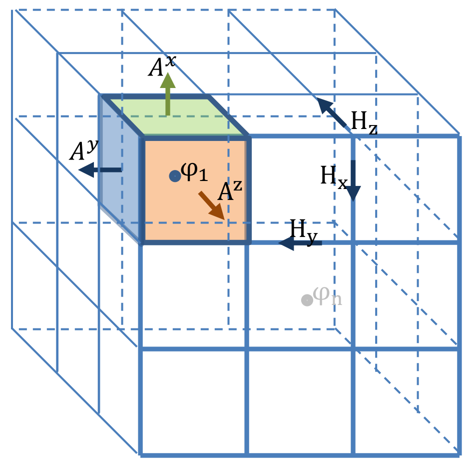
Model Parameters

Cell-center discretization



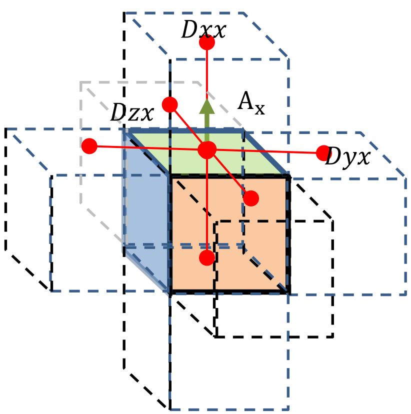
Vector Laplacian

$$\Delta\, A_x = \frac{\partial^2 A^x}{\partial x^2} + \frac{\partial^2 A^x}{\partial y^2} + \frac{\partial^2 A^x}{\partial z^2}$$

$$\frac{\partial^2 A_x}{\partial x^2} = \frac{\frac{(A^x{}_{n+1} - A^x)}{h_{n-1}} - \frac{(A^x - A^x{}_{n-1})}{h_n}}{h_{mid}}$$

$$\frac{\partial^2 A_x}{\partial y^2} = \frac{\frac{(A^x{}_{n+1} - A^x)}{h_{mid}} - \frac{(A^x - A^x{}_{n-1})}{h_{mid}}}{h}$$

$$\frac{\partial^2 A_x}{\partial z^2} = \ldots$$



In Matlab form…

$$\frac{\partial}{\partial x} = \begin{bmatrix} 1/\sqrt{h_1} & \cdots & 0 \\ -1/\sqrt{h_2} \quad 1/\sqrt{h_2} & \ddots & \vdots \\ 0 & \cdots & -1/\sqrt{h_{end}} \end{bmatrix}$$

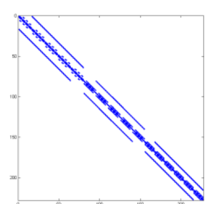
$$\frac{\partial^2}{\partial x^2} = \mathrm{H_{mid}} \left(\frac{\partial}{\partial x}\right)^{\mathrm{T}} \frac{\partial}{\partial x}$$

$$\frac{\partial^2 A^x}{\partial x^2} = \mathrm{kron}\ (\ \mathrm{kron}\ (\mathrm{speye(nz)}, \mathrm{speye(ny)}), \frac{\partial^2}{\partial x^2})$$

Same idea for the 8 other partial derivatives yielding…

Laplacian Operator

$$\mathbf{L} = \begin{matrix} \frac{\partial^2 Ax}{\partial x^2} + \frac{\partial^2 Ax}{\partial y^2} + \frac{\partial^2 Ax}{\partial z^2} & 0 & 0 \\ 0 & \frac{\partial^2 Ay}{\partial x^2} + \frac{\partial^2 Ay}{\partial y^2} + \frac{\partial^2 Ay}{\partial z^2} & 0 \\ 0 & 0 & \frac{\partial^2 Az}{\partial x^2} + \frac{\partial^2 Az}{\partial y^2} + \frac{\partial^2 Az}{\partial z^2} \end{matrix}$$



Boundary Conditions

(Primary Field)

x-y polarized

$$\frac{\partial A^x}{\partial x}\ and\ \frac{\partial A^y}{\partial x}\bigg|_{\partial\Omega} = 0$$

$$\frac{\partial A^x}{\partial y}\ and\ \frac{\partial A^y}{\partial y}\bigg|_{\partial\Omega} = 0 \qquad \Delta A^z\big|_{\partial\Omega} = 0$$

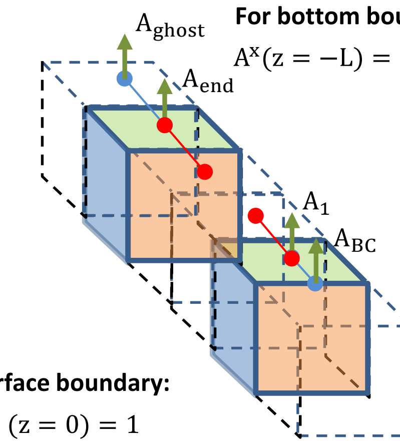
$$\frac{\partial A^x}{\partial z}\ and\ \frac{\partial A^y}{\partial z}\bigg|_{\partial\Omega} = g(x,y,z)$$

Defining g(x,y,z) as the 1D problem for the x and y components:

$$A^x(z) = e^{i\sqrt{i\,\omega\mu\sigma}}$$

For bottom boundary:

$A^x(z = -L) = i\sqrt{i\,\omega\mu\sigma}A^x$ - A^x



For surface boundary:

$A^x{}_{BC}\ (z = 0) = 1$