Probability Assignment 1

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1 Problem statement

Two balls are drawn at random with replacement from box containing 10 black and 8 red balls. Find the probability that :

1) both balls are red
2) one of them is black the other one is red

2 Answer

Assume random variable X_1 and X_2 :

- X_1: Colour of first ball picked

$$Pr_{X_1}(n) = \begin{cases} (1-p), & \text{n = 0, black ball} \\ p, & \text{n = 1, red ball} \end{cases} \tag{1}$$

$$p = \frac{8}{18} \tag{2}$$

- X_2: Colour of second ball picked

$$Pr_{X_2}(n) = \begin{cases} (1-p), & \text{n = 0, black ball} \\ p, & \text{n = 1, red ball} \end{cases} \tag{3}$$

$$p = \frac{8}{18} \tag{4}$$

Let us define a random variable X = X_1 + X_2

$$M_X(z) = \sum_{n=-\infty}^{+\infty} z^{-n} \Pr_X(n) \tag{5}$$

$$M_X(z) = M_{X_1+X_2}(z) \tag{6}$$

$$= M_{X_1}(z) \times M_{X_2}(z) \tag{7}$$

$$= (1-p)^2 + 2(1-p)(p)z^{-1} + p^2 z^{-2} \tag{8}$$

$$= \Pr(X=0) + \Pr(X=1)\, z^{-1} + \Pr(X=2)\, z^{-2} \tag{9}$$

$$= Pr_X(n) = \begin{cases} (1-p)^2, & \text{n = 0} \\ 2(1-p)p, & \text{n = 1} \\ p^2, & \text{n = 2} \end{cases} \tag{10}$$

- Both balls are red :

$$= \Pr(X=2) \tag{11}$$

$$= p^2 \tag{12}$$

$$= \left(\frac{16}{81}\right) \tag{13}$$

- One of them is black and other is red :

$$= \Pr(X=1) \tag{14}$$

$$= 2(1-p)p \tag{15}$$

$$= \left(\frac{40}{81}\right) \tag{16}$$