Transformers in Time Series: A Survey

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Abstract

Transformers have achieved superior performances in many tasks in natural language processing and computer vision, which also intrigues great interests in the time series community. Among multiple advantages of transformers, the ability of capturing long-range dependencies and interactions is especially attractive for time series modeling, leading to exciting progress in various time series applications. In this paper, we systematically review transformer schemes for time series modeling by highlighting their strengths as well as limitations through a new taxonomy to summarize existing time series transformers in two perspectives. From the perspective of network modifications, we summarize the adaptations of module level and architecture level of the time series transformers. From the perspective of applications, we categorize time series transformers based on common tasks including forecasting, anomaly detection, and classification. Empirically, we perform robust analysis, model size analysis, and seasonal-trend decomposition analysis to study how Transformers perform in time series. Finally, we discuss and suggest future directions to provide useful research guidance.

1 Introduction

The innovation of Transformer in deep learning [Vaswani et al., 2017] has brought great interests recently due to its excellent performances in natural language processing (NLP) [Kenton and Toutanova, 2019], computer vision (CV) [Dosovitskiy et al., 2021], speech processing [Dong et al., 2018], and other disciplines [Chen et al., 2021b]. Over the past few years, numerous Transformer variants have been proposed to advance the state-of-the-art performances of various tasks significantly. There are quite a few literature reviews from different aspects, such as in NLP applications [Qiu et al., 2020; Han et al., 2021], CV applications [Han et al., 2020; Khan et al., 2021; Selva et al., 2022], efficient Transformers [Tay et al., 2020], and attention models [Chaudhari et al., 2021; Galassi et al., 2020].

Transformers have shown great modeling ability for longrange dependencies and interactions in sequential data and thus are attractive in time series modeling. Due to the special characteristics of time series data and time series tasks, many Transformer variants have been proposed to adapt to time series data in various time series tasks, such as forecasting [Li et al., 2019; Zhou et al., 2021; Zhou et al., 2022], anomaly detection [Xu et al., 2022; Tuli et al., 2022], classification [Zerveas et al., 2021; Yang et al., 2021], and so on. For example, seasonality or periodicity is an important feature of time series [Wen et al., 2021a], how to effectively model long-range and short-range temporal dependency and capture seasonality simultaneously remains a challenge [Wu et al., 2021; Zhou et al., 2022]. As Transformer for time series is a newly emerging area in deep learning, a systematic and comprehensive survey on Transformers in time series would greatly benefit the time series community. We note that there exist some surveys related to deep learning for time series, include forecasting [Lim and Zohren, 2021; Benidis et al., 2020; Torres et al., 2021], classification [Ismail Fawaz et al., 2019], anomaly detection [Choi et al., 2021; Blázquez-García et al., 2021], and data augmentation [Wen et al., 2021b], but little or no in-depth analysis on Transformers for time series are discussed.

In this paper, we aim to fill the aforementioned gaps by summarizing existing time series Transformers. Specifically, we first give a brief introduction about vanilla Transformer, and then propose a new taxonomy from perspectives of network modifications and application domains for time series Transformers. For network modifications, we consider both low-level module adjustments and high-level architecture improvements of Transformers optimized for time series data. For applications, we summarize and analyze Transformers for popular time series tasks including forecasting, anomaly detection, and classification. For each time series Transformer, we analyze its insights, strengths as well as limitations. To provide practical guidelines on how to use Transformers for time series, we further conduct some empirical studies of time series Transformers, including robustness analysis, model size analysis, and seasonal-trend decomposition analysis. Lastly, we suggest and discuss possible future directions, including inductive biases for time series Transformers, Transformers and GNN for time series, pre-trained Transformers for time series, and Transformers with NAS for time series. To the best of our knowledge, this paper is the first work to comprehensively and systematically summarize

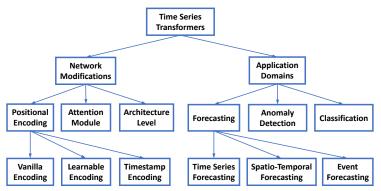


Figure 1: Taxonomy of Transformers for time series modeling from the perspectives of network modifications and application domains.

the recent advances of Transformers for modeling time series data. We hope this survey will ignite further research interests in time series Transformers.

2 Preliminaries of the Transformer

2.1 Vanilla Transformer

The vanilla Transformer [Vaswani *et al.*, 2017] follows most competitive neural sequence models with an encoder-decoder structure. Both encoder and decoder are composed of multiple identical blocks. Each encoder block consists of a multi-head self-attention module and a position-wise feed-forward network (FFN) while each decoder block inserts cross-attention models between the multi-head self-attention module and the position-wise feed-forward network (FFN).

2.2 Input Encoding and Positional Encoding

Unlike LSTM or RNN, Transformer has no recurrence and no convolution. Instead, it utilizes the positional encoding added in the input embeddings, to model the sequence information. We summarize some positional encodings below.

Absolute Positional Encoding

In vanilla Transformer, for each position index t, encoding vector is given by

$$PE(t)_i = \begin{cases} sin(\omega_i t) & i\%2 = 1\\ cos(\omega_i t) & i\%2 = 0 \end{cases}$$
 (1)

where ω_i is the hand-crafted frequency for each dimension. Another way is to learn a set of positional embeddings for each position which is more flexible [Kenton and Toutanova, 2019; Gehring *et al.*, 2017].

Relative Positional Encoding

Following the intuition that pairwise positional relationships between input elements is more beneficial than positions of elements, relative positional encoding methods have been proposed. For example, one of such methods is to add a learnable relative positional embedding to keys of attention mechanism [Shaw *et al.*, 2018].

Besides the absolute and relative positional encodings, there are methods using hybrid positional encodings that combine them together [Ke *et al.*, 2020]. Generally, the positional encoding is added to the token embedding and fed to Transformer.

2.3 Multi-head Attention

With Query-Key-Value (QKV) model, the scaled dot-product attention used by Transformer is given by

$$Attention(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = softmax(\frac{\mathbf{Q}\mathbf{K^T}}{\sqrt{D_k}})\mathbf{V} \qquad (2)$$

where queries $\mathbf{Q} \in \mathcal{R}^{N \times D_k}$, keys $\mathbf{K} \in \mathcal{R}^{M \times D_k}$, values $\mathbf{V} \in \mathcal{R}^{M \times D_v}$ and N, M denote the lengths of queries and keys (or values), D_k, D_v denote the dimensions of keys (or queries) and values. Transformer uses multi-head attention with H different sets of learned projections instead of a single attention function as

 $MultiHeadAttn(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = Concat(head_1, \dots, head_H)\mathbf{W}^O,$ where $head_i = Attention(\mathbf{Q}\mathbf{W}_i^Q, \mathbf{K}\mathbf{W}_i^K, \mathbf{V}\mathbf{W}_i^V).$

2.4 Feed-forward and Residual Network

The point-wise feed-forward network is a fully connected module as

$$FFN(\mathbf{H}') = ReLU(\mathbf{H}'\mathbf{W}^1 + \mathbf{b}^1)\mathbf{W}^2 + \mathbf{b}^2, \quad (3)$$

where \mathbf{H}' is outputs of previous layer, $\mathbf{W}^1 \in \mathcal{R}^{D_m \times D_f}$, $\mathbf{W}^2 \in \mathcal{R}^{D_f \times D_m}$, $\mathbf{b}^1 \in \mathcal{R}^{D_f}$, $\mathbf{b}^2 \in \mathcal{R}^{D_m}$ are trainable parameters. In a deeper module, a residual connection module followed by Layer Normalization Module is inserted around each module. That is,

$$\mathbf{H}' = LayerNorm(SelfAttn(\mathbf{X}) + \mathbf{X}), \tag{4}$$

$$\mathbf{H} = LayerNorm(FFN(\mathbf{H}') + \mathbf{H}'), \tag{5}$$

where SelfAttn(.) denotes self attention module and LayerNorm(.) denotes the layer normal operation.

3 Taxonomy of Transformers in Time Series

To summarize the existing time series Transformers, we propose a taxonomy from perspectives of network modifications and application domains as illustrated in Fig. 1. Based on the taxonomy, we review the existing time series Transformers systematically. From the perspective of network modifications, we summarize the the adaptations of module level and architecture level of the time series Transformers. From the perspective of applications, as different applications require different models, we categorize time series Transformers based on common tasks including forecasting, anomaly detection, classification and clustering. In the following two sections, we would delve into the existing time series Transformers from these two perspectives.

4 Network Modifications for Time Series

4.1 Positional Encoding

As Transformers are permutation equivalent and the ordering of time series matters, it is of great importance to encode the positions of input time series into Transformers. A common design is to first encode positional information as vectors and then inject them into the model as an additional input together with the input time series. How to obtain these vectors when modeling time series with Transformers can be divided into three main categories.

Vanilla Positional Encoding. A few works [Li *et al.*, 2019] simply introduce vanilla positional encoding (Section 2.2) used in [Vaswani *et al.*, 2017], which is then added to the input time series embeddings and fed to Transformer. This plain application can extract some positional information from time series, but do not fully utilize the time series data and adapt to different data.

Learnable Positional Encoding. As the vallina positional encoding is hand-crafted and less expressive and adaptive, some works find that representing position information by learning a set of positional embeddings for each position is much more expressive. Compared to the fixed vanilla positional encoding, learned embeddings are much more flexible and can adapt to the specific tasks. [Zerveas *et al.*, 2021] introduce an embedding layer to learn embeddings for each position index, jointly learnt with model parameters. To further exploit the ordering information of positions, [Lim *et al.*, 2019] use an LSTM network to encode positional embeddings, providing an appropriate inductive bias for the time ordering of the input time series.

Timestamp Encoding. When modeling time series in real-world scenarios, the timestamp information is commonly accessible, including calendar timestamps (e.g., second, minute, hour, week, month and year) and special timestamps (e.g., holidays and events). These timestamps are quite informative in real applications but hardly leveraged in vanilla Transformers. To mitigate the issue, Informer [Zhou *et al.*, 2021] proposed to encode timestamps as positional encoding, which embeds the timestamp features using learnable embedding layers. Autoformer [Wu *et al.*, 2021] and FEDformer [Zhou *et al.*, 2022] follow this timestamp encoding scheme due to its capacity of leveraging global temporal information.

4.2 Attention Module

As the central piece of the Transformer, self-attention uses a quite flexible mechanism to process the inputs, it can be viewed as a fully connected layer where the weights are dynamically generated from the pairwise multiplication from inputs. So, it has the same maximum path length as fully connected layers, but much less parameter, which makes it suitable for long-term dependencies modeling.

As we show in the previous section the self-attention module in vanilla Transformer has a time and memory complexity of $O(L^2)(L)$ is the input time series length) which is the calculation bottleneck when dealing with long sequences. Many efficient Transformers investigate ways to reduce its quadratic complexity. Here we summarize some main directions for

Table 1: Complexity comparisons of popular time series transformers with different attention modules.

Methods	Trai	Testing	
Wellods	Time	Memory	Steps
Transformer [Vaswani et al., 2017]	$\mathcal{O}\left(L^{2}\right)$	$\mathcal{O}\left(L^2\right)$	L
LogTrans [Li et al., 2019]	$\mathcal{O}(L \log L)$	$\mathcal{O}\left(L^2\right)$	1
Informer [Zhou et al., 2021]	$\mathcal{O}(L \log L)$	$\mathcal{O}(L \log L)$	1
Pyraformer [Liu et al., 2022]	$\mathcal{O}(L)$	$\mathcal{O}(L)$	1
FEDformer [Zhou et al., 2022]	$\mathcal{O}(L)$	$\mathcal{O}(L)$	1

improving the efficiency of attention: (1) Explicitly introducing a sparsity bias into the attention mechanism like Log-Trans [Li *et al.*, 2019] and Pyraformer [Liu *et al.*, 2022]; (2) Exploring the low-rank property of self-attention matrix. Some typical examples include Informer [Zhou *et al.*, 2021] and FEDformer [Zhou *et al.*, 2022]. Formally, we summarize the time and memory complexity of popular Transformers in time series in Table 1.

4.3 Architecture-Level Innovation

In addition of modifying individual modules in Transformers for modeling time series, a number of works [Zhou et al., 2021; Liu et al., 2022] seek to renovate Transformers in architecture level. Recent works modify Transformer into hierarchical architecture considering the multi-resolution of time series. Informer [Zhou et al., 2021] inserts max-pooling layers with stride 2 between attention blocks, which could downsample series into its half slice. Pyraformer [Liu et al., 2022] designs a C-ary tree based attention mechanism, in which nodes at the finest scale correspond to the time points in the original time series, while nodes in the coarser scales represent series with lower resolutions. Pyraformer conducts both intra-scale and inter-scale attentions, which capture temporal dependencies in an individual resolution and build a multiresolution representation of the original series, respectively. The advantages of hierarchical architectures are twofold: (1) Hierarchical architectures allow the model to handle long time series with less computational complexity; (2) Hierarchical modeling can generate multi-resolution representations that might be beneficial to specific tasks.

5 Applications of Time Series Transformers

5.1 Transformers in Forecasting

Time Series Forecasting

Forecasting is the most common and important application of time series. LogTrans [Li *et al.*, 2019] proposes convolutional self-attention by employing causal convolutions to produce queries and keys in the self-attention layer. Besides, it introduces sparse bias, a Logsparse mask to calculate $O(\log L)$ dot products as a concrete representation of all O(L) dot products to reduce the model's time and calculation complexity.

Without explicitly introducing sparse bias, Informer [Zhou et al., 2021] uses the Kullback-Leibler divergence distribution measurement between queries and keys to select $O(\log L)$ dominant queries, reducing the complexity of the attention module. It also designs a generative style decoder to produce long sequence output with only one forward step to avoid accumulation error.

AST [Wu et al., 2020] uses a generative adversarial encoder-decoder framework to train a sparse Transformer

model for time series forecasting. It shows that adversarial training can improve the time series forecasting by directly shaping the output distribution of the network to avoid the error accumulation through one-step ahead inference.

Autoformer [Wu et al., 2021] devises a simple seasonal-trend decomposition architecture with an auto-correlation mechanism working as an attention module. The auto-correlation block is not a traditional attention block. It measures the time-delay similarity between inputs signal and aggregate the top-k similar sub-series to produce the output with a reduced complexity of $O(L \log L)$.

FEDformer [Zhou et al., 2022] designs two attention modules which process the attention operation in the frequency domain with Fourier transform and wavelet transform, respectfully. It achieves a linear complexity through a random mode section in the Fourier transform. We would like to point out that since Autoformer and FEDformer, the unique proprieties of time series in frequency domain or time-frequency domain attract more attention in the community.

TFT [Lim *et al.*, 2021] designs a multi-horizon forecasting model with static covariate encoders, gating feature selection module and temporal self-attention decoder. It encodes and selects useful information from various covariates information to perform forecasting. It also preserves interpretability incorporating global, temporal dependency and event.

SSDNet [Lin *et al.*, 2021] and ProTran [Tang and Matteson, 2021] combine the Transformer architecture with state space models to provide probabilistic forecasts. SSDNet uses the Transformer part to learn the temporal pattern and estimate the parameters of SSM, and uses the SSM parts to perform the seasonal-trend decomposition and maintain the interpretable ability. ProTran designs a generative modeling and inference procedures based on variational inference.

Pyraformer [Liu *et al.*, 2022] designs a hierarchical pyramidal attention module with binary tree following path, to capture temporal dependencies of different ranges with linear time and memory complexity.

Aliformer [Qi *et al.*, 2021] makes the time-series sales forecasting using a Knowledge-guided attention with a branch to revise and denoise the the attention map.

Spatio-Temporal Forecasting

In spatio-temporal forecasting, we need to consider both the temporal dependency and spatio-temporal dependency to make accurate forecasting. Traffic Transformer designs an encoder-decoder structure using a self attention module to capture the temporal-temporal dependencies and a Graph neural network module to capture the spatial dependencies [Cai et al., 2020]. Spatial-temporal Transformer networks for traffic flow forecasting make a step further, besides introducing a temporal Transformer block to capture the temporal dependencies, it also design a spatial Transformer block to assist a graph convolution network to capture more spatial-spatial dependencies [Xu et al., 2020]. Further, spatio-temporal graph Transformer networks designs a attention-based graph convolution mechanism to learn a more complex temporal-spatial attentions pattern to improve pedestrian trajectory prediction [Yu et al., 2020].

Event Forecasting

Event sequence data with irregular and asynchronous timestamps are naturally observed in many real-life applications, which is in contrast to regular time series data with equal sampling intervals. Event forecasting or prediction aims to predict the times and marks of future events given the history of past events, and it is often modeled by temporal point processes (TPP) [Shchur *et al.*, 2021].

Recently, some neural TPP models start to incorporate Transformers to improve the performance of event prediction. Self-attentive Hawkes process (SAHP) [Zhang et al., 2020] and Transformer Hawkes process (THP) [Zuo et al., 2020] adopt Transformer encoder architecture to summarize the influence of history events and compute the intensity function for event prediction. They modify the positional encoding by translating time intervals into sinusoidal function such that the intervals between events can be utilized. Later, a more flexible named attentive neural Datalog through time (ANDTT) [Mei et al., 2022] is proposed to extend SAHP/THP schemes by embedding all possible events and times with attention as well. Experiments show that it can better capture sophisticated event dependencies than existing methods.

5.2 Transformers in Anomaly Detection

Deep learning also intrigues some novel time series anomaly detection algorithms [Ruff et al., 2021]. As deep learning is a kind of representation learning, reconstruction model plays a significant role in anomaly detection tasks. Reconstruction model aims to learn a neural network that maps vectors from a simple predefined source distribution \mathcal{Q} to the actual input distribution \mathcal{P}^+ . \mathcal{Q} is usually a Gaussian or uniform distribution. Anomaly score is defined by reconstruction error. Intuitively, the higher reconstruction error which means less likely to be from the input distribution, the higher anomaly score. A threshold is set to discriminate anomaly from normality.

Recently, [Meng et al., 2019] reveals the advantage of Transformer over other traditional temporal dependency models such as LSTM in anomaly detection by a quick comparison: 1) 80% time is saved compared with LSTM methods due to parallel computing in Transformer architecture, 2) higher F1 point-based score is achieved with range-based precision and recall. Besides, anomaly detection models combining Transformer with neural generative models, such as VAEs [Kingma and Welling, 2013] and GANs [Goodfellow et al., 2014] for better reconstruction models, are also proposed in TranAD [Tuli et al., 2022], MT-RVAE [Wang et al., 2022], and TransAnomaly [Zhang et al., 2021].

TranAD proposes an adversarial training procedure to amplify reconstruction errors as a simple Transformer-based network tends to miss small deviation of anomaly. GAN style adversarial training procedure is designed by two Transformer encoders and two Transformer decoders to gain stability. Ablation study shows that, if Transformer-based encoderdecoder is replaced, F1 score performance drops nearly 11%, which shows that Transformer architecture contributes a lot to TranAD.

While both MT-RVAE and TransAnomaly combine VAE with Transformer, TransAnomaly combines VAE with Trans-

former to allow more parallelization and training cost is reduced by nearly 80%, and MT-RVAE focuses on data with few dimensions or sparse relationships. In MT-RVAE, multiscale Transformer is designed to overcome the shortcomings of traditional Transformer that only local sequential information is extracted, where multiscale feature fusion is designed to better extract different levels of global time-series information.

Some other Transformer-based approaches designed for multivariate time series combine Transformer with graph structure learning architecture, such as GTA [Chen et al., 2021d]. Note that, MT-RVAE is also for multivariate time series but with few dimensions or insufficient close relationships among sequences where the graph neural network model does not work well. To deal with such challenge, MT-RVAE modifies positional encoding module and introduces feature-learning module. GTA contains graph convolution structure to model the influence propagation process. Similar with MT-RVAE, GTA also considers 'global' information, yet by replacing vanilla multi-head attention with multi-branch attention mechanism, that is, a combination of both global-learned attention and vanilla multi-head attention and neighborhood convolution.

AnomalyTrans [Xu et al., 2022] combines Transformer and Gaussian Prior-Association to make rare anomalies more distinguishable. Such motivation is similar with TranAD, yet AnomalyTrans achieves this goal in a very different way. The insight is that it is harder for anomalies to build strong associations with the whole series while easier with adjacent time points compared with normality. In AnomalyTrans, prior-association and series-association are modeled simultaneously. Besides reconstruction loss, anomaly model is optimized by the minimax strategy to constrain the prior- and series- associations for more distinguishable association discrepancy.

5.3 Transformers in Classification

Transformer is proved to be efficient in various time series classification tasks due to its prominent capability in capturing long-term dependency. Classification Transformers usually employ a simple encoder structure, in which self-attention layers performs representation learning and feed forward layers produce probability of each class.

GTN [Liu et al., 2021] uses a two-tower Transformer with each tower respectively working on time-step-wise attention and channel-wise attention. To merge the feature of the two towers, a learnable weighted concat (also known as 'gating') is used. The proposed extension of Transformer achieves state-of-the-art results on 13 multivariate time series classification. [Rußwurm and Körner, 2020] studies the self-attention based Transformer for raw optical satellite time series classification and obtained the best results comparing with recurrent and convolutional models.

Pre-trained Transformers are also investigated in classification tasks. [Yuan and Lin, 2020] studies the Transformer for raw optical satellite image time series classification. The authors use self-supervised pre-trained schema because of rareness of labeled data. [Zerveas *et al.*, 2021] introduces an unsupervised pre-trained framework and the model is pre-

Table 2: The MSE comparisons in robustness experiment of forecasting 96 steps for ETTm2 dataset with prolonging input length.

Model		Transformer	Autoformer	Informer	Reformer	LogFormer	
u	96	0.557	0.239	0.428	0.615	0.667	
Input Len	192	0.710	0.265	0.385	0.686	0.697	
nt	336	1.078	0.375	1.078	1.359	0.937	
du	720	1.691	0.315	1.057	1.443	2.153	
I	1440	0.936	0.552	1.898	0.815	0.867	

Table 3: The MSE comparisons in model size experiment of forecasting 96 steps for ETTm2 dataset with different number of layers.

Mo	odel	Transformer	Autoformer	Informer	Reformer	LogFormer
Num	3	0.557	0.234	0.428	0.597	0.667
Ž	6	0.439	0.282	0.489	0.353	0.387
	12	0.556	0.238	0.779	0.481	0.562
Layer	24	0.580	0.266	0.815	1.109	0.690
Γc	48	0.461	NaN	1.623	OOM	2.992

trained with proportionally masked data. The pre-trained models are then fine-tuned in downstream tasks such as classification. [Yang *et al.*, 2021] proposes to use large-scale pre-trained speech processing model for downstream time series classification problems and generates 19 competitive results on 30 popular time series classification datasets.

6 Experimental Evaluation and Discussion

In this section, we conduct empirical studies to analyze how Transformers work on time series data. Specifically, we test different algorithms with different configurations on a typical benchmark dataset ETTm2 [Zhou *et al.*, 2021].

Robustness Analysis

A lot of works we describe above carefully design attention module to lower the quadratic calculation and memory complexity, though they practically use a short fixed-size input to achieve the best result in their reported experiments. It makes us question the actual usage of such an efficient design. We perform a robust experiment with prolonging input sequence length to verify their prediction power and robustness when dealing with long-term input sequences.

As shown in Table 2, when we compare the prediction results with prolonging input length, various Transformer-based model deteriorates quickly. This phenomenon makes a lot of carefully designed Transformers impractical in long-term forecasting tasks since they cannot effectively utilize long input information. More works need to be done to fully utilize long sequence input other than simply running it.

Model Size Analysis

Before being introduced into the field of time series prediction, Transformer has shown dominant performance in NLP and CV community [Vaswani *et al.*, 2017; Kenton and Toutanova, 2019; Qiu *et al.*, 2020; Han *et al.*, 2021; Han *et al.*, 2020; Khan *et al.*, 2021; Selva *et al.*, 2022]. One of the key advantages Transformer holds in these fields is being able to increase prediction power through increasing model size. Usually the model capacity is controlled by Transformer's layer number, which is commonly set between 12 to 128 in CV and NLP.

But as shown in our experiments in Table 3, when we compare the prediction result with different Transformer models with various layer numbers, the shallowest Transformer with

Table 4: The MSE comparisons in ablation experiments of seasonal-trend decomposition analysis. 'Ori' means the original version without the decomposition. 'Decomp' means with decomposition. The experiment is performed on ETTm2 dataset with prolonging output length.

Model FED	Oformer Au	toformer	Inf	ormer	Log	gTrans	Ref	ormer	Tran	sformer	Promotion
MSE Ori	Decomp Ori	Decomp	Ori	Decomp	Ori	Decomp	Ori	Decomp	Ori	Decomp	Relative
8 96 0.457 192 0.841 ま 336 1.451 0 720 3.282	0.203 0.581 0.269 1.403 0.325 2.632 0.421 3.058	0.281 0.339	0.365 0.533 1.363 3.379	0.354 0.432 0.481 0.822	0.768 0.989 1.334 3.048	0.231 0.378 0.362 0.539	0.658 1.078 1.549 2.631	0.218 0.336 0.366 0.502	0.604 1.060 1.413 2.672	0.204 0.266 0.375 0.537	53% 62% 75% 82%

3 to 6 layers triumphs. It raises a question about how to design a proper Transformer architecture with deep layers to increase the model's capacity and achieve better forecasting performance.

Seasonal-Trend Decomposition Analysis

In the latest studies, researchers [Wu et al., 2021; Zhou et al., 2022; Lin et al., 2021; Liu et al., 2022] begin to realize that the seasonal-trend decomposition is a crucial part for Transformer's performance in time series forecasting. As a simple experiment shown in table 4, we use a moving average trend decomposition architecture proposed in [Wu et al., 2021] to test various attention modules. A seasonal-trend decomposition model can significantly boost model's performance by 50 % to 80%. It is a unique block and such performance boosting through decomposition seems a consistent phenomenon in time series forecasting for Transformer's application, which worth further investigating.

7 Future Research Opportunities

7.1 Inductive Biases for Time Series Transformers

The vanilla Transformer requires minimal inductive biases without assumptions on data patterns and characteristics, which makes it a general and universal network to learn long-range dependencies of various data types. However, it comes with cost that lots of training data are needed in order to avoid overfitting problem. For time series data, it has apparent temporal dependency and often exhibits complex seasonal/periodic and trend patterns [Wen et al., 2019; Cleveland et al., 1990]. Some recent studies have shown that incorporating series periodicity [Wu et al., 2021] or frequency processing [Zhou et al., 2022] into time series Transformer architectures would bring superior performance improvements. Therefore, an interesting future direction is to consider more effective ways to induce inductive biases into Transformers based on the understanding of time series data patterns as well as the characteristics of specific tasks, thus leading to more efficient and effective Transformer architectures for time series.

7.2 Transformers and GNN for Time Series

Multivariate and spatio-temporal time series are becoming more and more dominant across different scenarios, which calls for additional techniques to handle their high-dimensionality, especially for capturing the underlying relation among dimensions. Introducing graph neural networks (GNNs) is a natural way to model spatial dependency or relationships among dimensions. Recently, some investigations have demonstrated that the combination of GNN and

transformers/attentions could bring not only significant performance improvement like in traffic forecasting [Cai et al., 2020; Xu et al., 2020] and multi-modal forecasting [Li et al., 2021], but also in-depth understanding the dynamic spatiotemporal characteristics and latent casuality. Therefore, it is worthy further researching to exploit the strong modeling abilities of both Transformers and GNNs for time series data.

7.3 Pre-trained Transformers for Time Series

Large-scale pre-trained Transformer models have significantly boosted various tasks in NLP [Kenton and Toutanova, 2019; Brown et al., 2020] and CV [Chen et al., 2021a], since they can effectively learn universal representations and capture knowledge from massive labeled/unlabeled data and thus benefit various downstream tasks rather than learning models from scratch. However, there are limited works on pre-trained Transformers for time series, and existing studies mainly focus on time series classification [Zerveas et al., 2021; Yang et al., 2021]. Therefore, how to leverage the methodologies of pre-trained Transformer models from NLP and CV domains, which are less investigated for time series data, remain exciting future research opportunities.

7.4 Transformers with NAS for Time Series

The design of Transformer architecture is challenging, since all the hyper-parameters like embedding dimension, number of heads, and number of layers can largely affect the performance. Manual configuring and trying these hyperparameters are time-consuming and hard to obtain desirable results. Neural architecture search (NAS) [Elsken et al., 2019; Wang et al., 2020] has been a popular technique for discovering effective deep neural architectures, and automating Transformer design using NAS in NLP and CV can be found in recent studies [So et al., 2019; Chen et al., 2021c]. For industry-scale time series data which can be of both highdimension and long-length, discovering both memory- and computational-efficient Transformer architectures is of practical importance. Therefore, it is worthy investigating in future how to to design efficient Transformer architectures for time series by NAS automatically.

8 Conclusion

In this paper, we provide a comprehensive survey on time series Transformers in various tasks. We organize the reviewed methods in a new taxonomy consisting of network modifications and applications domains. We summarize representative methods in each category, discuss their strengths and limitations by experimental evaluation, and highlight future research directions.

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