1.

Let $G_{1,2}$ be the graph on two vertices with one edge.

$$L_{G_{1,2}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

As we see $x^T\ L_{G_{1,2}}\ x = (x_1 - x_2)^2$, where $x = (x_1, x_2)$.

For any graph $G_{u,v}$ we define:

$$L_{G_{u,w}}\ (\mathrm{i,j}) = \begin{cases} 1 & if \quad i = j \quad and \quad i \in \{\mathrm{u,w}\} \\ -1 & if \quad i = u \quad and \quad j = w, \quad or \quad i = w \quad and \quad j = u \\ 0 & otherwise \end{cases}$$

For a graph G=(V, E) we define the Laplacian matrix as follows:

$$L_G = \sum_{\{u,w\} \in E} L_{G_{u,w}}.$$

2.

As with circulations, the set $\mathcal{B}$ of all potential differences in D is closed under addition and scalar multiplication and, hence, is a vector space.

Analogous to the function f_c associated with a cycle C, there is a function g_B associated with a bond B. Let B=$[S,\ \overline{S}\ ]$ be a bond of D. We define g_b by

$$g_B\ (\mathrm{a}) = \begin{cases} 1 & if \quad a \in (S, \overline{S}) \\ -1 & if \quad a \in (\overline{S}, S) \\ 0 & if \quad a \notin B \end{cases}$$

It can be verified that $g_B = \delta$p where

$$\mathrm{p(v)} = \begin{cases} 1 & if \quad v \in \mathrm{S} \\ 0 & if \quad v \in \mathrm{B} \end{cases}$$