1 problem 1

L={abcab, abcabab, abcabcab, ...}
#L={(2,2,1), (3,3,1), (3,3,2), ...}
Presburger formula: $\#a=\#b \wedge \#a>\#c \wedge \#a\geq 2 \wedge \#c\geq 1$

2 problem 2

Let M be the given PDA, we construct a PDA M1 to simulate M as follow. For the instructions of M, M1 simulates the M faithfully. However, when M "pushes a" to the stack, M1 will do the same and additional M1 makes sure that it reads an input symbol 'push a' from M1 own input. When M "push b" to the stack, M1 will also push b to it own stack while reading input symbol 'push b' from M1's own input. When M says yes, M1 also says yes.
I define L1 to be the set of all push(a) and push(b) sequences of M1's instructions.
For example: If M1's instructions is {push(b), push(c), push(a), pop(c), push(b), push(b)} L1={b, ba, bab, babb}
Clearly we know L1 is semilinear language. Besides, $\{3\#push(b)-2\#push(a)\geq 0 \wedge 9\#push(b)-6\#push(a)\geq \#a+\#b\ \}$ is semilinear and comutative.
So $L=L1 \cap 3\#push(b)-2\#push(a)\geq 0 \wedge 9\#push(b)-6\#push(a)\geq \#a+\#$
} is semilinear. so it is decidable whether M is stable.

3 problem 3

we construct a Presburger formula as follow:
P(x1, x2, x3; x4, x5, x6)
1. (x1, x2, x3) are the counts of a, b, c when M moves from p0 to p;
2. (x4, x5, x6) are the counts of a, b, c when M moves from p0 to p;
3. x1=x2=x3 & x4=x5=x6 & x4>x1 & x5>x2 & x6>x3

4 problem 4

α is a sequence from start to end. So, all such #(α) are defined by a Presburger formula P(#a, #b, #c).
Suppose that:
a: (1, 2)
b: (2, 3)
c: (3, 4).
$\exists$#a, #b, #c, ta, tb, tc
P(#a, #b, #c) $\wedge$ —ta-tb—$\leq$ 100 $\wedge$ —ta-tc—$\leq$100 $\wedge$ —ta-tc—$\leq$ 100 $\wedge$ $\#a<ta<2\#a$ $\wedge$ $2\#b<tb<3\#b$ $\wedge$ $3\#c<tc<4\#c$
This formula is decidable, so M is time-fair is decidable.

5 problem 5

L(M) is context free language and hence semilinear. Pre(L(M)) is the set of all prefixes of all word in L(M). And claearly

Pre(L(M)) is semilinear. Besides, w: #(w) satisfy $\neg$ L is semilinear and comutative. So L = Pre(L(M)) $\cap$ w: #(w) satisfy $\neg$ L is semilinear language.

6 problem 6

For this question, we just need to show the decidability of:
For given p, q, P, Q. $\exists$u, v satisfy (p, u) $\longrightarrow$ (g, v) & P(u) Q(u).
P(u) define a semilinear set on u's. So, it's a linear set; For example
u: $\binom{1}{2}$ + a1$\binom{2}{3}$ + a2$\binom{3}{1}$, a1, a2$\geq$0
Q(v) define a semilinear set on v's. So it's a linear set; For example
v: $\binom{2}{0}$ + a1$\binom{1}{2}$, a1$\geq$0
For the upper example, I construct VASS1 as follow:
q0 is the start state. q0 $\longrightarrow$(q1, $\binom{1}{2}$);
q1 $\longrightarrow$(q1, $\binom{2}{3}$);
q1 $\longrightarrow$(q2, $\binom{0}{0}$);
q2 $\longrightarrow$(q2, $\binom{3}{1}$);
q2 $\longrightarrow$(p, $\binom{0}{0}$);
VASS2:
q0 $\longrightarrow$(q1, $\binom{2}{0}$);
q1 $\longrightarrow$(q1, $\binom{1}{2}$);
q1 $\longrightarrow$(p, $\binom{0}{0}$);