1.

a. $P(BuyJersey = yes) = \frac{7}{11} = 0.636$

$P(BuyJersey = no) = \frac{4}{11} = 0.364$

b. $P(Weather = clear|BuyJersey = yes) = \frac{3}{7} = 0.429$

$P(Weather = cloudy|BuyJersey = yes) = \frac{2}{7} = 0.286$

$P(Weather = rainy|BuyJersey = yes) = \frac{2}{7} = 0.286$

$P(Weather = clear|BuyJersey = no) = \frac{1}{4} = 0.25$

$P(Weather = cloudy|BuyJersey = no) = \frac{1}{4} = 0.25$

$P(Weather = rainy|BuyJersey = no) = \frac{2}{4} = \frac{1}{2} = 0.5$

c. $P(Uniform = crimson|BuyJersey = yes) = \frac{6}{7} = 0.857$

$P(Uniform = gray|BuyJersey = yes) = \frac{1}{7} = 0.143$

$P(Uniform = crimson|BuyJersey = no) = \frac{0}{4} => \frac{0+1}{4+2} = \frac{1}{6} = 0.1667$

$P(Uniform = gray|BuyJersey = no) = \frac{4}{4} = 1$

d. $P(Win = yes|BuyJersey = yes) = \frac{4}{7} = 0.571$

$P(Win = no|BuyJersey = yes) = \frac{3}{7} = 0.429$

$P(Win = yes|BuyJersey = no) = \frac{1}{4} = 0.25$

$P(Win = no|BuyJersey = no) = \frac{3}{4} = 0.75$

e. $let\ y = yes, n = no, c = cloudy, g = gray$

$$= \alpha < \begin{Bmatrix} P(Weather = c, Uniform = g, Win = y|BuyJersey = y) \\ * P(BuyJersey = y) \end{Bmatrix}, \begin{Bmatrix} P(Weather = c, Uniform = g, Win = y|BuyJersey = n) \\ * P(BuyJersey = n) \end{Bmatrix} >$$

$$= \alpha < \begin{Bmatrix} P(Weather = c|BuyJersey = y)P(Uniform = g|BuyJersey = \\ y)P(Win = y|BuyJersey = y)P(BuyJersey = y) \end{Bmatrix}, \begin{Bmatrix} P(Weather = c|BuyJersey = n)P(Uniform = g|BuyJersey = \\ n)P(Win = y|BuyJersey = n)P(BuyJersey = n) \end{Bmatrix} >$$

$= \alpha < 0.015, 0.023 > = < 0.39, 0.61 >$

$P(BuyJersey = yes | Weather = cloudy, Uniform = gray, Win = yes)$

$= 0.39$

$P(BuyJersey = no | Weather = cloudy, Uniform = gray, Win = yes)$

$= 0.61$

f. Naïve Bayes would choose BuyJersey=no.

2.

a.

Weather	**Uniform**	**Win**	**BuyJersey**
0	0	1	1
0	0	0	1
0	1	1	1
0	1	0	0
1	0	1	1
1	0	0	1
1	1	0	0
2	0	1	1
2	0	0	1
2	1	1	0
2	1	0	0

b.

1	1	1

Since $w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 + 1 + 1 + 1 = 4$ which is bigger than 0, the perceptron (before and training) would choose BuyJersey=1(yes).

c.

example 1:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 + 0 + 0 + 1 = 2 \geq 0 => 1(correct)$$

example 2:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 + 0 + 0 + 0 = 1 \geq 0 => 1(correct)$$

example 3:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 + 0 + 1 + 1 = 3 \geq 0 => 1(correct)$$

example 4:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 + 0 + 1 + 0 = 2 \geq 0 => 1(incorrect)$$

Update weights:

$$w_0 = 1 + 0.5 * (0 - 1) * 1 = 0.5$$

$$w_1 = 1 + 0.5 * (0 - 1) * 0 = 1$$

$$w_2 = 1 + 0.5 * (0 - 1) * 1 = 0.5$$

$$w_3 = 1 + 0.5 * (0 - 1) * 1 = 1$$

example 5:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 + 1 + 0 + 1 = 2.5 \geq 0 => 1(correct)$$

example 6:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 + 1 + 0 + 0 = 1.5 \geq 0 => 1(correct)$

example 7:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 + 1 + 0.5 + 0 = 2 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0.5 + 0.5 * (0 - 1) * 1 = 0$

$w_1 = 1 + 0.5 * (0 - 1) * 1 = 0.5$

$w_2 = 0.5 + 0.5 * (0 - 1) * 1 = 0$

$w_3 = 1 + 0.5 * (0 - 1) * 0 = 1$

example 8:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 + 1 + 0 + 1 = 2 \geq 0 => 1(correct)$

example 9:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 + 1 + 0 + 0 = 1 \geq 0 => 1(correct)$

example 10:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 + 1 + 0 + 1 = 2 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0 + 0.5 * (0 - 1) * 1 = -0.5$

$w_1 = 0.5 + 0.5 * (0 - 1) * 2 = -0.5$

$w_2 = 0 + 0.5 * (0 - 1) * 1 = -0.5$

$w_3 = 1 + 0.5 * (0 - 1) * 1 = 0.5$

example 11:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 - 1 - 0.5 + 0 = -2 < 0 => 0(correct)$

d. <1,1,1>

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 - 0.5 - 0.5 - 0.5 = -2 < 0 => 0$$

3.

Input:

@relation homework10

@attribute Weather {clear, cloudy, rainy}

@attribute Uniform {crimson, gray}

@attribute Win {yes, no}

@attribute BuyJersey {yes, no}

@data

clear,crimson,yes,yes

clear,crimson,no,yes

clear,gray,yes,yes

clear,gray,no,no

cloudy,crimson,yes,yes

cloudy,crimson,no,yes

cloudy,gray,no,no

rainy,crimson,yes,yes

rainy,crimson,no,yes

rainy,gray,yes,no

rainy,gray,no,no

Output:

=== Run information ===

Scheme: weka.classifiers.bayes.NaiveBayes

Relation: homework10

Instances: 11

Attributes: 4

Weather

Uniform

Win

BuyJersey

Test mode: evaluate on training data

=== Classifier model (full training set) ===

Naive Bayes Classifier

Class

Attribute yes no

(0.62) (0.38)

==========

Weather

clear	4.0	2.0
cloudy	3.0	2.0
rainy	3.0	3.0
[total]	10.0	7.0

Uniform

crimson	7.0	1.0
gray	2.0	5.0
[total]	9.0	6.0

Win

yes	5.0	2.0
no	4.0	4.0
[total]	9.0	6.0

Time taken to build model: 0 seconds

=== Evaluation on training set ===

Time taken to test model on training data: 0 seconds

=== Summary ===

Correctly Classified Instances	10	90.9091 %
Incorrectly Classified Instances	1	9.0909 %
Kappa statistic	0.8136	
Mean absolute error	0.2047	
Root mean squared error	0.2399	
Relative absolute error	43.6901 %	
Root relative squared error	49.8209 %	
Total Number of Instances	11	

=== Detailed Accuracy By Class ===

	TP Rate	FP Rate	Precision	Recall	F-Measure	MCC	ROC Area	PRC Area	Class
	0.857	0.000	1.000	0.857	0.923	0.828	1.000	1.000	yes
	1.000	0.143	0.800	1.000	0.889	0.828	1.000	1.000	no
Weighted Avg.	0.909	0.052	0.927	0.909	0.911	0.828	1.000	1.000	

=== Confusion Matrix ===

a b <-- classified as

6 1 | a = yes

0 4 | b = no

4.

pass 2

1:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(\text{correct})$

2:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$

Update weights:

$w_0 = 0, w_1 = -0.5, w_2 = -0.5, w_3 = 0.5$

3:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$

4:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$

5:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$

6:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$

Update weights:

$w_0 = 0.5, w_1 = 0, w_2 = -0.5, w_3 = 0.5$

7:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0, w_1 = -0.5, w_2 = -1, w_3 = 1$

8:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$

Update weights:

$w_0 = 0.5, w_1 = 0.5, w_2 = -1, w_3 = 1$

9:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1.5 \geq 0 => 1(correct)$

10:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1.5 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0, w_1 = -0.5, w_2 = -1.5, w_3 = 0.5$

11:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -2.5 < 0 => 0(correct)$

Pass 3

1:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$

2:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$

3:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1 < 0 => 0(incorrect)$

Update weights:

$w_0 = 0.5, w_1 = -0.5, w_2 = -1, w_3 = 1$

4:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$

5:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 \geq 0 => 1(correct)$

6:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$

7:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1 < 0 => 0(correct)$

8:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$

9:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$

Update weights:

$w_0 = 1, w_1 = 0.5, w_2 = -1, w_3 = 1$

10:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 2 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0.5, w_1 = -0.5, w_2 = -1.5, w_3 = 0.5$

11:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -2 < 0 => 0(correct)$

Pass 4

1:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 \geq 0 => 1(correct)$

2:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$

3:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$

Update weights:

$w_0 = 1, w_1 = -0.5, w_2 = -1, w_3 = 1$

4:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(incorrect)$

Update weights:

$w_0 = 0.5, w_1 = -0.5, w_2 = -1.5, w_3 = 1$

5:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 \geq 0 => 1(correct)$

6:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$

7:

$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1.5 < 0 => 0(correct)$

8:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$$

9:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(incorrect)$$

Updated weights:

$$w_0 = 1, w_1 = 0.5, w_2 = -1.5, w_3 = 1$$

10:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1.5 \geq 0 => 1(incorrect)$$

Update weights:

$$w_0 = 0.5, w_1 = -0.5, w_2 = -2, w_3 = 0.5$$

11:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -2.5 < 0 => 0(correct)$$

Pass 5

1:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 \geq 0 => 1(correct)$$

2:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$$

3:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1 < 0 => 0(incorrect)$$

Update weights:

$$w_0 = 1, w_1 = -0.5, w_2 = -1.5, w_3 = 1$$

4:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$$

5:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1.5 \geq 0 => 1(correct)$$

6:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$$

7:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1.5 < 0 => 0(correct)$$

8:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$$

9:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$$

10:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$$

11:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -2.5 < 0 => 0(correct)$$

Pass 6

1:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 2 \geq 0 => 1(correct)$$

2:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1 \geq 0 => 1(correct)$$

3:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$$

4:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$$

5:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 1.5 \geq 0 => 1(correct)$$

6:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$$

7:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -1.5 < 0 => 0(correct)$$

8:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0.5 \geq 0 => 1(correct)$$

9:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = 0 \geq 0 => 1(correct)$$

10:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -0.5 < 0 => 0(correct)$$

11:

$$w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 = -2.5 < 0 => 0(correct)$$

The final perceptron weights are:

$$w_0 = 1, w_1 = -0.5, w_2 = -1.5, w_3 = 1$$