

Class Discussion

Unit 7 Topic 7 Determinant of a square matrix

Objective: students will understand what is the determinant of an $n \times n$ matrix

Assume that
$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

Minor : $M_{ij} = \det(A')$ where A' is a matrix of $(n-1) \times (n-1)$ by removing the i -th row and j -th column from A .

Cofactor: $C_{ij} = (-1)^{i+j} M_{ij}$

$$\det A = \sum_{i=1}^n a_{ik} C_{ik} = \sum_{j=1}^n a_{kj} C_{kj}$$

Ex 1 find
$$\det \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & 0 & -3 & 1 \\ 3 & 9 & 0 & -1 \\ -1 & 2 & 1 & 1 \end{bmatrix}$$

Ex 2 Given
$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$
 if $\det(A - \lambda I) = 0$

(1) Find λ

(2) For every λ , there exists a nontrivial vector
$$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
 such that $(A - \lambda I)v = 0$, find v for each λ