Solutions to Cpt S 516 Homework # 2

1. use the template.

2. in class.

3. Done in class.
4. Let TM M recognize L. Then, the following M' will recognize L^r (hence L^r is recursive):
input x;
change x to x^r;
Run M on x^r;
accept/reject according to M.

5. Many ways to prove. Easiest is to use two-counter machines – the x-coordinate corresponds to counter x and y-coordinate corresponds to counter y. In below, I will show another prove, which is in a hard way:

Let M be a TM and w be an input word. I construct a 2-TM (read-only) M' as follows:
On an input of M',
M' checks the input is in the following format:
in the lowest row, it stores C_0;
on one row above, it stores C_1;
$\vdots$
the topest row stores C_n for some n;
All the strings C_i's starts from the first cell at the left boundary on each row;
C_0 is the initial configuration of M on w;
C_n is a final configuration of M on w;
each C_i to C_{i+1} is the result of running one move of M from configuration C_i to C_{i+1}. (this can be checked by the read-only machine M', why? think)
M' accepts the input if all above check out.
Clearly, the language accepted by M' is empty iff M does not accept w. Hence, the result follows.

6. In two papers:

Zhe Dang, Oscar H. Ibarra and Pierluigi San Pietro. Liveness Verification of Reversal-bounded Multicounter Machines with a Free Counter. Proceedings of the 20th International Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2001) Lecture Notes in Computer Science vol 2245, pp. 132-143, Springer. 2001,
and a generalized form in
Zhe Dang and Oscar H. Ibarra. The Existence of Omega-Chains for Transitive Mixed Linear Relations and Its Applications. International Journal of Foundations of Computer Science, Vol. 13, No. 6 (2002) 911-936.