Problem:

This is a problem for optimizing the supply chain of a company. The company operates as a manufacturer and distributor of products.

The raw material is supplied through four different supplier locations. The material is received and processed into finished products at two factories. Each supplier supplies to only one factory. The factories then supply the products to eight different warehouses between them. Each warehouse is supplied by only one factory. The warehouses are strategically placed around the country catering to the demands of the customer base of the company. Each customer base is covered by one warehouse. The two factories also serve as warehouses for their customer region.

There is a cost associated with supply of raw material from suppliers to factory, supply of products from factory to warehouses and the supply of products from warehouses to customers.

Our objective function is to minimize the total cost of transportation subject to the constraints.

The constraints are related to supply capacity, production capacity, warehouse storage capacity, customer demand constraint, and the exclusivity of each entity pair i.e., supplier supplies to only one factory, factory supplies to only some warehouses, and warehouses supply to only one customer region.

Formulation:

Objective Function:

$$Min\,\{\sum_{i=1}^{n} F_i Y_i + \sum_{e=1}^{t} F_e Y_e + \sum_{h=1}^{l}\sum_{i=1}^{n} C_{hi} X_{hi} + \sum_{i=1}^{n}\sum_{e=1}^{t} C_{ie} X_{ie} + \sum_{e=1}^{t}\sum_{j=1}^{m} C_{ej} X_{ej}\}$$

Constraints:

Total amount shipped from supplier cannot exceed supplier's capacity.

$$\sum_{i=1}^{n} X_{hi} \qquad where \ \leq \ S_h \, for \; h = 1,2,3. \dots L$$

Amount shipped out of factory cannot exceed the quantity of raw material received.

$$\sum_{h=1}^{l} X_{hi} - \sum_{e=1}^{t} X_{ie} \qquad \geq \ 0 \, for \; i = 1,2,3 \dots n$$

Units produced in factory cannot exceed factory capacity.

$$\sum_{e=1}^{t} X_{ie} \qquad \leq \ K_i Y_i \, for \; i = 1,2,3 \dots n$$

Amount shipped out of warehouse cannot exceed quantity received from factories.

$$\sum_{e=1}^{t} X_{ie} - \sum_{j=1}^{m} X_{ej} \qquad \geq \ 0 \, for \; e = 1,2,3 \dots t$$

Amount shipped through warehouses cannot exceed its capacity.

$$\sum_{j=1}^{m} X_{ej} \qquad \leq \; W_e Y_e \; for \; e = 1,2,3 \ldots t$$

Amount shipped to customer must equal the customer demand; and

$$\sum_{e=1}^{t} X_{ej} \qquad = \; D_j \; for \; e = 1,2,3 \ldots m$$

$$\boldsymbol{Y_i Y_e} \in \{\boldsymbol{0}, \boldsymbol{1}\}$$
$$\boldsymbol{X_{ie} X_{ej}} \geq 0$$

Where:
m = number of markets or demand points
n = number of potential factory locations
l = number of suppliers
t = number of potential warehouse locations
Dj = annual Demand from customer j Ki = potential Capacity of factory at site i Sh = supply capacity at supplier h
We = potential warehouse capacity at site e
Fi = fixed Cost of locating a Plant at site i
Fe = fixed cost of locating a warehouse at site e
Chi = cost of shipping one unit from supply source h to factory i
Cie = cost of shipping one unit from factory i to warehouse e
Cej = cost of shipping one unit from warehouse e to customer j
Yi = 1 if plant is located at site i, 0 otherwise Ye = 1 if warehouse is located at site e, 0 otherwise
xej = quantity transported from warehouse e to market j
Xie = quantity transported from plant i to warehouse e
Xhi = quantity shipped from supplier h to factory at site i

Explanation of Formulation:

1. We have defined the sets n, l, m, t which are the number of factories, suppliers, warehouses, and customers (markets) or demand points.
2. Then, for indexing their range has been defined.
3. The parameters have been defined then on the format of Xij where X is the parameter from i to j.
4. We have five decision variables. Two are Boolean identifying whether a factory and a warehouse is utilized or not. The other three decision variables describe the amount of raw material from supplier to factory, the quantity of products from factory to warehouse and the quantity of products from warehouse to customer (market).

Model File Code:

```
int n = ...; //Number of factory locations
int l = ...; //Number of suppliers
int t = ...; //Number of warehouse locations
int m = ...; //Number of markets or demand points

range Factories = 1..n;
range Suppliers = 1..l;
range Warehouses = 1..t;
range Markets = 1..m;

//Parameters
float Ki[Factories] = ...; //capacity of factory at site i
float Sh[Suppliers] = ...; //Supply capacity at supplier h
float We[Warehouses] = ...; //warehouse capacity at site e
float Dj[Markets] = ...; //Annual demand from customer j
float Fi[Factories] = ...; //Fixed cost of locating a plant at site i
float Fe[Warehouses] = ...; //Fixed cost of locating a warehouse at site e
float Chi[Suppliers][Factories] = ...; //Cost of shipping one unit from supply source h to factory i
float Cie[Factories][Warehouses] = ...; //Cost of shipping one unit from factory i to warehouse e
float Cej[Warehouses][Markets] = ...; //Cost of shipping one unit from warehouse e to customer j

//decision variables
dvar boolean Yi[Factories]; //1 if plant is located at site i, 0 otherwise
dvar boolean Ye[Warehouses]; //1 if warehouse is located at site e, 0 otherwise
dvar float+ Xhi[Suppliers][Factories]; //Quantity shipped from supplier h to factory i
dvar float+ Xie[Factories][Warehouses]; //Quantity transported from plant i to warehouse e
dvar float+ Xej[Warehouses][Markets]; //Quantity shipped from warehouse e to market j
```

```
//objective function
minimize
  sum(i in Factories) Fi[i] * Yi[i] +
  sum(e in Warehouses) Fe[e] * Ye[e] +
  sum(h in Suppliers, i in Factories) Chi[h][i] * Xhi[h][i] +
  sum(i in Factories, e in Warehouses) Cie[i][e] * Xie[i][e] +
  sum(e in Warehouses, j in Markets) Cej[e][j] * Xej[e][j];

//constraints
subject to {
  //Supply capacity constraint
  forall(h in Suppliers)
    sum(i in Factories) Xhi[h][i] <= Sh[h];

  //Factory production constraint
  forall(i in Factories)
    (sum(h in Suppliers) Xhi[h][i] - sum(e in Warehouses) Xie[i][e]) >= 0;

  //Factory capacity constraint
  forall(i in Factories, e in Warehouses)
    Xie[i][e] <= Ki[i] * Yi[i];

  //Warehouse throughput constraint
  forall(e in Warehouses)
    sum(i in Factories) Xie[i][e] - sum(j in Markets) Xej[e][j] >= 0;

  //Market demand constraint
  forall(j in Markets)
    sum(e in Warehouses) Xej[e][j] == Dj[j];
}
```

Data File Code:

```
SheetConnection file("Data Sets.xlsx");

n from SheetRead(file, "'Data Set 1'!j43:j43");//Number of potential factory locations
l from SheetRead(file, "'Data Set 1'!j44:j44");//Number of suppliers
t from SheetRead(file, "'Data Set 1'!j45:j45");//Number of potential warehouse locations
m from SheetRead(file, "'Data Set 1'!j46:j46");//Number of markets or demand points

Ki from SheetRead(file, "'Data Set 1'!i4:i5"); //Potential capacity of factory at site i
Sh from SheetRead(file, "'Data Set 1'!f44:f47"); //Supply capacity at supplier h
We from SheetRead(file, "'Data Set 1'!b12:i12"); //Potential warehouse capacity at site e
Dj from SheetRead(file, "'Data Set 1'!b69:i69"); //Annual demand from customer j
Fi from SheetRead(file, "'Data Set 1'!b72:b73"); //Fixed cost of locating a plant at site i
Fe from SheetRead(file, "'Data Set 1'!b40:i40"); //Fixed cost of locating a warehouse at site e
Chi from SheetRead(file, "'Data Set 1'!b44:c47"); //Cost of shipping one unit from supply source h to factory i
Cie from SheetRead(file, "'Data Set 1'!b51:i52"); //Cost of shipping one unit from factory i to warehouse e
Cej from SheetRead(file, "'Data Set 1'!b57:i64"); //Cost of shipping one unit from warehouse e to customer j
```

Data Generation and Adoption:

The data for this problem has been given in the article. However, for some parameters we had to create synthetic datasets.

The given data included:

- production capacity of the factories
- supply sources and their destinations
- warehouse capacity (we assumed the customer demand also the same)
- cost of transportation per km and the distances between each warehouse and their supply regions from which we calculated cost of transportation from warehouse to customer.
- annual warehouse fixed cost

The data which we generated from Excel random array function includes:

- annual plant fixed cost of locating a plant at site i.
- supplier capacity
- cost of transportation from supplier to factory
- Cost of Transportation from Warehouse to Markets

Things we can variate for synthetic data sets.

- Production Capacity of the Factory
- Annual Demands at Depots
- Cost of Transportation from Supplier to Factory
- Supply Capacity
- Cost of Transportation from Factory to Warehouse
- Cost of Transportation from Warehouse to Markets
- Annual Demand from Customers

Experimental Results:

Result No. 1:

Change in Data Set: The values already given in the paper were used. The values not given were generated by random array function of MS Excel.

```
// solution (optimal) with objective 2094413
// Quality Incumbent solution:
// MILP objective                                   2.0944130000e+06
// MILP solution norm |x| (Total, Max)              2.13610e+04  7.12000e+03
// MILP solution error (Ax=b) (Total, Max)          4.66116e-10  1.55524e-10
// MILP x bound error (Total, Max)                  0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)            0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)              4.54747e-13  4.54747e-13
//

Yi = [1
         0];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 0]
             [7120 0]];
Xie = [[2373.3 0 0 0 0 0 2373.3 2373.3]
             [0 0 0 0 0 0 0 0]];
Xej = [[0 0 0 0 398.33 945 515 515]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 1091.7 1165 116.67 0 0 0]
             [515 1635 223.33 0 0 0 0 0]];
```

Figure 1: Result of Data Set 1

Decision Variable	Result
Yi = Location of Factory	[1 0] = Addis Ababa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	7120 cases from supplier Metehara → Addis Ababa factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	It has picked the warehouses with zero cost (located at 2x plants (AA) and 1 WH)
Xej = Cases shipped from warehouse to market	Warehouse (WH) 1 → Markets (M) 5,6,7,8 WH 7 → M 3,4,5 WH 8 → M 1,2,3
Transportation Cost	**2,094,413 Birr**

Result No. 2:

Change in Data Set:

- Changed the factory production capacity to random values.
- Changed the cost of supplier to factory to a random array between 50-100.
- Changed the supplier capacity to a random array and weighted it with the amount going to each factory.
- Changed the annual demand of the customers to a random array between 200-300 cases.
- Changed the fixed cost of factory to a random array.

```
// solution (optimal) with objective 226788.5
// Quality Incumbent solution:
// MILP objective                                 2.2678850000e+05
// MILP solution norm |x| (Total, Max)            6.14200e+03  2.04700e+03
// MILP solution error (Ax=b) (Total, Max)        0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)          0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)            0.00000e+00  0.00000e+00
Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 2047]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 269 1778]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 269]
             [266 293 244 287 225 257 206 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	2047 cases from supplier Ziway → Dire Dawa factory (Lowest Cost) It is just picking the supplier-factory pair with lowest cost
Xie = Cases shipped from factory to warehouse	269 cases from Dire Dawa factory to Awash warehouse 1778 cases from Dire Dawa factory to Dire Dawa warehouse
Xej = Cases shipped from warehouse to market	WH 7 →M 8 WH 8 → all M except 8
Transportation Cost	**226,788 Birr**

Result No. 3:

Change in Data Set: The values already given in the paper were used. We tried to match the values given in the results section of paper to verify our result with the author.

```
// solution (optimal) with objective 6232103
// Quality Incumbent solution:
// MILP objective                                6.2321030000e+06
// MILP solution norm |x| (Total, Max)           1.62820e+04  5.42700e+03
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 5427]
             [0 0]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 730 4697]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 730]
             [699 711 758 518 676 786 549 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	5427 cases from supplier Wonji → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	730 cases from Dire Dawa factory to Awash warehouse 4697 cases from Dire Dawa factory to Dire Dawa warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**2,094,413 Birr**

Result No. 4:

Change in Data Set:

- Changed the factory production capacity to random value below 300,000.
- Changed the supplier capacity to a random array below 150,000 and weighted it with the amount going to each factory.
- Changed the annual demand of the customers to a random array between 7000-7500 cases.
- Changed the fixed cost of factory to a random array between 200,000-300,000

```
// solution (optimal) with objective 3434772.05025035
// Quality Incumbent solution:
// MILP objective                                 3.4347720503e+06
// MILP solution norm |x| (Total, Max)            1.73656e+05  5.78850e+04
// MILP solution error (Ax=b) (Total, Max)        0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)          0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)            0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 57885]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 18213 39672]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 3693.5 0 7230 0 7290]
             [7033 7271 7389 3631.5 7345 0 7002 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	57885 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	18213 cases from DD factory to Awash warehouse 39672 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 4,6,8 WH 8 → all M
Transportation Cost	**3,434,772 Birr**

Result No. 5:

Change in Data Set:

- Changed the factory production capacity to originally calculated value in DS 1.

```
// solution (optimal) with objective 3254535
// Quality Incumbent solution:
// MILP objective                                3.2545350000e+06
// MILP solution norm |x| (Total, Max)           1.73656e+05  5.78850e+04
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 57885]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 7290 50595]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 7290]
             [7033 7271 7389 7325 7345 7230 7002 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	57885 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	7290 cases from DD factory to Awash warehouse 50595 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**3,434,772 Birr**

Result No. 6:

Change in Data Set:

- Changed the factory fixed cost to a random array between 500,000-1,000,000.

```
// solution (optimal) with objective 3603110
// Quality Incumbent solution:
// MILP objective                                  3.6031100000e+06
// MILP solution norm |x| (Total, Max)             1.73656e+05  5.78850e+04
// MILP solution error (Ax=b) (Total, Max)         0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                 0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)           0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)             0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 57885]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 7290 50595]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 7290]
             [7033 7271 7389 7325 7345 7230 7002 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	57885 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	7290 cases from DD factory to Awash warehouse 50595 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**3,603,110 Birr**

Result No. 7:

Change in Data Set:

- Changed the supply capacity to a random array between 1000,000-1,500,000 and weighted it by the factory. No change observed.

```
// solution (optimal) with objective 3603110
// Quality Incumbent solution:
// MILP objective                                 3.6031100000e+06
// MILP solution norm |x| (Total, Max)            1.73656e+05  5.78850e+04
// MILP solution error (Ax=b) (Total, Max)        0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)          0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)            0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 57885]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 7290 50595]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 7290]
             [7033 7271 7389 7325 7345 7230 7002 0]];
```

Decision Variable	Result
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	57885 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	7290 cases from DD factory to Awash warehouse 50595 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**3,603,110 Birr**

Result No. 8:

Change in Data Set:

- Changed the supplier to factory cost to a random array between 150-1,000.

```
// solution (optimal) with objective 20505530
// Quality Incumbent solution:
// MILP objective                                2.0505530000e+07
// MILP solution norm |x| (Total, Max)           1.73656e+05  5.78850e+04
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 57885]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 7290 50595]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 7290]
             [7033 7271 7389 7325 7345 7230 7002 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	57885 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	7290 cases from DD factory to Awash warehouse 50595 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**20,505,530 Birr**

Result No. 9:

Change in Data Set:

- Changed the customer demand to a random array between 1400-1,420.

```
// solution (optimal) with objective 4462743.5
// Quality Incumbent solution:
// MILP objective                                  4.4627435000e+06
// MILP solution norm |x| (Total, Max)             3.37810e+04  1.12600e+04
// MILP solution error (Ax=b) (Total, Max)         0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                 0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)           0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)             0.00000e+00  0.00000e+00
//

Yi = [0
         1];
Ye = [0 0 0 0 0 0 0 0];
Xhi = [[0 0]
             [0 0]
             [0 11260]
             [0 0]];
Xie = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 1409 9851]];
Xej = [[0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 1409]
             [1403 1402 1415 1408 1413 1402 1408 0]];
```

Decision Variable	Result
Yi = Location of Factory	[0 1] = Dire Dawa
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	11260 cases from supplier Ziway → DD factory (Lowest Cost)
Xie = Cases shipped from factory to warehouse	1409 cases from DD factory to Awash warehouse 9851 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 8 WH 8 → all M except 8
Transportation Cost	**4,462,743 Birr**

Result No. 10:

Change in Data Set:

- Used the data set 1 and changed the supplier-factory cost to a random array between 705-720.

```
// solution (optimal) with objective 5711373
// Quality Incumbent solution:
// MILP objective                               5.7113730000e+06
// MILP solution norm |x| (Total, Max)          2.13610e+04  7.12000e+03
// MILP solution error (Ax=b) (Total, Max)      4.67026e-10  1.55978e-10
// MILP x bound error (Total, Max)              0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)        0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)          4.54747e-13  4.54747e-13
//

Yi = [1
         0];
Ye = [0 0 0 0 0 0 0 0 0];
Xhi = [[7120 0]
             [0 0]
             [0 0]
             [0 0]];
Xie = [[2373.3 0 0 0 0 0 2373.3 2373.3]
             [0 0 0 0 0 0 0 0]];
Xej = [[0 0 913.33 0 0 945 0 515]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0]
             [515 291.67 401.67 1165 0 0 0 0]
             [0 1343.3 0 0 515 0 515 0]];
```

Decision Variable	**Result**
Yi = Location of Factory	[1 0] = AA
Ye = Location of Warehouse	[0 0 0 0 0 0 0 0 0] = No preference
Xhi = Cases shipped from supplier to factory	7120 cases from supplier USA → AA factory
Xie = Cases shipped from factory to warehouse	2373 cases from AA factory to AA, Shashemene and Awash warehouse 9851 cases from DD factory to DD warehouse
Xej = Cases shipped from warehouse to market	WH 7 → M 1,2,3,4 WH 8 → M 2,5,7
Transportation Cost	**5,711,373 Birr**

Conclusion:

After running the code for ten different data sets and analyzing the results, following are our conclusions:

1. The objective function (minimization of transportation cost) is dependent on 5 decision variables namely:
    a. Choice of factory
    b. Choice of warehouse
    c. Quantity from supplier to factory
    d. Quantity from factory to warehouse
    e. Quantity from warehouse to markets
2. Factory cost, warehouse fixed cost is the deciding factor for the first two variables, which are Boolean. For quantity, it can be any, it just must satisfy the constraints.
3. The project was completed by randomly generated data and a definite conclusion did not present itself.

Reflections:

Working on this project was a unique experience. The problem under consideration was directly related to one of the assignment questions for CPLEX. From this project we learnt a lot about the transportation optimization problem.

Additional Paper Comparison of Formulations

Paper1:

Minimize: $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to $$\sum_{j=1}^{n} x_{ij} \le a_i \quad ; \quad i = 1, 2, \ldots \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad ; \quad j = 1, 2, \ldots \ldots, n$$

$$x_{ij} \ge 0 \quad \text{for all } i \text{ and } j.$$

where

$i = 1, 2, \ldots\ldots, m$ is the set of origins.

$j = 1, 2, \ldots\ldots, n$ is the set of destinations.

x_{ij} = the quantity transported from the i-th origin to the j-th destination.

c_{ij} = per unit cost in transporting goods from i-th origin to the j-th destination.

a_i = the amount available at the i-th origin.

b_j = the demand of the j-th destination.

Figure 1 Transportation problem: a cost minimization approach

In this paper, they have formulated a transportation cost for (i) origins and (j) destinations. The cost of transportation of (Xij) quantity between (i) and (j) is given by (Cij). It is subject to the supply constraint and the demand constraint.

Paper 2:

$$Min \sum_{i}^{m} \sum_{j}^{n} Cij * Sij$$

Constraints:

$\sum_{j}^{n} Sij \leq Fi$	$i = 1,\ldots,m$	C_{ij} - is cost per unit from factory i to shop j
$\sum_{i}^{m} Sij = Dj$	$j = 1,\ldots,n$	S_{ij} - Shipment amount from factory i to the shop j
$Sij \leq \max T$	$i = 1,\ldots,m$ $j = 1,\ldots,n$	F_i - Production capacity for factory F_1, F_2 or F_3 D_j - Shipment amount received in the shop j
$Sij \geq 0$	$i = 1,\ldots,m$ $j = 1,\ldots,n$	Max T - is maximal loading truck capacity

Figure 2 Distribution Network problem

This paper also has a similar transportation problem in which objective function is cost multiplied by shipment amount. The shipment amount is subject to demand constraint. It is also subject to truck loading capacity, and it has a non-negativity constraint too. The indexes are origins and destinations denoted by (i) and (j).

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