Semester 1 Final Practice Test

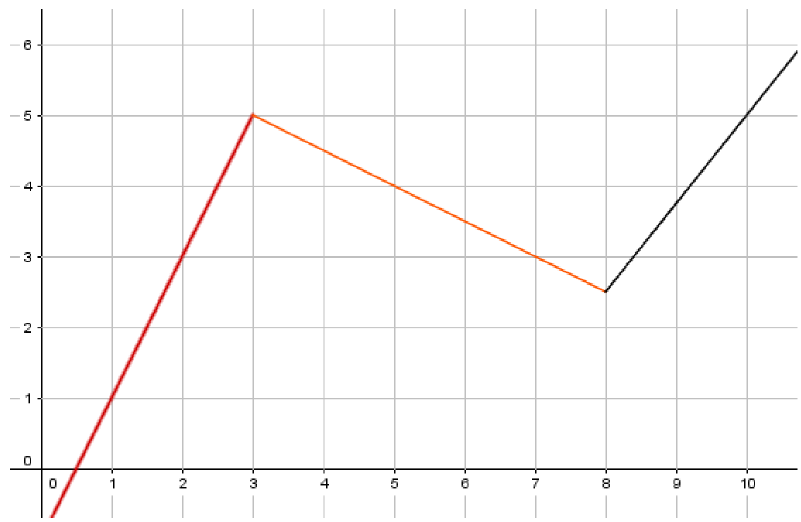
Unit 1:

A. Determine where in the domain will   be increasing? (use interval notation to write your answers.)

1. $f(x)=|x-3|-\frac{1}{2}|x+4|$

2. $f(x)=|2x-1|+|2x-5|$

B. Given a graph below, write a piecewise defined function (assumed each segment can be represented by a linear function), the entire real number line is the domain.



C. Use transformation to graph each function below. Identify the transformations between the given function and its parent function.

1. $f(x)=\dfrac{2}{x-3}+4$

2. $f(x)=1-\sqrt{x+3}$

D. Find the intersection of $f(x)$ and $g(x)$ algebraically and verify your result graphically.

$f(x)=|x+2|-1$, $g(x)=x^2+1$

E. For each function below, find its inverse function (if possible). Graph the function as well as its inverse on the same coordinate plane. Find the intersections between the function and its inverse (if possible) algebraically, and verify your result graphically.

1. $f(x)=\dfrac{2x-8}{x-5}$, $x>\dfrac{5}{2}$

2. $f(x)=\dfrac{4-x}{x-2}$, $x<2$

Unit 2:

A. Assume $f(x)$ is a 4th degree polynomial and all coefficients of $f(x)$ are real numbers. If $f(x)$ has

$1-i$ and $4-2i$ as zeros and $f(0)=-10$. Find f(x) (in general form)? What is the reminder if $f(x)\div(x-2)$?

B. Assume z_1 , z_2 are complex numbers, and a,b are real numbers if

$$
\begin{aligned}
z_1 &= a+bi, \\
z_2 &= 2a-3bi, \\
z_1+2z_2 &= -5+10i
\end{aligned}
$$

Find 1) z_1 , z_2, 2) $\left|\frac{z_2}{z_1}\right|$

C. Graph the rational function, identify its x and y intercepts, holes, as well as possible asymptotes.

1. $f(x)=\dfrac{2x^2-9x+9}{2x^3-3x^2-2x+3}$

2. $f(x)=\dfrac{x^2-5x-6}{x+2}$

D. Describe the ending behaviors for

1. $f(x)=x^3-4x$

2. $g(x)=(x^2-4)(2x-1)+(2x^2-5x+2)$

E. Continue from question D,

1. on what domain will $f(x)>0$

2. on what domain will $g(x)<0$

Unit 3:

A. Evaluate

$$2+\cfrac{\sqrt{e}}{1+\cfrac{\sqrt{e}}{2+\cfrac{\sqrt{e}}{1+\cfrac{\sqrt{e}}{\cdots}}}}$$

B. Assume that x is a real number, Solve for x if

$$\frac{2}{x+e}=\frac{e}{x}-\frac{2}{x-e}$$

C. Solve the following logarithmic equation (x is the variable)

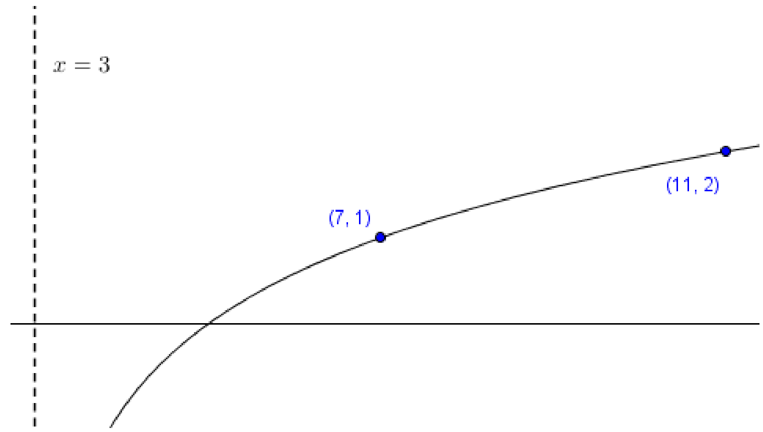
1. $2\log_6(2x+1)=\log_{36}4+\log_6(6x-1)$
2. $\log_2\sqrt{2x-3}=\log_4(x+4)$

D. For the following question assume $\log 2=\dfrac{3}{10}$, $\log 3=\dfrac{12}{25}$, $\log 7=\dfrac{21}{25}$ and $\ln 10=\dfrac{23}{10}$. Evaluate

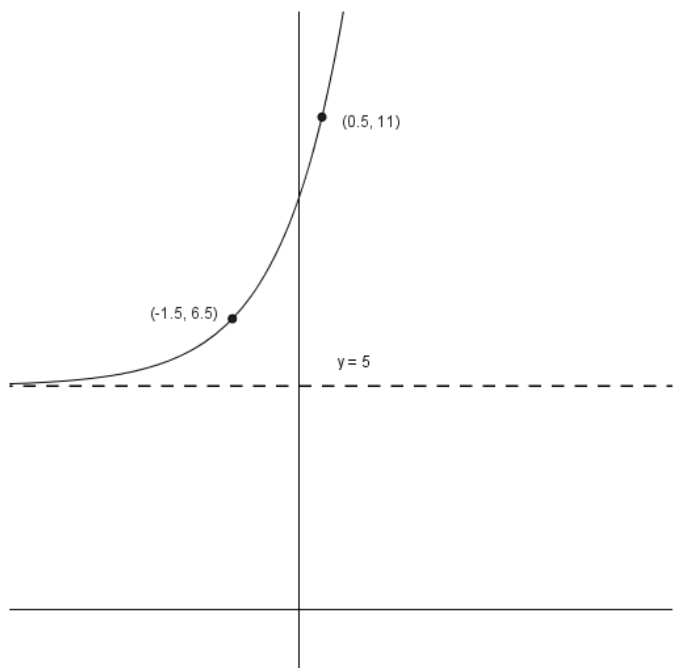
$$\log_2\frac{e}{\sqrt{5}}$$

E. Use the given graph below to find its function

(1) The graph shown is below a logarithm function. x=3 is the vertical asymptote. Two given points that the function passes through are (7, 1) and (11, 2)



(2) The graph shown below is an exponential function. y = 5 is the horizontal asymptote. Two given points that the function passes through are $(\frac{-3}{2},\frac{13}{2})$ and $(\frac{1}{2},11)$



F. **(Transparency)** Ms. King uses transparencies in her art class to show students how to draw a painting in different layers. However, light intensity will decrease as the transparency overlaid. The intensity decrease follows the model below:

$$I_n = I_0(1-x)^n$$

Where

n: number of transparency overlaid

x: intensity reduction per sheet of transparency, in percent

I_0: intensity of light before going through the transparency, in lumens

I_n: intensity of light after going through the transparency, in lumens

(5 pts) Ms. King has a Panasonic Projector produces 1600 lumens of light when it is in fully operation. If Ms. King overlaid 20 transparencies on the projector, the light intensity dropped to 1250 lumens. What is the light intensity reduction (in percent, round to the hundredth percent) if only one transparency is on the projector?

(10 pts) Assuming Ms. King is using the same projector, she now needs to overlay at most 25 transparencies in a project. She knows that if light intensity cannot be lower than 1400 lumens, her current transparencies will not be adequate. Which of the following brands at Office Depot can be recommended for her, if you were a sales assistant? If her budget to get 25 transparencies cannot be more than $25 dollars, which brands can she consider?

Brands of the transparency	price per package	sheets per package	Light intensity reduction
ClearPix	$5.00	10	0.57%
As Air	$6.00	8	0.51%
Ultra	$7.00	15	0.55%
No Lost	$9.00	5	0.52%
Last Long	$10.00	10	0.48%

G. **(Yeast Growth)** Following model represents a yeast population when a sour dough bread was rising:

$$Y(t) = \frac{180}{1+35e^{-.85t}}, 0 \le t \le 10$$

Where t represents the time (in hours), Y(t) represent the numbers of yeast in millions. And the domain is called the rising period.

(a) (5 pts) What is the initial population when the yeast was just added into the dough?

(b) (5 pts) Make a graph of Y(t) over the whole domain [0, 10] by evaluating the yeast population at whole hours.

(c) (10 pts) What is the final population of the yeast in the dough? (when $t \to \infty$)

(d) (10 pts) When the population of the yeast reaches 97% of the final population, we said the dough is fully risen. In this case, when will the dough be fully risen? (Round to tenth hour)

(e) (10 pts) The yeast growth rate function can be defined as $Y'(t) = Y(t+1) - Y(t)$. Based on the function, which hour in the entire rising period, the yeast population grew the fastest?

H. (Pollution) The concentration of a pollutant from a source can be modelled by the following function:

$$C(x) = 2000e^{-\left(\frac{x^2}{25}\right)}$$

where x is the distances from the pollution source in km and C is the concentration of pollutant at ground level in ppm.

(a) what is the concentration of the pollutant when it is just out of the source?

(b) if the pollutant is considered as undetectable when its concentration is lower than 10 ppm, than how far away from the source of pollution is considered to be undetectable?

I. (Altitude) The altitude h (in meters) above the sea level is related to the atmospheric pressure at the level. It can be modeled by

$$h = -8000 \ln\left(\frac{P}{P_0}\right)$$

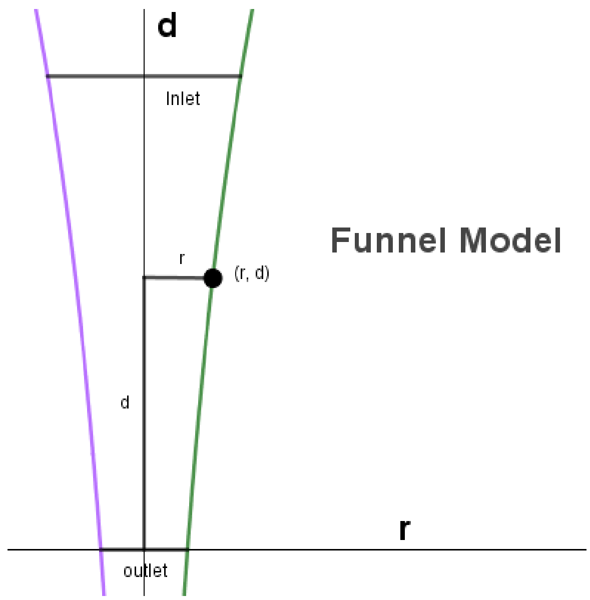
Where P_0 is the reference pressure (in Kpa).

(a) Given that the pressure is 65Kpa at the altitude of 3,500 m, What is the height if the pressure is 57Kpa?

(b) Near the sea level of the earth the air pressure is usually at 101.3 Kpa, estimate the air pressure at the top of the Mt. Everest (8,848 m)?

J. (Funnel) A funnel is 4" tall. The diameter of the outlet is $\frac{1}{2}$ ", and the diameter of the inlet is 5" . (See sketch)A horizontal cross section of the funnel is a circle parallel to the inlet and outlets. Let the distance between the any horizontal cross section to the outlet be d and the radius of any cross section be r. The relationship between the r and d can be modeled by the following expression,

$$d(r) = a + b\ln r$$



where a and b are constants.

(a) what is the valid domain of the function $d(r)$?

(b) what is the diameter of the midway cross section? (midway is defined as equidistant from both ends of the funnel)?

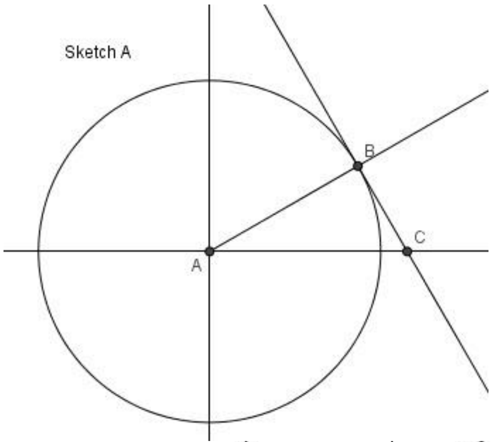
Unit 4:

A. Find the exact value of the trigonometric ratios.

$$\sec\frac{13\pi}{2}, \csc\left(-\frac{13}{3}\pi\right), \cot\left(-\frac{5}{6}\pi\right)$$

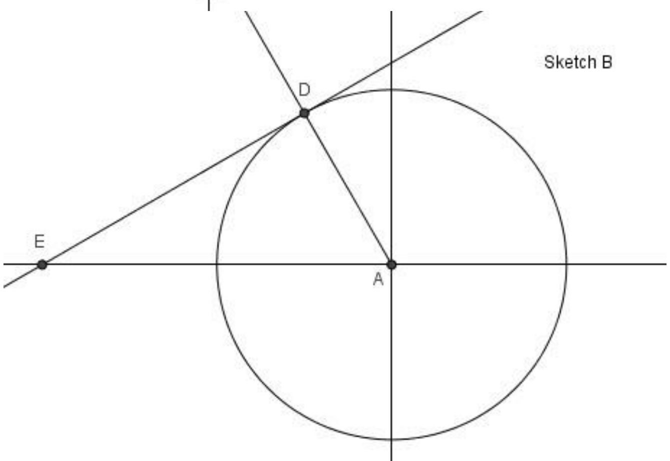
B. Form proper responses for questions asked for each sketch.

Sketches may not be drawn to scale, use the given angle value for calculation



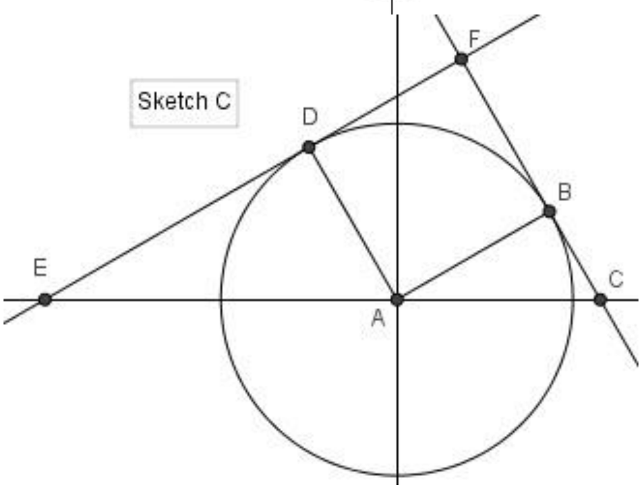
A is the origin and the radius of circle A is 2 and $m\angle BAC = 30^\circ$ and $\overleftrightarrow{CB}$ is a tangent line of circle A passes through B. Find

(1) $m\angle BCA$
(2) Coordinates of B?
(3) The equations of $\overleftrightarrow{BC}$ and $\overleftrightarrow{AB}$



A is the origin and the radius of circle A is 2 and $m\angle DAE = 60^\circ$ and $\overleftrightarrow{DE}$ is a tangent line of circle A passes through D. Find

(4) $m\angle DEA$
(5) Coordinates of E?
(6) The equations of $\overleftrightarrow{DE}$ and $\overleftrightarrow{AD}$



A is the origin and the radius of circle A is 2 and $m\angle CAB = 30^\circ$, $m\angle DAE = 60^\circ$. If $\overleftrightarrow{CB}$ is a tangent line of circle A passes through B and $\overleftrightarrow{DE}$ is a tangent line of circle A passes through D, and F is the intersection of these two tangent lines (as shown in the sketch). Find

(7) Coordinates of F?
(8) $m\angle CFE$?
(9) Area of $\triangle FEC$
(10) the length of BD

C. Find

(1) $\csc\theta$ if $\dfrac{2+\cot\theta}{\cot\theta} = \dfrac{3}{\tan\theta}$ (2) $\sin x$ if $2\cos^2 x - 5\cos x + 2 = 0$ (3) $\csc\varphi$ if $6\sec^2\varphi = 7\tan\varphi + 9$

D. For the following functions:

Function	domain requirement of the graph	Tasks besides the graph

$f(x) = -\frac{2}{3}\csc\left(\pi x - \frac{\pi}{2}\right) + \frac{4}{3}$	at least 2 full periods, x>0	(1) Find period (2) Find vertical asymptotes (3) Identify range (4) find possible x-intercepts and y-intercept
$f(x) = \sqrt{3}\tan\left(\frac{1}{2}x - \frac{\pi}{3}\right) + 1$	at least 2 full periods, x>0	(1) Find period (2) Find vertical asymptotes (3) find possible x-intercepts and y-intercepts
$f(x) = \sqrt{2}\sec\left(\frac{3}{2}x - \frac{3\pi}{4}\right) + 2$	at least 2 full periods, x>0	(1) Find period (2) Find vertical asymptotes (3) Identify range (4) find possible x-intercepts and y-intercept
$f(x) = -\cot\left(\frac{1}{4}x - \frac{\pi}{6}\right) + \frac{1}{\sqrt{3}}$	at least 2 full periods, x>0	(1) Find period (2) Find vertical asymptotes (3) find possible x-intercepts and y-intercepts
$f(x) = -\sqrt{3}\sin\left(\frac{1}{3}x - \frac{\pi}{2}\right) + \frac{3}{2}$	at least 2 full periods, x>0	(1) Find period (2) Identify range (3) find possible x-intercepts and y-intercept
$f(x) = 2\cos\left(\frac{2}{3}x - \frac{\pi}{4}\right) + \sqrt{2}$	at least 2 full periods, x>0	(1) Find period (2) Identify range (3) find possible x-intercepts and y-intercept

E. $\csc\phi = \sqrt{5}$ and ϕ is at the 2nd quadrant, Find $\cos\phi$

F. Verify the following identity:

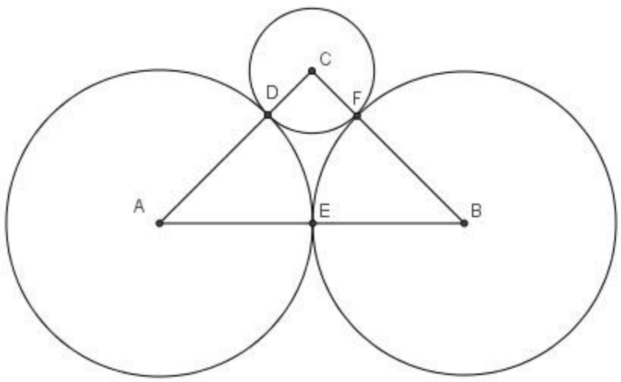
(1) $\tan^2 x + 1 + \tan x \sec x = \frac{1+\sin x}{\cos^2 x}$

(2) $\frac{\cos x}{1-\tan x} + \frac{\sin x}{1-\cot x} = \sin x + \cos x$

(3) $\frac{1-\sin x}{1+\sin x} = \left(\sec x - \tan x\right)^2$

(4) $\frac{\sec x - \cos x}{\sec x + \cos x} = \frac{\sin^2 x}{1+\cos^2 x}$

G.



(1) Show that

$$\frac{AD}{DC} = \sqrt{2} + 1$$

(2) Find the area bounded by DE, EF, and FD.

(3) Construct $\overline{DF}$ and $\overline{CE}$, show that $\overline{DF} \perp \overline{CE}$

(4) Construct $\triangle DEF$, show that

area of $\triangle ABC = 2(\sqrt{2}+1)$ area of $\triangle DEF$

Given $\bigcirc A \cong \bigcirc B$. $\triangle ABC$ is an isosceles right triangle with $\overline{BC} \perp \overline{AC}$. The radius of $\bigcirc A$ is r. If $\bigcirc A$, $\bigcirc B$ and $\bigcirc C$ are tangent to each other as shown in the diagram above

H. A forest fire has been spotted by two fire towers (A, B). Tower A is 30 miles due east of B. The bearing of the fire from tower A is S 35° W. The bearing of the fire from tower B is S14° E. Now a helicopter from a rescue center C 10 miles due east of tower A is ready for taking off to the fire.

(1) As a staff at traffic control tower to help the pilot to set the course to the fire, how are you going to tell the pilot to set his course?

(2) How long (in time) will the helicopter take to reach the fire if the maximum speed of the helicopter is 250 mph?

I. Evaluate

(1) $\arcsin\left(\sqrt{3}\right)$ (2) $\cot(\arcsin\left(\frac{5}{6}\right))$ (3) $\arccos\left(\csc\frac{7\pi}{6}\right)$

J. if $0 \le x < 2\pi$,solve for x.

1. $2\sin^2 x - 1 = 0$
2. $\sin\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{2}$

K. Rewrite the following trigonometric statements into algebraic expressions, if x>0

1. $\sin(\arccos x) + \cos(\arcsin x)$
2. $\csc(\arctan x) + \cot(\arccos\frac{x}{\sqrt{x^2+1}})$