Analysis of a simple wireless communication network

Festo Owiny, Mat. number 174213
festo.owiny@studenti.unitn.it

I. Introduction

In recent years, wireless networks has enjoyed a great and steady growth with high popularity in both research and industry. However, the development of adequate formalism for modeling and analyzing wireless networks has not kept pace with the growing pattern. Traditionally, the main method for analyzing wireless networks is simulation. In this report we build a small discrete event network simulator and a simplified queue model implementing and analyzing a simple ALOHA protocol.

The network topology in Figure 1 defines the given ten node points communicating with one another within the specified radius. In addition, we are provided with transmission queue buffer size of 64 and Uniform Distribution for both Inter arrival time and Packet size. Consequently, we are required to evaluate network parameters for each node;
The goal is to estimate values of different performance measures Θ such as throughput, loss rate and collision rate. Since we are looking at discrete events through time, we employ the discrete-event simulation for the operation of our queuing system. The discrete event simulator is termed terminating and runs over a simulated time interval $[0, T_E]$ under the specified initial conditions.

II. Fundamentals

A. Uniform Distribution

It is a symmetric probability distribution where finite number of values are equally likely to be observed. U(a, b) is defined by;

$$P(X = x) = \begin{cases} \dfrac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The mean; $\mu = \dfrac{a+b}{2}$ and Var; $\sigma^2 = \dfrac{(b-a)^2}{12}$

B. Simulation

Discrete event simulator has a natural time for its replications that is inherent in the system under well specified initial conditions, Figure 2; The number of replications is the critical parameter of the associated output analysis; the only way of controlling the sample size of any estimator; Sample size directly affects estimator's variance and its statistical accuracy. The run is guaranteed to halt hence no interest in long-term performance metrics requiring longer time to arrive at steady state.
The simulator comprises three process event types; sample queue level, packet arrive and Packet Sent.

C. Measures of Performance

Performance measures are used in our simulation model to assess the system performance under different possible operational scenarios of the network. We perform output analysis with the purpose of examining the data generated from our simulation and estimate system performance via simulation regulations. If ϕ is the system performance, the precision of the estimator $\hat{\phi}$ can be measured by standard error of $\hat{\phi}$ and width of a confidence interval for ϕ. The purpose of statistical analysis in this case is to estimate the standard error or *CI* and also to figure out the number of observations (replications) required to achieve desired error.

Point Estimator: Consider the estimation of a performance parameter θ of a simulated system with discrete time data $[Y_1, Y_2, ..., Y_n]$ with ordinary mean value θ. The point estimation for discrete time data,

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n}(y_i) \tag{2}$$

It is said to be unbiased if $\mathbf{E}(\hat{\theta}) = \theta$ and biased if $\mathbf{E}(\hat{\theta}) \neq \theta$

Confidence-Interval Estimation: The Confidence Interval (*CI*) is used to measure error for the parameter estimates of our simulation result where Y_i (replications) are normally distributed (central limit theorem).

$$CI = \overline{Y} \pm t_{\alpha/2,R-1}(\frac{S}{\sqrt{R}}) \tag{3}$$

Where *R* is the number of replications and *S*, standard deviation.

Due to sampling variation, the *CI* for a particular sample might not contain the parameter. A 95% CI means that if you collect a large number of samples and construct the corresponding *CIs*, then about 95% of the intervals will contain the parameter. Therefore the more replications we make, the less error in $\overline{Y}$. Collecting very large data size requires enormous amount of time and may last several days; Standard Normal Distribution is used to estimate optimal R value as below;

For *CI* with specified precision, assuming an initial sample size R_0 (independent) replications, obtain an initial estimate

S_0^2, then choose R s.t. $R \geq R_0$. Since $t_{\alpha/2,R-1} \geq Z_{\alpha/2}$, an initial estimate of R with required precision ε is;

$$R \geq \frac{Z_{\alpha/2} S_0}{\varepsilon} \tag{4}$$

D. Analysis

For the simplified model we solve analytically and compare the results with that obtained from the simulation. For the theoretical model we use M/M/1 rather than G/G/1 for approximation. This not the best approximation model for our case but much easier to manipulate than the other models. We make assumptions for the simplified mode such as; Inter-arrival rate and service rate are Markovian, a node starts transmitting without checking its current receiving status; and probability of losing packet due to collision, $P_{loss\_at\_col} = \frac{(Time\_without\_overlapping)}{(Time\_of\_sending)}$

Every node buffer is a queue and every queue is independent from another; therefore we have ten M/M/1/B independent queues corresponding to the ten nodes of the network Figure 3. We can then compute required parameters from the steady state distribution. P(k packets in queue)= π_k, and π_0 (initial).
Given average arrival rate λ and the service rate of a single server μ, the load of the queue $\rho = \frac{\lambda}{\mu}$; $0 < \rho < \infty$

$$\pi_k = \rho^i \pi_0, \;\; 0 < i < n, \;\; \text{n is buffer size} \tag{5}$$

$$\pi_0 = \begin{cases} \frac{1-\rho}{1-\rho^{n+1}} & \text{if } \rho \neq 1, \\ \frac{1}{n+1} & \text{if } \rho = 1, \end{cases} \tag{6}$$

Inter arrival time and packet size distribution follow Uniform, therefore λ and μ can be calculated using Equation (1.
The loss probability is same for all queues due to independence, and equal to the probability of customers in full queue. The loss probability $Loss\_rate = P_{loss}$ defined by;

$$Loss\_rate = \frac{1-\rho}{1-\rho^{n+1}} \rho^n \tag{7}$$

$$Collision\_rate = (1 - \pi_0)^2 \tag{8}$$

Throughput of the queue, the Probability that the transmitted message is successfully received by the receiver without any collision also referred, the rate of (successful) transmission of packets.

$$Th = \lambda(1 - \pi_n) \;\; = \;\; \mu(1 - \pi_0) \tag{9}$$

$$Th_{bits} = P(X_r) * speed, \tag{10}$$

$$Th_{packets} = P(X_r) * \mu \tag{11}$$

E. Pure Aloha

The pure Aloha protocol follows that whenever a node has packet(s) to send it transmits the packet and if while transmitting, you receive any packet from another station, there has been a message collision. All transmitting nodes will need to try resending "later", all packets have same length, nodes cannot generate a packet while transmitting, nodes transmit according to Poisson distribution. Let G be the average number of nodes that begin transmission within period T , probability that exactly x nodes begin transmission during period T is

$$P[X = x] = \frac{G^x e^{-G}}{x!} \tag{12}$$

Probability that exactly one node will begin transmission is $P[X = 1]$, no node begins transmission with $P[X = 0]$ BUT for successful transmission of a packet, both the events should occur simultaneously i.e, $P = P(0)XP(1)$. This is also the throughput hence;

$$Th_{pureAloha} = Ge^{-2G} \tag{13}$$

III. Results

Since it is only possible to analytically model complex protocols using several simplifications and assumptions, approximate techniques have been used, Figure 2, Figure 6, Figure 4. Consequently, theoretical results need to be validated by comparing them with those obtained from simulation. A simulation program was written using Python whereas the simplified analytical model with R language. A total of 10 experiments were performed, each with 50 replications identified by a separate seed. The parameter value 10 is the T_{max} value. The simulations were run for $818s$. Graphical representation of the simulation and analytical results for throughput, collision rate and loss rate are shown in Figure 8, Figure 9, Figure 10 and Figure 11 respectively. The plots for individual nodes can be found in folder *"results"*. The average queue level for T_{max} is $0.0175s$ can be found in Figure 5 and the corresponding queue levels for other nodes in the *"result"* folder.

A. Comparison of simulator with analytical results

From the graphs we infer that the simulated analytical model behaviour closely relate. Comparing the throughput of both models, we see that the analytical model tend to provide maximum throughput than the simulated model Figure 8, Figure 9. This is because though we calculate collision in analytical formula, we keep the model simple by not adding the restriction that the node cannot send a packet while it is receiving another packet from its neighbours. Collision rate and loss rate of the simulated model show similar behaviour with the analytical model (Figure 10, Figure 11). The loss rate of both models show a constant behavior.

B. Comparison of Analytical model with Pure Aloha protocol

We can infer from Figure 7 that the throughput of analytical model is higher than that of pure Aloha. This is due to the fact that in case of collision, Aloha protocol re-transmits the packet till it is successful whereas model broadcasts the packet only once. In addition, the broadcast in Aloha is to all nodes irrespective of their positions while in the analytical model is limited to the neighbor nodes only.

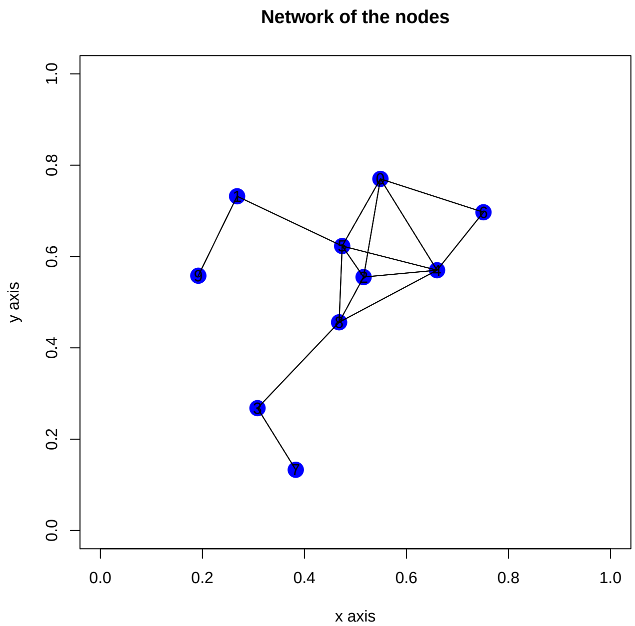

Figure 1. Network Topology.

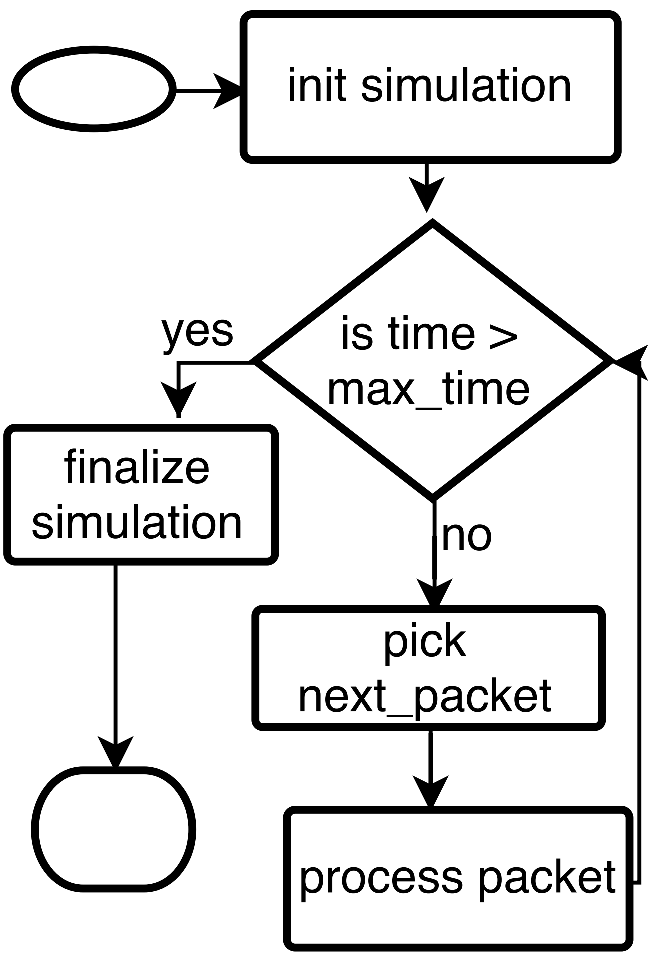

Figure 2. Flow chart: DES.

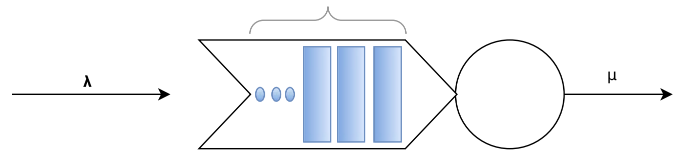

Figure 3. Queue model.

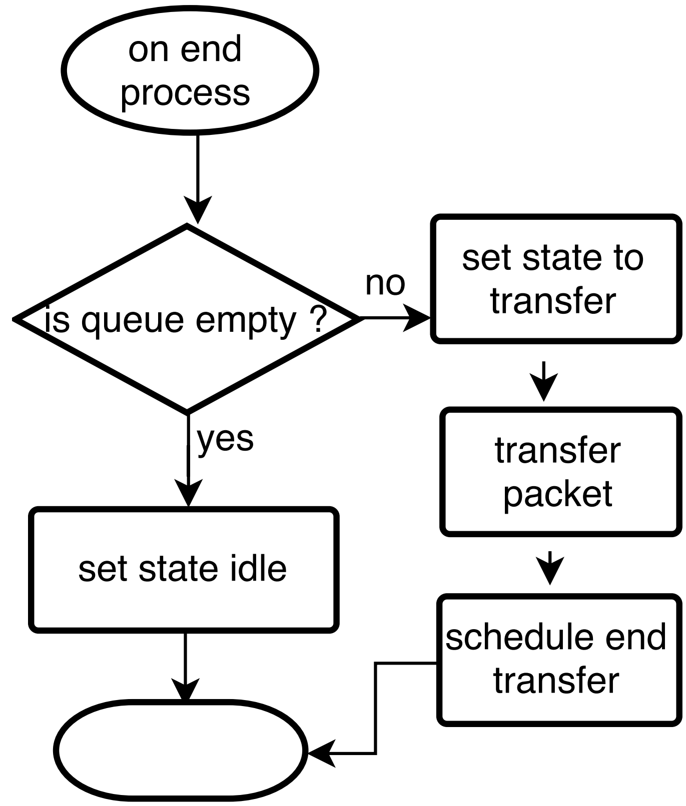

Figure 4. Flow chart: end process.

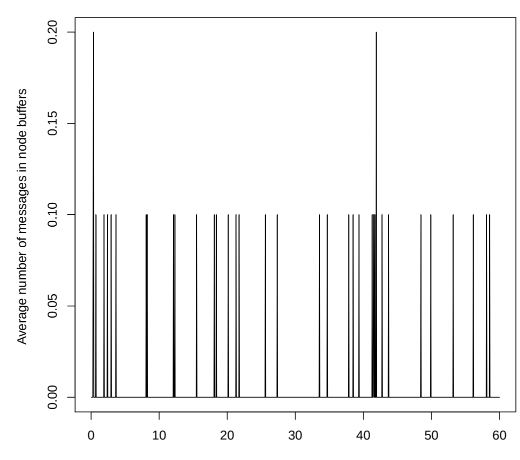

Figure 5. Average number of packets in node.

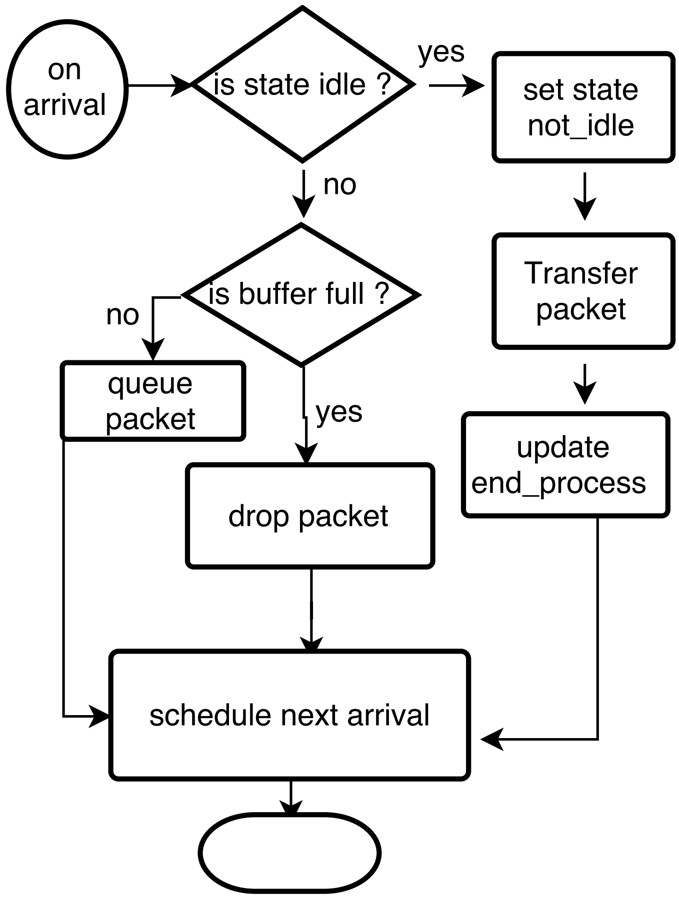

Figure 6. Flow chart: arrival event.

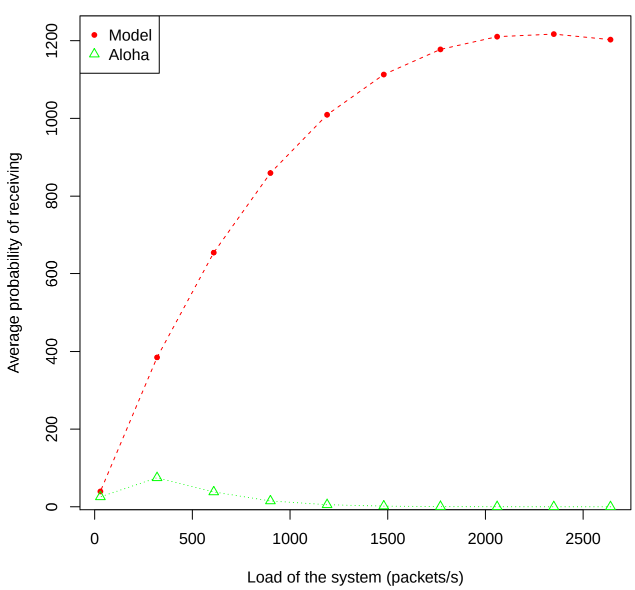

Figure 7. Analytical model vs (trivial) model of pure Aloha protocol.

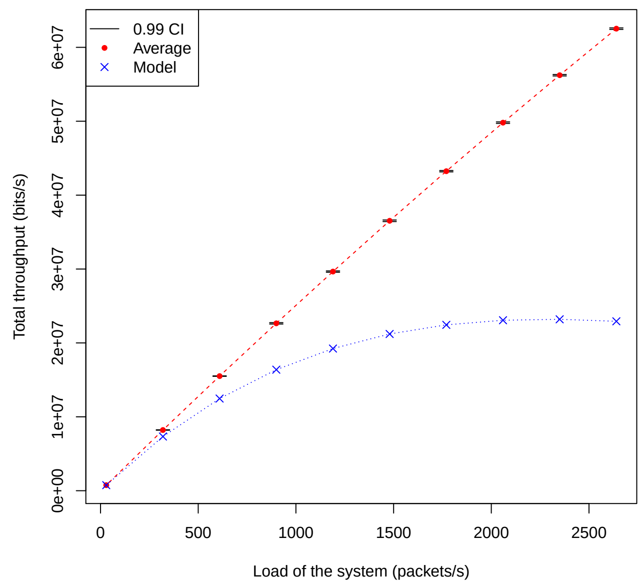

Figure 8. Throughput comparison for simulated and analytical results (bits).

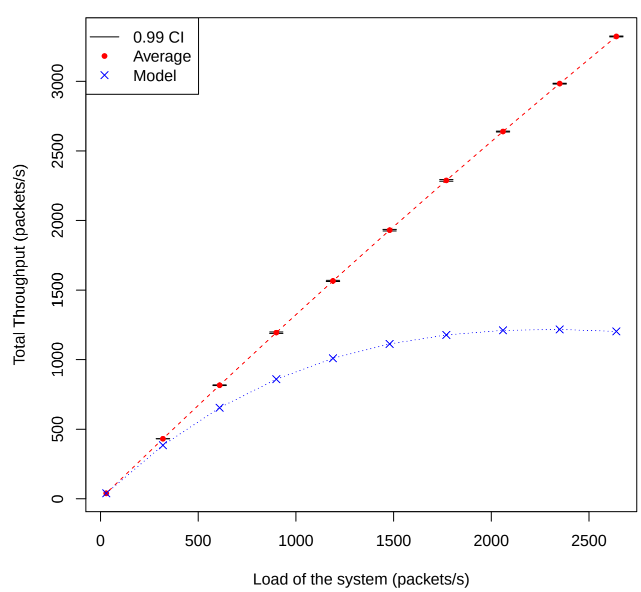

Figure 9. Throughput comparison for simulated and analytical results (packets).

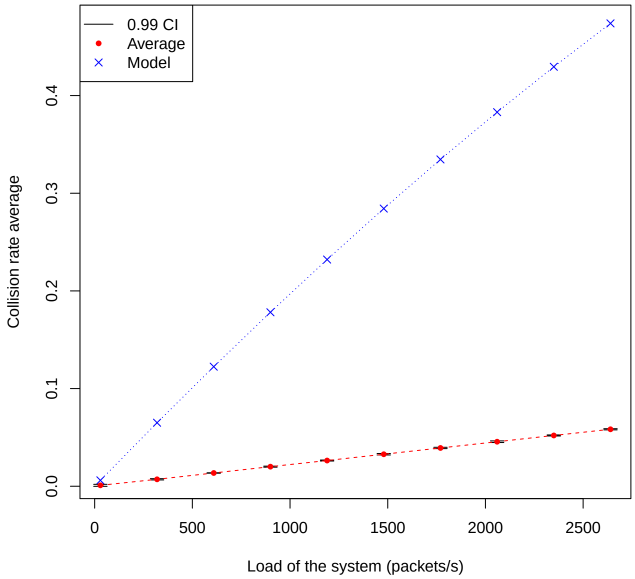

Figure 10. Average collision rate of the simulated model and analytical results.

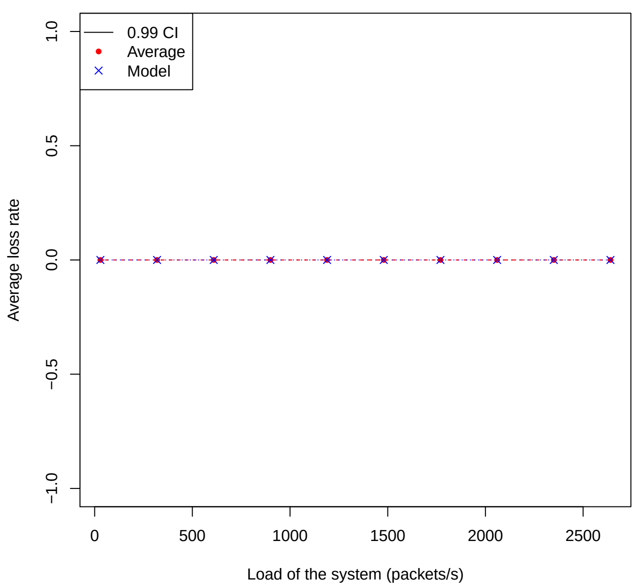

Figure 11. Average collision rate of the simulated model and analytical results.

IV. Conclusion

Numerical results showed generally good agreement with simulation ones, especially when collision rate is low. However, even at high collision less error is observed suggesting a good building block for modeling the protocol. Future work will focus on studying other functionalities of pur Aloha protocol, where some of the results and techniques used in our method execution will be reapplied. In particular, we will focus our attention on a more detailed model of the backoff technique. The quality of the backoff scheme chosen significantly influences the efficiency of the protocol, the ultimate channel capacity, and the predictability of its behavior. Finally, our model parameters can be improved for better accuracy.

References

[1] K.S. Trivedi. *Probability and Statistics with Reliability, Queuing, and Computer Science Applications.* , 2001.

[2] Boudewijn R. Haverkort,Henrik C. Bohnenkamp,Connie U. Smith. *Computer Performance Evaluation. Modeling Techniques and Tools: 11th.* , March 2000.

[3] https://en.wikipedia.org/wiki/ALOHAnet. *ALOHA protocol.* , March 2017.