1. $\forall x\ \text{Stench}(x) \Rightarrow \exists y\ \text{Adjacent}(x, y) \wedge \text{At}(\text{Wumpus}, y)$

$$(\forall x\ \neg\text{Stench}(x)) \vee (\exists y\ \text{Adjacent}(x, y) \wedge \text{At}(\text{Wumpus}, y))$$

$$(\forall x\ \neg\text{Stench}(x)) \vee (\exists y\ \text{Adjacent}(x, y) \wedge \text{At}(\text{Wumpus}, y))$$

$$(\neg\text{Stench}(x)) \vee (\text{Adjacent}(x, y) \wedge \text{At}(\text{Wumpus}, y))$$

$$(\neg\text{Stench}(x) \vee \text{Adjacent}(x, y)) \wedge (\neg\text{Stench}(x) \vee \text{At}(\text{Wumpus}, y))$$

2.

a. $\forall x\ \text{Likes}(x, \text{Apples}) \Rightarrow \text{Plays}(x, \text{Chess})$

b. $\forall x\ \text{Likes}(x, \text{Oranges}) \Rightarrow \text{Plays}(x, \text{Go})$

c. $\forall x\ (\text{Likes}(x, \text{Apples}) \vee \text{Likes}(x, \text{Oranges})) \wedge \neg(\text{Likes}(x, \text{Apple}) \wedge \text{Likes}(x, \text{Oranges}))$

d. $\text{Likes}(\text{John}, \text{Apples})$

e. $\forall y\ \text{Likes}(\text{John}, y) \Rightarrow \neg\text{Likes}(\text{Marry}, y)$

3.

a. $\forall x\ \neg\text{Likes}(x, \text{Apples}) \vee \text{Plays}(x, \text{Chess})$

$C1$: $\neg\text{Likes}(x, \text{Apples}) \vee \text{Plays}(x, \text{Chess})$

b. $\forall x\ \neg\text{Likes}(x, \text{Oranges}) \vee \text{Plays}(x, \text{Go})$

C2: $\neg\text{Likes}(x, \text{Oranges}) \vee \text{Plays}(x, \text{Go})$

c. $\forall x\ (\text{Likes}(x, \text{Apples}) \vee \text{Likes}(x, \text{Oranges})) \wedge \neg(\text{Likes}(x, \text{Apple}) \wedge \text{Likes}(x, \text{Oranges}))$

$\forall x\ (\text{Likes}(x, \text{Apples}) \vee \text{Likes}(x, \text{Oranges})) \wedge (\neg\text{Likes}(x, \text{Apple}) \vee$

¬Likes(x, Oranges))

(Likes(x, Apples) ∨ Likes(x, Oranges)) ∧ (¬Likes(x, Apple) ∨

¬Likes(x, Oranges))

$C3$: (Likes(x, Apples) ∨ Likes(x, Oranges))

$C4$: (¬Likes(x, Apple) ∨ ¬Likes(x, Oranges))

d. $C5$: Likes(John, Apples)

e. ∀y ¬Likes(John, y) ∨ ¬Likes(Marry, y)

$C6$: ¬Likes(John, y) ∨ ¬Likes(Marry, y)

4. $C1$: ¬Likes(x, Apples) ∨ Plays(x, Chess)

C2: ¬Likes(x, Oranges) ∨ Plays(x, Go)

$C3$: (Likes(x, Apples) ∨ Likes(x, Oranges))

$C4$: (¬Likes(x, Apple) ∨ ¬Likes(x, Oranges))

$C5$: Likes(John, Apples)

$C6$: ¬Likes(John, y) ∨ ¬Likes(Marry, y)

query: $Plays(Mary, Go)$

Assume the query is false: $C0$: $\neg Plays(Mary, Go)$

i. Resolve: $C5$ *and* $C6$

Variable substitutions: $C6$ (y/Apples)

$C7$: $\neg Likes(Marry, Apples)$

ii. Resolve: $C7$ *and* $C3$

Variable substitutions: $C3$ (x/Mary)

$C8$: $Likes(Mary, Oranges)$

iii. Resolve: $C8$ *and* $C2$

Variable substitutions: $C2$ (x/Mary)

$C9$: $Plays(Mary, Go)$

iv. Resolve: $C0$ *and* $C9$

$\neg Plays(Mary, Go)$ *and* $Plays(Mary, Go)$: {}

$\neg Plays(Marry, Go)$ is not true. Marry doesn't play game Go is False. Hence, $Plays(Mary, Go)$ is true.

5.

Input File:

```
fof(a1, axiom,
    ! [X] : (likes(X,apples) => plays(X,chess))).
fof(a2, axiom,
    ! [X] : (likes(X,oranges) => plays(X,go))).
fof(a3, axiom,
    ! [X] : ((likes(X,apples) | likes(X,oranges)) & (~likes(X,apples) |
    ~likes(X,oranges)))).
fof(a4, axiom,
    likes(john,apples)).
fof(a5, axiom,
    ! [Y] : ((~likes(john,Y)) | (~likes(mary,Y)))).
```

fof(c1, conjecture, plays(mary,go)).

Output:

% Refutation found. Thanks to Tanya!

% SZS status Theorem for problem

% SZS output start Proof for problem

2. ! [X0] : (likes(X0,oranges) => plays(X0,go)) [input]

3. ! [X0] : ((~likes(X0,oranges) | ~likes(X0,apples)) & (likes(X0,oranges) | likes(X0,apples))) [input]

4. likes(john,apples) [input]

5. ! [X1] : (~likes(mary,X1) | ~likes(john,X1)) [input]

6. plays(mary,go) [input]

7. ~plays(mary,go) [negated conjecture 6]

8. ! [X0] : (~likes(mary,X0) | ~likes(john,X0)) [rectify 5]

9. ~plays(mary,go) [flattening 7]

11. ! [X0] : (plays(X0,go) | ~likes(X0,oranges)) [ennf transformation 2]

13. ~likes(X0,oranges) | plays(X0,go) [cnf transformation 11]

14. likes(X0,oranges) | likes(X0,apples) [cnf transformation 3]

16. likes(john,apples) [cnf transformation 4]

17. ~likes(mary,X0) | ~likes(john,X0) [cnf transformation 8]

18. ~plays(mary,go) [cnf transformation 9]

20. plays(X0,go) | likes(X0,apples) [resolution 14,13]

24. 1 <=> likes(mary,apples) [avatar definition]

26. likes(mary,apples) <- (1) [avatar component clause 24]

33. likes(mary,apples) [resolution 20,18]

34. 1 [avatar split clause 33,24]

35. ~likes(john,apples) <- (1) [resolution 26,17]

37. $false <- (1) [subsumption resolution 35,16]

38. ~1 [avatar contradiction clause 37]

39. $false [avatar sat refutation 34,38]

% SZS output end Proof for problem

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% Version: Vampire 4.5.1 (commit 57a6f78c on 2020-07-15 11:59:04

+0200)

% Termination reason: Refutation

% Memory used [KB]: 4861

% Time elapsed: 0.157 s

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