Practice Test, Unit 6 Assessment

Group 1

Find all unique nth roots for the following complex numbers. Write your final answers in standard form with exact values.

(A) Cubic Roots of $-27i$

(B) Sixth roots of $-i$

Group 2

Given

1) $x \in (0, \frac{\pi}{2})$, $y \in \left(\frac{\pi}{2}, \pi\right)$ and $\tan x = \frac{3}{4}$, $\sec y = -\frac{13}{12}$

2) $\vec{u} = \cos x\vec{i} + \sin x\vec{j}$ and $\vec{v} = \cos y\vec{i} + \sin y\vec{j}$

3) $\vec{w} = -\vec{i} - \vec{j}$

(A) Find the component form of vector $\vec{u}$ and $\vec{v}$

(B) directional angles of vector $\vec{u}$ and $\vec{v}$

(C) Show that $\cos\varphi = \vec{u}\cdot\vec{v}$ if φ is the angle between vector $\vec{u}$ and $\vec{v}$.

(D) Let $\vec{w} = a\vec{u} + b\vec{v}$, find real numbers a and b?

Group 3

Given $x \in [0, \pi)$,

(A) Solve $\sin 2x = \cos^2 x$

(B) from (A), Assume the possible 2 solutions of the equation are α and β ($\alpha < \beta$).

Let α be the directional angle of a unit vector $\vec{a}$ and β be the directional angle of another unit vector $\vec{b}$. Find the component forms both vector $\vec{a}$ and vector $\vec{b}$.

(C) if $\vec{s} = 3\vec{i} + 2\vec{j}$, and $\vec{s} = \mathrm{Proj}_{\vec{a}}\vec{s} + \vec{n_a}$, find $\vec{n_a}$

(D) Continue from (C), if $\vec{s} = \mathrm{Proj}_{\vec{b}}\vec{s} + \vec{n_b}$,find $\vec{n_b}$

(E) Let ϕ be the angle between $\vec{n_a}$ and $\vec{n_b}$, find ϕ

Group 4

(A) let $\vec{u}$ and $\vec{v}$ be two vectors with non-zero magnitudes. If the angle between $\vec{u}$ and $\vec{v}$ is θ , show that $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$

(B) Use mathematical induction to prove DeMoivre Theorem

(C) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, Show that $z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$ and $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$

Group 5

Let z_i ($i = 1,2,3,4,5$) be the 5th roots of 1 and $\vec{a_i} = \mathrm{Re}(z_i)\vec{i} + \mathrm{Im}(z_i)\vec{j}$ where $\mathrm{Re}(z)$ is the real part of the complex number z and $\mathrm{Im}(z)$ is the imaginary part of the complex number z.

(A) Locate z_i on the complex plane.

(B) Find the angle between $\vec{a_1}$ and $\vec{a_2}$

(C) Let vector $\vec{m} = \vec{a_3} - \vec{a_1}$ and $\vec{n} = \vec{a_4} - \vec{a_2}$, find the angle between vector $\vec{m}$ and $\vec{n}$

Group 6

Solve the following triangles

(A) $m\angle A = 35^\circ$, b = 15, a = 12

(B) $a = 12, b = 9, c = 6$

(C) $m\angle A = 45^\circ, b = 4, c = 6$

Group 7

Given $\vec{u} = 4\vec{i} + 3\vec{j}$, where $\vec{i} = <1,0>$ and $\vec{j} = <0,1>$

(a) Rotate the $\vec{i}$ and $\vec{j}$ about the origin counter clockwise 30° where $\vec{i_1}$ and $\vec{j_1}$ are the transformed images of $\vec{i}$ and $\vec{j}$ after the rotation respectively.

(b) if $\vec{u} = \alpha\vec{i_1} + \beta\vec{j_1}$, find (α, β)

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Group 8

Given $f(x)=x^5+x^4+x^3-x^2-2$, if $x=i$ is a zero of $f(x)$,

(a) find all other zeros for $f(x)$

(b) Write all zeros into its trigonometric forms.

(d) Graph all zeros on a complex plane.

(e) Let z_1, z_2, z_3, z_4, z_5 be all the zeros. $\theta_i, i=1,2,3,4,5$ are the arguments for each zero, if $\theta_1<\theta_2<..<\theta_5$, evaluate $z_1+z_2^{\ 2}+z_3^{\ 3}+z_4^{\ 4}+z_5^{\ 5}$

Group 9

Aerodynamics

A plane flies 500 km with a bearing of 316° from Naples to Elgin, the same plane than set a course 750 miles to Canton. If the bearing of Naples from Canton is N 86° E. Find the bearing of Elgin from Canton?

Forest Fire

Two watch towers spotted the same forest fire with bearings N 42° E (from tower A) and N 45° W (from tower B). If two watch towers are 12 miles apart, and the bearing of tower A from tower B is S 75° W. If the rescue center C is 7miles away from tower B and the bearing of center C from tower B is S 38° W

Find the bearing a helicopter pilot should set from center C to the fire. If the average speed of the helicopter is 40 mph, how long in time would it take the helicopter to reach the fire?

Height of a Tree

A tree is on a hillside of slope 28° (from horizontal). 75 feet downhill from where the tree is, the angle of elevation at the top of the tree is 45° . Find the height of the tree.

Camp Fire

In a camp site 3 tents(A, B and C) set up in the following fashion: tent C is 30 yards away from tent A and 40 yards away from tent B. From Tent A, Tent C is in the direction with bearing $N25^\circ E$ and from Tent B, Tent C is the direction with bearing $N45^\circ W$. If the location of the campfire is equidistant from all 3 tents, how far away is the campfire from each tent?(round to the tenth yard)

Cannon ball

A cannon ball was fired at an angle θ (measured from the horizon) with an initial velocity of v_0. We know that the trajectory of the flying cannon ball is a parabola. Let the location where the cannon ball was fired be the origin, the x and y coordinates of the cannon ball can be modeled as $\left(v_0 t\cos\theta, v_0 t\sin\theta-\frac{1}{2}gt^2\right)$ where t is the time in second when the cannon was flying and g is the acceleration caused by gravity. Use the model to

(a) Show that the flying time for a cannon ball is T , and $T=\dfrac{2v_0\sin\theta}{g}$

(b) Show that for a given initial velocity, the maximum horizontal distance a cannon ball can travel may occur when the cannon ball was fired at 45°

(c) Write the function of the trajectory of the cannon ball, $y=f(x)$. Show that

$$f(x)=(\tan\theta)x-\left(\frac{g\sec^2\theta}{2v_0^2}\right)x^2$$