

# Probability Assignment 1

EE22BTECH11217 - Sayan Biswas

## 1 PROBLEM STATEMENT

Two balls are drawn at random with replacement from box containing 10 black and 8 red balls. Find the probability that :

- 1) both balls are red
- 2) one of them is black the other one is red

- One of them is black and other is red :

$$= \Pr(X = 1) \quad (14)$$

$$= 2(1 - p)p \quad (15)$$

$$= \left(\frac{40}{81}\right) \quad (16)$$

## 2 ANSWER

Assume random variable  $X_1$  and  $X_2$  :

- $X_1$ : Colour of first ball picked

$$Pr_{X_1}(n) = \begin{cases} (1 - p), & n = 0, \text{ black ball} \\ p, & n = 1, \text{ red ball} \end{cases} \quad (1)$$

$$p = \frac{8}{18} \quad (2)$$

- $X_2$ : Colour of second ball picked

$$Pr_{X_2}(n) = \begin{cases} (1 - p), & n = 0, \text{ black ball} \\ p, & n = 1, \text{ red ball} \end{cases} \quad (3)$$

$$p = \frac{8}{18} \quad (4)$$

Let us define a random variable  $X = X_1 + X_2$

$$M_X(z) = \sum_{n=-\infty}^{+\infty} z^{-n} P_X(n) \quad (5)$$

$$M_X(z) = M_{X_1+X_2}(z) \quad (6)$$

$$= M_{X_1}(z) \times M_{X_2}(z) \quad (7)$$

$$= (1 - p)^2 + 2(1 - p)(p)z^{-1} + p^2z^{-2} \quad (8)$$

$$= \Pr(X = 0) + \Pr(X = 1)z^{-1} + \Pr(X = 2)z^{-2} \quad (9)$$

$$= Pr_X(n) = \begin{cases} (1 - p)^2, & n = 0 \\ 2(1 - p)p, & n = 1 \\ p^2, & n = 2 \end{cases} \quad (10)$$

- Both balls are red :

$$= \Pr(X = 2) \quad (11)$$

$$= p^2 \quad (12)$$

$$= \left(\frac{16}{81}\right) \quad (13)$$