nat}, suc_{nat,nat}, +_{nat,nat,nat}, not_{bool,bool}\}$

$(T_\Sigma)_s =$

zero
true
not(false)
+(zero, zero)
not(true)
suc(zero)
false
+(suc(zero), zero)
+(zero, +(suc(zero), suc(suc(zero)))
+(suc(zero), suc(suc(zero))
suc(suc(suc(zero)))

$(T_{\Sigma,X})_s =$

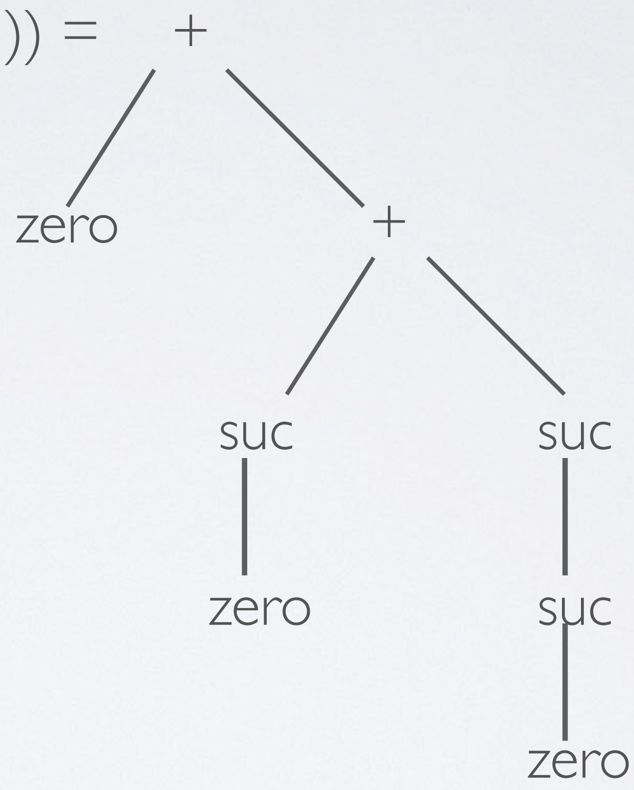
b
zero
true
x
not(false)
+(zero, zero)
not(not(b))
not(true)
suc(zero)
not(b)
false
+(suc(zero), zero)
+(suc(x), zero)
+(zero, +(suc(zero), suc(suc(zero)))
+(suc(zero), suc(suc(zero))
suc(suc(suc(zero)))

SYNTACTIC DERIVATION

- Two types of terms: atoms and composite terms.
  - atoms: true, false, zero; composite: suc(zero), +(zero, zero), etc.)

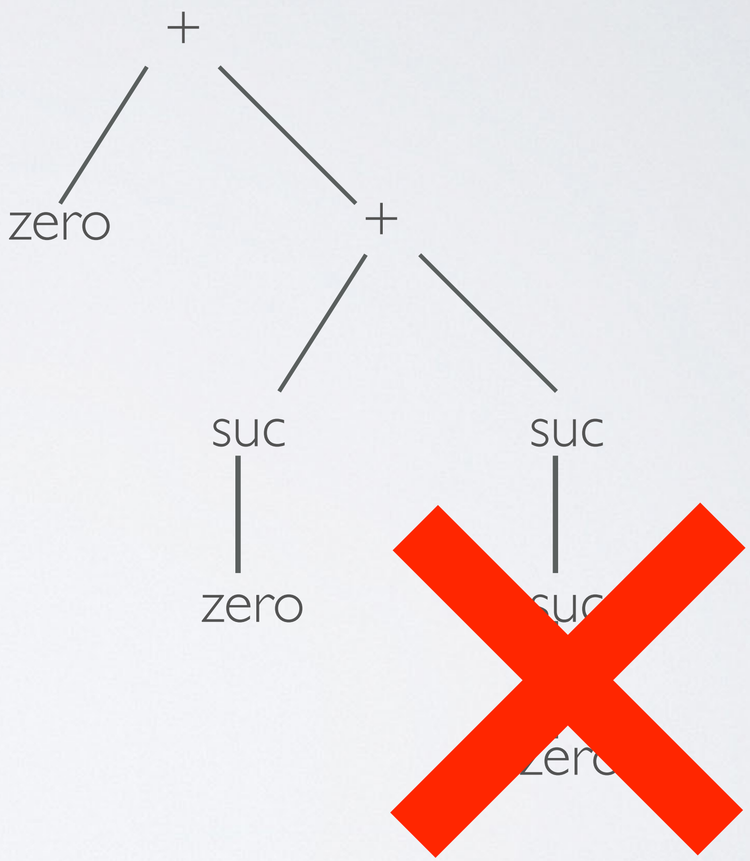
COMPOSITE TERM AS TREE

+(zero, +(suc(zero), suc(suc(zero)))) =



CONTEXT

+(zero, +(suc(zero), suc(_))) =

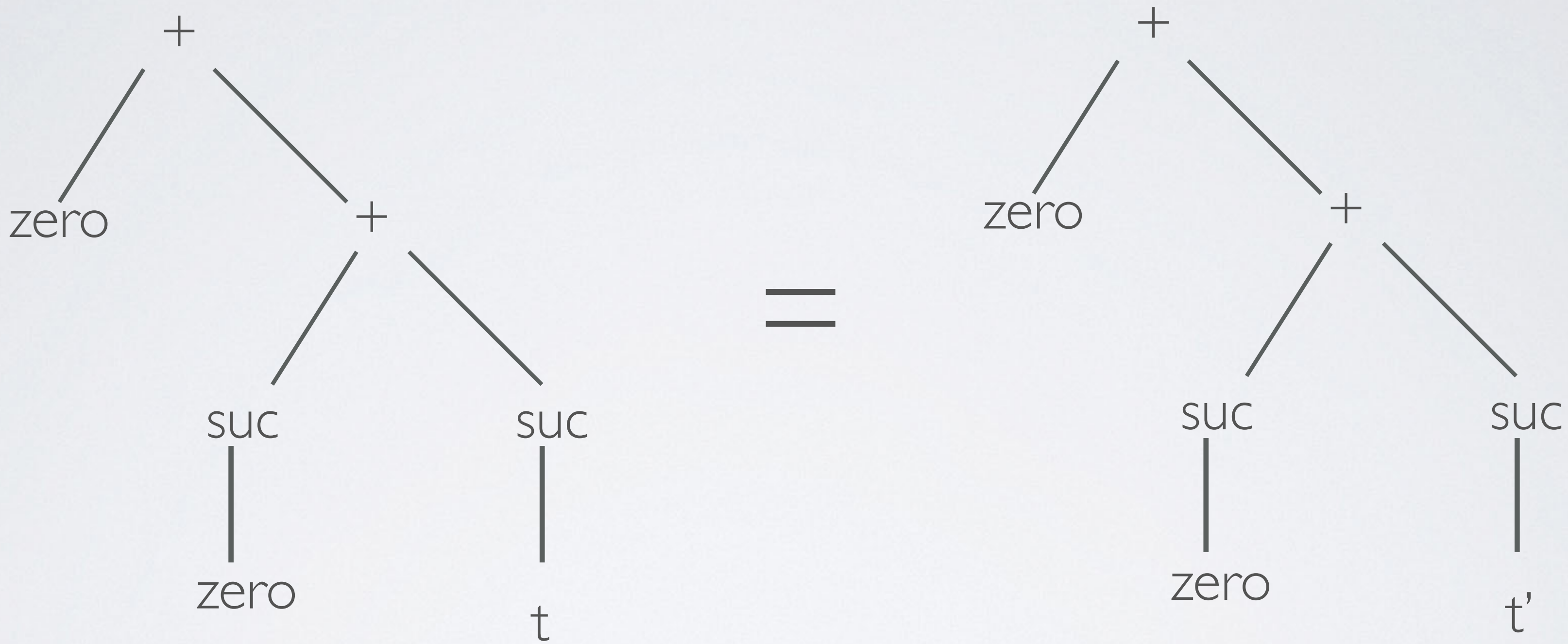


c[_] = +(zero, +(suc(zero), suc(_)))

SYNTACTIC DERIVATION PRINCIPLE

- c[t] = c[t'] iff t = t', which is the same as
    - c[t] = c[t'] implies t = t', and
    - t = t' implies c[t] = c[t']
- Example: if not(true) = false is the equation, and c[_] = not(_) the context, we can **deduce by syntactic derivation that:**
    - not(true) = false $\vdash$ not(not(true)) = not(false)

GRAPHICALLY



if and only if, $t = t'$

REPLACEMENT

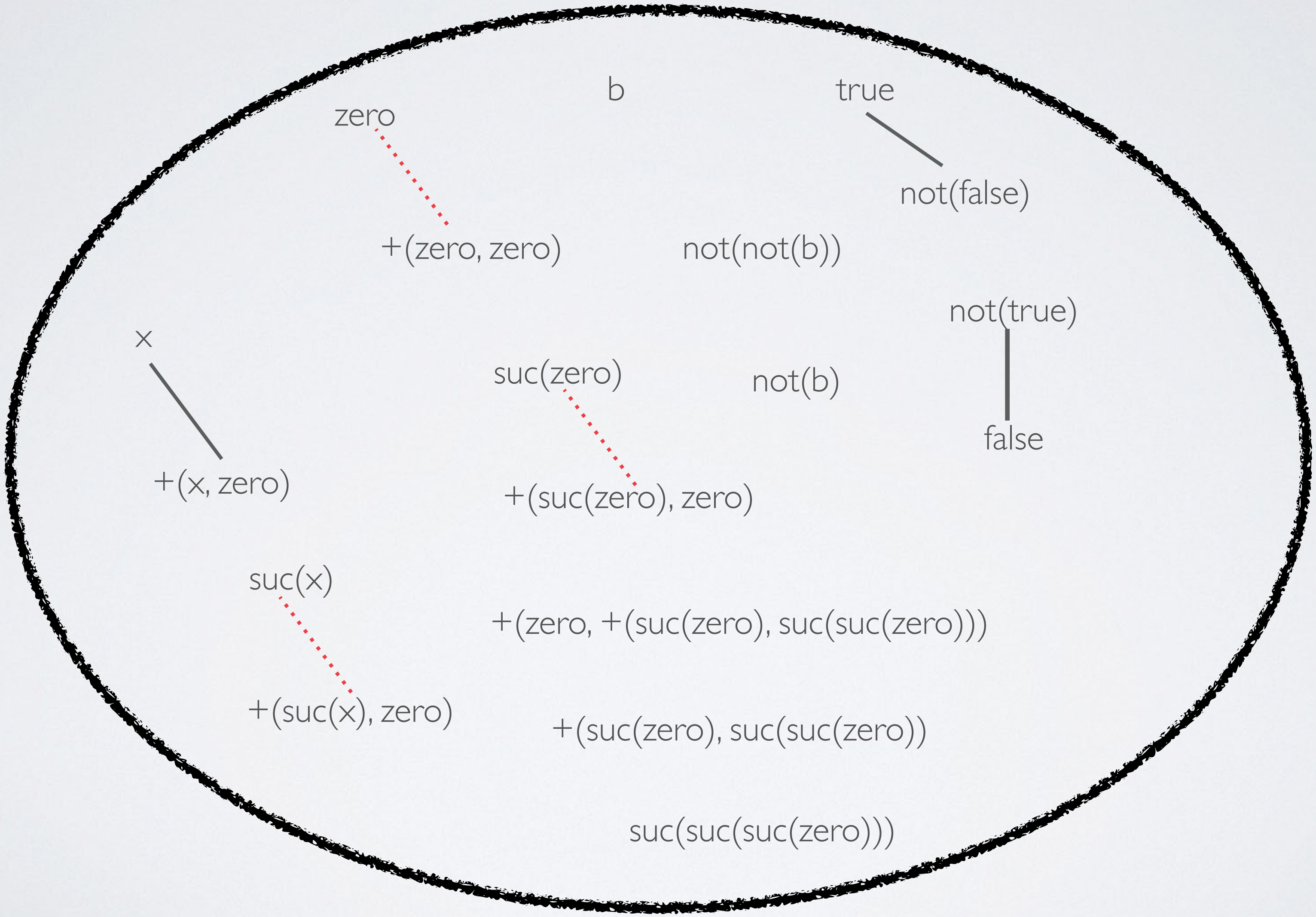
- The operation that takes a term c[t] and a new term t' and returns a c[t'] is called a **replacement**.

$\Sigma = \langle S, F \rangle$ $\qquad S = \{nat, bool\}$ $\qquad X_s = \{x_{nat}, b_{bool}\}$

$F = \{true_{\varepsilon,bool}, false_{\varepsilon,bool}, zero_{\varepsilon,nat}, suc_{nat,nat}, +_{nat,nat,nat}, not_{bool,bool}\}$

$Axioms = \{not(true) = false, not(false) = true, +(x, zero) = x\}$

$(T_{\Sigma,X})_s =$



Systèmes axiomatiques:

1. $f(\$a) = g(\$a)$
2. $g(\$a) = 0$

Systèmes de réécriture:

1. $f(\$a) \rightarrow g(\$a)$
2. $g(\$a) \rightarrow 0$

Orientation!

Definition (Rewrite step)

Let $\Sigma = \langle S, F \rangle$ be a signature and X be a S-sorted set of variables and $l \rightsquigarrow r$,$l, r \in T_\Sigma(X)$ a rewrite rule.

- $filter(t, l) =< \sigma, c > \Leftrightarrow \exists \sigma \in S\ \exists c, c[\sigma l] = t$ [a]
- $t' = c[\sigma r]$
- $< t, t' > \in Rew_{l \rightsquigarrow r}$ a rewrite step

---

a. $c[\_]$ denotes the context of a term, i.e. a term with a place holder

Rew. rule: $f(\$a) \rightsquigarrow \$a$

Rew. step: $g(f(0)) \rightsquigarrow g(0)$

$l \Leftrightarrow f(\$a)$	$t \Leftrightarrow g(f(0))$	$\sigma \Leftrightarrow [0/a]$
$r \Leftrightarrow \$a$	$t' \Leftrightarrow g(0)$	$c[t] \Leftrightarrow g(t)$

Notations non-officielles!

Definition (Rewrite step)

Let $\Sigma = \langle S, F \rangle$ be a signature and X be a S-sorted set of variables and $l \rightsquigarrow r$,$l, r \in T_\Sigma(X)$ a rewrite rule.

- $filter(t, l) =< \sigma, c > \Leftrightarrow \exists \sigma \in S \ \exists c, c[\sigma l] = t$ [a]
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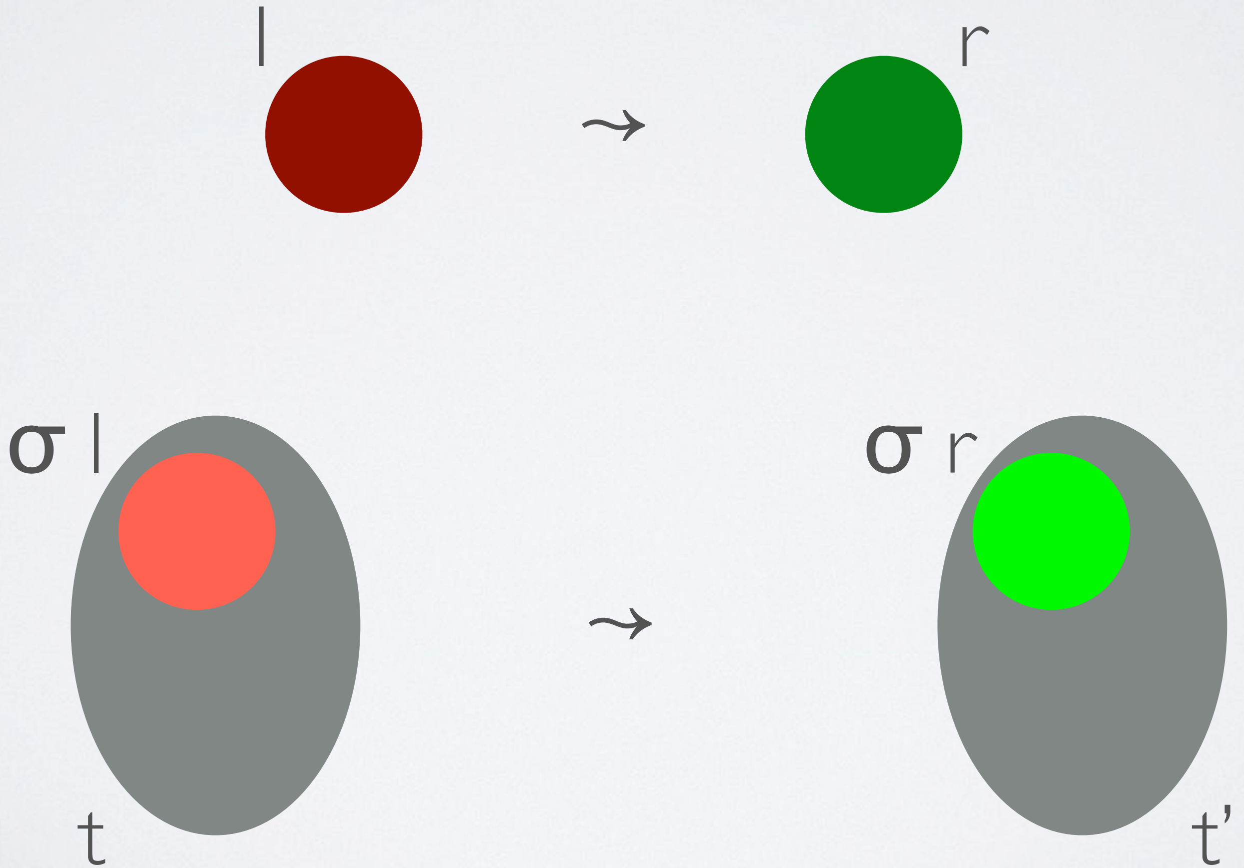

$\Sigma = \langle S,\ F \rangle \qquad S = \{nat, bool\} \qquad X_s = \{x_{nat}, b_{bool}\}$

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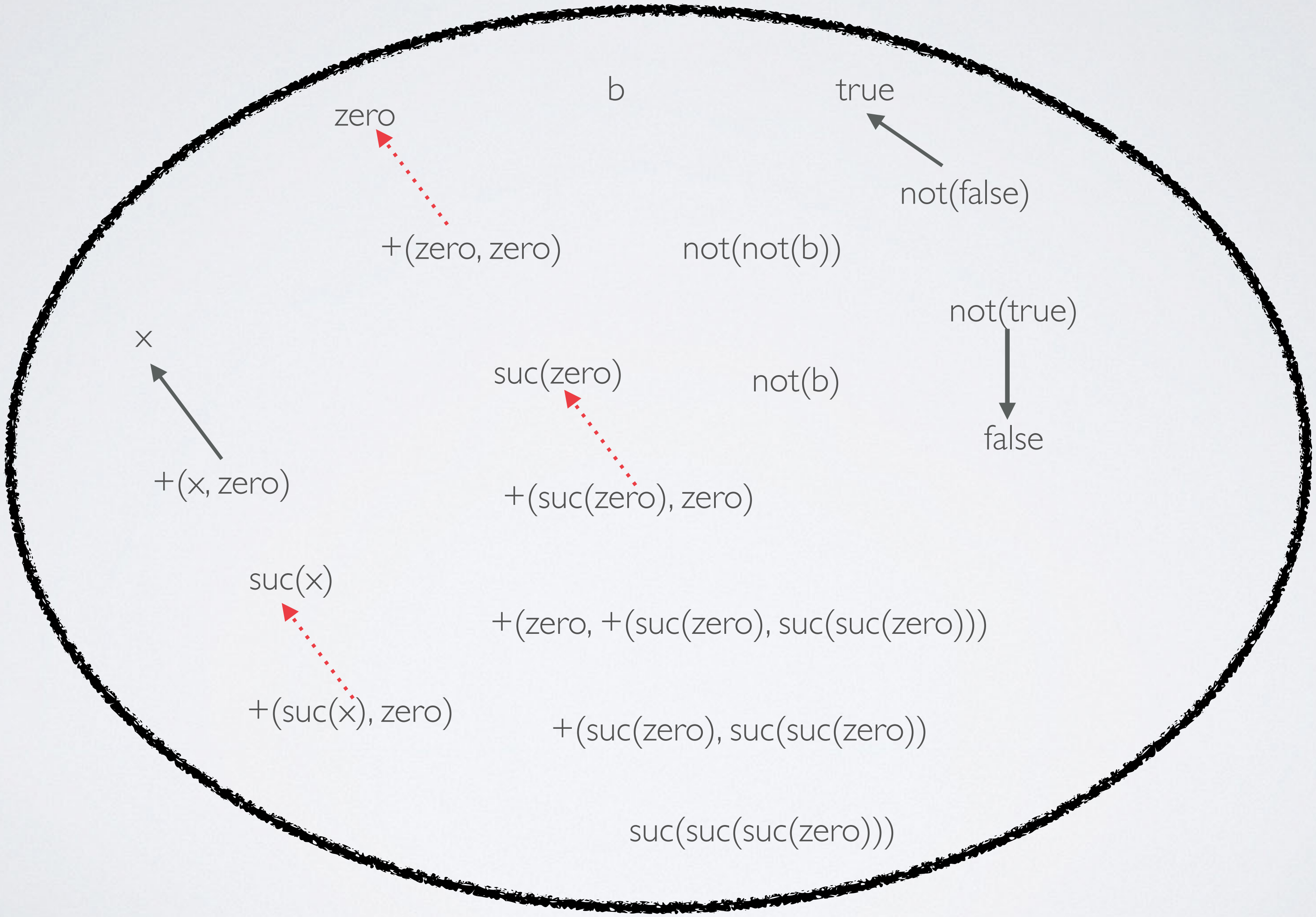
$Axioms = \{not(true) \rightsquigarrow false, not(false) \rightsquigarrow true, +(x, zero) \rightsquigarrow x\}$

$(T_{\Sigma,X})_s =$



1 $f(0) \rightsquigarrow s(0)$

2 $g(f(\$x)) \rightsquigarrow g(\$x)$

3 $h(\$x) \rightsquigarrow h(s(\$x))$

$$g(f(0)) \overset{1}{\rightsquigarrow} g(s(0))$$

$$g(f(0)) \overset{2}{\rightsquigarrow} g(0)$$

Problème : confluence

1 $f(0) \rightsquigarrow s(0)$

2 $g(f(\$x)) \rightsquigarrow g(\$x)$

3 $h(\$x) \rightsquigarrow h(s(\$x))$

$$h(0) \overset{3}{\rightsquigarrow} h(s(0)) \overset{3}{\rightsquigarrow} h(s(s(0))) \overset{3}{\rightsquigarrow} \ldots$$

Problème: terminaison

Confluence / terminaison

$\rightsquigarrow^*$

Clôture transitive

Confluence :

Knuth-Bendix

Terminaison :

Critical Pairs - Knuth-Bendix theorem

Let $l_1 \rightsquigarrow r_1$ and $l_2 \rightsquigarrow r_2$ be two rules of a term rewriting system. we suppose that these rules have no variables in common. If l_1^{sub} is a subterm (and not a variable) of l_1 (or the term itself) with $l_1^{context}[l_1^{sub}] = l_1$ and there exist a most general unifier σ such that $l_1^{sub}\sigma = l_2\sigma$, then $r_1\sigma$ and $l_1^{context}[r_2\sigma]$are called a critical pair.

1 $g(f(x, y), x) \rightsquigarrow h(x)$

2 $f(x, x) \rightsquigarrow x$

$$\begin{array}{ccc} & g(f(x,x),x) & \\ \swarrow^{2} & & \searrow^{1} \\ g(x,x) & \rightsquigarrow & h(x) \end{array}$$

$l_1^{sub} \Leftrightarrow f(x, y)$

$l_1^{context}[t] \Leftrightarrow g(t, x)$

$l_2 \Leftrightarrow f(x, x)$

$\sigma \Leftrightarrow [x/y]$

$r_1\sigma \Leftrightarrow h(x)$

$l_1^{context}[r_2\sigma] \Leftrightarrow g(x, x)$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

Objectif:

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $x.e \rightsquigarrow x$

5 $x.I(x) \rightsquigarrow e$

6 $x.(y.z) \rightsquigarrow (x.y).z$

7 $I(e) \rightsquigarrow e$

8 $I(I(x)) \rightsquigarrow x$

9 $I(x.y) \rightsquigarrow I(y).I(x)$

1 $e.x \rightsquigarrow x$

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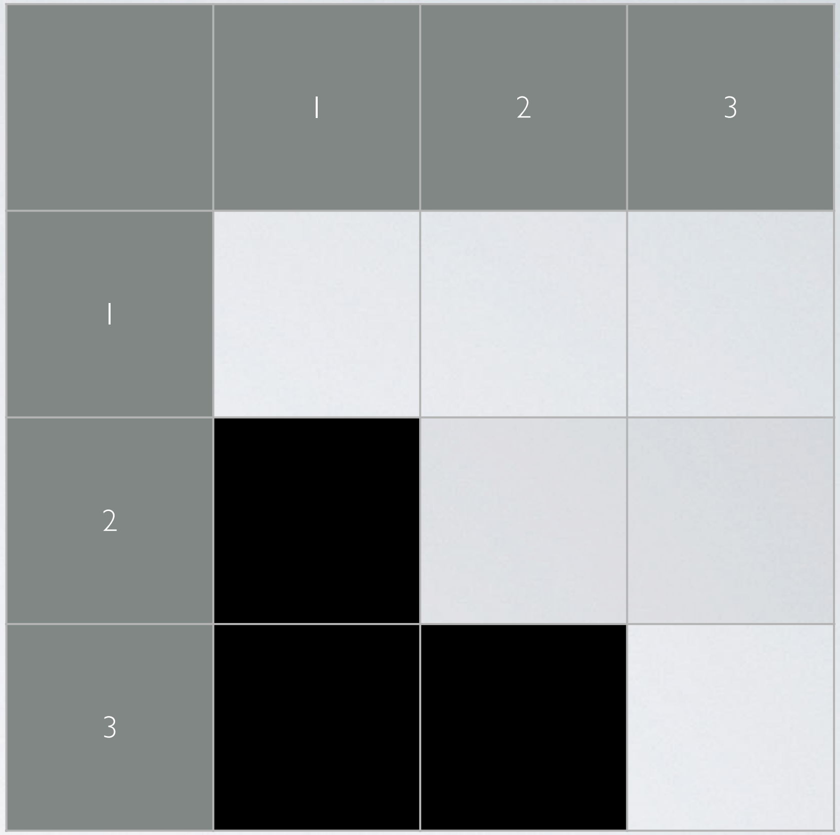
3 $(x.y).z \rightsquigarrow x.(y.z)$



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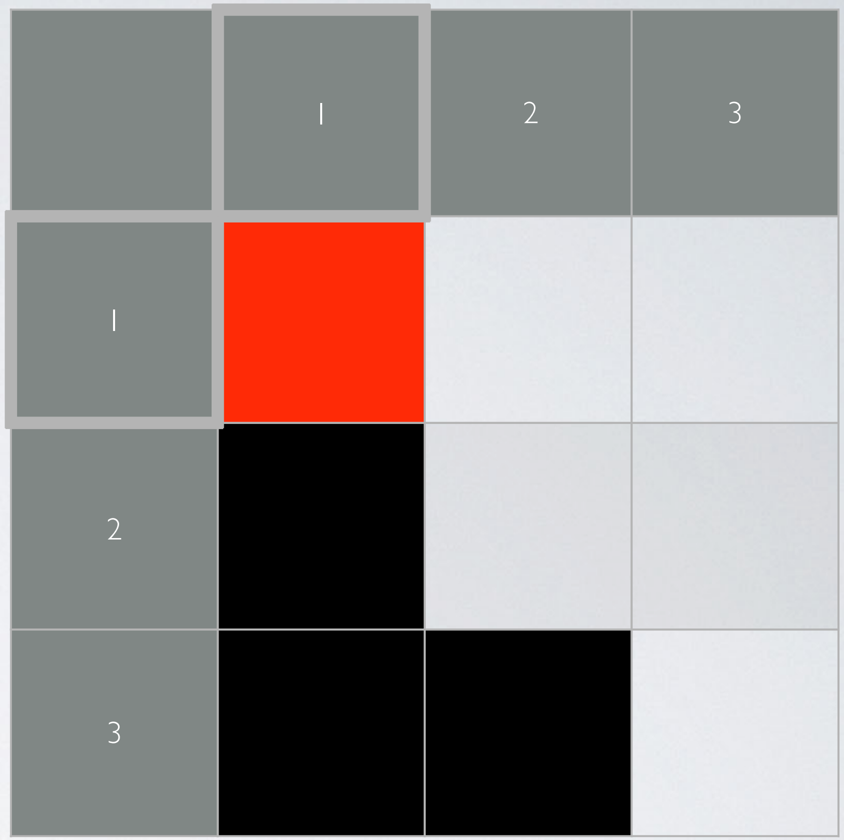

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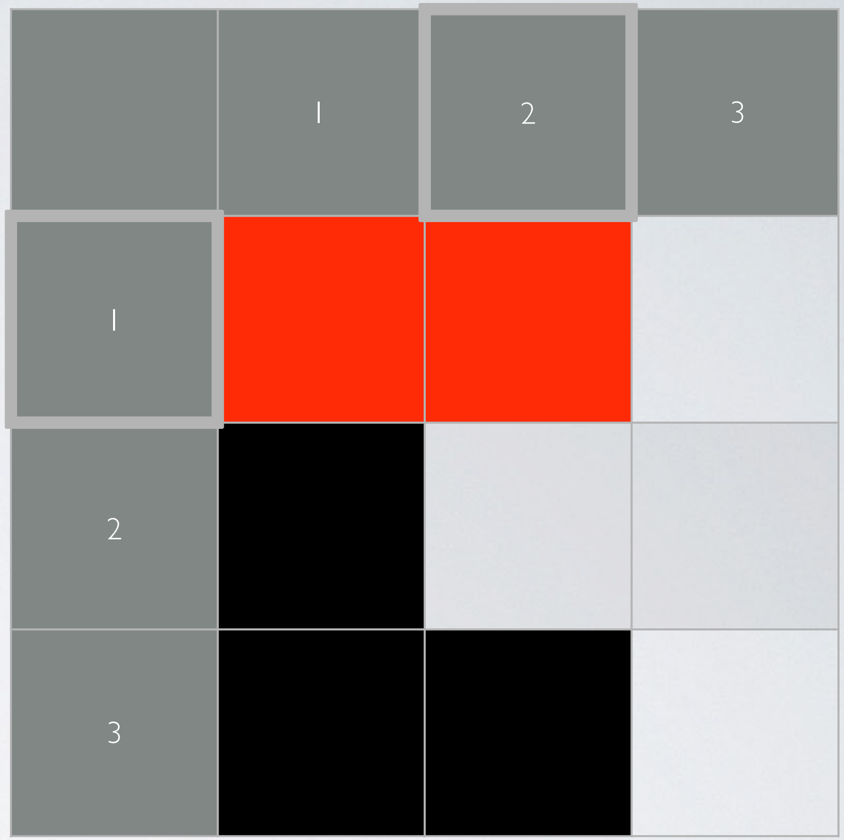

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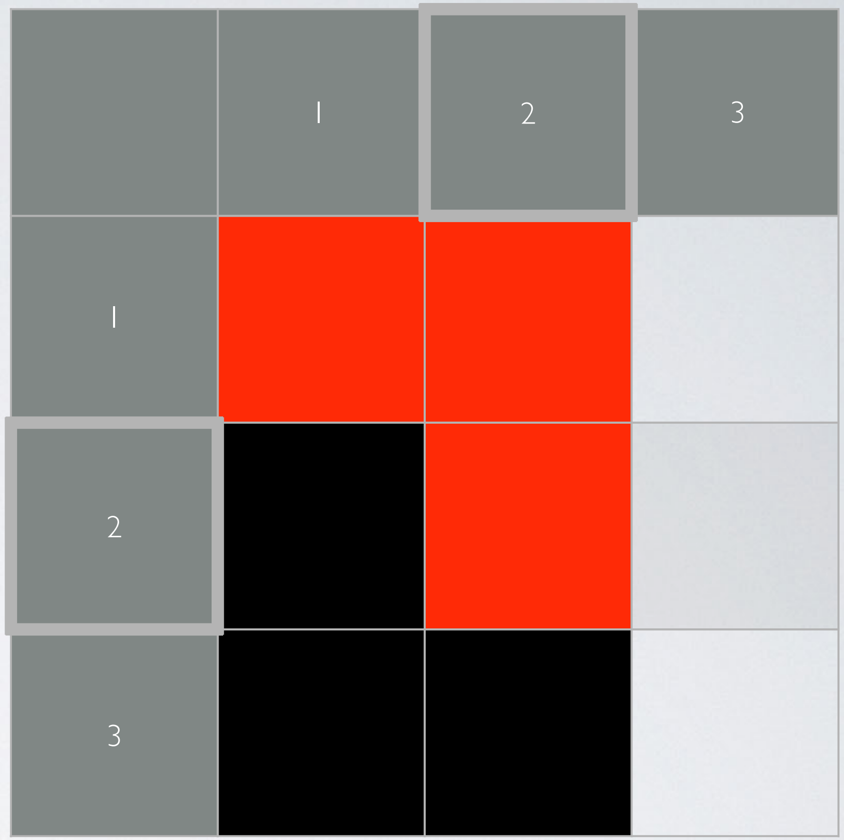

1 $e.x \rightsquigarrow x$

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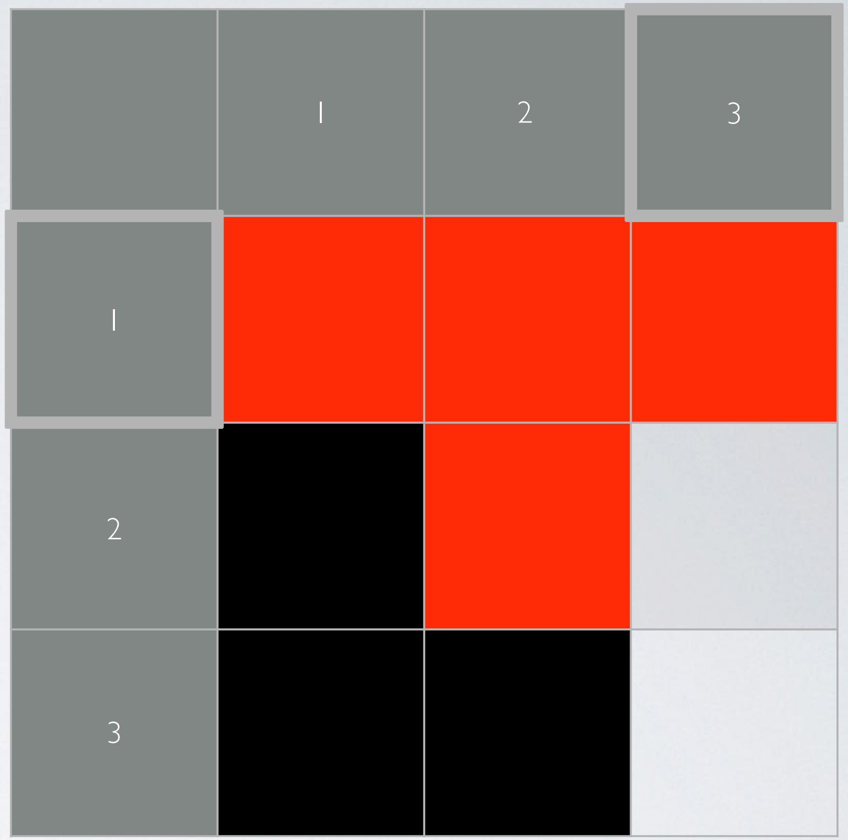
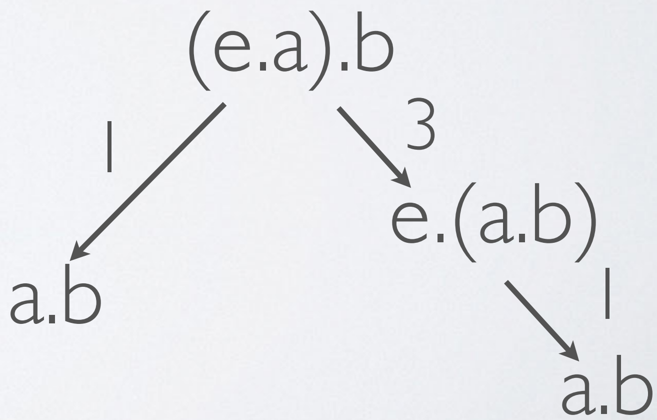
3 $(x.y).z \rightsquigarrow x.(y.z)$



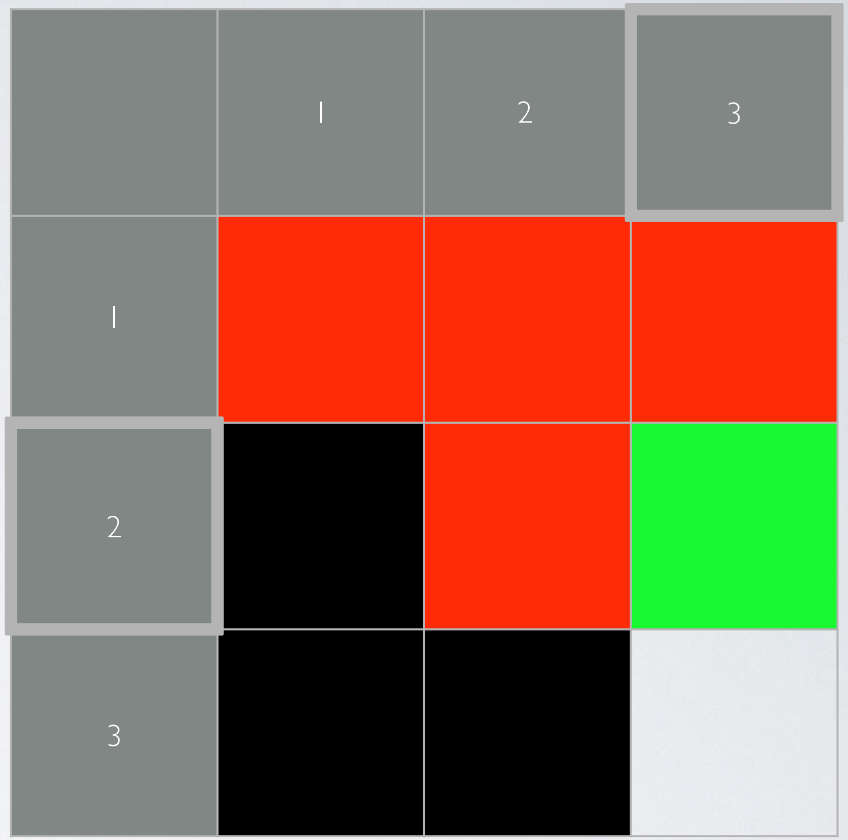


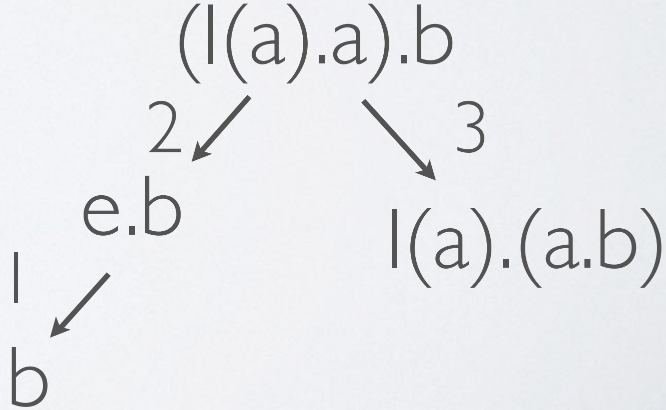
1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$





1. $e.x \rightsquigarrow x$
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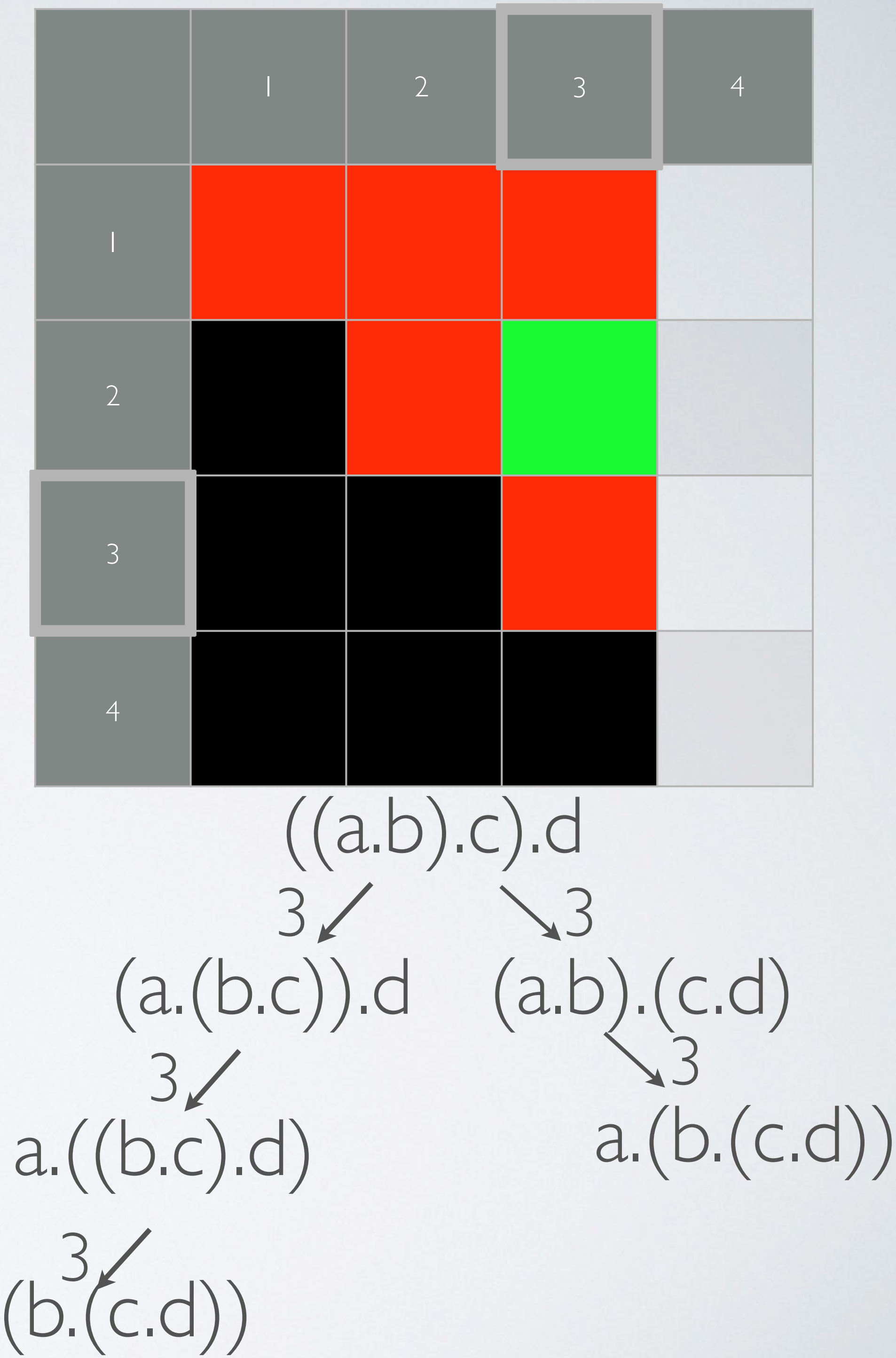
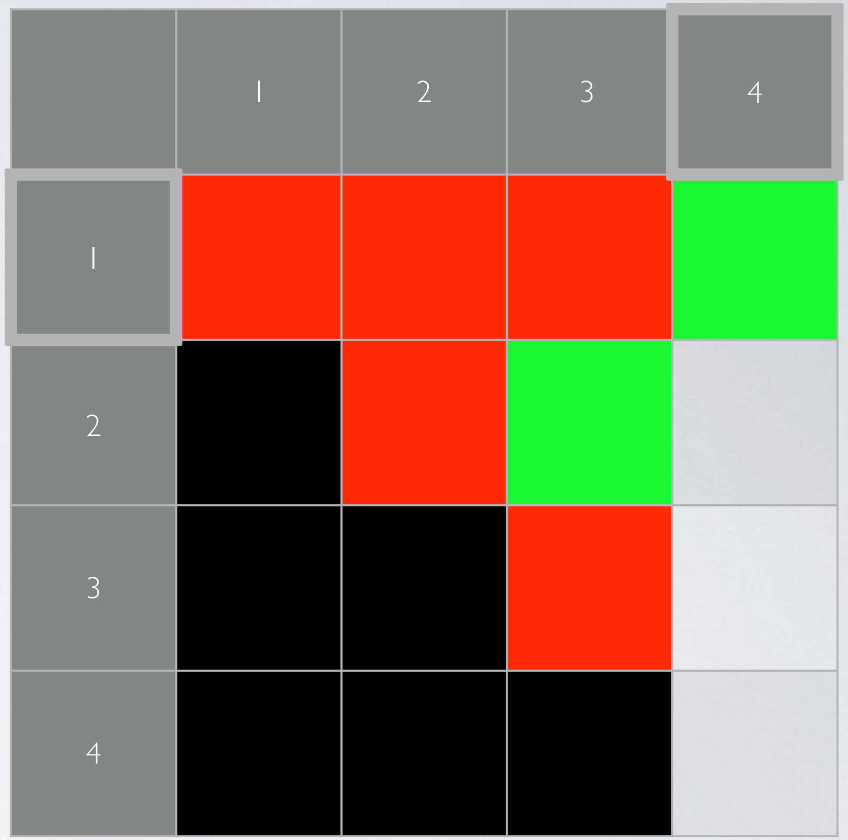
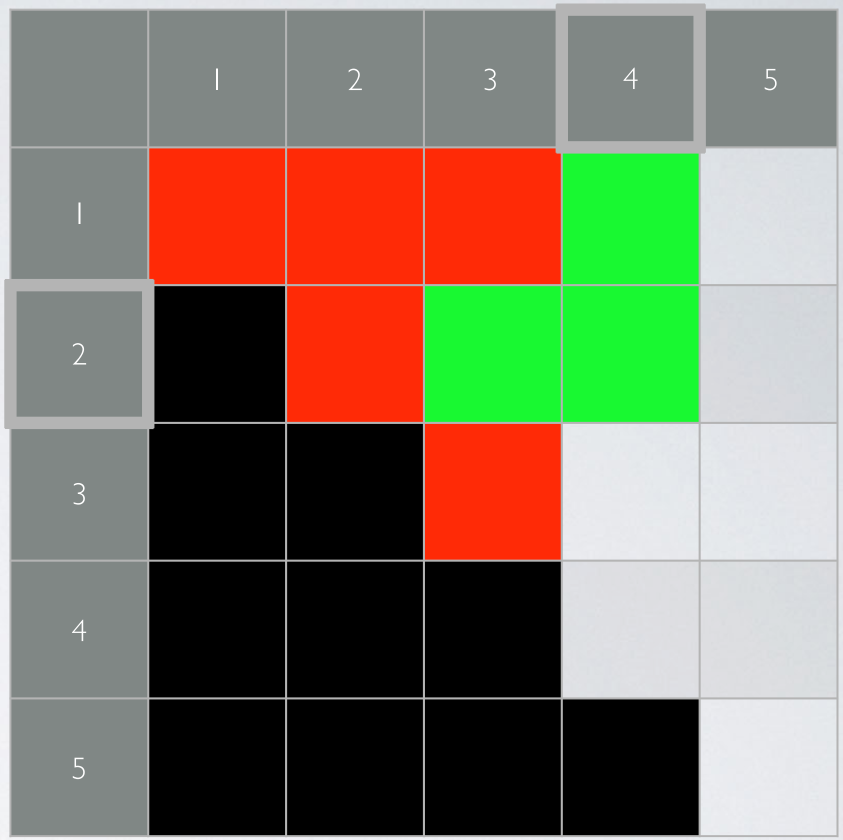


1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$



$$\begin{tikzcd} & I(e).(e.a) \arrow[dl, "1"'] \arrow[dr, "4"] & \\ I(e).a & & a \end{tikzcd}$$

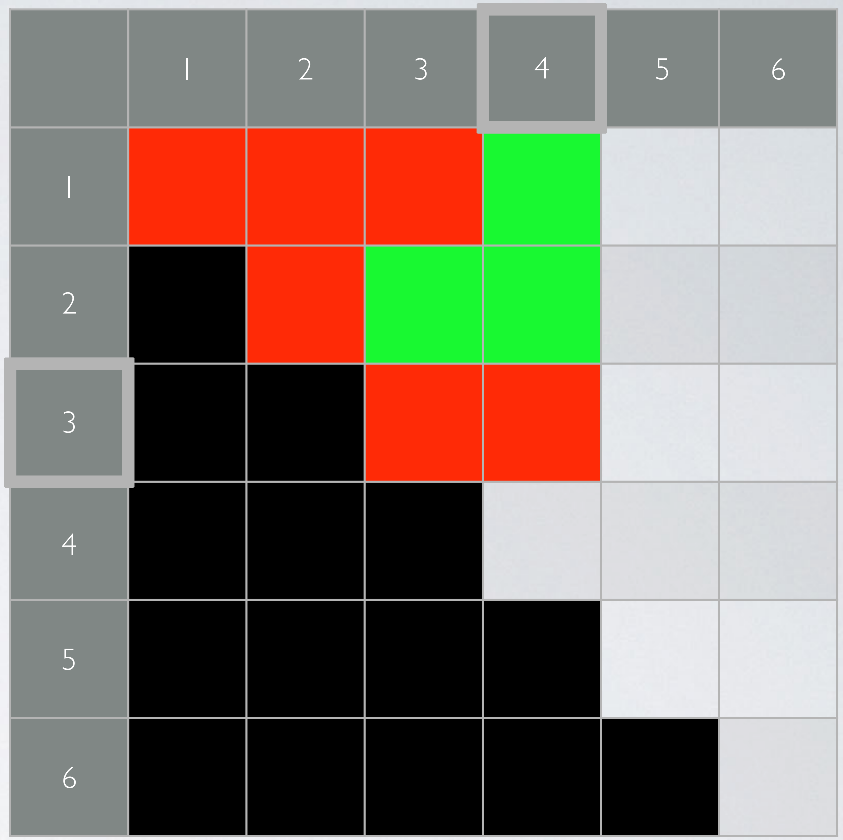
1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$



$$I(I(a)).(I(a).a)$$

$$I(I(a)).e \xleftarrow{2} I(I(a)).(I(a).a) \xrightarrow{4} a$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$



$(I(a).(a.b)).c$

$\swarrow 3 \qquad \searrow 4$

$I(a).((a.b).c) \qquad b.c$

$\swarrow 3$

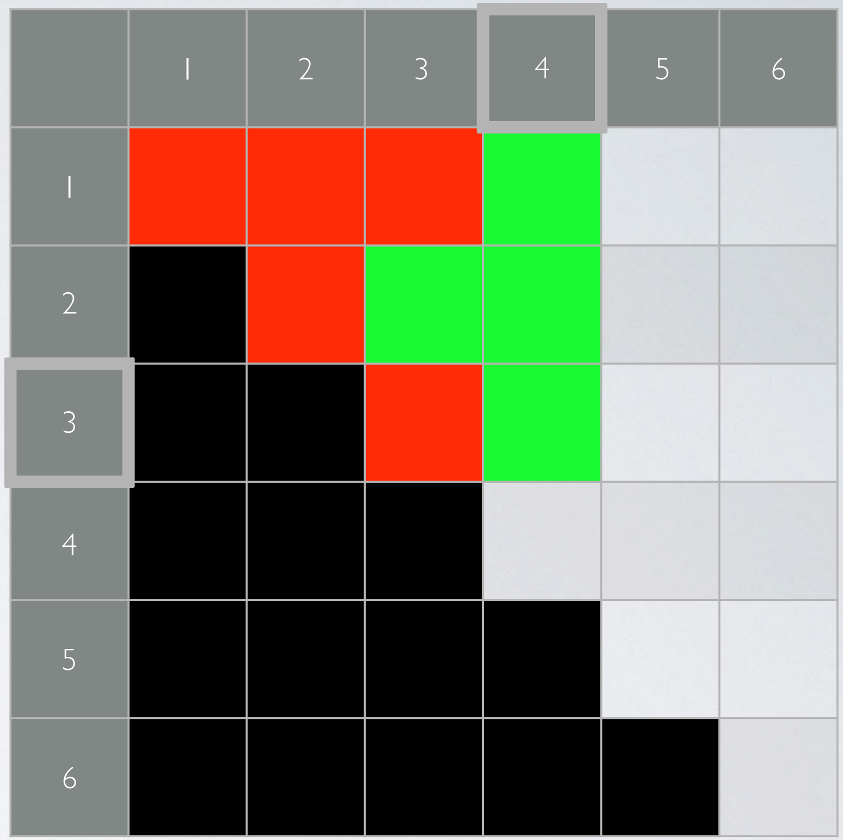
$I(a).(a.(b.c))$

$\swarrow 4$

$b.c$

Mêmes règles, autre substitution!

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$



$$
\begin{array}{ccc}
 & I(a.b).((a.b).c) & \\
\swarrow^{3} & & \searrow^{4} \\
I(a.b).(a.(b.c)) & & c
\end{array}
$$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

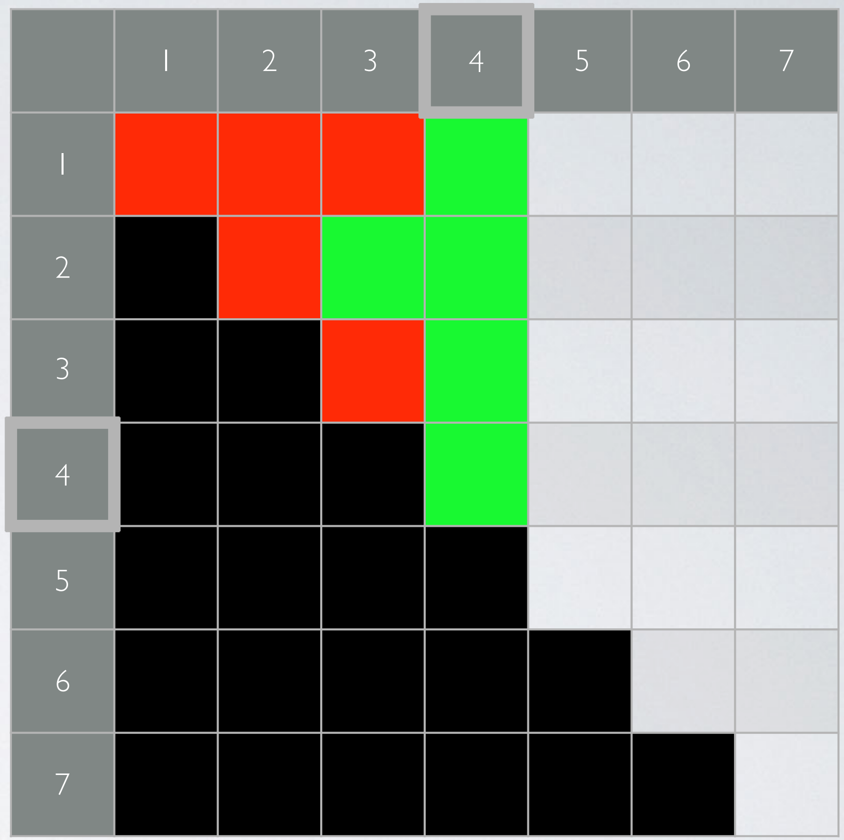
4 $I(x).(x.z) \rightsquigarrow z$

5 $I(e).z \rightsquigarrow z$

6 $I(I(x)).e \rightsquigarrow x$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

8 $I(I(x)).y \rightsquigarrow x.y$



$$I(I(a)).(I(a).(a.b))$$

$$I(I(a)).b \xleftarrow{4} I(I(a)).(I(a).(a.b)) \xrightarrow{4} (a.b)$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
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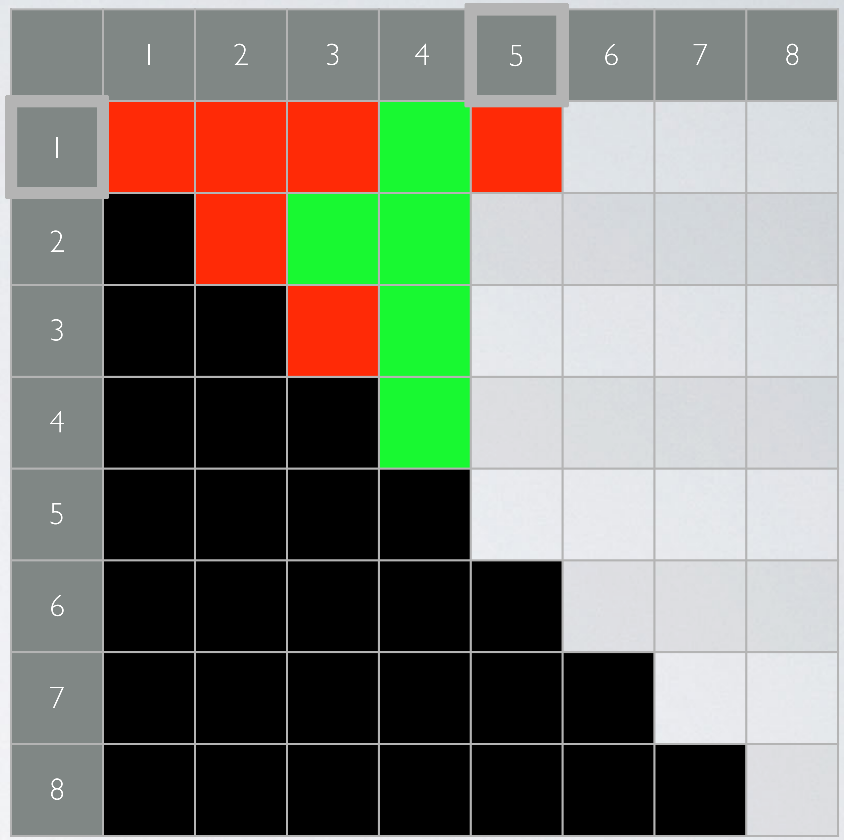

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
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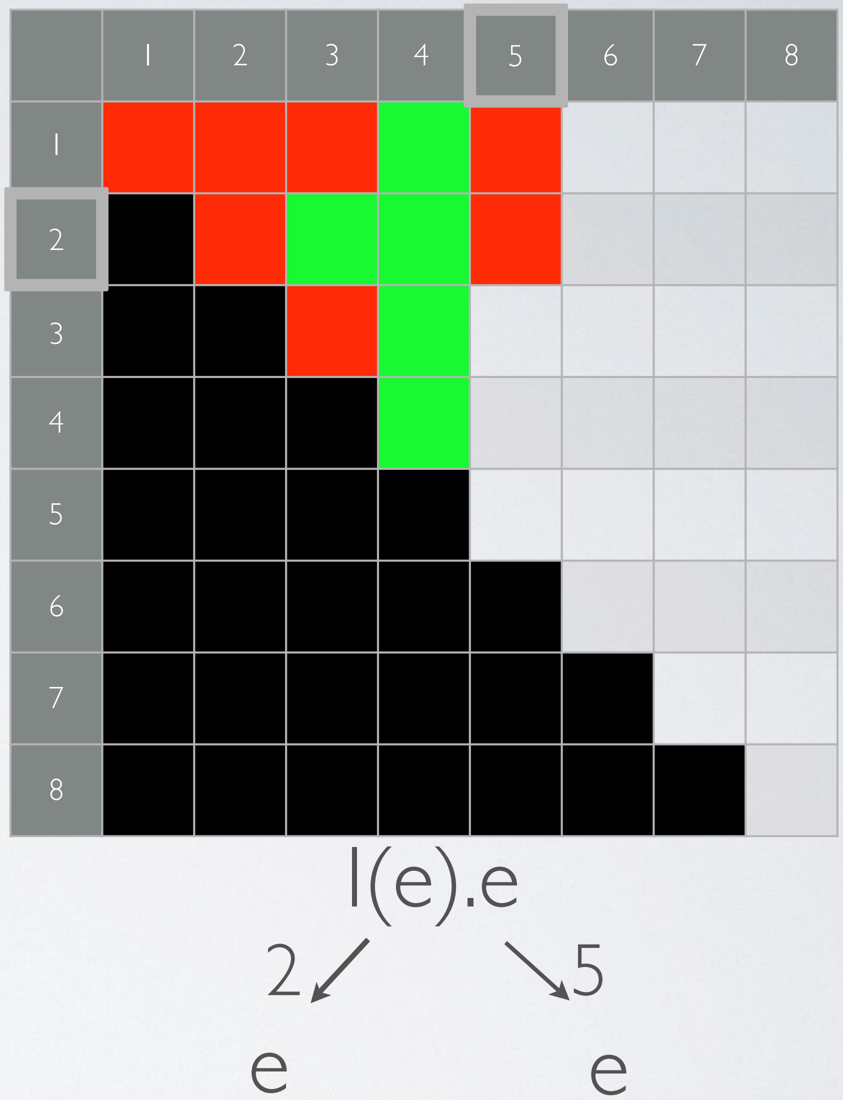

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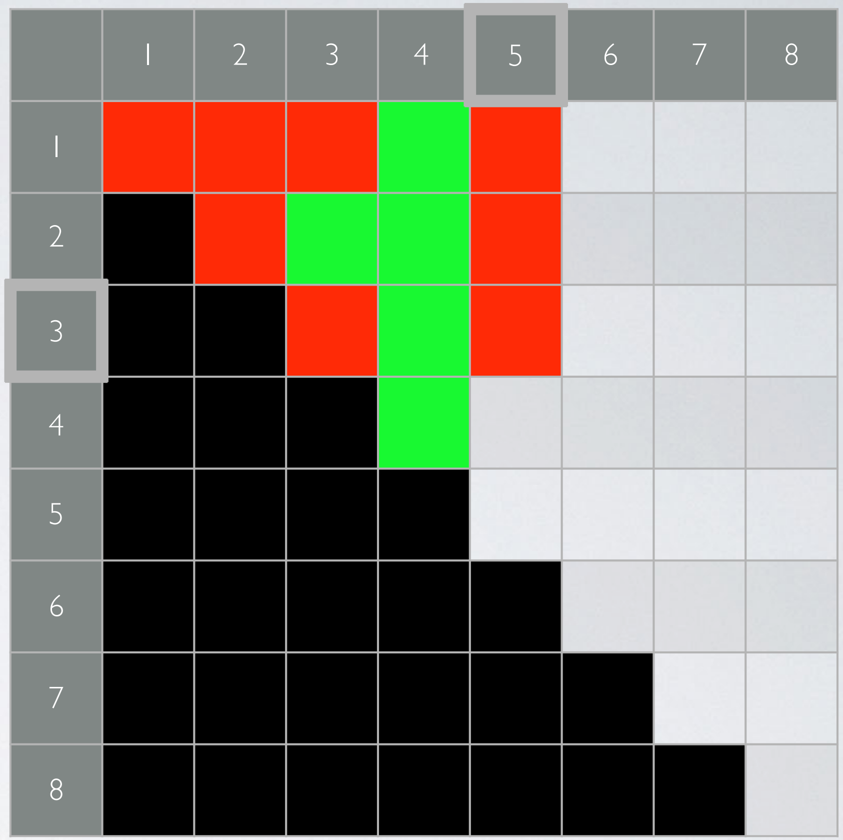
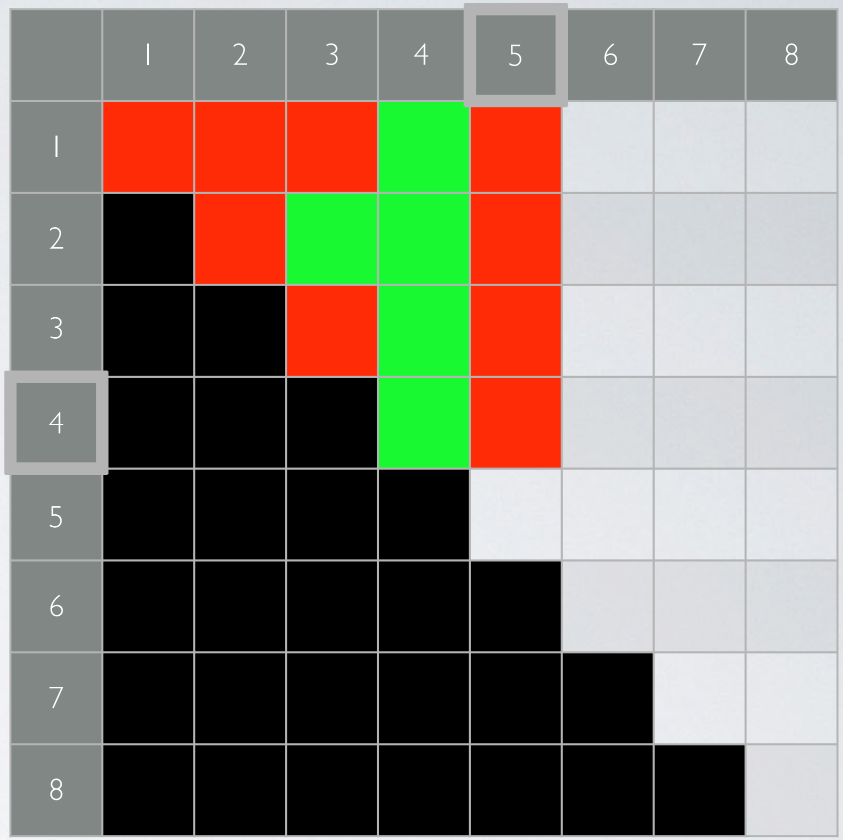


$$\begin{array}{ccc} & (I(e).a).b & \\ \swarrow_{3} & & \searrow^{5} \\ I(e).(a.b) & & (a.b) \\ \swarrow_{5} & & \\ (a.b) & & \end{array}$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$



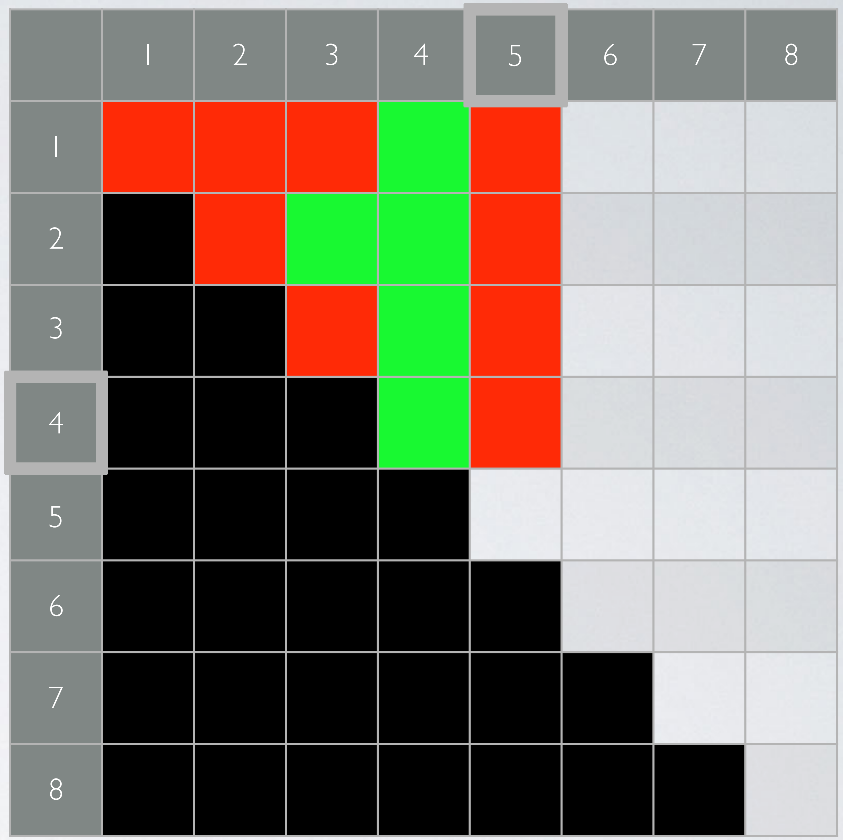
$I(I(e)).(I(e).a)$

$\xrightarrow{4} a$

$\xrightarrow{5} I(I(e)).a \xrightarrow{8} e.a \xrightarrow{1} a$

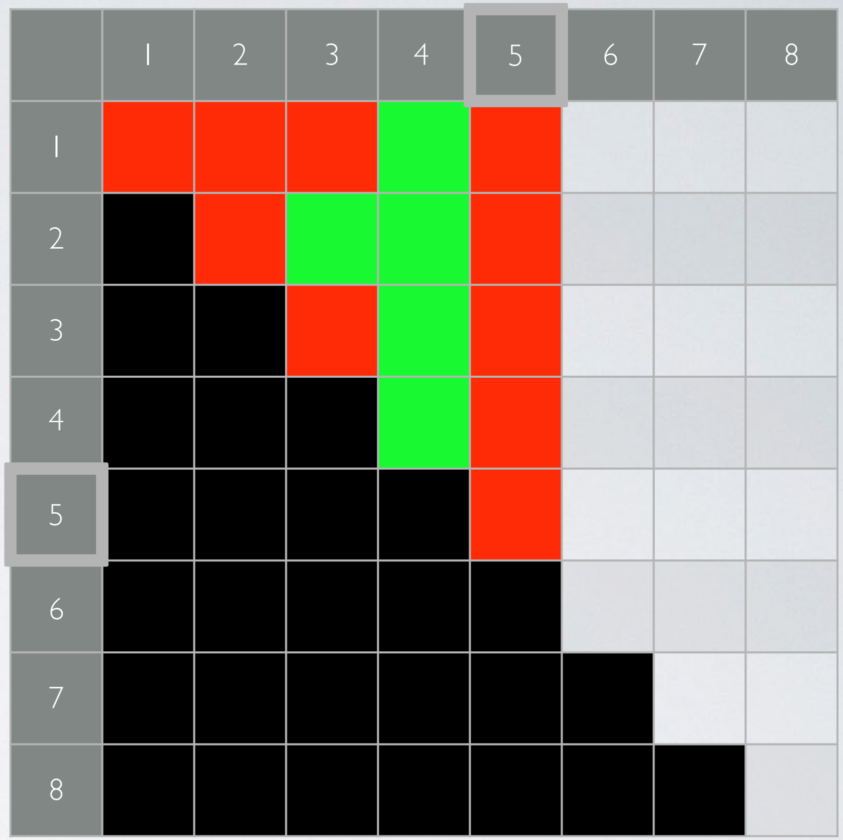
Mêmes règles, autre substitution!

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$



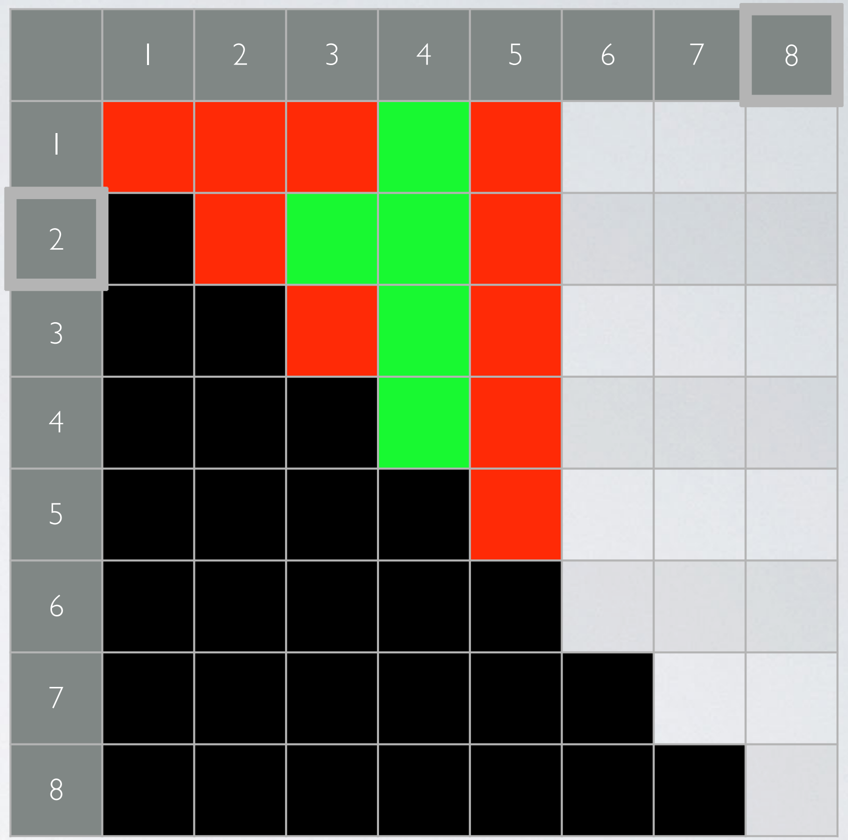
$$b \xleftarrow{4} I(e).(e.b) \xrightarrow{5} e.b \xrightarrow{1} b$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$



Trichons un peu pour accélérer le processus...

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$



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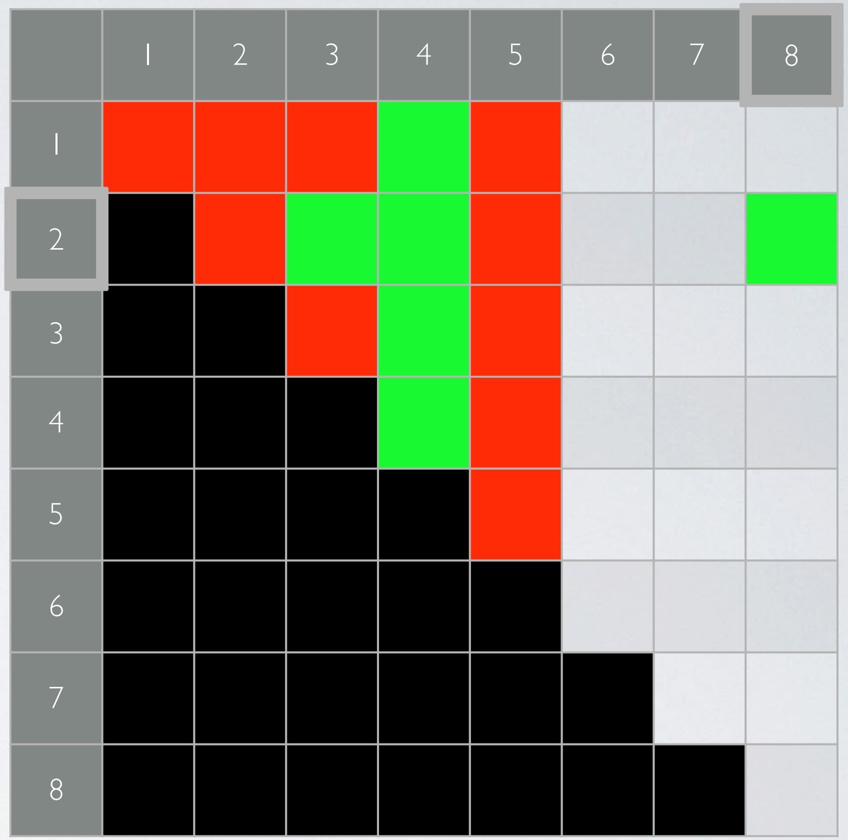
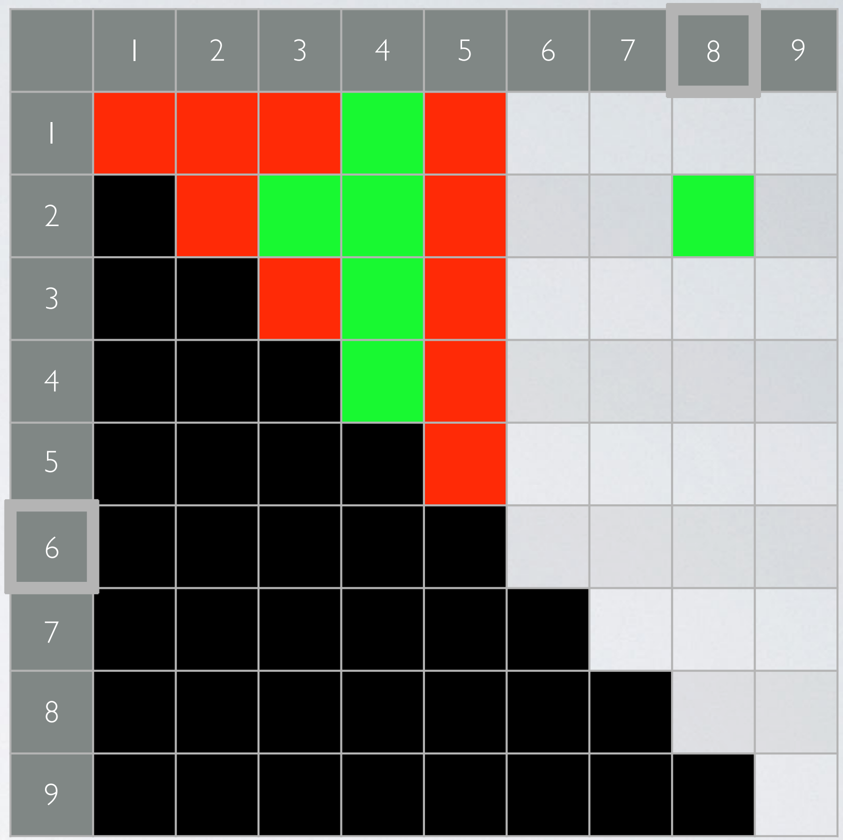
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$
9. $x.I(x) \rightsquigarrow e$

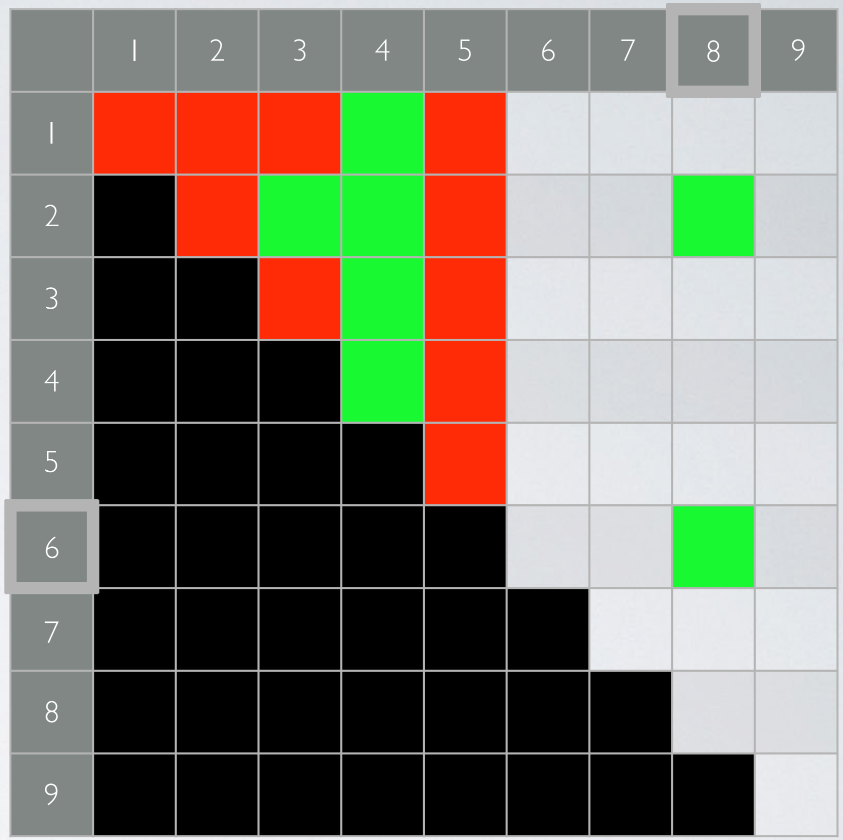


$$e \xleftarrow{2} I(I(a)).I(a) \xrightarrow{8} a.I(a)$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$
9. $x.I(x) \rightsquigarrow e$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(I(x)).e \rightsquigarrow x$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
8. $I(I(x)).y \rightsquigarrow x.y$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$



$$
\begin{array}{ccc}
 & I(I(a)).e & \\
\swarrow^{6} & & \searrow^{8} \\
a & & a.e
\end{array}
$$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

5 $I(e).z \rightsquigarrow z$

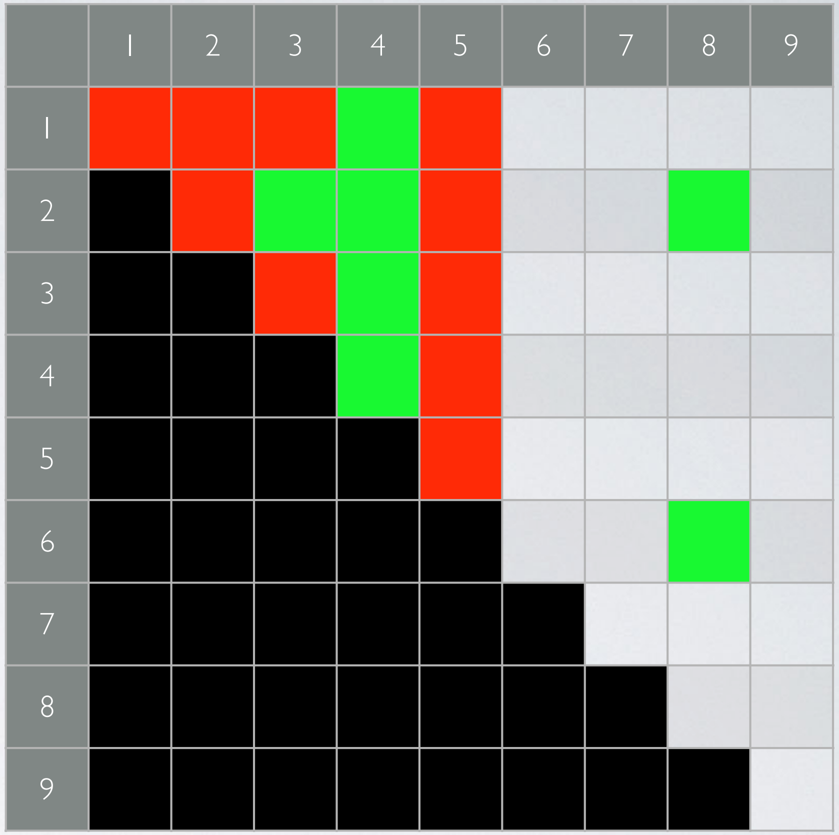
~~6~~ $\cancel{I(I(x)).e \rightsquigarrow x}$ 8 + 10

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

8 $I(I(x)).y \rightsquigarrow x.y$

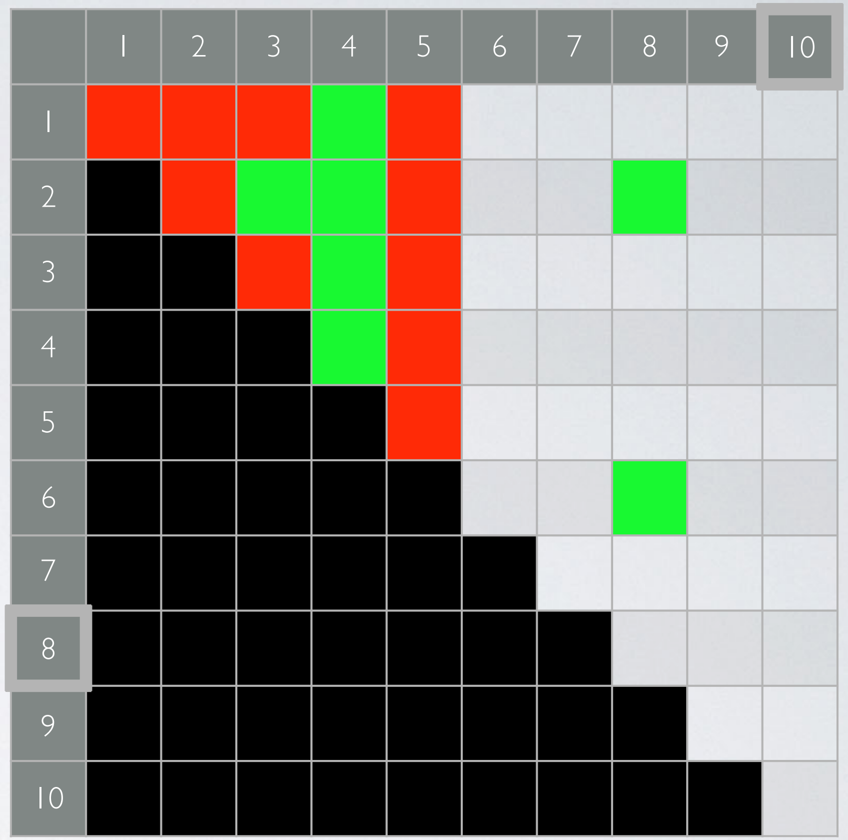
9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

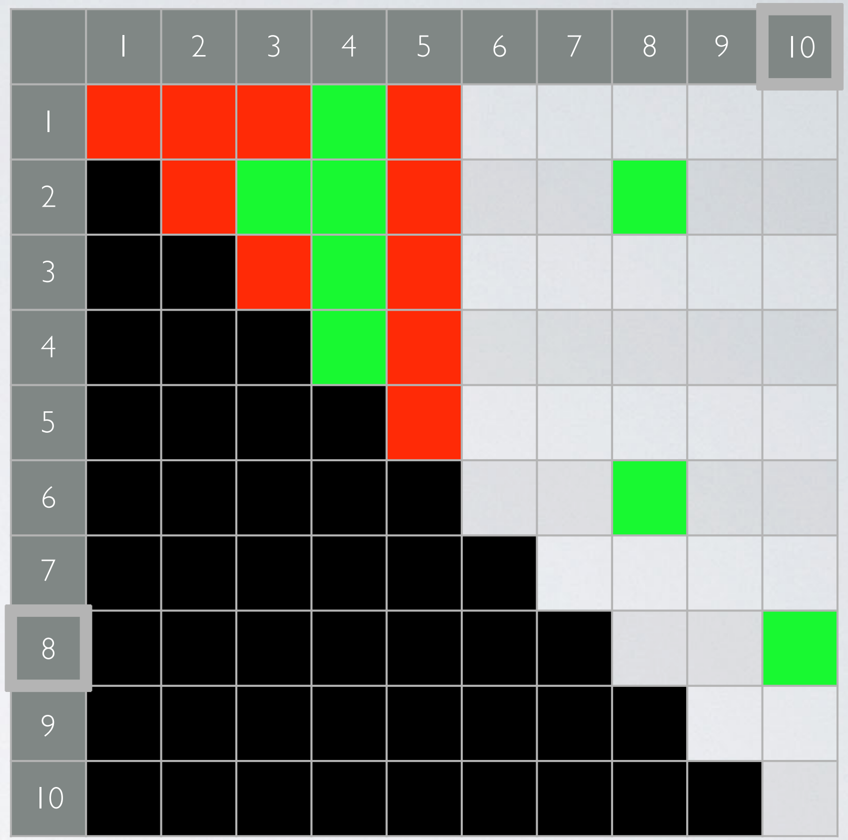


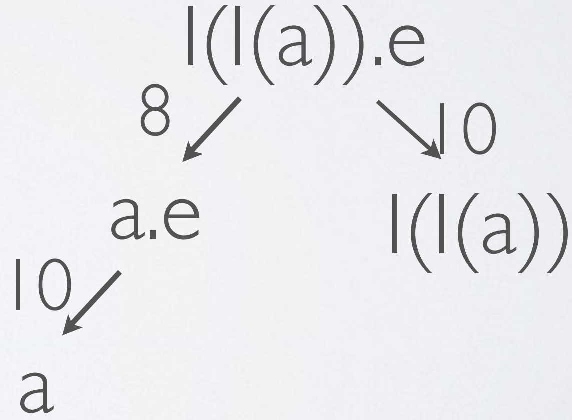
Useless rule!

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(x.y).(x.(y.z)) \rightsquigarrow z$
7. $I(I(x)).y \rightsquigarrow x.y$
8. $x.I(x) \rightsquigarrow e$
9. $x.e \rightsquigarrow x$

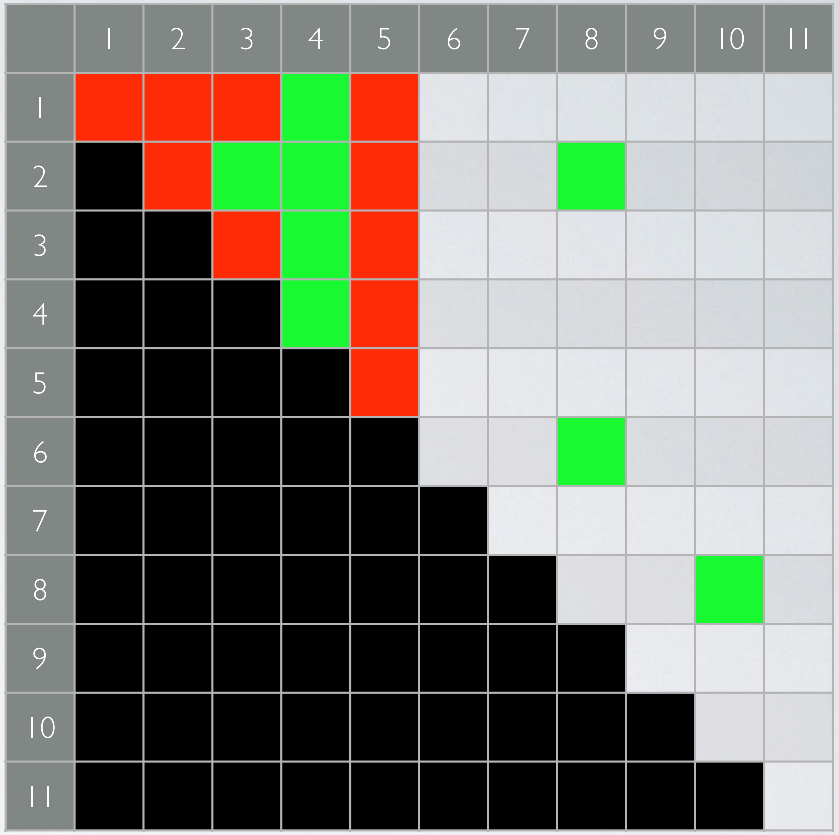


1. $e.x \leadsto x$
2. $I(x).x \leadsto e$
3. $(x.y).z \leadsto x.(y.z)$
4. $I(x).(x.z) \leadsto z$
5. $I(e).z \leadsto z$
6. $I(x.y).(x.(y.z)) \leadsto z$
7. $I(I(x)).y \leadsto x.y$
8. $x.I(x) \leadsto e$
9. $x.e \leadsto x$
10. $I(I(x)) \leadsto x$

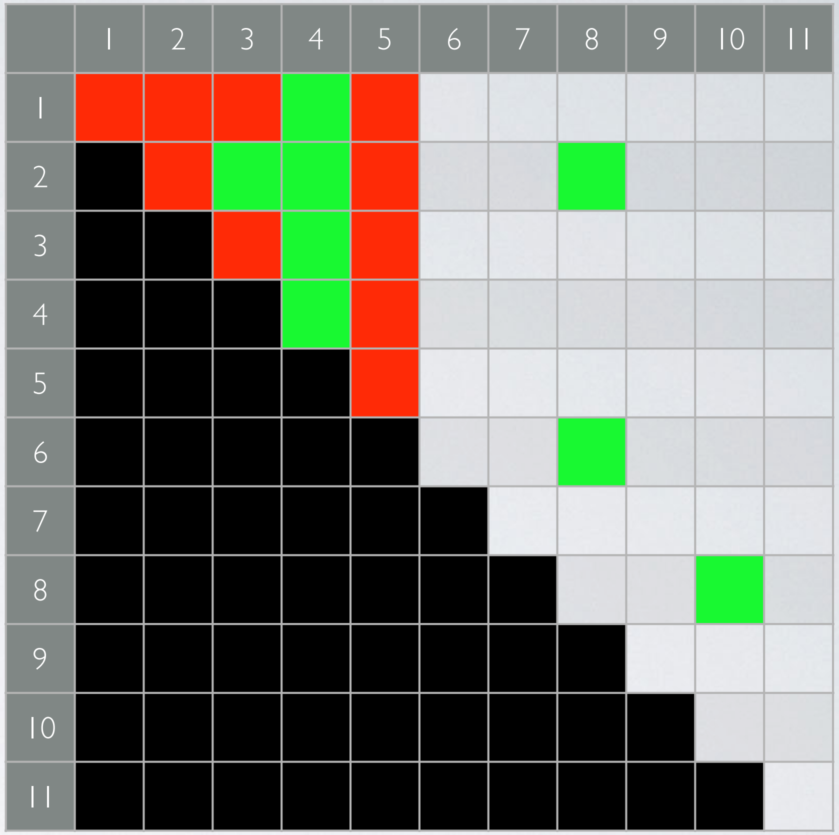




1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(x.y).(x.(y.z)) \rightsquigarrow z$
7. $I(I(x)).y \rightsquigarrow x.y$
8. $x.I(x) \rightsquigarrow e$
9. $x.e \rightsquigarrow x$
10. $I(I(x)) \rightsquigarrow x$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
5. $I(e).z \rightsquigarrow z$
6. $I(x.y).(x.(y.z)) \rightsquigarrow z$
7. $\cancel{I(I(x)).y \rightsquigarrow x.y}$ 11
8. $x.I(x) \rightsquigarrow e$
9. $x.e \rightsquigarrow x$
10. $I(I(x)) \rightsquigarrow x$



1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

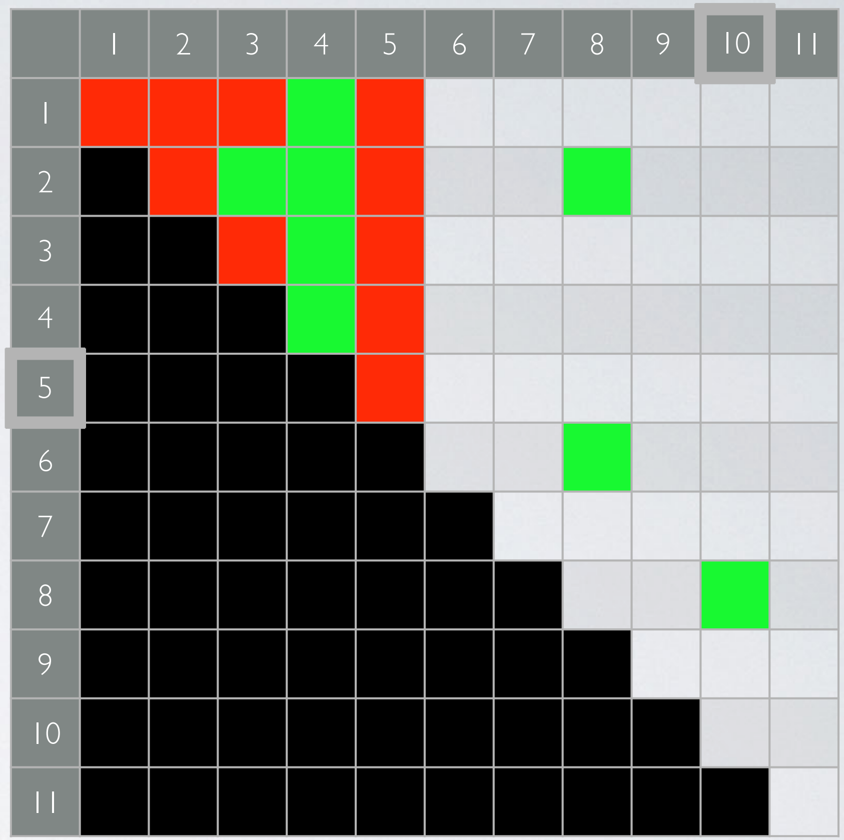
5 $I(e).z \rightsquigarrow z$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$



1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

5 $I(e).z \rightsquigarrow z$

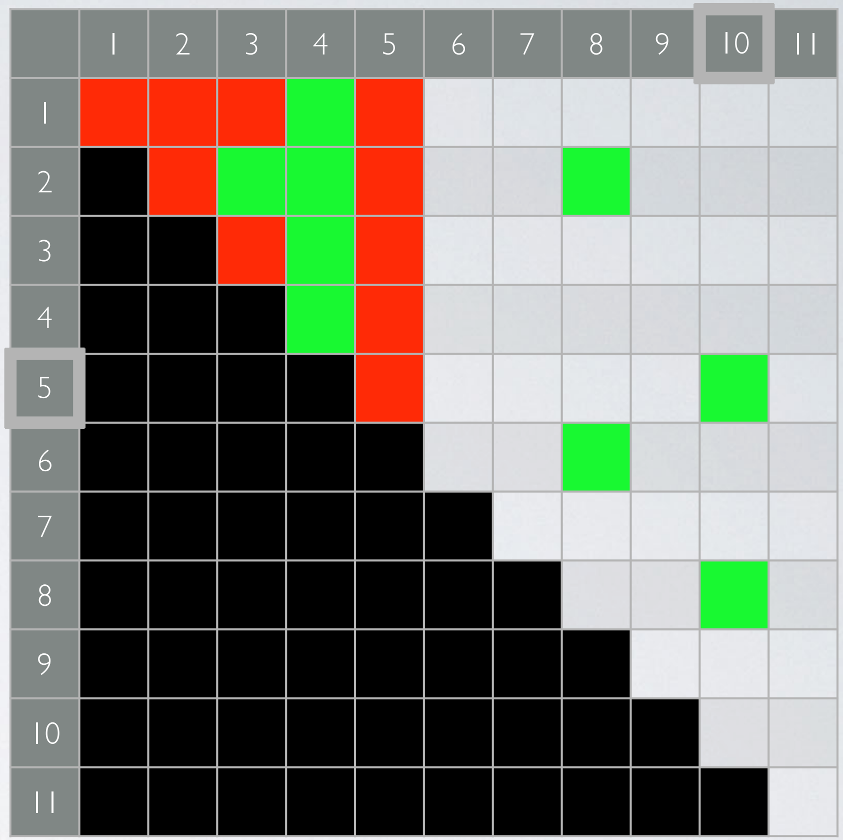
7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$

12 $I(e) \rightsquigarrow e$



$$\begin{tikzcd} & I(e).e \arrow[dl, "5"'] \arrow[dr, "10"] & \\ e & & I(e) \end{tikzcd}$$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

~~5 $I(e).z \rightsquigarrow z$~~ 12 + 1

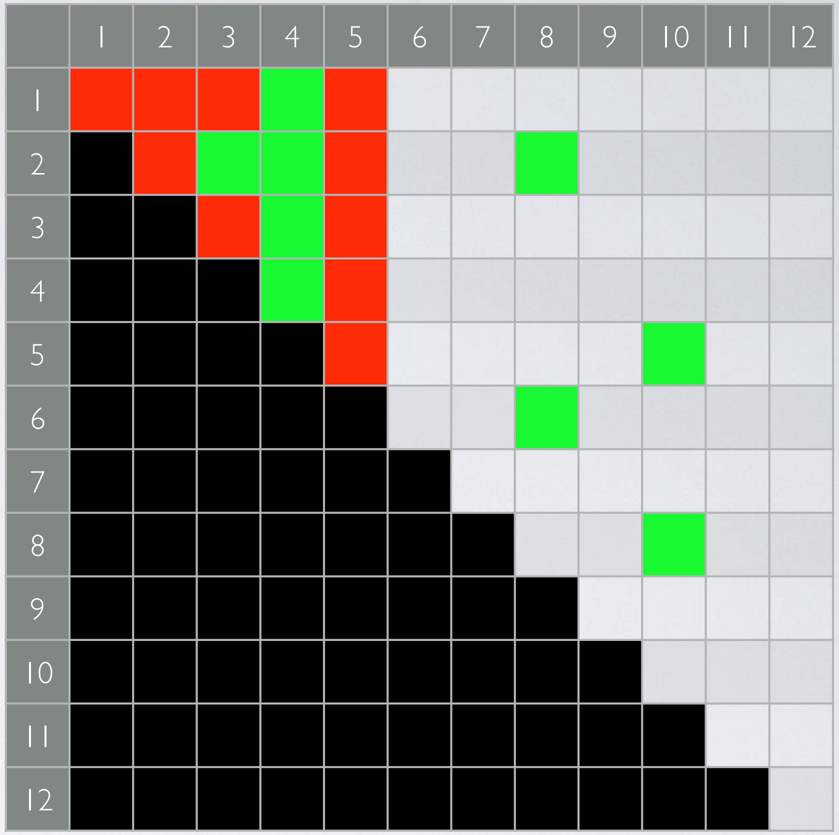
7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

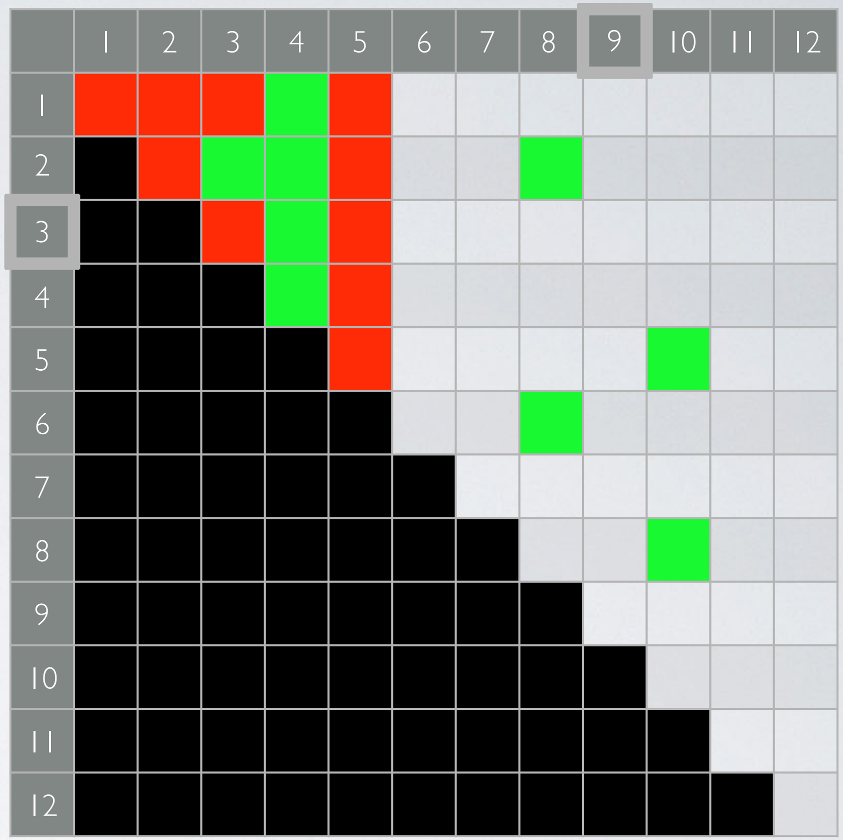
10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$

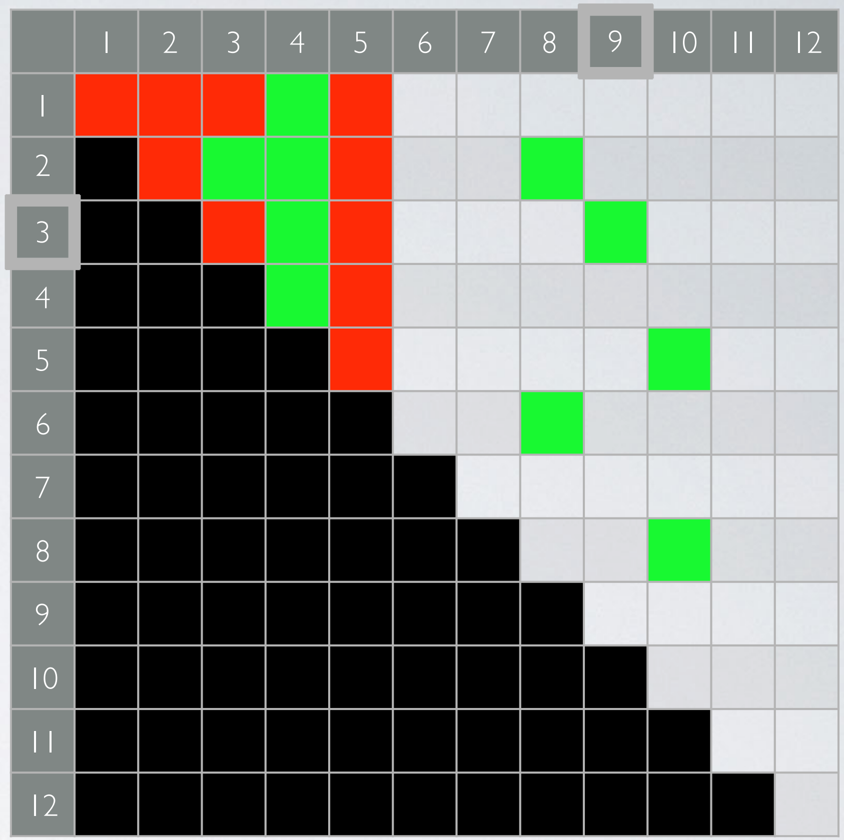
12 $I(e) \rightsquigarrow e$

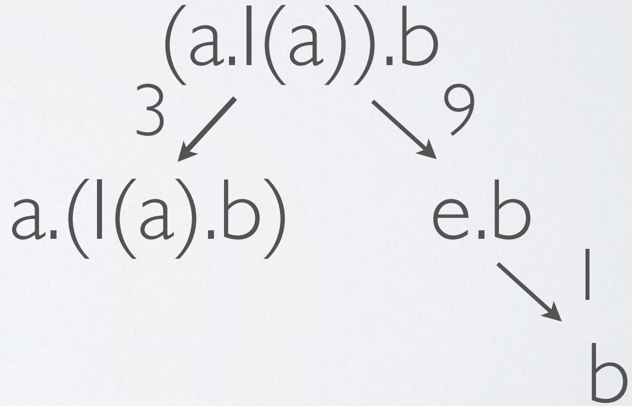


1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$





1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

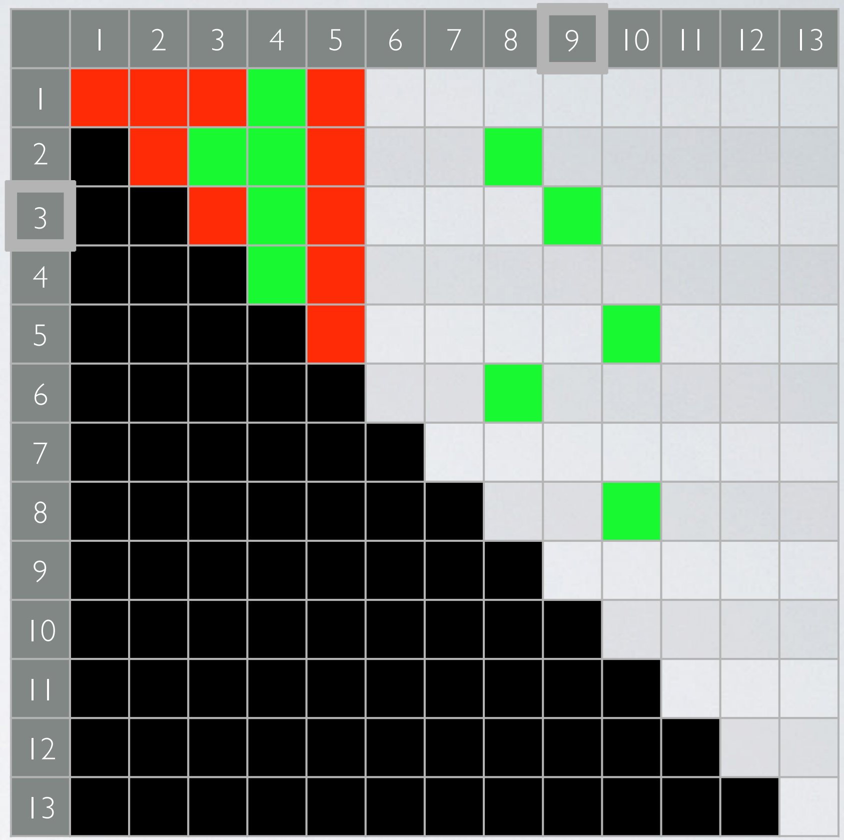
9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$

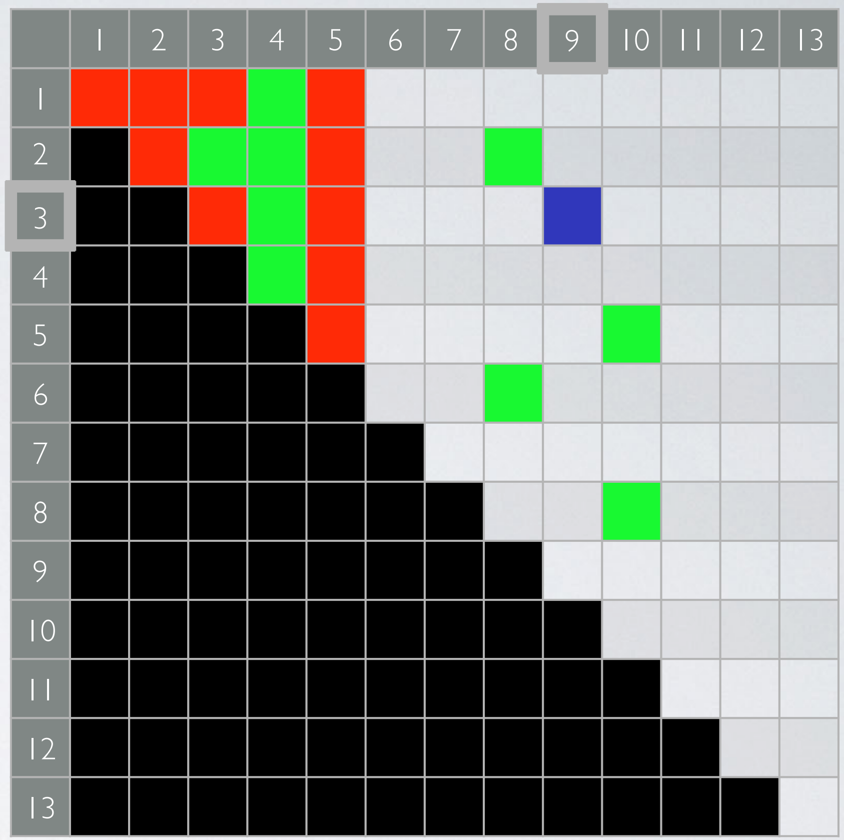
12 $I(e) \rightsquigarrow e$

13 $y.(I(y).x) \rightsquigarrow x$



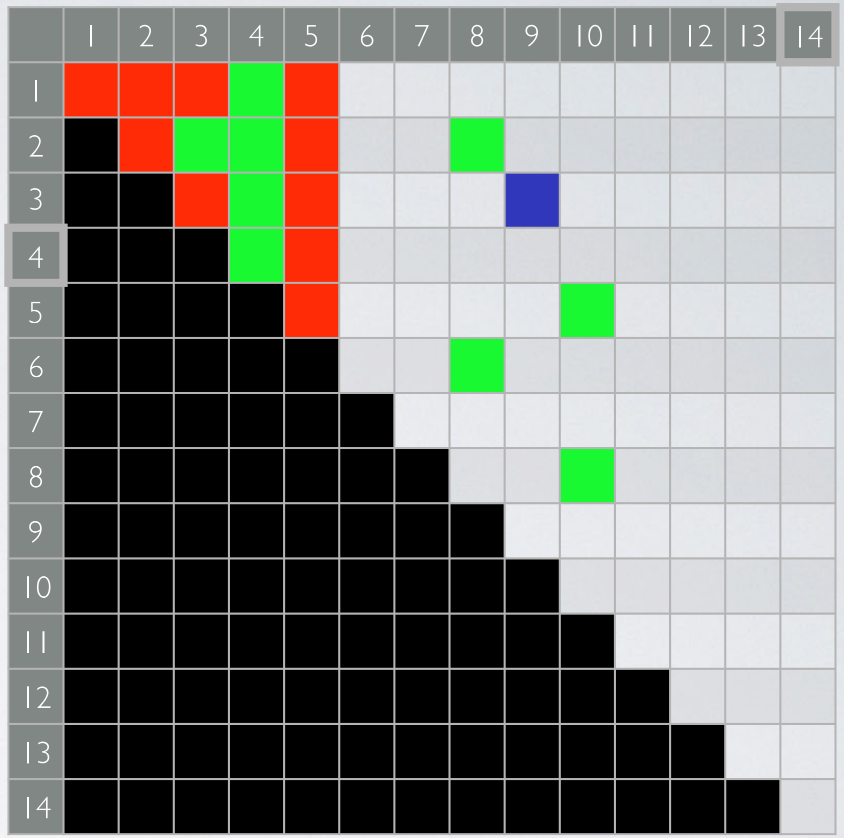
Mêmes règles, autre substitution!

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$
14. $x.(y.I(x.y)) \rightsquigarrow e$

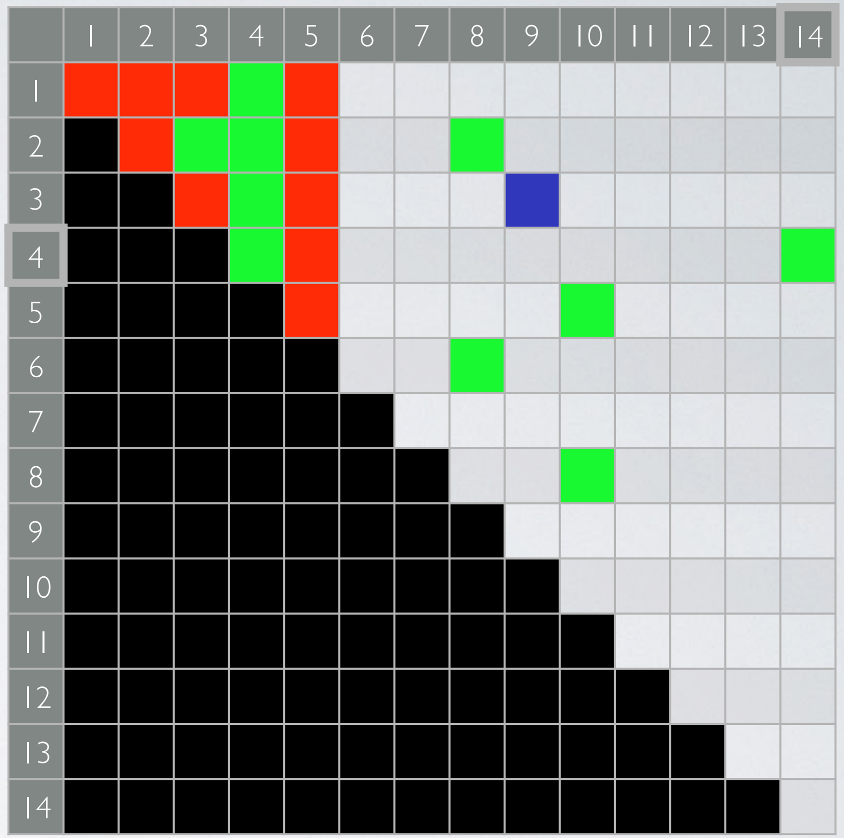


$$a.(b.(I(a.b))) \xleftarrow{3} (a.b).I(a.b) \xrightarrow{9} e$$

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$
14. $x.(y.I(x.y)) \rightsquigarrow e$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$
14. $x.(y.I(x.y)) \rightsquigarrow e$
15. $x.I(y.x) \rightsquigarrow I(y)$



$$I(a).(a.(b.I(a.b)))$$

$$b.I(a.b) \xleftarrow{4} I(a).(a.(b.I(a.b))) \xrightarrow{14} I(a)$$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

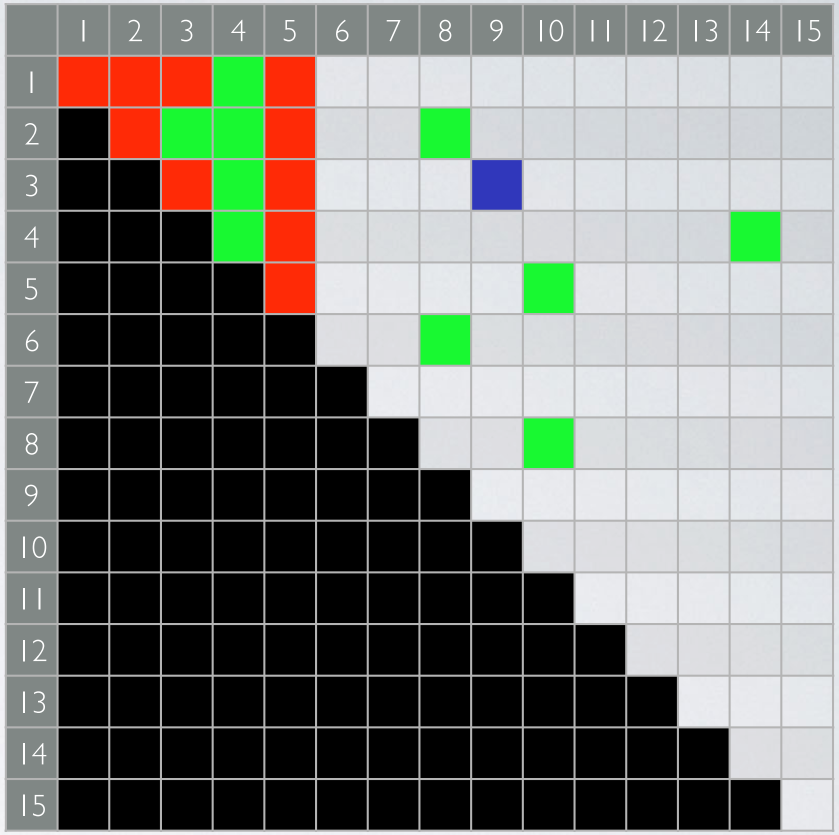
11 $I(I(x)) \rightsquigarrow x$

12 $I(e) \rightsquigarrow e$

13 $y.(I(y).x) \rightsquigarrow x$

~~14 $x.(y.I(x.y)) \rightsquigarrow e$~~ 15 + 9

15 $x.I(y.x) \rightsquigarrow I(y)$



1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

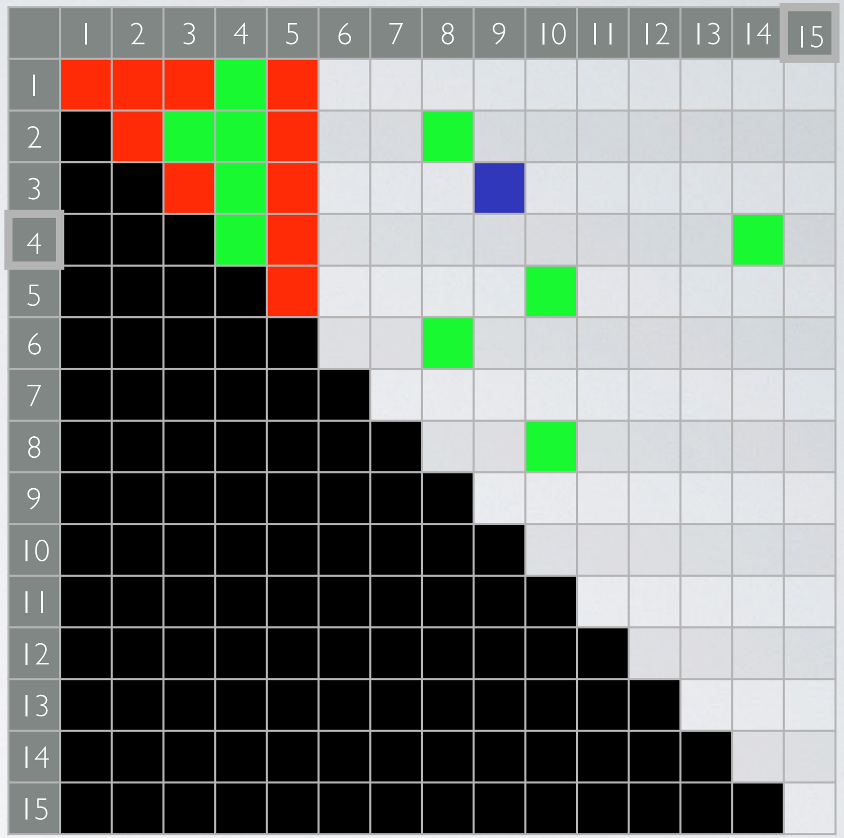
10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$

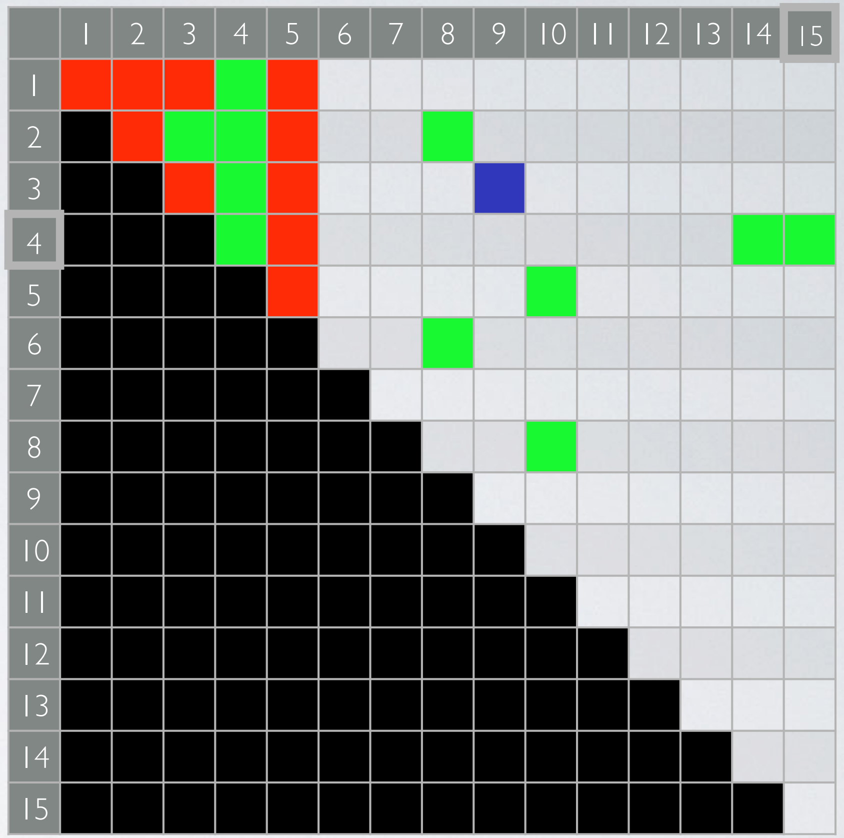
12 $I(e) \rightsquigarrow e$

13 $y.(I(y).x) \rightsquigarrow x$

15 $x.I(y.x) \rightsquigarrow I(y)$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. $I(x.y).(x.(y.z)) \rightsquigarrow z$
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$
15. $x.I(y.x) \rightsquigarrow I(y)$
16. $I(x.y) \rightsquigarrow I(y).I(x)$



$$I(a).(a.I(b.a))$$

$$I(a.b) \xleftarrow{4} I(a).(a.I(b.a)) \xrightarrow{15} I(b).I(a)$$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

7 $I(x.y).(x.(y.z)) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

10 $x.e \rightsquigarrow x$

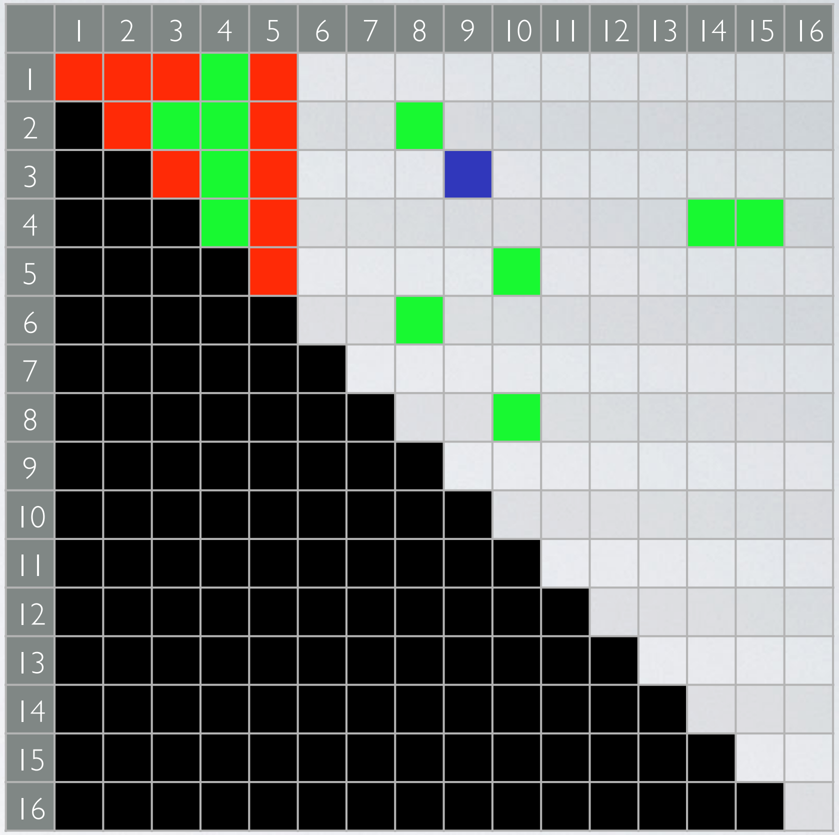
11 $I(I(x)) \rightsquigarrow x$

12 $I(e) \rightsquigarrow e$

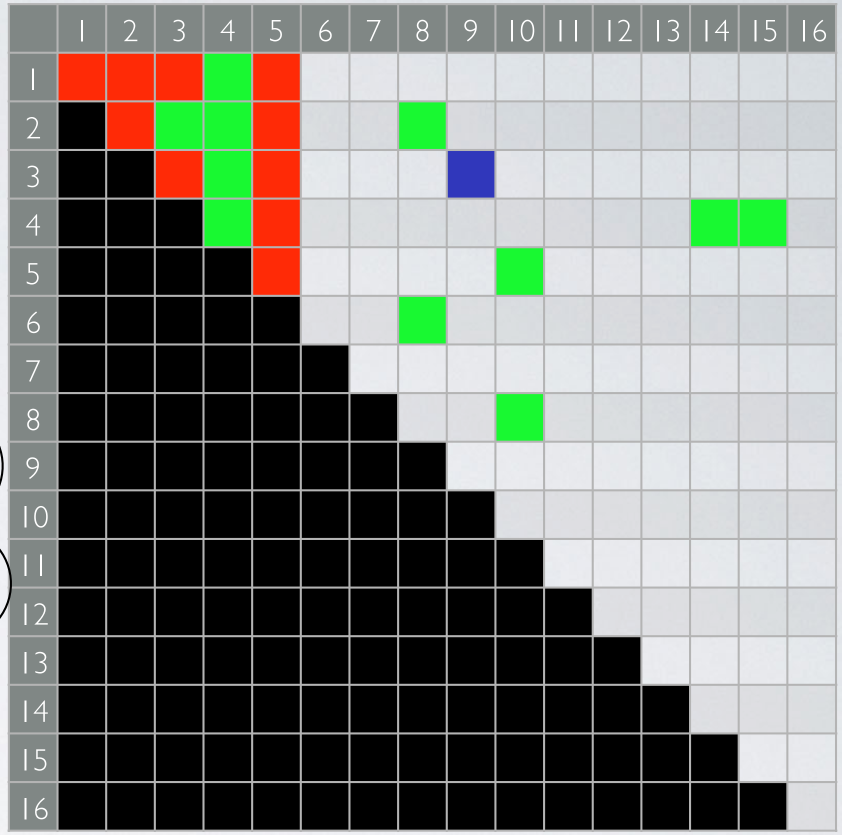
13 $y.(I(y).x) \rightsquigarrow x$

~~15 x.I(y.x) → I(y)~~ 16 + 13

16 $I(x.y) \rightsquigarrow I(y).I(x)$



1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $I(x).(x.z) \rightsquigarrow z$
7. ~~$I(x.y).(x.(y.z)) \rightsquigarrow z$~~ 16 + 3 + 4 + 4
9. $x.I(x) \rightsquigarrow e$
10. $x.e \rightsquigarrow x$
11. $I(I(x)) \rightsquigarrow x$
12. $I(e) \rightsquigarrow e$
13. $y.(I(y).x) \rightsquigarrow x$
16. $I(x.y) \rightsquigarrow I(y).I(x)$



1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $I(x).(x.z) \rightsquigarrow z$

9 $x.I(x) \rightsquigarrow e$

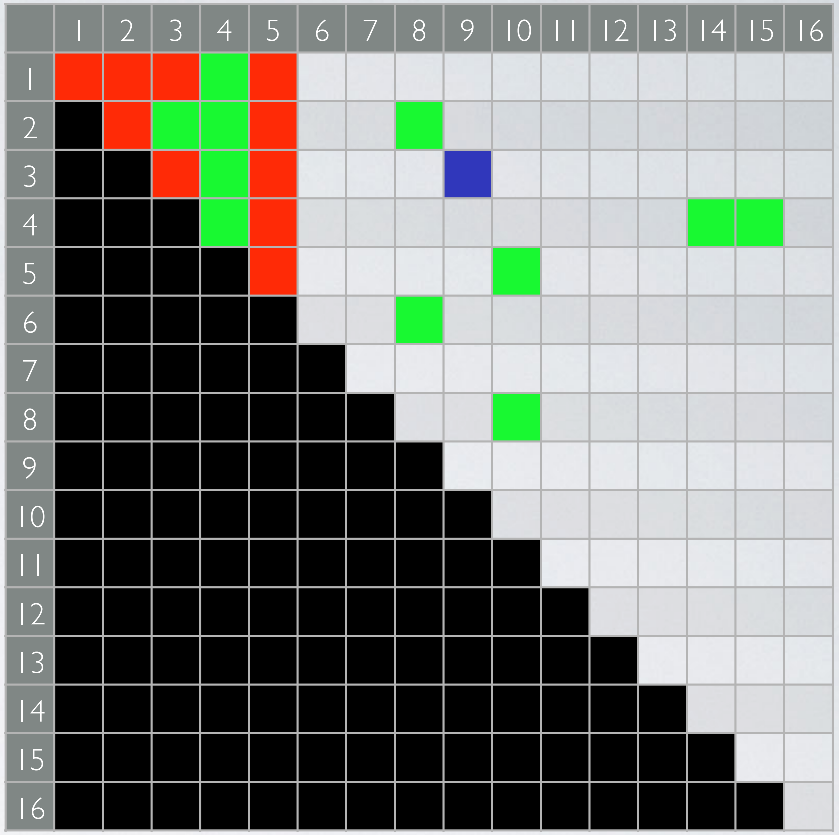
10 $x.e \rightsquigarrow x$

11 $I(I(x)) \rightsquigarrow x$

12 $I(e) \rightsquigarrow e$

13 $y.(I(y).x) \rightsquigarrow x$

16 $I(x.y) \rightsquigarrow I(y).I(x)$



$$e.x \rightsquigarrow x$$

$$I(x).x \rightsquigarrow e$$

$$(x.y).z \rightsquigarrow x.(y.z)$$

$$I(x).(x.z) \rightsquigarrow z$$

$$x.I(x) \rightsquigarrow e$$

$$x.e \rightsquigarrow x$$

$$I(I(x)) \rightsquigarrow x$$

$$I(e) \rightsquigarrow e$$

$$y.(I(y).x) \rightsquigarrow x$$

$$I(x.y) \rightsquigarrow I(y).I(x)$$

Objectif:

1. $e.x \rightsquigarrow x$
2. $I(x).x \rightsquigarrow e$
3. $(x.y).z \rightsquigarrow x.(y.z)$
4. $x.e \rightsquigarrow x$
5. $x.I(x) \rightsquigarrow e$
6. $x.(y.z) \rightsquigarrow (x.y).z$
7. $I(e) \rightsquigarrow e$
8. $I(I(x)) \rightsquigarrow x$
9. $I(x.y) \rightsquigarrow I(y).I(x)$

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

$I(x).(x.z) \rightsquigarrow z$

5 $x.I(x) \rightsquigarrow e$

4 $x.e \rightsquigarrow x$

8 $I(I(x)) \rightsquigarrow x$

7 $I(e) \rightsquigarrow e$

$y.(I(y).x) \rightsquigarrow x$

9 $I(x.y) \rightsquigarrow I(y).I(x)$

Objectif:

1 $e.x \rightsquigarrow x$

2 $I(x).x \rightsquigarrow e$

3 $(x.y).z \rightsquigarrow x.(y.z)$

4 $x.e \rightsquigarrow x$

5 $x.I(x) \rightsquigarrow e$

6 $x.(y.z) \rightsquigarrow (x.y).z$

7 $I(e) \rightsquigarrow e$

8 $I(I(x)) \rightsquigarrow x$

9 $I(x.y) \rightsquigarrow I(y).I(x)$