1.

a. We know the graph G is a connected graph, and the degree of all vertices in G is even. The Euler circuit of G is $00 \to 01 \to 02 \to 00 \to 10 \to 11 \to 12 \to 10 \to 20 \to 21 \to 22 \to 20 \to 00$. Hence, G is Eulerian.

b. We can produce a Hamilton cycle of G: $20 \to 21 \to 11 \to 10 \to 00 \to 01 \to 02 \to 02 \to 12 \to 22 \to 20$. Hence G is Hamiltonian.

2.

a. If either n or q is at least 3, there must be at least one vertex between start point and end point of P_n *or* P_q with an odd degree. Hence, $P_{n,q}$ is not Eulerian. For example, in $P_{3,3}$ the degree of $01, 10, 12, 21$ are odd. Thus, $P_{3,3}$ is not Eulerian.

b. For any $q \geq 2$, we can produce a Hamilton circle of $P_{2,q}$:

$$00 \to 01 \to 02 \to 03 \to \cdots \to 0q \to 1q \to \cdots \to 13 \to 12 \to 11 \to 10 \to 00$$

Hence, $P_{2,q}$ is Hamiltonian.

c. We can produce a Hamilton circle of $P_{4,4}$:

$$00 \to 01 \to 02 \to 03 \to 13 \to 12 \to 11 \to 21 \to 22 \to 23 \to 33 \to 32 \to 31 \to 30 \to 20 \to 10 \to 00$$

Hence, $P_{4,4}$ is Hamiltonian.

d. If we delete the vertices S = $\{01, 10, 12, 21\}$ from $P_{3,3}$, then we get 5 components. Since, If G is Hamiltonian, then for any non-empty subset S of $V(G)$, $W(G-S) \leq |S|$.

For $P_{3,3}$ $W(G-S) = 5$ $is\ bigger\ than$ $|S| = 4$. Hence H is not Hamiltonian.

e. If n and q are both odd, then we can get $n = 2k+1, q = 2n+1$.

If $k = 0, n = 1$ or $n = 0, k = 1$, we know $P_{n,q}$ is a path and is not Hamiltonian.

If $k \geq 1, n \geq 1$, $P_{n,q}$ $has\ n * q\ vertices.$ $nq = (2k+1)*(2n+1) = 4kn + 2k + 2n + 1$. Let $S = \{uv \mid (u+v)\ is\ odd, 0 \leq u \leq 2k, 0 \leq v \leq 2n\}$. Then $|S| = 2kn + n + k$. $W(P_{nq} - S) = 4kn + 2k + 2n + 1 - (2kn + n + k) = 2kn + n + k + 1$, which is bigger than $|S|$. Hence, $P_{n,q}$ is not Hamiltonian.

f. We suppose n is even, then we can get $n = 2k$.

If $k = 1$, $P_{n,q} = P_{2,q}$ is Hamiltonian. (we have proved in **b**)

If $k > 1$, we can produce a Hamilton circle of $P_{n,q}$:

$$00 \to 01 \to 02 \to \cdots \to 0(q-1) \to 1(q-1) \to \cdots \to 12 \to 11 \to \cdots \to$$
$$(2k-2)1 \to \cdots \to (2k-2)(q-1) \to (2k-1)(q-1) \to \cdots \to$$
$$(2k-1)0 \to \cdots \to 20 \to 10 \to 00$$

Since n is even, we can always find the Hamilton circle. $P_{n,q}$ is Hamiltonian.

3.

a. If by deleting any 1 link of G turns every other link into a bridge, then the relationship among k, m, n is: $m - 1 = n - k$

b. If by deleting l links G turns an all-bridge graph, then the relationship among k, m, n is: $m - l = n - k$

c. If there are two such values l_1 and l_2, then we get $l_1 = l_2 = m - n + k$. Hence, there is a unique value l that works in part b.

d. The value of l in "theta" graph is 2, because we need to delete 2 edges to make it into a link less graph.

e. The value of l in K_5 is 6, because we need to delete six edges to make it into a link less graph.