# 1 3.1-4

Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

$$\lim_{n\to\infty} \frac{2^{n+1}}{2^n} = \lim_{n\to\infty} \frac{2 \times 2^n}{2^n} = \lim_{n\to\infty} 2 = 2$$

Since $\lim_{n\to\infty} \frac{2^{n+1}}{2^n}$ evaluates to a constant $2^{n+1}$ is $\Theta(2^n)$ which means it is also $O(2^n)$.

$$\lim_{n\to\infty} \frac{2^{2n}}{2^n} = \lim_{n\to\infty} \frac{2^{2^n}}{2^n} = \lim_{n\to\infty} 4^n = \infty$$

Since $\lim_{n\to\infty} \frac{2^{2n}}{2^n} = \infty$ it is $\omega(2^n)$ which means it is not $O(2^n)$

# 2 3.2-3

Prove equation $\lg(n!) = \Theta(n \lg n)$. Also prove that $n! = \omega(2^n)$ and

# 3 3-2

| | A | B | $O$ | $o$ | $\Omega$ | $\omega$ | $\Theta$ |
|---|---|---|---|---|---|---|---|
| **a.** | $\lg^k n$ | $n^\epsilon$ | yes | yes | no | no | no |
| **b.** | $n^k$ | $c^n$ | yes | yes | no | no | no |
| **c.** | $\sqrt{n}$ | $n^{\sin n}$ | | | | | |
| **d.** | $2^n$ | $2^{n/2}$ | no | no | yes | yes | no |
| **e.** | $n^{\lg c}$ | $c^{\lg n}$ | | | | | |
| **f.** | $\lg n!$ | $\lg n^n$ | | | | | |

work:

**a.**

$$\lim_{n\to\infty} \frac{\lg^k n}{n^\epsilon} = \lim_{n\to\infty} \frac{k \lg^{k-1} n \times \frac{1}{n \ln 2}}{\epsilon n^{\epsilon - 1}} = \lim_{n\to\infty} \frac{k \lg^{k-1} n}{\ln(2)\epsilon n^\epsilon} = \lim_{n\to\infty} \frac{k!}{(\ln(2)\epsilon)n^\epsilon} = 0$$

**b.**

$$\lim_{n\to\infty} \frac{n^k}{c^n} = \lim_{n\to\infty} \frac{kn^{k-1}}{c^n \ln c} = \lim_{n\to\infty} \frac{k!}{c^n \ln^k c} = 0$$

**c.**

$$\lim_{n\to\infty}$$

**d.**

$$\lim_{n\to\infty} \frac{2^n}{2^{n/2}} = \lim_{n\to\infty} (\frac{2}{\sqrt{2}})^n = \infty$$

**e.**

$$\lim_{n\to\infty}$$

**f.**

$$\lim_{n\to\infty}$$

# 4 17.1-3