# RIM Research Project – Dual minimum support solution to the attenuation tomography problem.

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Analysis of borehole georadar data has proven to be a valuable tool in mining exploration. Several inversion schemes have been proposed to solve the attenuation tomography problem. Building upon the work of Aldridge & Oldenburg (1993)[i] and Ajo-Franklin *et al.* (2002)[ii], we propose a *dual minimum support* inversion algorithm. We seek a solution that can allow for both sharp and gradual changes in physical properties while remaining stable as noise level increases. Our new objective function can be expressed in compact notation as:

$$minimize\ \boldsymbol{\varphi} = \|\mathbf{G\,m} - \mathbf{d}\|_2 + \alpha_\nabla \left\| \frac{\boldsymbol{\nabla m}}{(\nabla m)^{2-p} + \boldsymbol{\varepsilon}} \right\|_2 + \alpha_c \left\| \frac{\mathbf{m}}{(m)^{2-q} + \boldsymbol{\varepsilon}} \right\|_2$$

where **G** is the forward operator, **m** is the discretized model, **d** the observed data and [β, α, ε] are constants. The first term controls the misfit between observed and predicted data, the second term imposes a constraint on the gradient and the last term penalizes small model values. The inversion can recover sharp boundaries between anomalies while preserving continuous features.

To illustrate the capability of the algorithm we have inverted a synthetic example. The model consists of four anomalies over a uniform background (Fig-1). The survey design mimics a crosswell experiment with 10 transmitters on one side, and 25 vertical receivers on the other. Data were corrupted with 10% Gaussian noise and corrected for the effect of geometrical spreading and source receiver pattern as prescribed by Holliger *et al* (2001)[iii].

For comparison, three solutions were computed using different inversion techniques. All three solutions have the same final data misfit. The model recovered from the *dual minimum support* scheme is superior on various levels.

1. **The geometry and physical property of individual anomalies are better recovered.**
2. **The algorithm has reproduced almost perfectly the raw data, suppressing only the random noise.**

[i] Aldridge D.F. Oldenburg, D.W. 1993 Two-dimensional tomography inversion with finite-difference traveltimes. Journal of Seismic Exploration **2**: 257-274

[ii] Ajo-Franklin, J. Minsley, B.J. Daley, T.M. 2007 Applying compactness constraints to differential traveltime tomography. Geophysics **72**: 67-75

[iii] Holliger, K. Musil, M. Maurer, H.R. 2001 Ray-based amplitude tomography for crosshole georadar data. Journal of Applied Geophysics **47**: 285-298
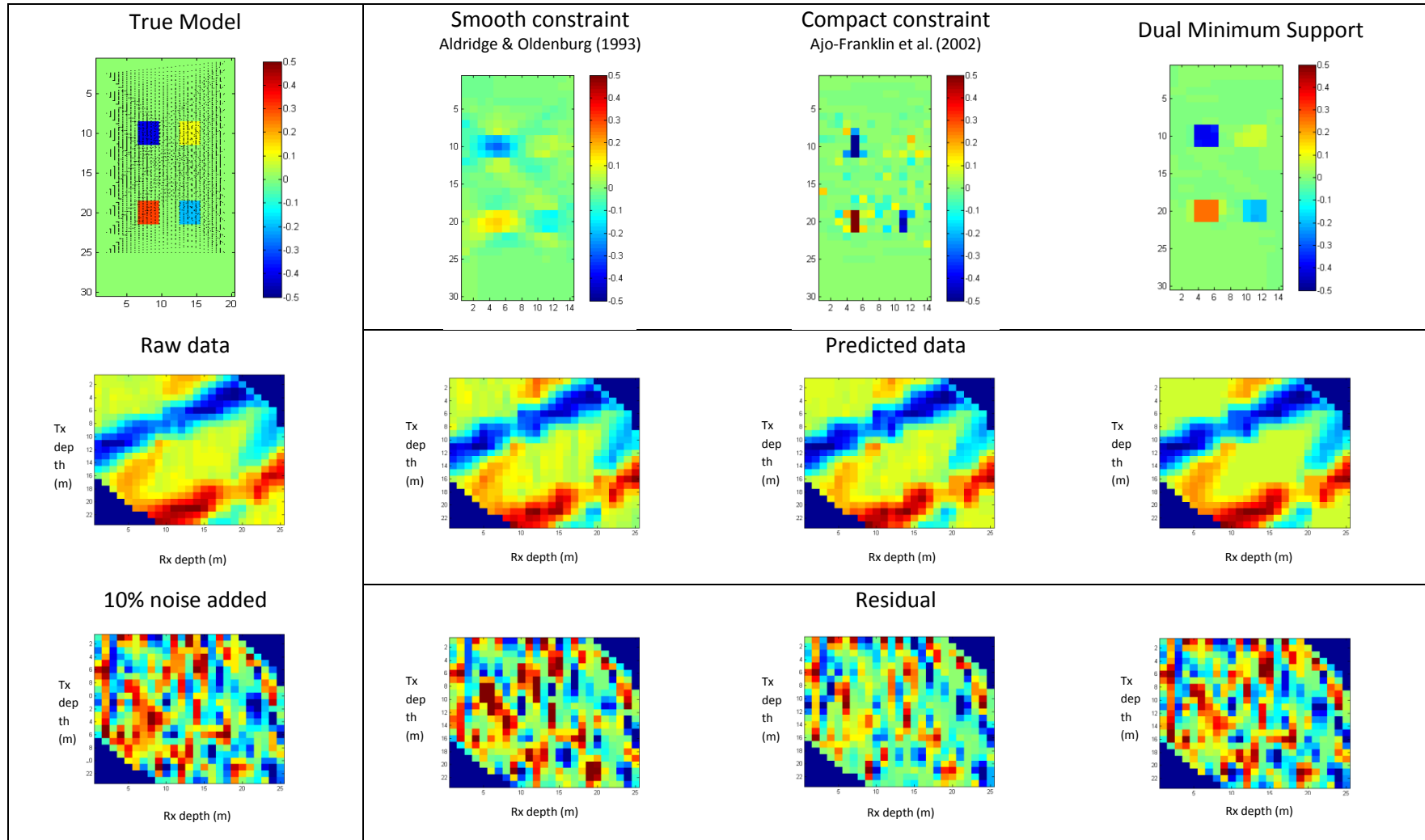
Figure 1: True and recovered models using three different inversion algorithms.