Optimization of the Transformer's attention

Linear Algebra (1.22-23.PKN22/M)

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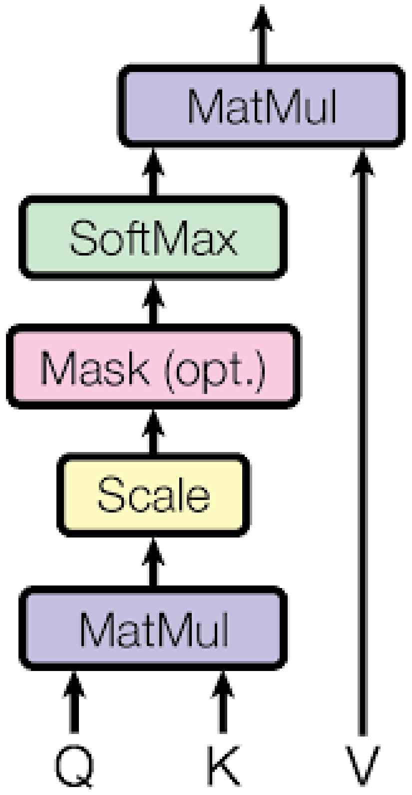
Project goals

1. Investigate method of imrovement for dot-product (softmax) attention.
2. Implement and evaluate method on CIFAR-10 image classification benchmark.



CIFAR-10 banchmark: Learning multiple layers of features from tiny images. Technical report, University of Toronto, 2009

Image source: https://arxiv.org/abs/1706.03762

Attention Is All You Need

$$\text{Attention}(Q,K,V) = \text{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$

$$X \in R^{batch \times tokens \times d_k}, Q = XW^Q, K = XW^K, V = XW^V$$

- d_{model} is the size of the embedding vector of each input element from our sequence.
- d_k is the inner dimension of that is specific to each self-attention layer.
- $batch$ is the batch size
- $tokens$ is the number of elements that our sequence has, e.g. number of pixels.

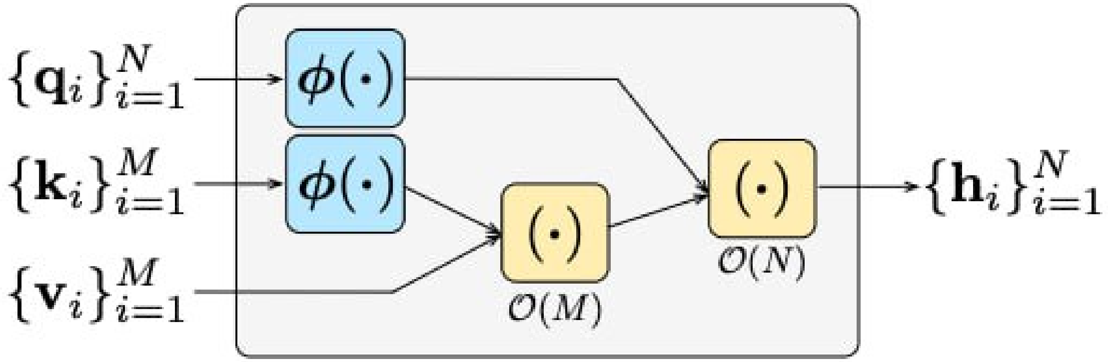
Source: https://theaisummer.com/self-attention

Kernel-based attention optimization methods

Random feature attention (RFA)

$$\mathrm{RFA}(Q,K,V) = \mathrm{Softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \approx$$

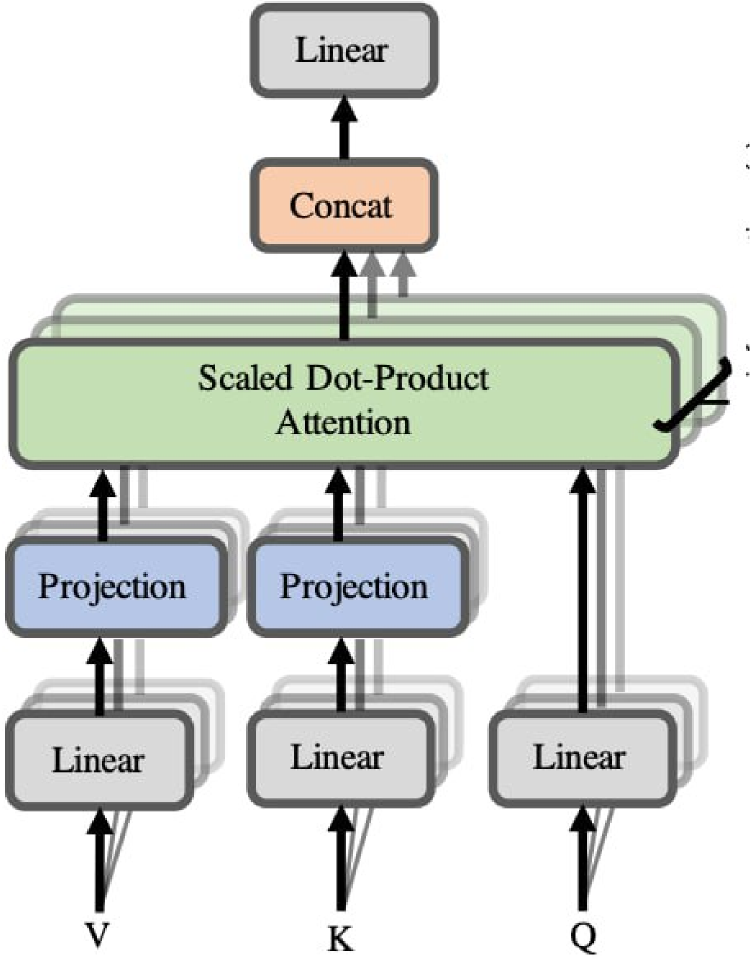
$$\approx \frac{\phi(q)^T \sum_i \phi(k_i)\otimes v_i}{\phi(q)\cdot\sum_j \phi(k)}$$

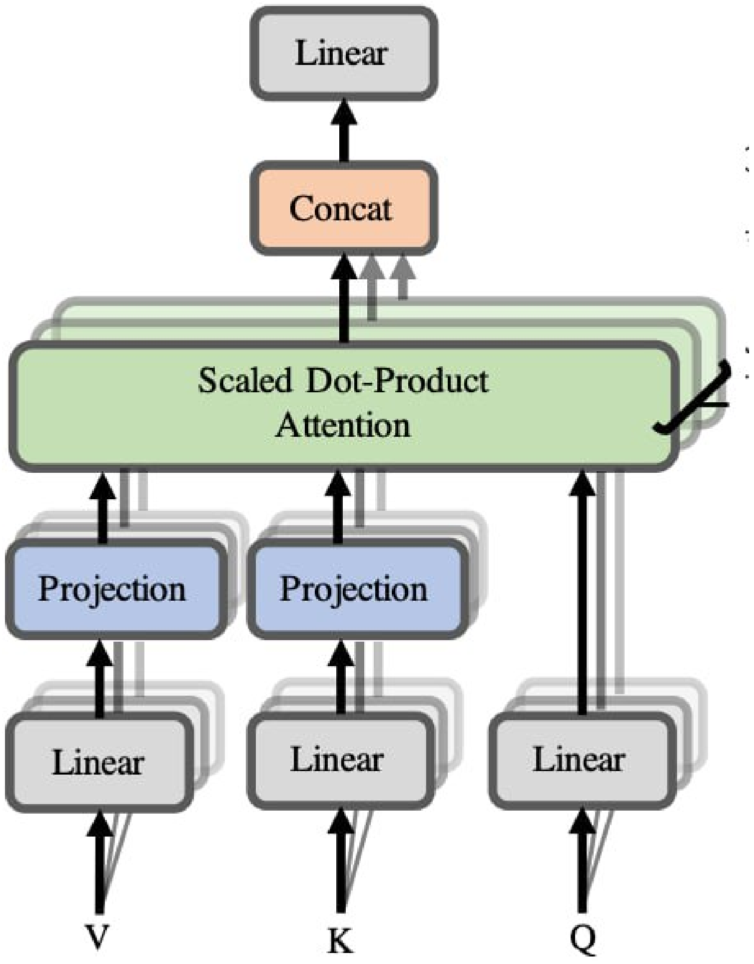


Low-rank attention optimization methods

Linear Attention

$$\text{Linear Attention}(Q, K, V) =$$
$$= \left(\frac{Q}{\sqrt{(d_k)}}\right)\left(\frac{K^T}{\sqrt{(d_k)}}V\right) =$$
$$= \frac{1}{d_k}Q\left(K^T V\right) = \frac{1}{d_k}\left(QK^T\right)V =$$
$$= \text{Attention}(Q, K, V)$$





Low-rank attention optimization methods

Linformer Attention

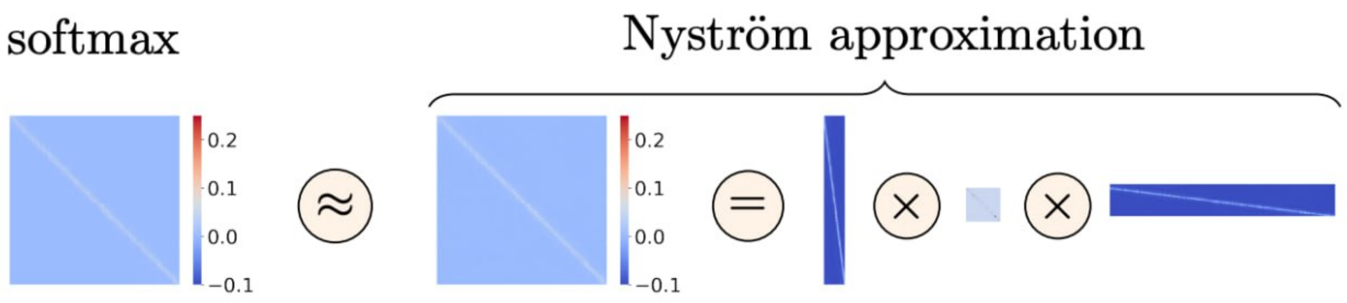
$$\text{Linformer}(Q, K, V) =$$
$$= \text{Softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \approx$$
$$\approx \text{Softmax}\left(\frac{Q\left(EK\right)^T}{\sqrt{d_k}}\right)\left(FV\right)$$

Low-rank attention optimization methods

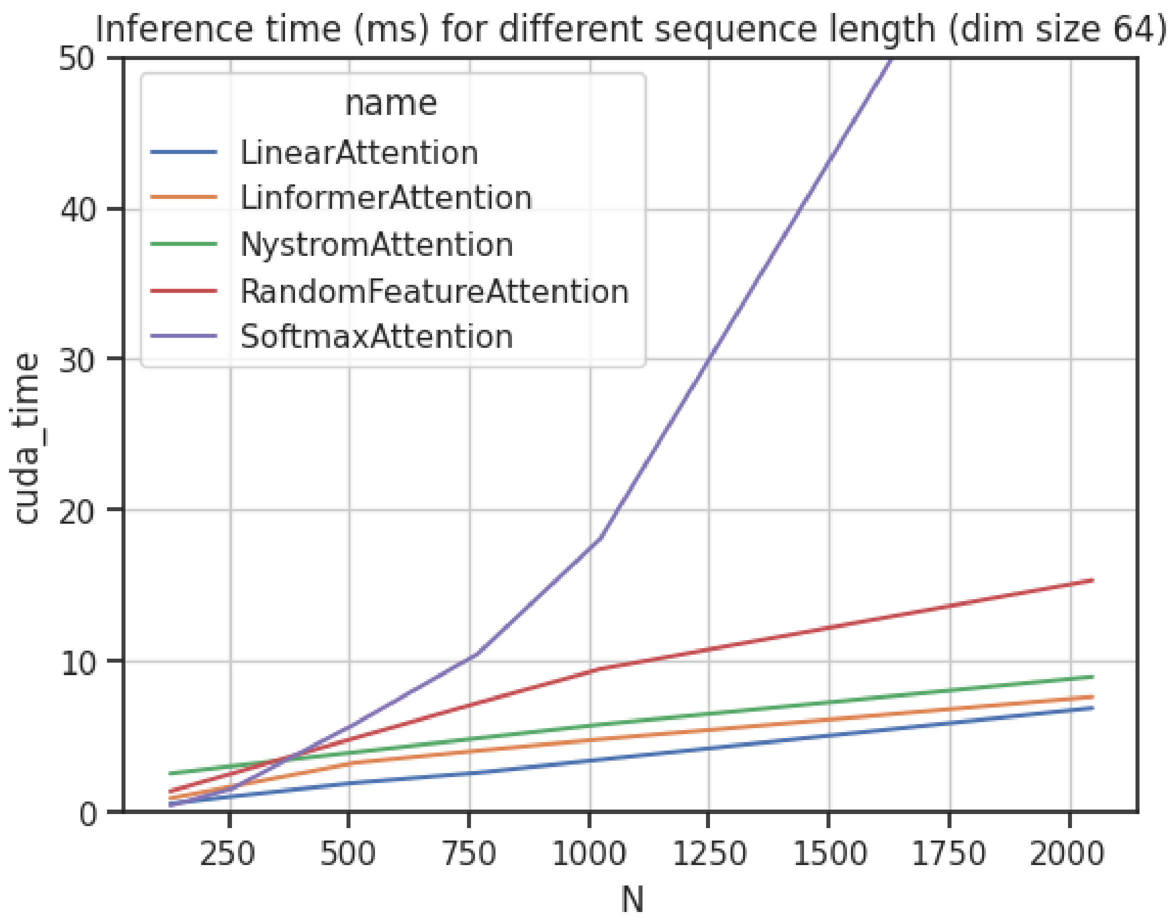
Nystrom Attention

$$\text{Attention}(Q,K,V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_q}}\right)V = \begin{bmatrix} A_S & B_S \\ F_S & C_S \end{bmatrix}$$

$$\text{Nystrom } \widehat{\text{Attention}}(\text{Q, K,V}) = \left[\text{softmax}\left(\frac{Q\tilde{K}^T}{\sqrt{d_q}}\right)\left(\text{softmax}\left(\frac{\tilde{Q}\tilde{K}^T}{\sqrt{d_q}}\right)\right)^{\dagger}\text{softmax}\left(\frac{\tilde{Q}K^T}{\sqrt{d_q}}\right)\right]V$$



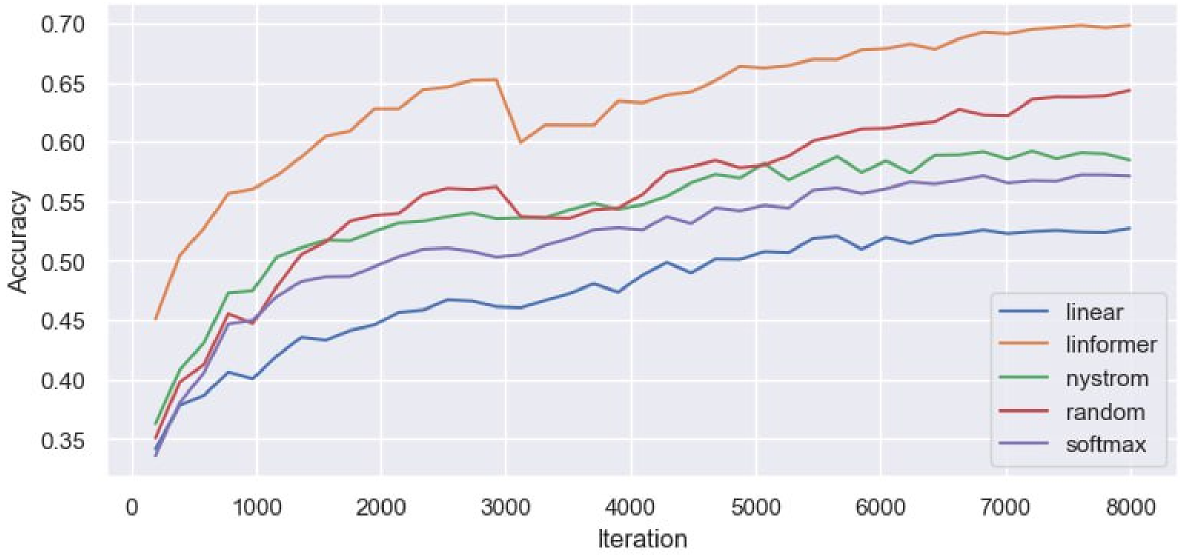
Attention Complexity



Attention Complexity

Method	Computational	Memory
Dot-product (softmax) attention	$O(d^2n + dn^2)$	$O(nd^2)$
Linear attention	$O(d^2n)$	$O(dn + d^2)$
Linformer Attention	$O(n)$	$O(n)$
Random Feature Attention	$O(nd)$	$O(4D + 2Dd)$
Nystrom Attention	$O(n)$	$O(n)$

Attention accuracy



Conclusions

- Investigated dot-product attention mechanism optimization using **linear algebra** matrix transformation techniques.
- Empirically demonstrated that linear attention approach has lower computational complexity than observed methods.
- Linformer attention mechanism showed best classification accuracy among observer models with similar hyper-parameter sets.

Thanks for your attention!