EVHS Algebra II 2013-14Unit 1 Handouts

Linear Relations and Systems

Contents

Topic 1 The Basics

Objective:

In this topic, I will review the properties of the real numbers, how to evaluate an expression, and the meaning of “solve for”.

The Lesson

Properties of the real numbers:

Define: Real numbers are numbers can be found on a ___________________________.

Properties:

Properties	Addition	Multiplication
Closure	a+b	ab
Commucative	a+b = b+a	ab=ba
Associative	a+(b+c) = (a + b) +c	a(bc)=(ab)c
Identity	a+0=a, 0+a=a	$a \cdot 1 = 1 \cdot a = a$
inverse	$a+(-a)=0$	$a \cdot \frac{1}{a} = 1, a \neq 0$
Distributive	a(b+c) = ab+ac	

Ex1: Order the real numbers

1. $6,\ -\sqrt{5},\ 2.7,\ -2,\ \frac{7}{3}$	2. $-3, \frac{5}{2}, 2, -\frac{9}{4}, 4$

Ex2: Unit Analysis,

3. The speed limit of Highway 101 is 65 mph. What is the speed limit in km/hr?	4. If you travel 75 km in 2.5 hrs, what is your average speed in mph?

Evaluating the algebraic expressions:

Ex3:

5. $(k^3 - \frac{1}{2}) - 2(k^2 - 1)$, if $k = \frac{3}{2}$	6. $2x^2 + 4x - 1$, if $x = -\frac{3}{4}$

Ex4: Combine the like terms

7. $4p^2 - 12p - 9p^2 + 3(4p + 7)$	8. $10(n^2 + n) - 6(n^2 - 2)$

Ex5: Solve for y

9. $xy = x + y$	10. $xyz = x + y + z$

Exit Tickets:

1 Solve for x:

$$4x + 7y + 5xy = 0$$

2. Simplify:

(a) $x(2y+x)-y(2x+y)$

(b) $(my+x)^2+(mx-y)^2$

Topic 2 Linear Equations and Absolute value Equations

Objective:

In this topic I will review how to solve the linear equations (1 var) and linear equations involves absolute value.

The Lesson

Objective 1: Solve Linear Equations:

Define “Solve”: Isolate the variable in an equation.

1. $\frac{2}{5}k + \frac{1}{6} = \frac{3}{10}k + \frac{1}{3}$	2. $3(2x - 5) - x = -7(x + 3)$
3. $10(w - 4) = 4(w + 4) + 4w$	4. $\frac{4}{7}z + \frac{2}{3}z = 13$
5. $4(\frac{3}{2} + 5x) = -2(-2x - 4) + 6(x - \frac{1}{3})$	6. $x - 6 = 2(x - 1) - x$

Objective 2: Solve the absolute value equation: Slay a dinosaur

3 steps:

1. Chop off the tail (remove the constant term, if any)

2. Chop off the head (remove the leading coefficient, if any)

3. Split the heart (+/- solutions of the absolute value equations)

1. $\lvert 7h - 10 \rvert = 4$	2. $\lvert 3n - 7 \rvert = 4$
3. $\lvert 3x - 4 \rvert = x$	4. $\lvert 8 + 5x \rvert = 7 - x$
5. $\lvert 2x - 1 \rvert + 4 = -x - 4$	6. $3\lvert x - 3 \rvert - 5 = 7(2 - x) + 1$

Exit Tickets:

Solve the following equations:

1. $2x+1+4(x-2)=6x+3$

2. $\frac{2}{3}x-\frac{1}{4}\left(1-\frac{2}{3}x\right)=\frac{3}{4}\left(\frac{2}{5}x+\frac{7}{6}\right)$

3. $\left|\frac{2}{7}x-11\right|+x=-4$

4. $2\left|4x-\frac{2}{7}\right|+11=\frac{2}{3}\left(6-\frac{9}{4}x\right)$

Topic 3 Solving Linear Inequalities

Objective:

In this topic, you will review the skills on how to solve linear inequalities, and the linear inequalities with absolute value signs.

The Lesson

Solve and Graph the following inequalities:

1. $3x-1<2x+5$	2. $7x-1\geq 14x+6$
3. $-3\leq 4c+5<7$	4. $6<6-5p\leq 21$
5. $4x>2$ or $-2x\geq x+3$	6. $5(y+2)<3y$ or $4-2y<-2$

Solve and graph the inequalities with absolute value (Slay the dinosaur in the **L**and of **G**or)

7. $4\|3-x\| \le 8$	8. $\|2x+1\| < 3$
9. $3\left\|\frac{1}{2}x+1\right\| > 6$	10. $-\frac{7}{2}\|x-4\|+\frac{5}{6} \le -\frac{1}{3}$

Exit tickets:

Solve and graph

1.

$|6x-9| \geq 33$

2.

$|7x+2| < 5$

Challenge Questions: (Luigi's Gold Coin:1)

Consider that the solution of $|ax+b| \le 5$ is $-3 \le x \le 4$, Find a and b in the absolute value inequality?

Topic 4 Functions and Linear Models

Objective:

In this topic, you will review the definition of a function, the mapping diagram, meanings of domain and range, eventually, you will review how to classify if a relation is a function (linear function).

The Lesson

Define: In mathematics, a function is a ________________________ between TWO (2) sets of quantities.

The first set is called the ____________________ while the other group is called the _________________.

There are several different ways to represent a function: **A map**, A **table**, a **graph** or an **expression**.

Ex1: Given the order pairs, identify (a) domain (b) range (c) present the relation as a map.

1. (1,1), (2,3), (3,6),(-1, 5), (0, 4)	2. (2,3), (-3, 3), (-1, 3), (0,3),(1, 3)

The Rules of a function: Every element from ___________________ finds _________________ mapping in

the _____________.

Ex2: Check if the following relation is a function?

3. (2,1), (-2, 3), (1, 3), (-3, -2)	4.(4,1), (-2,1) ,(3, -1), (4,-1)

Linear relation: when the graph of a function is a line, then the relation is a linear relation.

3. You are shopping online. The cost of shipping is $6 flat rate and each T-shirt costs $12.
(a)Write a relation between the number of T-shirts (t) and the total cost (including the shipping), c.

(b)if you spent $111, how many t-shirts has did you buy?

4. In order to fix your bike, you brought your broken bike to the shop. You have to pay for both the parts and the hours of labor. If the cost of the parts was $105 and the labor was $25 per hour,
(a) write a relationship between the hours (h) and the total bill (b).

(b) if you finally pay $155 for your bike to be fixed. How long did the technician take to fix your car?

Evaluate a function:

5. If $f(x) = -2x+3$, evaluate f if x=-2,	6. if $f(x) = 3x+4$, evaluate f if x = $\frac{2}{3}$

Exit tickets:

Given the ordered pairs, (i) identify the domain, (ii) the range of the relation. (iii)Construct a mapping diagram between inputs and the outputs. (iv)If the relation is a function, classify if the function is linear. (v) if it is linear, write the function in the form of $f(x) = ax + b$

(a) (2,-1), (3, -4), (5, -10), (1, 2)

(b) (6, 1), (12, 2), (24, 4), (-4, $-\frac{2}{3}$)

(c) (2, -2), (7, -2), (-4, -2), (-3, -2)

(d) (1, 5), (2, 10), (1, -5), (2, -10)

Topic 5 Graphing and Writing linear relations

Objective:

In this topic, you will review how to graph a linear equation $y = mx + b$; find a linear relation with given information, including finding parallel lines and perpendicular lines.

The Lesson

Objective 1: Graph a linear relation

1. Solve for y, if the given equation is not.

2. graph y = mx+b

Practice:

1. $5x - y = 3$	2. $y - 2 = 2x$
3. $y = -\dfrac{2}{3}x + 3$	4. $y = \dfrac{1}{3}x + 2$

Objective 2: Write an equation of a line

1. Find slope (m) of the equation with given information

2. Assume the equation is $y = mx + b$

3. Use a (given information) point that is on the line to solve for b.

About parallel and perpendicular lines

Given that L_1 with slope of m_1, L_2 with slope of m_2 :

$L_1 \parallel L_2 \Rightarrow m_1 = m_2$ the slopes are the same

$L_1 \perp L_2 \Rightarrow m_1 m_2 = -1$ the slopes are negative reciprocal

Ex2: Write the equation of a line that

1. m = 3, passes through (2,-3)	2. $m = -\frac{2}{3}$, passes through (-1, 4)
3. passes (2,1) and (4, 6)	4. passes (-3, 1) and (3, 2)

5. passes (4,5) and parallel to $y = 2x + 2$	6. passes (-4,3) and parallel to $y = -\frac{1}{2}x + 3$
7. passes (4,5) and perpendicular to $y = 2x + 2$	8. passes (-4,3) and perpendicular to $y = -\frac{1}{2}x + 3$

Exit tickets:

Given $L: 3x + 2y = -5$ and $P(6, 2)$

(a) Find the equation of M so that $M \parallel L$ and passes P

(b)Find the equation of N so that $N \perp L$ and passes P

(c) on a coordinate plane, graph lines L, M and N

Topic 6 Direct Variation and linear models

Objective:

In this topic you will learn when two variables are in direct variation. and what is the constant of variation.

The Lesson

Direct variation: if y is varied directly with x, then the ratio of y and x are constant.

$\frac{y}{x} = m$, m is called the constant of variation(and all solutions on the the relation are proportional.)

Ex1: Given (x, y) and assume that y is varied directly with x, Find the constant of variation, m

1. (5, 9)	2.(4, 11)
3. (6, 2)	4. (7,1)

Ex2: Check if the data in the following tables varies directly

6.

x	-2	-1	0	1	2
y	4	3	2	1	0

7.

x	-3	-1	1	3	5
y	-2	$-\frac{2}{3}$	$\frac{2}{3}$	2	$\frac{10}{3}$

Ex 3:

Reading The number of pages p a student can read varies directly with the amount of time t in minutes spent reading. The student can read 90 pages in 60 minutes. Write an equation that relates p and t. Predict the number of pages the student can read if 90 minutes is spent reading.

Ex 4:

Movies The cost c of going to the movies varies directly with the number n of people attending. A group of four paid $14 to go to the movies on Friday. Write an equation that relates c and n. How much would it cost for 7 people to go to the movies?

Ex 5:

MULTIPLE REPRESENTATIONS The table shows the numbers of countries that participated in the Winter Olympics from 1980 to 2002.

Year	1980	1984	1988	1992	1994	1998	2002
Countries	37	49	57	64	67	72	77

(a)

Making a List Use the table to make a list of data pairs (x, y) where x represents years since 1980 and y represents the number of countries.

(b) **Drawing a Graph** Draw a scatter plot of the data pairs from part (a)

(c)

Writing an Equation Write an equation that approximates the best-fitting line, and use it to predict the number of participating countries in 2014.

Exit Ticket:

Softball The table shows the number of adult softball teams for the years 1999 to 2003.

Year	1999	2000	2001	2002	2003
Number of teams (in thousands)	163	155	149	143	119

(a)

Draw a scatter plot for the data. Let t represent the number of years since 1999.

(b) Use the data to find a best fitting line (use calculator or not)

(c) Use the model to predict the adult softball team in 2010.

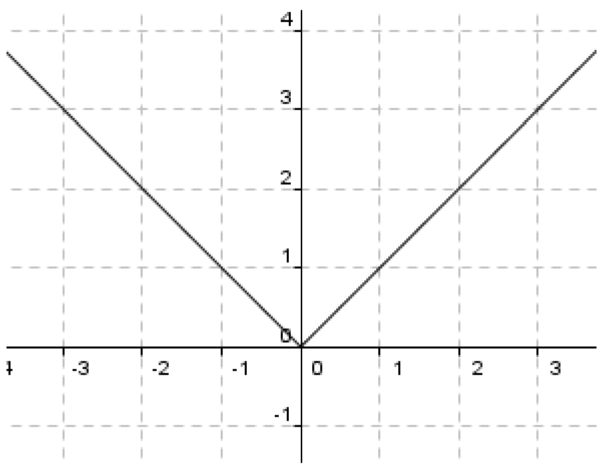
Topic 7 Function transformation and absolute value functions

Objective:

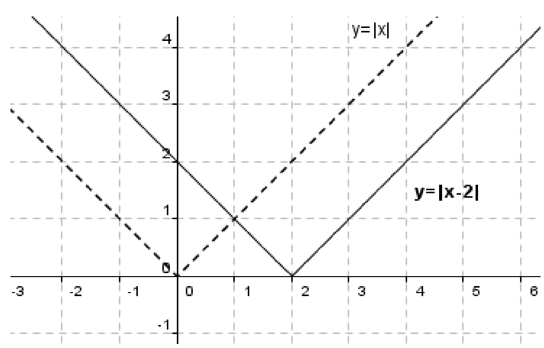
In this topic, you will review graph of the absolute value functions, and how to use function transformations to do translation, reflection and scaling.

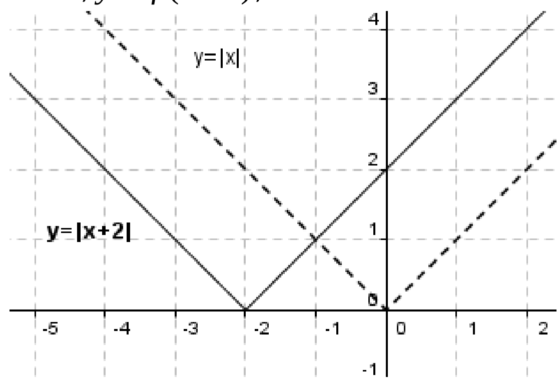
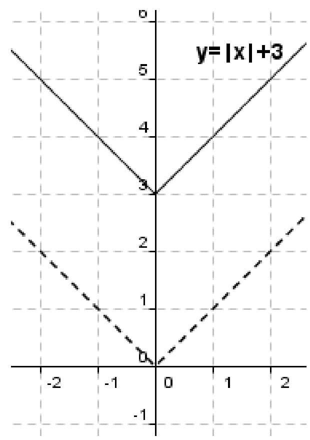
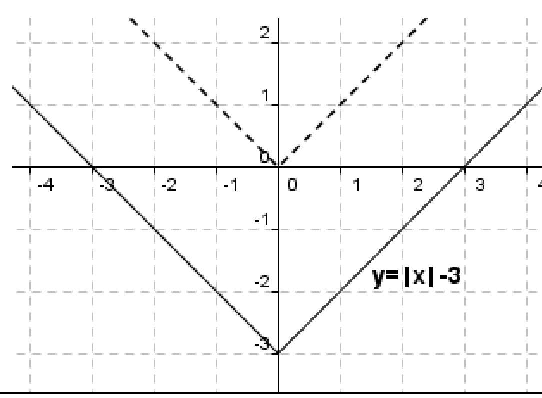
The Lesson

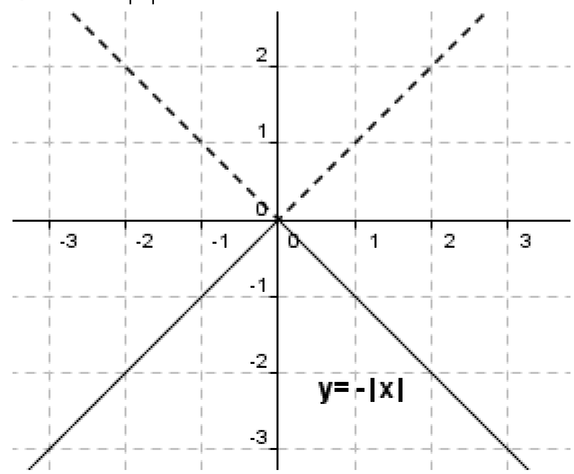
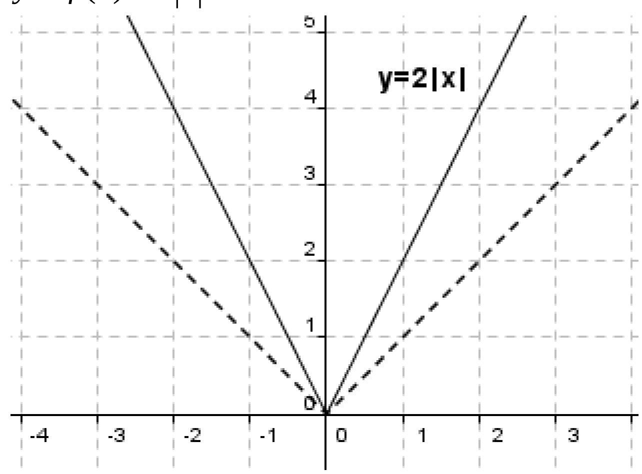
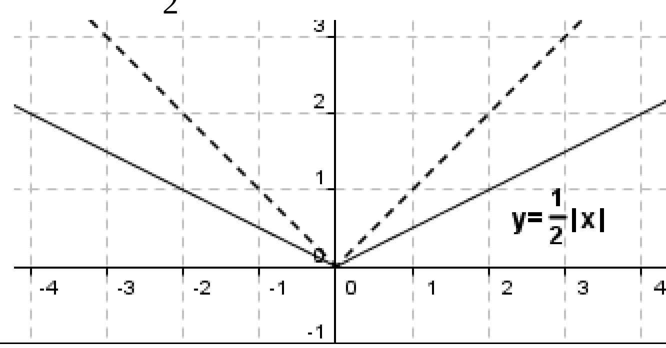
Graph of $y = |x|$



Transformations and the graphs: $y = f(x) = |x|$

Transformation	change in the function	graph change
Horizontal Translation	$y = f(x-h)$ $y = \|x-h\|$	$h > 0$, $y = f(x-2)$, $h = 2$ 

		$h<0$, $y=f(x+2)$, $h=-2$ 
Vertical Translation	$y-k=f(x)$,or $y=f(x)+k$ $y=\|x\|+k$	$y=f(x)=\|x\|+3$  $y=f(x)=\|x\|-3$ 

Reflection	$y = -\lvert x \rvert$	$y = f(x) = -\lvert x \rvert$ 
Scaling	Stretching $y = a\lvert x \rvert,\ a > 1$ Shrinking (flattened) $y = a\lvert x \rvert, 0 < a < 1$	$y = f(x) = 2\lvert x \rvert$  $y = f(x) = \frac{1}{2}\lvert x \rvert$ 

Now put the transformations together, you will find most of the graphing are combinations of the transformations of the parent function, y = |x|.

Ex1: Graph the following functions (Setup your own coordinate system in the grid provided)

1. $y = |x-1| + 4$

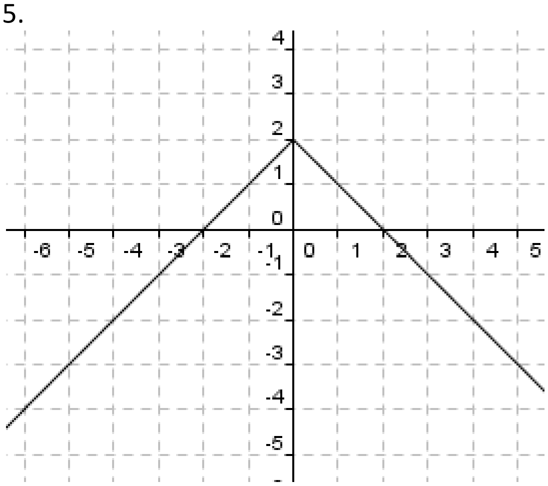
2. $y = -|x+2| - 3$

3. $y = 2|x| - 1$

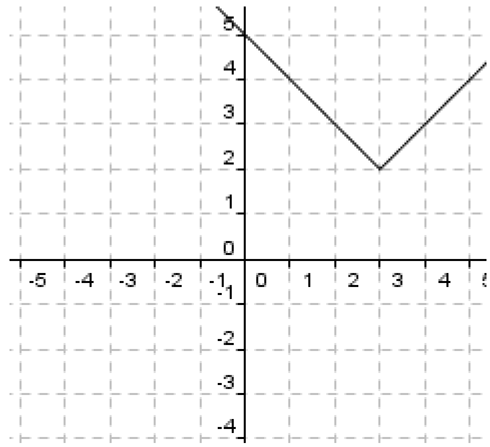
4. $y = -\frac{1}{3}|x| + 2$

Ex2: Write an absolute function for each of the following graphs

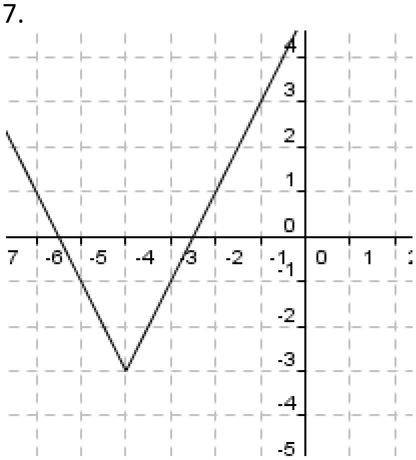
5.



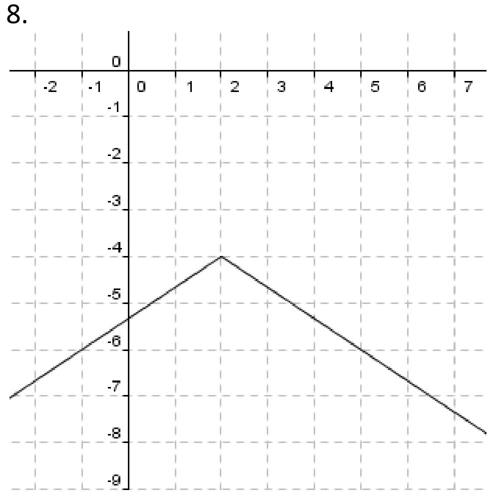
6.



7.



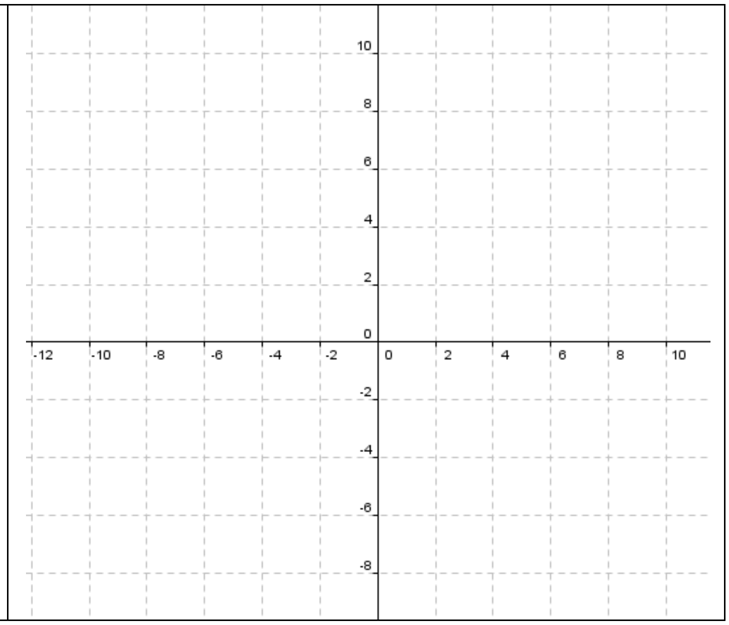
8.



Exit Tickets

Consider the system $\begin{cases} y = -\dfrac{1}{2}|x-2|+4 \\ y = \dfrac{1}{2}|x-2|-4 \end{cases}$

(1) Graph each function in the system
(2) Prove that the quadrilateral bounded by the functions is a rhombus.
(3) Find the area of the rhombus.
(4) Find the perimeter of the rhombus.



Topic 8 Inequalities in two variables

Objective:

In this topic, you will review the graph of the inequalities in two variables, as well as writing the inequality (or inequalities) based on a given graph.

The Lesson

3-step to graph a linear inequality (2-variables)

1. Rewrite the inequality so that the inequality is in the "Solve for y form" [isolate the y on the left hand-side of the inequality]

   ex: $2x + y \geq 4 \rightarrow y \geq -2x + 4$

   And the boundary of the solution is

   $y = -2x + 4$

2. Graph the linear equation. (??: what if the inequality cannot be rewritten into the solve for y form?)

3. Use the symbol of the inequality to find the type of the boundary, and the territory of the solution.

Inequality Symbol	Boundary Type	Territory
$\geq$	solid	Top half plane
$\leq$	solid	Bottom half plane
$>$	open	Top half plane
$<$	open	Bottom half plane

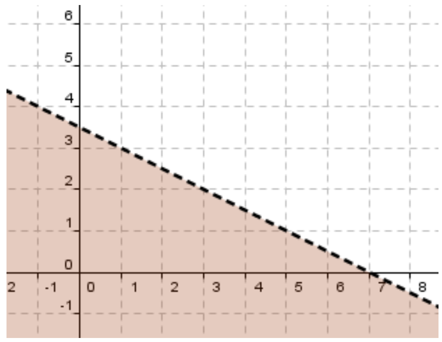
To identify an inequality based on a graph, reverse the steps for graphing the inequality.

Graph the solutions for the inequalities below:

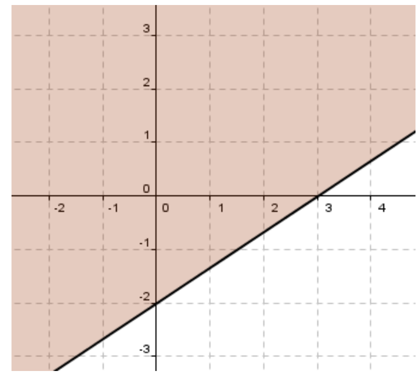
1. $y \leq -2x - 1$	2. $y > \frac{3}{4}x + 1$
3. $y < 3\lvert x\rvert + 2$	4. $y \geq -2\lvert x + 1\rvert + 3$
5. $x \geq 6$	6. $x < 3$

Find the inequalities of the following graphs

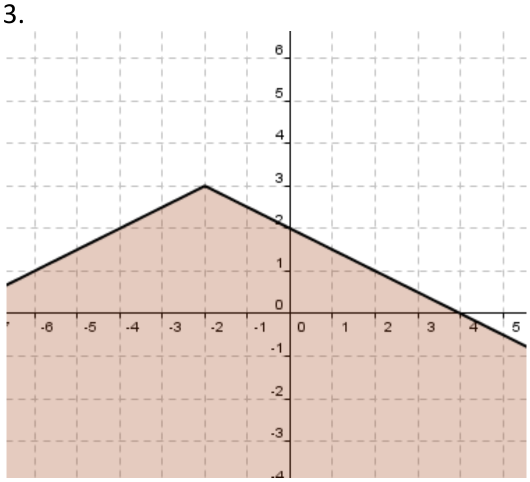
1.



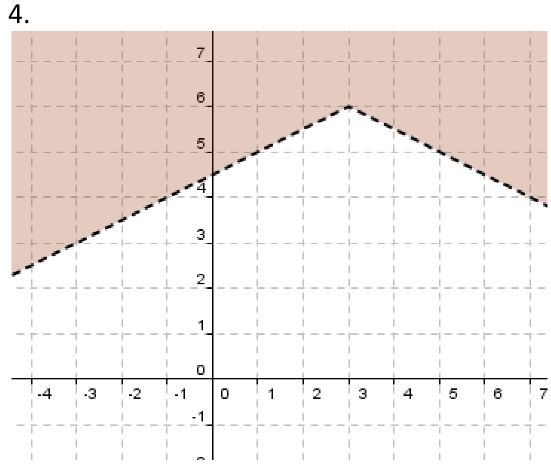
2.



3.



4.

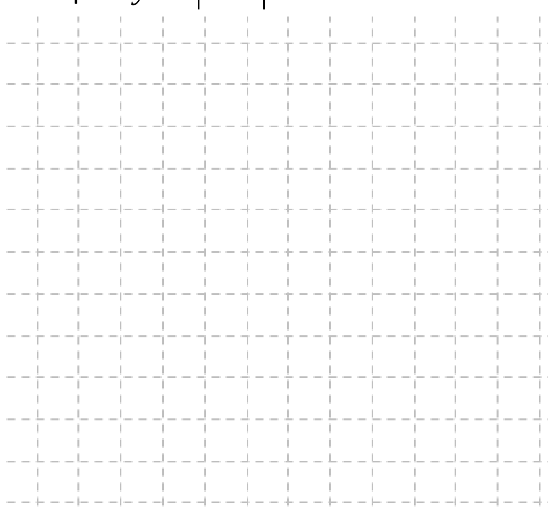


Exit Tickets

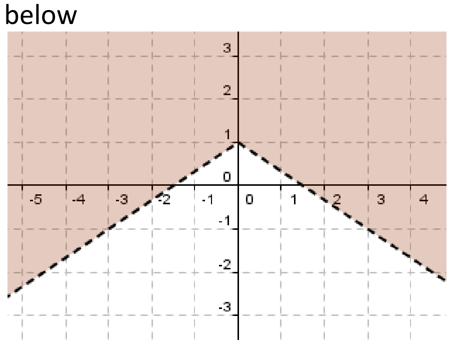
1. Graph $y < -\frac{3}{4}x + 4$

2. Graph $-4x - 6y \le 3$

3. Graph $2y - 4|x+3| \le 3$



4. Find the inequality whose graph is the graph below



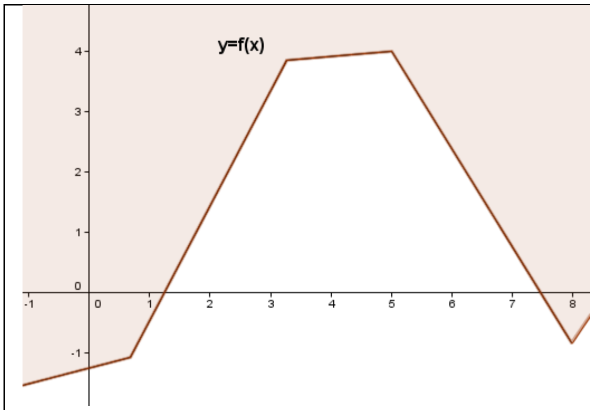
(Luigi’s Gold Coin: 2)

Consider a general inequality:

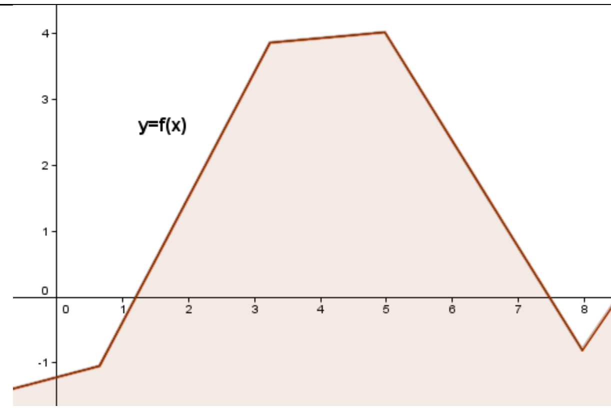
$$y * f(x)$$

where $*$ can be one of the inequality symbols ($>, <, \geq, \leq$)

Describe that for any functions with real domain, the following graphs are true.



for $y \geq f(x)$



for $y \leq f(x)$

Topic 9 Solving Systems of equations with 2 variables

Objective:

In this topic, I will review the skills of solving the system of linear equations with 2 variables.

The Lesson

In general, there are two different approaches to solve a system of linear equations:

1. Graph the two equations, find the intersection.

2. Use elimination to find the solution.

You will review them through the following examples in class.

Graph the system and find the intersections of the linear equations.

1. $\begin{cases} 2x-y=5 \\ x+3y=10 \end{cases}$	2 $\begin{cases} -3x+2y=-8 \\ 4x-3y=10 \end{cases}$

Solve the following systems

3. $3x - y = 2$ $6x + 3y = 14$	4. $2x + 5y = 13$ $6x + 2y = -13$
5 $3x + 4y = 18$ $6x + 8y = 18$	6. $6x - 2y = 5$ $-3x + y = 7$
7. $4x - 3y = 10$ $8x - 6y = 20$	8. $6x - 3y = 15$ $-2x + y = -5$

9.	10.
$\frac{1}{2}x + \frac{2}{3}y = \frac{5}{6}$ $\frac{5}{12}x + \frac{7}{12}y = \frac{3}{4}$	$\frac{x-1}{2} + \frac{y+2}{3} = 4$ $x - 2y = 5$

Exit tickets:

Solve the following systems

1.

$$\begin{cases} 0.25x + \frac{5}{6}y = \frac{2}{5} \\ \frac{5}{8}x - 0.5y = 1 \end{cases}$$

2.

$$\begin{cases} \frac{x+y}{2} - \frac{2x-y}{4} = 3 \\ \frac{x-2y}{3} + \frac{x-y}{5} = \frac{2}{3} \end{cases}$$

Luigi's coin: 1

Consider that $a_1b_2 \neq a_2b_1$, find the solution of the system $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$

Topic 10 Model your World

Objective:

In this topic, you will use GRASP method to model word problems and use the skills of solving systems to solve them

The Lesson

1. The sum of the digits of a two-digit (10-based) number is 15. If you switch the places of the number, the difference of the new number and the original number is 9. Find the original two-digit number.

2. Find the values of two numbers if their sum is 12 and their difference is 4.

3. Flying to Neverland with a tailwind a plane averaged 158 km/h. On the return trip the plane only averaged 112 km/h while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.

4. A boat travel 210 miles downstream and back. The trip downstream took 10 hours. The trip back took 70 hours. What is the speed of the boat in still water? (still water: the water does not move) What is the speed of the current?

5. The school that Hannah goes to is selling tickets to a show. On the first day of the ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

6. The senior classes at Santa Teresa High School and Mt. Pleasant High School planned separate trips to

Oregon. The senior class at STHS rented and filled 1 van and 6 buses with 372 students. MPHS rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

Exit tickets:

In Hannah’s piggy bank there are 20 coins. Hannah puts only quarters and dimes in the piggy bank. last Saturday she counted the coins in the piggy bank and found that the coins worth $5.70. How many quarters and dimes are in the bank?

Topic 11 Graph the systems of inequalities with 2 variables

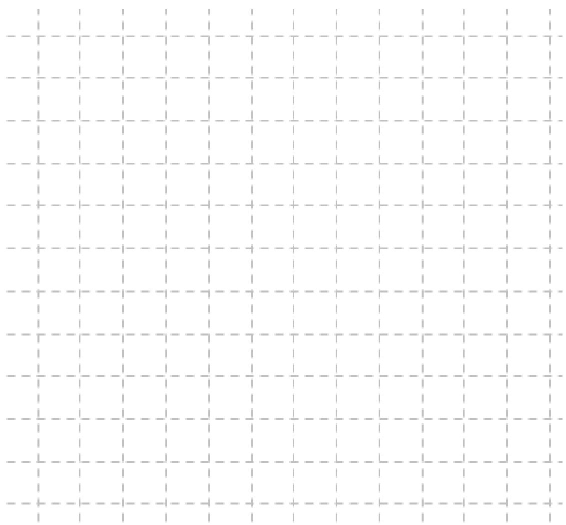
Objective:

In this topic, you will review how to graph the solutions of the inequalities with two variables.

The Lesson:

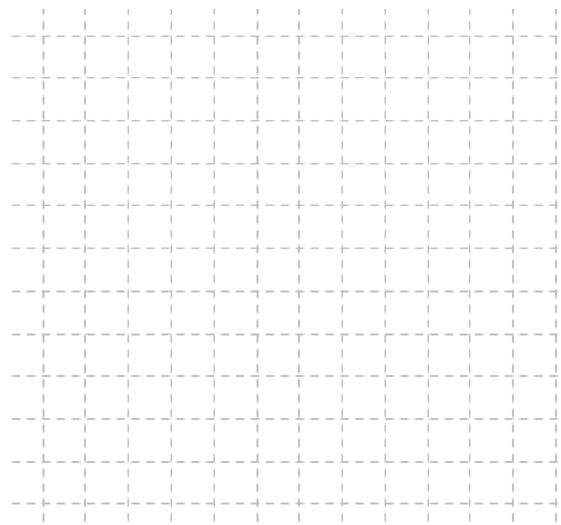
1. Graph the system

$$\begin{cases} x+y>1 \\ -2x-3y \geq -6 \end{cases}$$

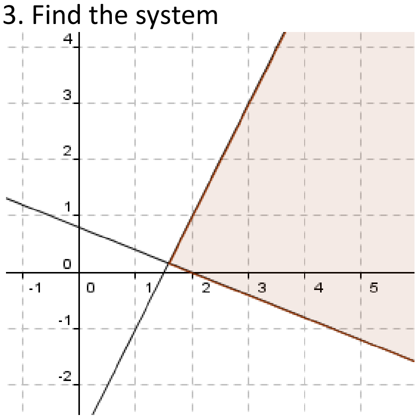


2. Graph the system

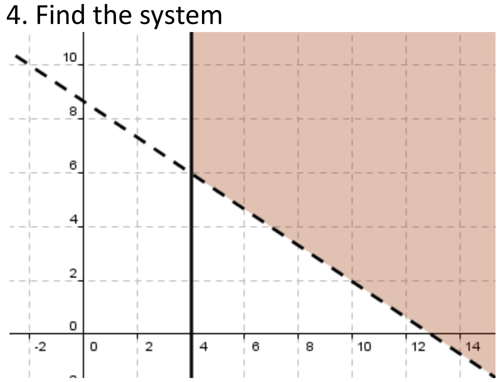
$$\begin{cases} y<1 \\ x-y<2 \end{cases}$$



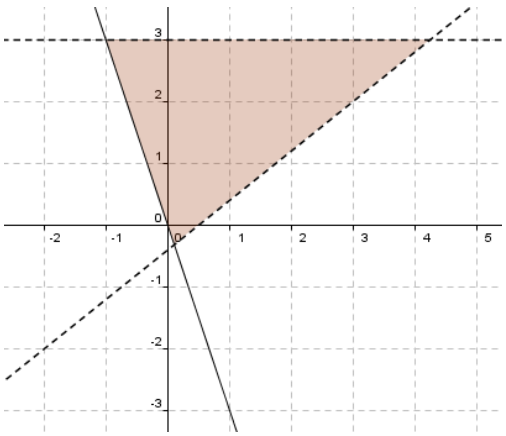
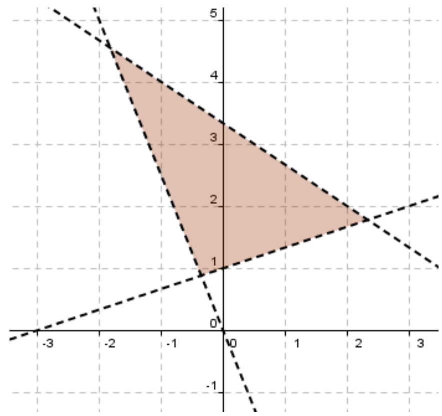
3. Find the system



4. Find the system



Practice 1: For each graph below, find the intersections of the boundaries and the system that represent the solution half-planes (or regions, if closed).

5. 	6. 

Practice 2: Graph the solutions of the system of the inequalities below.

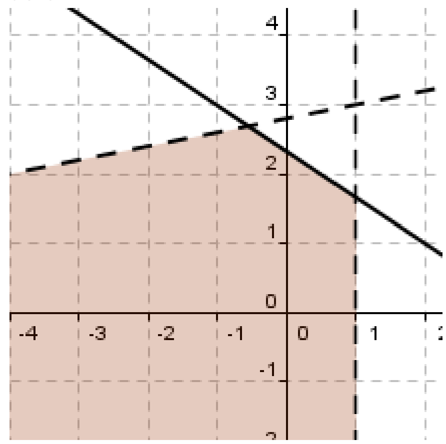
7. $\begin{cases} y+2<-\dfrac{3}{4}(x-4) \\ 2x-3y\geq 7 \\ x+1\leq -\dfrac{1}{2}(y+2) \end{cases}$	8. $\begin{cases} x<4 \\ x>-2 \\ x+y\geq 1 \end{cases}$

Exit Tickets:

1. Graph the systems

$$\begin{cases} x \geq 0 \\ y > 1 \\ 2x + 4y < 7 \\ 3x + y \leq 6 \end{cases}$$

2. Find a system represents the solution territory below:



Topic 12 The Linear Programming

Objective:

In this topic, you will learn the linear programming, which is an application to the solution of a system of linear inequality.

The Lesson

Consider the following problem:

Mr. Chen likes to hit the gym. Besides working out, he also watches carefully what he eats in order to build muscles. After some research he finds out the following facts about chicken and beef:

per lb	Protein (grams)	Fat (grams)	Price (dollar)
chicken breast	120	20	12
ground beef	150	50	6

Based on a scientific research, a person weight 180 – 200 lb should consume at least 900 grams of protein and no more than 500 grams of fat in a week to keep his muscle growing. How should Mr. Chen buy the chicken breast and ground beef every week to meet the requirement by the research and minimize the cost?

Practice:

Find Max and Min of each question

1. constraints $\begin{cases} x+2y\le 14 \\ 3x-y\ge 0 \\ x-y\le 2 \end{cases}$ Objective Function $z=3x+4y$	2 . constraints $\begin{cases} x\ge 0 \\ y\ge 0 \\ x+y\le 7 \\ x+2y\ge 4 \\ y\le x+5 \end{cases}$ Objective Function $z=-0.4x+3.2y$

Exit Tickets:

You need to buy some filing cabinets. There are two brands to choose, say, brand X and brand Y. You know that X brand costs $10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Brand Y costs $20 per unit , requires eight square feet of floor space, and holds twelve cubic feet of files. You have been given $140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinates. How many of which model should you buy in order to maximize storage volume?

Topic 13 Solving Systems of Equations with 3 variables

Objective:

In this topic, I will learn how to solve linear systems with 3 variables.

The Lesson

Objective 1: Check if a triplet (x, y, z) is a solution of a given system

1. Given $\begin{cases} 2x - y + z = -5 \\ 5x + 2y - 2z = 19 \\ x - 3y + z = -5 \end{cases}$; (1, 4, -3)	2. Given $\begin{cases} 4x - y + 3z = 13 \\ x + y + z = 2 \\ x + 3y - 2z = -17 \end{cases}$; (-1, -2, 5)

Objective 2: Solve the following systems:

The key is to eliminate 1 variable from the system and produce a system with 2 variables first.

3. $\begin{cases} 6x + y - z = -2 \\ x + 6y + 3z = 23 \\ -x + y + 2z = 5 \end{cases}$	4. $\begin{cases} 2x + y - 2z = 23 \\ 3x + 2y + z = 11 \\ x - y + z = -2 \end{cases}$

5. $\begin{cases} x+\frac{1}{2}y+\frac{1}{2}z=\frac{5}{2} \\ \frac{3}{4}x+\frac{1}{4}y+\frac{3}{2}z=\frac{7}{4} \\ \frac{1}{3}x+\frac{3}{2}y+\frac{2}{3}z=\frac{13}{6} \end{cases}$	6. $\begin{cases} x-\frac{1}{2}y+z=-\frac{21}{2} \\ \frac{1}{5}x+y-\frac{1}{5}z=5 \\ -\frac{1}{4}x+\frac{1}{6}y+\frac{1}{3}z=\frac{1}{2} \end{cases}$

Exit tickets:

Solve the system.

$$\begin{cases} x+3y-z=12 \\ 2x+4y-2z=6 \\ -x-2y+z=-6 \end{cases}$$

Luigi's Coin: (1 coin)

Solve $\begin{cases} \frac{1}{2}(x+y)+\frac{3}{2}(y+z)=3 \\ \frac{1}{3}(x-2y)+\frac{1}{4}(3y-2z)=-\frac{5}{2} \\ \frac{2}{3}(2x+3y)-(y+x)+\frac{1}{4}z=-2 \end{cases}$

Topic 14 Calculate Determinants of 3 x 3 matrices

Objective:

In this topic, you will learn how to calculate the determinant of a 3x3 matrix

The Lesson

Given a 3 x 3 matrix $M = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$, the determinant of M is defined as

$$\det M = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3$$

Objective 1: Use the rule to calculate the determinant of each 3x3 matrix

1. $\begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix}$	2. $\begin{pmatrix} 1 & \frac{2}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{1}{2} & -1 \\ \frac{1}{3} & -1 & \frac{1}{4} \end{pmatrix}$

3. $\begin{pmatrix} -3 & 8 & 5 \\ -2 & 1 & -1 \\ -4 & -7 & 2 \end{pmatrix}$	4. $\begin{pmatrix} 1 & 0 & -1 \\ -2.1 & 0.2 & -1.3 \\ 4.1 & -0.3 & 0.2 \end{pmatrix}$
5. $\begin{pmatrix} -1 & -2 & -8 \\ 2 & -2 & -1 \\ 8 & 1 & 1 \end{pmatrix}$	6. $\begin{pmatrix} 0.01 & 6 & 1.25 \\ 7.21 & 2 & 0.01 \\ 0.1 & 0 & 0 \end{pmatrix}$

Objective 2:

Given 3 points in a coordinate plane $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$: the area of the triangle formed by $P_1P_2P_3 = \frac{1}{2}\left|\det A\right|$, where A is a matrix $A = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$

Example: Find the area of ΔABC

7. A(2, -3), B(4, -1), C(1, -1)	8. A(0, 2), B(0, 5), C(15, 2)
9. A(4, 2), B(9, 0), C(0, -3)	10. A(1, 1), B(-1, 1), C(6, 1)

Exit Ticket:

If the determinant of $\begin{pmatrix} 1 & 0 & 3 \\ -3 & 2x & 4 \\ 1 & x-2 & 1 \end{pmatrix}$ is -8, find the value of x.

Topic 15 Cramer's Rule

Objective:

In this topic you will use Cramer's rule to solve a system with 3 variables.

The Lesson

In the history of mathematics, it was Gabriel Cramer who proposed his rule in 1750 to solve a system of equations with n-variables. Although his rule can be used to solve a system of any variables, we will see in this topic how it works in a system of 3 variables.

Consider a system

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

One can construct four 3 x 3 matrices

$$\Delta = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix},\ \Delta_x = \begin{pmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{pmatrix},\ \Delta_y = \begin{pmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{pmatrix},\ \Delta_z = \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{pmatrix}$$

and the solution of the system will be (if exist)

$$(x, y, z) = \left(\frac{\det \Delta_x}{\det \Delta}, \frac{\det \Delta_y}{\det \Delta}, \frac{\det \Delta_z}{\det \Delta}\right)$$

Ex: Solve

$$\begin{cases} 4x - y + 3z = 13 \\ x + y + z = 2 \\ x + 3y - 2z = -17 \end{cases}$$

To use Cramer's rule, you need to first construct all delta matrices, in this case,

$$\Delta = \begin{pmatrix} 4 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -2 \end{pmatrix},\ \Delta_x = \begin{pmatrix} 13 & -1 & 3 \\ 2 & 1 & 1 \\ -17 & 3 & -2 \end{pmatrix}, \Delta_y = \begin{pmatrix} 4 & 13 & 3 \\ 1 & 2 & 1 \\ 1 & -17 & -2 \end{pmatrix}, \Delta_z = \begin{pmatrix} 4 & -1 & 13 \\ 1 & 1 & 2 \\ 1 & 3 & -17 \end{pmatrix}$$

Then you will need to find the determinant of each matrix, in this case,

$\det\Delta = -17$, $\det\Delta_x = 17$, $\det\Delta_y = 34$, $\det\Delta_z = -85$

Therefore,

$$x = \frac{\det\Delta_x}{\det\Delta} = \frac{17}{-17} = -1,$$
$$y = \frac{\det\Delta_y}{\det\Delta} = \frac{34}{-17} = -2,$$
$$z = \frac{\det\Delta_z}{\det\Delta} = \frac{-85}{-17} = 5$$

Practice:

Use Cramer's rules to solve the following systems

1. $\begin{cases} 4x + y + 3z = 7 \\ 2x - 5y + 4z = -19 \\ x - y + 2z = -2 \end{cases}$	2. $\begin{cases} 3x - y + z = 25 \\ -x + 2y - 3z = -17 \\ x + y - 2z = 0 \end{cases}$

3. $\begin{cases} 5x - y - 2z = -6 \\ 2x + 3y + 4z = 16 \\ 2x - 4y + z = -15 \end{cases}$	4. $\begin{cases} x + 2y - 2z = 3 \\ 2x - y - z = 2 \\ 2x + 4y + 2z = 3 \end{cases}$

No Exit ticket for the topic.

End of the unit Handout.