

## Unit 9 Assessment

Before you start your response, please read the information here:

You may attempt as many questions as you can.

Your Summative Assessment grade will be based on the performance you did here, following the curve below:

$$f(x) = \begin{cases} \frac{5}{2}x, x \leq 40 \\ -\frac{1}{20}(x-60)^2 + 120, x > 40 \end{cases}$$

1. (10 points)  $f(x) = \frac{9x^3 + 36x^2 - 33x - 6}{x(x-2)(x+1)^2}$  can be uniquely written into as  $f(x) = \frac{b}{x} + \frac{a}{x+1} + \frac{c}{(x+1)^2} + \frac{d}{x-2}$

Where  $a, b, c, d \in \mathbb{R}$ ,

(1) find a, b, c and d

(2) evaluate  $\sqrt{\frac{a^4 + b^4}{c^2 + d^2}}$

2. (5 points) Let  $g(x) = ax^3 + bx^2 + cx + d$  where  $a, b, c$  and  $d$  are from question 1.

(1) find all possible x-intercepts

(2) find the y-intercept

(3) describe the ending behavior of  $g(x)$

(4) evaluate  $g(x)$  with given x in the table below

x	-3	-1	3	5
g(x)				

(5) properly choose the viewing window and scale, sketch  $g(x)$  on a coordinate plane with the information found from (a) – (d)

3. (5 points) Solve the trigonometric equation: (If solution exists, write the solution in its exact value)

$$3 \tan^2 x - 1 = 2 \tan x, \text{ if } x \in \left[0, \frac{\pi}{2}\right)$$

4. (5 points) Solve the logarithmic equation: (If solution exists, write the solution in its exact value)

$$\log_2(x-2) + \log_2(2x-3) = 2$$

5. (5 points) Graph the polar equation, identify the possible symmetry and zeros

$$r = 1 + 2 \cos \theta, 0 \leq \theta < 2\pi$$

6. (5 points) Graph the parametric equation

$$\begin{cases} x = 2 \cos \theta \\ y = 1 + \sin \theta \end{cases}, \frac{\pi}{2} \leq \theta < \frac{7\pi}{4}$$

7. (5 points) write the following conic in its standard form, classify the conic and find its possible x and y intercepts.

$$4x^2 + y^2 - 16x + 8y + 16 = 0$$

8. (5 points) Find the possible tangent lines for a parabola  $y^2 - 4x + 8y = -28$  that passes through a point off the parabola  $(1, -5)$

9. (10 points) Given a conic in its general form below, there exists  $\theta$  such that when rotated the axes of x and y about the origin for  $\theta$ , the xy term in the general can be eliminated in the new coordinate  $(x', y')$ . (1) Find the exact value of the angle of rotation of  $\theta$  between the original coordinate and the rotated coordinate. (2) Write the general form into the standard form in the rotated coordinate  $(x', y')$ , and (3) classify the conic.

$$9x^2 - 6xy + y^2 - x - 3y - 10\sqrt{10} = 0$$

10. (5 points) Find a parabola with the directrix  $x = -2$ , x-intercept @  $(-12, 0)$ , the distance between its focus and  $x = -2$  is 2.

Standard forms of conics

<p>circle:</p> $(x-h)^2 + (y-k)^2 = r^2$	<p>parabola:</p> $4c(x-h) = (y-k)^2$ $4c(y-k) = (x-h)^2$
<p>ellipse:</p> $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	<p>hyperbola:</p> $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $-\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$