I.

A. Use the matrix calculator we can find the eigenvalues of L is:

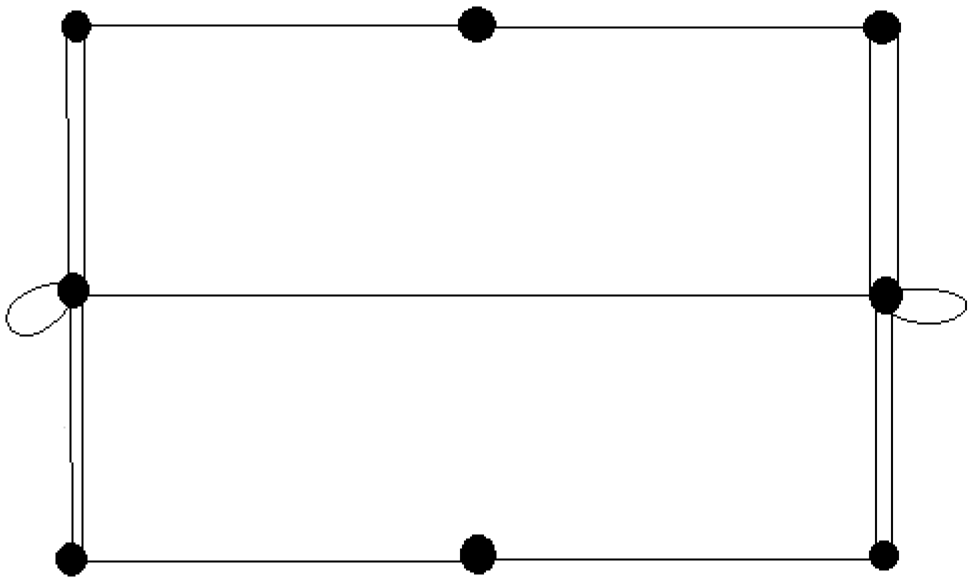
$$0 = \lambda_1 \leq 0.875 \leq 1 \leq 2 \leq 3 \leq 4 \leq 4 \leq 4.363 \leq 5 \leq 5.762$$

According to this, we can find the λ_2 *and* λ_3 together with eigenvectors v_2 *and* v_3 as below:

$$\lambda_2 = 0.875 \;\; v_2 = \begin{pmatrix} -1 \\ -1.32 \\ 0 \\ -0.805 \\ -1 \\ 0.805 \\ 0 \\ 1 \\ 1.32 \\ 1 \end{pmatrix} \qquad \lambda_3 = 1 \;\; v_3 = \begin{pmatrix} -1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \\ 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

B. According to part A, we can get the points:

$(-1,-1), (-1,0), (0,-1), (-1,0), (-1,1), (1,0), (0,1), (1,-1), (1,0), (1,1)$



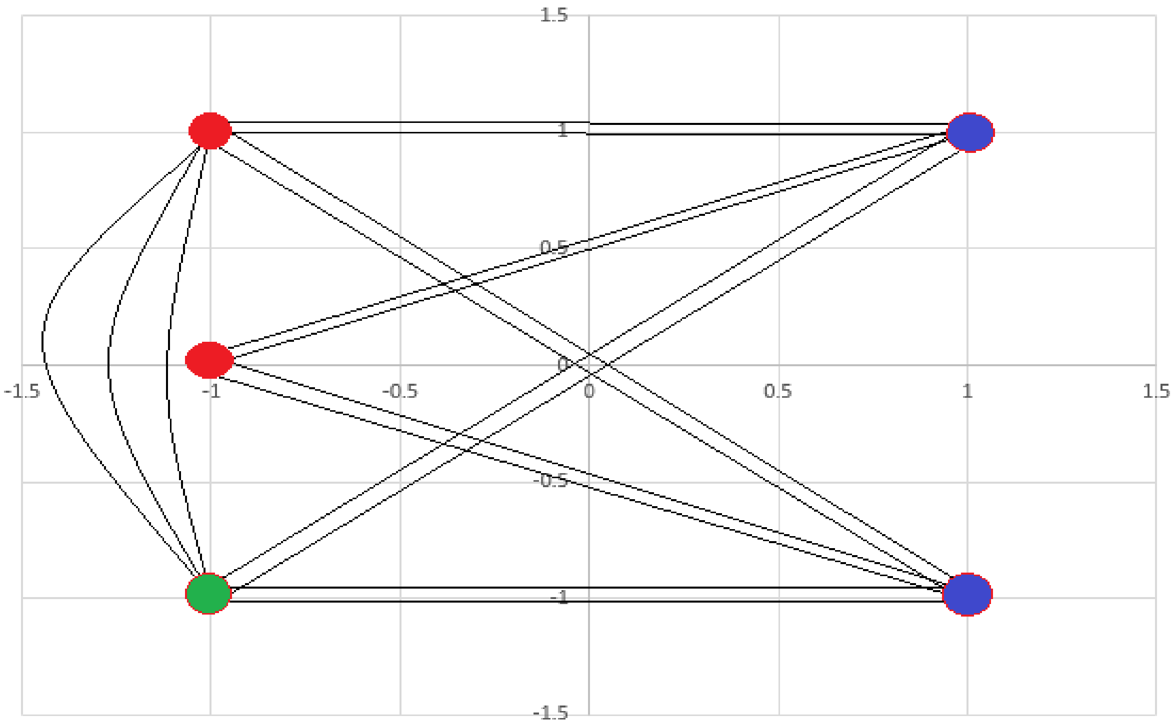
C. We can find λ_9 *and* λ_{10} along with corresponding eigenvectors

v_9 *and* v_{10} is:

$$\lambda_9 = 5 \;\; v_9 = \begin{pmatrix} 1 \\ -0.667 \\ -0.667 \\ -0.667 \\ 1 \\ -0.667 \\ -0.667 \\ 1 \\ -0.667 \\ 1 \end{pmatrix} \qquad \lambda_{10} = 5.76 \;\; v_{10} = \begin{pmatrix} -1 \\ -0.432 \\ 0 \\ 3.194 \\ -1 \\ -3.194 \\ 0 \\ 1 \\ 0.432 \\ 1 \end{pmatrix}$$

D. We can get the points as below:

$$(1,-1), (-1,-1), (-1,0), (-1,1), (1,-1), (-1,-1), (-1,0), (1,1), (-1,1), (1,1)$$



Hence, the chromatic number of G is 3.