CptS 540 Aritificial Intelligence

Hoemwork 7

Tengyang Zhang

10/21/2021

1.

a. $P(Win = true, Uniform = crimson, Weather = clear) = 0.18$

b. $P(Weather = clear) = 0.18 + 0.08 + 0.06 + 0.08 = 0.4$

c. $P(Uniform = crimson) = 0.18 + 0.08 + 0.05 + 0.06 + 0.07 + 0.08 =$

0.52

d. $P(Win = true|Weather = clear) = \frac{P(Win=true \wedge Weather=clear)}{P(weather=clear)} =$

$\frac{0.18+0.08}{0.4} = 0.65$

e. $P(Win = true|Weather = cloudy \vee Weather = rainy) =$

$\frac{P(Win=true \wedge (Weather=cloudy \vee Weather=rainy))}{P(Weather=cloudy \vee Weather=rainy)} =$

$\frac{(P(Win=true \wedge Weather=cloudy) \vee P(Win=true \wedge Weather=rainy))}{P(Weather=cloudy) \vee P(Weather=rainy)} =$

$\frac{(0.08+0.10)+(0.05+0.09)}{(0.08+0.10+0.07+0.09)+(0.05+0.09+0.08+0.04)} =$

$\frac{0.32}{0.6} = 0.533$

2. $P(Win|Practice = true \wedge Healthy = true)$

$= \frac{P(Pratice = true \wedge Healthy = true|Win)P(Win)}{P(Practive=true \wedge Healthy=true)}$

$= P(Pratice = true \wedge Healthy = true|Win)P(Win) * \alpha$

$= < P(Pratice = true \wedge Healthy = true|Win = true)P(Win =$

$true), P(Pratice = true \wedge Healthy = true|Win = false)P(Win =$

$false) > * \alpha$

$= < 0.8 * 0.7, 0.4 * 0.3 > * \alpha$

$= < 0.56, 0.12 > * \ \alpha$

$Since, \alpha = \frac{1}{0.56+0.12} = \frac{1}{0.68} = 1.47,$

$P(Win|Pratice = true \wedge Healthy = true) =< 0.56 * 1.47, 0.12 * 1.47 >=$

$< 0.82, 0.18 >$

3.

a. $breeze: \neg b_{1,1} \wedge b_{2,1} \wedge b_{1,2}$

$known: \neg p_{1,1} \wedge \neg p_{2,1} \wedge \neg p_{1,2} \wedge p_{3,1}$

$frontier: \{Pit_{1,3}\}$

$other: \{Pit_{2,3}, Pit_{3,3}, Pit_{3,2}\}$

b. $P\left(Pit_{2,2} \middle| breeze, known\right)$

$= \frac{P\left(Pit_{2,2}, breeze, known\right)}{P(breeze, known)}$

$= \alpha * P(pit_{2,2}, breeze, known)$

$= \alpha * \sum_{other} P\left(pit_{2,2}, breeze, known, other\right)$

$= \alpha * \sum_{frontier} \sum_{other} P\left(pit_{2,2}, breeze, known, frontier, other\right)$

$= \alpha * \sum_{f} \sum_{o} P\left(breeze \middle| pit_{2,2}, known, f, o\right) P(pit_{2,2}, known, f, o)$

$= \alpha * \sum_{f} \sum_{o} P\left(breeze \middle| pit_{2,2}, known, f\right) P\left(pit_{2,2}\right) P(known) P(f) P(o)$

$= \alpha * P(known) P\left(pit_{2,2}\right) \sum_{f} P(f) \sum_{o} P\left(breeze \middle| pit_{2,2}, known, f\right) P(o)$

$= \ \alpha * P(known) P\left(pit_{2,2}\right) \sum_{f} P(f)\ P\left(breeze \middle| pit_{2,2}, known, f\right)$

$$Let\ \alpha' = \alpha * P(known)$$

$$= \alpha' * P(pit_{2,2}) \sum_f P(f)\, P(breeze|pit_{2,2}, known, f)$$

$$= \alpha' * < P(p_{2,2}) \sum_f P(f)\, P(breeze|p_{2,2}, known, f),$$

$$P(\neg p_{2,2}) \sum_f P(f)\, P(breeze|p_{2,2}, known, f) >$$

$$= \alpha' * < P(p_{2,2})[P(pit_{1,3})P(breeze|p_{2,2}, known, pit_{1,3})],$$

$$P(\neg p_{2,2})[P(pit_{1,3})P(breeze|p_{2,2}, known, pit_{1,3})] >$$

$$= \alpha' * < P(p_{2,2})[P(p_{1,3})P(breeze|p_{2,2}, known, p_{1,3}) +$$

$$P(\neg p_{1,3})P(breeze|p_{2,2}, known, \neg p_{1,3})],$$

$$P(\neg p_{2,2})[P(p_{1,3})P(breeze|p_{2,2}, known, p_{1,3}) +$$

$$P(\neg p_{1,3})P(breeze|p_{2,2}, known, \neg p_{1,3})] >$$

$$= \alpha' * < (0.2)(0.2 * 1 + 0.8 * 1), (0.8)(0.2 * 1 + 0.8 * 0) >$$

$$= \alpha' * < 0.2, 0.16 >$$

$$Since\ \alpha' = \frac{1}{0.2+0.16} = 2.78, P(Pit_{2,2}|breeze, known) =< 0.56, 0.44 >$$

4. If there is a breeze in (3,3), It will not change the probability of a pit in (2,2). If there is a breeze in (3,3), It's means that the probability of pit in (2,3) and (3,2) will be change. For problem 3, the set of other is $\{Pit_{2,3}, Pit_{3,3}, Pit_{3,2}\}$, and we don't care about the other. Hence, $b_{3,3}$ will not change the probability of a pit in (2,2).