

- You may attempt any of the questions.
- School loop grade is calculated in the following manner:

$$f(x) = \begin{cases} \frac{x}{38} \times 100, & x \leq 38 \\ 100 + 15 \times \sqrt{\frac{x-38}{22}}, & x > 38 \end{cases}$$

- You may use a graphing calculator when necessary.
- If a question states: "write your answer with exact value" and the answer of the question is $\sqrt{3}$, then the answer of 1.732 is considered incorrect.
- Unless otherwise stated in the question, all your answers need to be written in the **standard rationalized simplest form**. for example:

if an answer is $\frac{6}{\sqrt{5}-i}$, you are required to

write your answer to be $\sqrt{5} + i$

- Some helpful formulas

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

Group 1 (6 pts)

Find all unique nth roots for the following complex number. Write your final answers in standard form with exact values.

Cubic roots of $64i$

Group 2(6 pts)

Let $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$ and

1) $x \in (0, \frac{\pi}{2})$, $y \in (\frac{\pi}{2}, \pi)$ and $\tan x = \frac{4}{3}$,

$\sec y = -\sqrt{2}$

2) $\vec{u} = \cos x \vec{i} + \sin x \vec{j}$ and $\vec{v} = \cos y \vec{i} + \sin y \vec{j}$

3) $\vec{w} = -3\vec{j}$

Find

(a) the exact values of directional angles of vector \vec{u} and \vec{v}

(b) Let $\vec{w} = a\vec{u} + b\vec{v}$, find the exact values of real numbers a and b?

Group 3 (6 pts)

Given $x \in [0, \pi)$,

(A) Solve $\cos 2x = \sin x$ (exact value)

(B) from (A), Assume the possible 2 solutions of the equation are α and β ($\alpha < \beta$).

Let α be the directional angle of a unit vector \vec{a} and β be the directional angle of another unit vector \vec{b} .

Find (exact values of) the component forms both vector \vec{a} and vector \vec{b} .

(C) if $\vec{s} = 3\vec{i} + 2\vec{j}$, and $\vec{s} = \text{Proj}_{\vec{a}} \vec{s} + \vec{n}_a$, find the exact value of \vec{n}_a

Group 4 (6 pts)

Use mathematical induction to prove DeMoivre Theorem

SA6 Version A

Group 5(12 pts)

Let z_i ($i = 1, 2, 3, 4, 5$) be the 5th roots of 1 and the arguments of each root is θ_i , $\theta_1 < \dots < \theta_5$

$\overline{a_i} = \text{Re}(z_i)\overline{i} + \text{Im}(z_i)\overline{j}$ where $\text{Re}(z)$ is the real part of the complex number z and $\text{Im}(z)$ is the imaginary part of the complex number z .

If vector $\overline{m} = \overline{a_3} - \overline{a_1}$ and $\overline{n} = \overline{a_4} - \overline{a_2}$, find the exact value of the angle between vector \overline{m} and \overline{n}

Group 6(3 pts)

Solve the following triangle

$$m\angle A = 40^\circ, b = 14, a = 12$$

Group 7 (6 pts)

Given $\vec{u} = 4\vec{i} + 3\vec{j}$, where $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$

And rotate the \vec{i} and \vec{j} about the origin clockwise 30° where \vec{i}_1 and \vec{j}_1 are the transformed images of \vec{i} and \vec{j} after the rotation respectively. If $\vec{u} = \alpha\vec{i}_1 + \beta\vec{j}_1$, find exact values of (α, β)

Group 8(9 pts)

Given $f(x) = x^5 + x^4 + 3x^2 - x + 2$, if $x = i$ is a zero of $f(x)$,

(A) find all other zeros for $f(x)$

(B) Graph all zeros on a complex plane.

(C) Let z_1, z_2, z_3, z_4, z_5 be all the zeros of $f(x)$ in its trigonometric forms. $\theta_i, i = 1, 2, 3, 4, 5$ are the arguments for each zero, if $\theta_1 < \theta_2 < \dots < \theta_5$, evaluate the exact value of $z_3^4 + z_1^6 + z_4^8 + z_2^{10} + z_5^{12}$

Group 9(6 pts)

Forest Fire

Two watch towers spotted the same forest fire with bearings N 22° E (from tower A) and N 45° E (from tower B). If two watch towers are 60 miles apart, and the bearing of tower B from tower A is S 75° W. If the rescue center C is 40 miles away from tower B and the bearing of center C from tower B is S 25° E

Find the bearing (to the whole degree) a helicopter pilot should set from center C to the fire. If the average speed of the helicopter is 180 mph, how long in time would it take the helicopter to reach the fire (round your answer to the whole minute)?