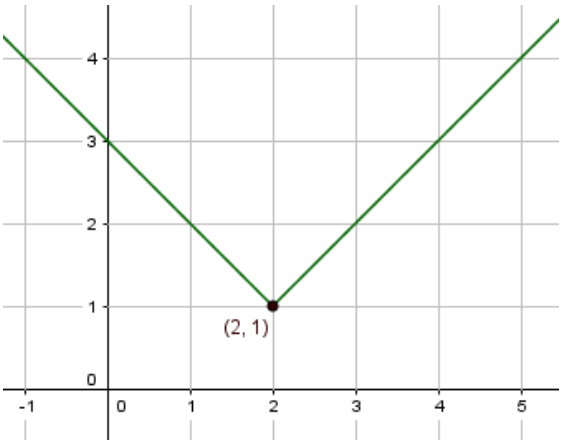
EVHS Math Analysis Semester 2 Final (H)
Multiple Choice

This is your final for Math Analysis 2018-19. Thanks for being with me for the past 10 months. I hope this is a 10 month worth of learning.

1. You will have 2 hours to finish this assessment.
2. This is a multiple choice test. Choose one of the 4 options you believe is the most appropriate for each question. Nothing other than your choices on the electronic answer document will be graded.
3. All calculators are welcome in this assessment.
4**. Be ROAR appropriate. Do not call or email me after this assessment for fixing your grade.** After the final, your grade is also final. Enjoy your summer, as I will enjoy mine. I will not respond to any parent/student communication regarding grades until you are on my roster again.
5. Good luck and have fun for the last 2 hours of math analysis. ☺

1. Given the graph of f(x).



What is the range of $f(x)$?(Write your answer in the interval notation).

(A) $y \in [2,\infty)$

(B) $x \in [2,\infty)$

(C) $y \in [1,\infty)$

(D) $x \in [1,\infty)$

2. If $g(x) = f(x-2) - f(x-1)$ and

$f(x)$ is the function from question 1. Which of the following is the function of g(x)?

(A) $g(x) = (x-3)^2 - (x-2)^2$

(B) $g(x) = |x-3| - |x-2|$

(C) $g(x) = (x-4)^2 - (x-3)^2$

(D) $g(x) = |x-4| - |x-3|$

3. Which of the following coordinates can be the y-intercept of the function in question 2?

(A) $(0, f(-3) - f(-4))$

(B) $(0, f(4) - f(3))$

(C) $(0, |f(3) + f(4)|)$

(D) $(0, f(3) - f(4))$

4. Given the rational function $f(x) = \dfrac{x^3 - x^2}{x^2 + x - 2}$, which of the following coordinates are the possible hole for the function $f(x)$?

(A) $(-2,1)$

(B) $\left(1, \dfrac{1}{3}\right)$

(C) $(1,1)$

(D) $\left(-2, \dfrac{1}{3}\right)$

5. Which of the following solutions is the solution to the logarithmic equation?

$$\log_4(4x-1)=\log_2 9$$

(A) $x=\frac{41}{2}$

(B) $x=\frac{19}{4}$

(C) $x=1$

(D) No Solution

6. Given $k\in\mathbb{R}$ and e is the base of the natural logarithm. If $\frac{k}{x+e}=\frac{e}{x}-\frac{k}{x-e}$, which of the following is the value interval of k so that the solution of the equation is a real number?

(A) $k\in\left[\frac{e}{2},\infty\right)$

(B) $k\in\left(\frac{e}{2},\infty\right)$

(C) $k\in\left(-\infty,\frac{e}{2}\right]$

(D) $k\in\left(-\infty,\frac{e}{2}\right)$

7. Which types of the conic does the following general form represent?

$$2x^2-3xy+2y^2-2x+6y-11=0$$

(A) A hyperbola

(B) An ellipse

(C) A circle

(D) A parabola

8. Which of the following angle θ can be the angle of rotation of a coordinate system (x',y') so that the conic in the question 7 can be written in its standard form in the (x',y') coordinate system?

(A) $\theta=\frac{\pi}{2}$

(B) $\theta=\frac{\pi}{3}$

(C) $\theta=\frac{\pi}{4}$

(D) $\theta=\frac{\pi}{6}$

9. Which of the following trigonometric ratio is correct, If $\theta\in\left[\pi,\frac{3\pi}{2}\right)$ and $\tan 2\theta=\frac{2}{3}$?

(A) $\sin\theta=\frac{3+\sqrt{13}}{\sqrt{26+6\sqrt{13}}}$

(B) $\cos\theta=\frac{-2}{\sqrt{26+6\sqrt{13}}}$

(C) $\cos\theta=\frac{2}{\sqrt{26-6\sqrt{13}}}$

(D) $\sin\theta=\frac{3-\sqrt{13}}{\sqrt{26-6\sqrt{13}}}$

10. Which of the following θ is the exact solution of $\tan\theta=\frac{4}{3}$ if $\theta\in\left[\pi,\frac{3\pi}{2}\right)$?

(A) $\arccos\left(-\frac{3}{5}\right)$

(B) $\arctan\left(\frac{4}{3}\right)$

(C) $\pi+\arccos\left(-\frac{3}{5}\right)$

(D) $\pi+\arctan\left(\frac{4}{3}\right)$

11. Which of the following complex numbers is **not** a cubic root of the complex number $z=i$?

(A) $\frac{-\sqrt{3}+i}{2}$

(B) $-i$

(C) $\frac{\sqrt{3}-i}{2}$

(D) $\frac{\sqrt{3}+i}{2}$

12. If the 3 unique cubic roots of the complex number from question 11 are u_1,u_2,u_3 , which of the following numbers is the value of $\left|u_1^{\,2}+u_2^{\,2}+u_3^{\,2}\right|$?

(A) 2

(B) 1

(C) 0

(D) -1

13. Given $f(x)=1-\frac{1}{1+\frac{1}{x}}$, $g(x)=\frac{2-x}{x}$, which of the following statements of the correct statement of composite function $(g\circ f)(x)$?

(A) $2x+1, x\in(-\infty,\infty)$

(B) $2x+1, x\in(-\infty,-1)\cup(-1,\infty)$

(C) $2x+1, x\in(-\infty,0)\cup(0,\infty)$

(D) $2x+1, x\in(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

14. Consider $f(x)=(x-2)(x^2-4)-(x^2-4)$ and its graph, which of the following descriptions about the characteristics of $f(x)$ is **not true**?

(A) $x\to\infty, f\to\infty$

(B) f has 3 distinct zeros.

(C) the y-intercept of $f(x)$ is $(0,12)$

(D) $x=2$ is a zero of $f(x)$, and the slope of its tangent line passes through $(2,0)$ is positive.

15. Which of the following equations is the directrix of the conic?

$$r=\frac{4}{3+3\sin\theta}$$

(A) $x=\frac{4}{3}$

(B) $x=-\frac{4}{3}$

(C) $y=\frac{4}{3}$

(D) $y=-\frac{4}{3}$

16. Which of following coordinates are the coordinates of a focus of the conic?

$8x^2+18y^2+24x-108y+108=0$

(A) $\left(\frac{3}{2}+\sqrt{5},3\right)$

(B) $\left(-\frac{3}{2}+\sqrt{5},3\right)$

(C) $\left(\frac{3}{2}+\sqrt{5},-3\right)$

(D) $\left(-\frac{3}{2}+\sqrt{5},-3\right)$

17. Given $f(x)=4-\frac{1}{2}x^2$, if $g(x)=-x+b$ a tangent line of $f(x)$, and b is a real number. Which of the following numbers is the value of b?

(A) $\frac{7}{2}$

(B) $-\frac{7}{2}$

(C) $-\frac{9}{2}$

(D) $\frac{9}{2}$

18. Which of the following coordinates are the coordinates of the point of tangency from <u>question 17</u>?

(A) $\left(1,\frac{11}{2}\right)$

(B) $\left(1,\frac{7}{2}\right)$

(C) $\left(-1,\frac{7}{2}\right)$

(D) $\left(-1,\frac{11}{2}\right)$

19. Which of the following polar equations is **<u>not</u>** an ellipse?

(A) $r=\frac{-1}{3\sin\theta+\sqrt{11}}$

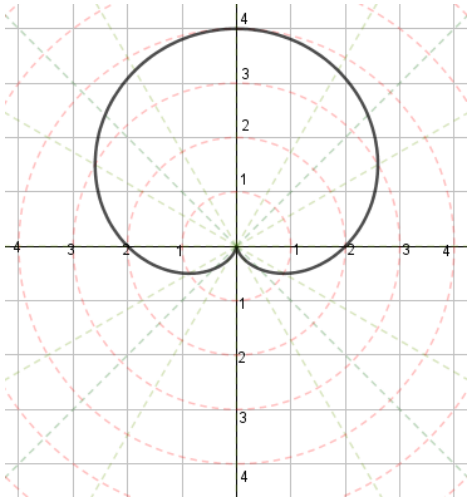
(B) $r=\frac{\frac{5}{2}}{3+\sqrt{5}\cos\theta}$

(C) $r=\frac{2}{\sqrt{2}\cos\theta-3}$

(D) $r=\frac{4}{\sqrt{2}-2\cos\theta}$

20. Which of the following graphs is the graph for $r = 2 + 2\sin\theta$?

(A)



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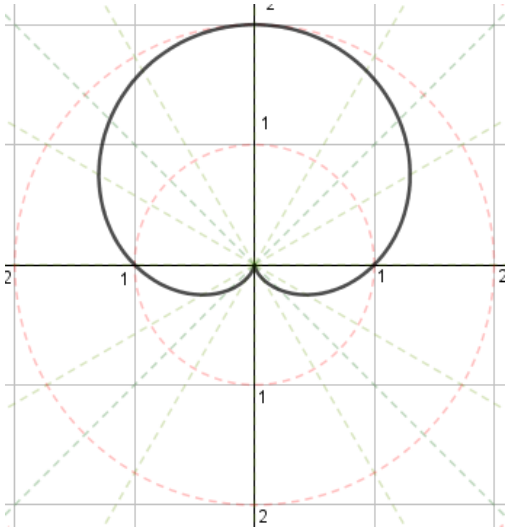

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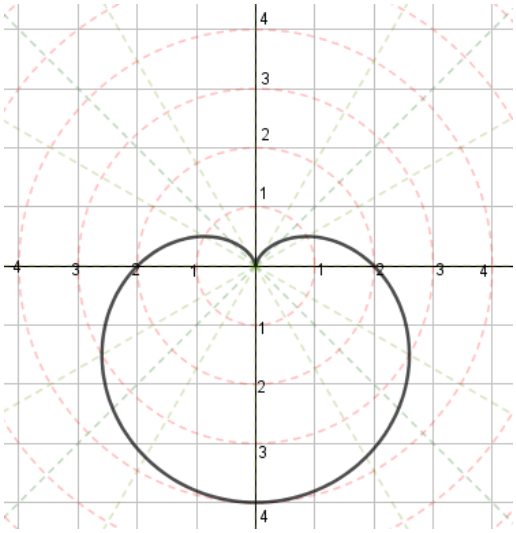

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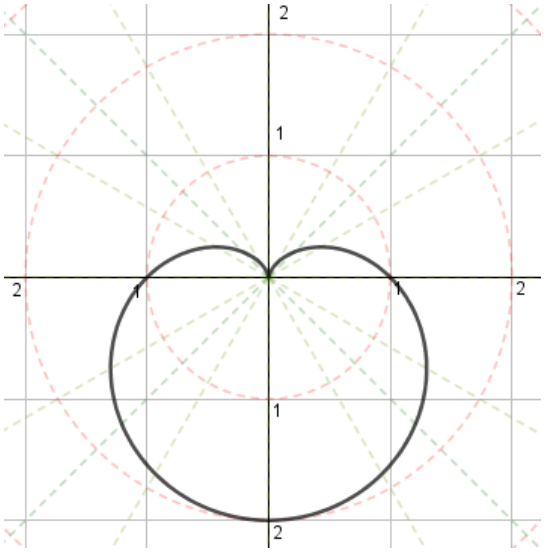


21. Evaluate $\sec\left(\arctan\left(-\frac{4}{3}\right)+\arccos\left(\frac{5}{13}\right)\right)$.

(A) $\frac{65}{56}$

(B) $-\frac{65}{36}$

(C) $\frac{65}{36}$

(D) $-\frac{65}{56}$

22. Given $\vec{w} = < -5, 3 >$ and $\vec{a} = -2\vec{i} + \vec{j}$, which of the following vectors is $\text{Proj}_{\vec{a}}\vec{w}$?

(A) $\left\langle \frac{-26}{5}, \frac{13}{5} \right\rangle$

(B) $\left\langle \frac{26}{5}, \frac{-13}{5} \right\rangle$

(C) $\left\langle \frac{-626}{25}, \frac{169}{25} \right\rangle$

(D) $\left\langle \frac{-169}{25}, \frac{626}{25} \right\rangle$

23. Which of the following equations is one of the equations of the asymptotes of the hyperbola?

$$9x^2 - 4y^2 - 18x + 16y = 43$$

(A) $y = \frac{2}{3}x + \frac{1}{2}$

(B) $y = -\frac{3}{2}x + \frac{1}{2}$

(C) $y = -\frac{2}{3}x + \frac{1}{2}$

(D) $y = \frac{3}{2}x + \frac{1}{2}$

24. The Richter scale is used for measuring the magnitude of an earth quake. The Richter scale is given by

$$R = \frac{2}{3}\log(0.37E) + 1.46$$

Where E is the energy in (Kwh). If two successive earthquakes are recorded with 0.4 difference on the scale, which of the following ratios is the closest estimate to the ratio of the energy released between the larger earthquake to the smaller earthquake?

(A) 2.7

(B) 2.5

(C) 2.3

(D) 2.1

25. If $\sec\theta = 8$ and $\tan\theta < 0$, which of the following values is the value of $\cos\left(\frac{\theta}{2}\right)$?

(A) $\frac{3}{4}$

(B) $\frac{1}{8}$

(C) $-\frac{3}{4}$

(D) $-\frac{1}{8}$

26. If the range of $y = f(x)$ is $y \in (0, \infty)$, which of the following sets **does not** have intersection with the domain of the function $f(x) = \tan\left(\arccos\left(\frac{x}{a}\right)\right)$, assume $a > 0$?

(A) $\{x \mid x \in (a, 2a)\}$

(B) $\{x \mid x \in (-2a, a)\}$

(C) $\{x \mid x \in [0, a]\}$

(D) $\{x \mid x \in [-a, a]\}$

27. Which of the following expressions is the equivalent expression of $f(x)$ in question 26?

(A) $\frac{\sqrt{a^2 + x^2}}{a}$

(B) $\frac{\sqrt{a^2 + x^2}}{x}$

(C) $\frac{\sqrt{a^2 - x^2}}{x}$

(D) $\frac{\sqrt{a^2 - x^2}}{a}$

28. Which of the following rectangular equations is equivalent to the polar equation $r = \frac{1}{1 + \cos\theta}$?

(A) $y^2 = -2\left(x - \frac{1}{2}\right)$

(B) $y^2 = 2\left(x + \frac{1}{2}\right)$

(C) $x^2 = -2\left(y - \frac{1}{2}\right)$

(D) $x^2 = 2\left(y + \frac{1}{2}\right)$

29. Which of the following coordinates are **not** a polar coordinate representation of $(x, y) = (-3\sqrt{3}, 3)$?

(A) $\left(-6, -\frac{\pi}{6}\right)$

(B) $\left(6, \frac{5\pi}{6}\right)$

(C) $\left(-6, \frac{11\pi}{6}\right)$

(D) $\left(-6, -\frac{5\pi}{6}\right)$

30. Which of the following statements is **always** true?

(A) Every matrix has an inverse matrix.

(B) If A, B and C are matrices, and $C = A + B$, then all 3 matrices have the same dimensions.

(C) If A, B and C are matrices, and $C = AB$, then all 3 matrices have the same dimensions.

(D) The determinant of a matrix is never trivial. (trivial means zero)

31. Define $A=\begin{bmatrix}0 & 1 & -5\\ 2 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, if $E=A-cI$, and c is a real number. Which of the following values **cannot** be c if $\det(E)=-10$?

(A) -1

(B) 0

(C) 1

(D) 2

32. If $\log 2=a, \log 3=b, \log 7=c$, $a,b,c\in\mathbb{R}$, which of the following expressions is $\log_7 15$?

(A) $\frac{1+c-b}{a}$

(B) $\frac{1+a-c}{b}$

(C) $\frac{1+b-a}{c}$

(D) $\frac{a+b+c}{2}$

33. Given $f(x)=\frac{2x^2-x+9}{x(x-1)(x-3)}=\frac{a}{x}+\frac{b}{x-1}+\frac{c}{x-3}$, which of the following statements is true?

(A) $b^2-4ac<0$

(B) $b^2-4ac=0$

(C) $b^2-4ac>0$

(D) b^2-4ac cannot be uniquely defined.

34. The concentration of a pollutant from a source can be modelled by the following function

$$C(x)=1600e^{-\left(\frac{x^2+2}{12}\right)}$$

Where

- x is how far away from the center of pollution (in Km) and
- $C(x)$ is the concentration of the pollutant (in ppm) detected at the location.

According to the "safe air" law, any given location near the source of pollution can be categorized into 3 different zones.

- Green Zone (Safe): pollutant concentration level is **less than or equal to 450 ppm.**
- Yellow Zone (Concerned): pollutant concentration level is **greater than 450 ppm, and less than or equal to 600 ppm**.
- Red Zone (Dangerous): pollutant concentration level is **greater than 600 ppm**.

Which of the following locations is considered to be in the Yellow Zone?

(A) 3.1 Km away from the source

(B) 3.4 Km away from the source

(C) 3.7 Km away from the source

(D) 4.1 Km away from the source

35. If $\pm i, 2\pm i$ are the zeros of a polynomial $f(x)$, which of the following polynomial expressions can be $f(x)$?

(A) $x^5-\frac{7}{2}x^4+5x^3-\frac{9}{2}x^2+6x+5$

(B) $x^5-7x^4+12x^3-12x^2+11x-5$

(C) $x^5-3x^4+8x^2-x-5$

(D) $x^5-5x^4+10x^3-10x^2+9x-5$

36. Which of the following expressions is equivalent to

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}?$$

(A) $\frac{\cos x}{1-\sin x}$

(B) $1+\sin x\cos x$

(C) $1-\sin x\cos x$

(D) $\frac{\cos x}{1+\sin x}$

37. Which of the following expressions is equivalent to

$$\cos^4\frac{x}{2}?$$

(A) $\frac{1}{8}+\frac{1}{2}\cos x+\frac{3}{8}\cos 2x$

(B) $\frac{3}{8}+\frac{1}{2}\cos x+\frac{1}{8}\cos 2x$

(C) $\frac{3}{8}+\frac{1}{8}\cos x+\frac{1}{2}\cos 2x$

(D) $\frac{1}{8}+\frac{3}{8}\cos x+\frac{1}{2}\cos 2x$

38. Which of the following values is the exact value of directional angle of a vector $\vec{w}=-2\vec{i}+\vec{j}$?

(A) $2\pi-\arctan\left(\frac{1}{2}\right)$

(B) $\frac{\pi}{2}+\arcsin\left(\frac{2}{\sqrt{5}}\right)$

(C) $\pi-\arctan\left(-\frac{1}{2}\right)$

(D) $\arccos\left(-\frac{2}{\sqrt{5}}\right)$

39. Two watch towers spotted the same forest fire with bearings N 52° W (from tower A) and N 30° W (from tower B). Further, two watch towers are 10 miles apart, and the bearing of tower A from tower B is N 65° E. Which of the following estimates is the closest estimate of **the difference** in distances from the forest fire to A and from the forest fire to B?

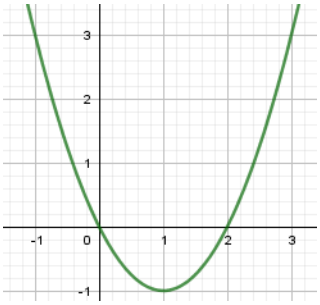
(A) 2.90 miles

(B) 3.43 miles

(C) 3.79 miles

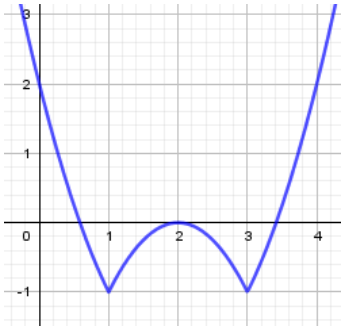
(D) 1.25 miles

40. Given the graph of $f(x)$



Which of the following graphs is $g(x) = -|f(x-1)|+1$?

(A)



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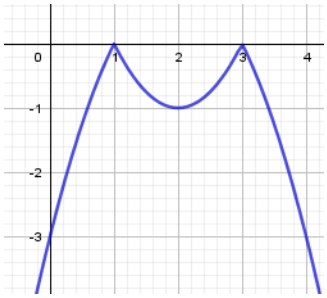

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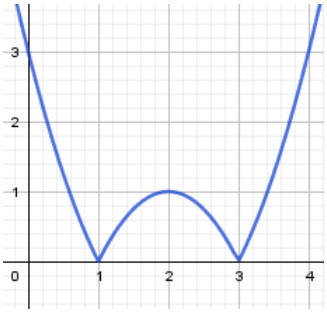

(D)