EVHS Algebra II Unit 5

Rational, Radical and Exponential Relations

Contents

Topic 1 Multiply and Divide Rational Expressions 5
Topic Objective: 5
Prerequisite: 5
The Lesson 5
Topic 2 Add and Subtract Rational Expressions 9
The Lesson 9
Topic 3 Solve Rational Equations 15
The Lesson 15
Practice: 15
Topic 4 Graph Simple Rational Functions 17
The Lesson 17
Topic 5 Function operation: composition of functions 25
The Lesson 25
Topic 6 Evaluating Nth Root 28
The lesson 28
Topic 7 Inverse function 32
The Lesson 32
Topic 8 Solve radical equations 34
The Lesson 34
Topic 9 Sketch the square root and cubic root functions 36
The Lesson 36
Topic 10 Graph Exponential Growth Functions 41
The Lesson 41
Practice: 43
Topic 11 Graph Exponential Decay Functions 46
The Lesson 46
Practice: 48
Topic 12 The Natural Base e 51
The Lesson 51

Topic 13 Exponential Models 55
Topic Objective: 55
Prerequisite: 55
The Lesson 55

Topic 1 Multiply and Divide Rational Expressions

Topic Objective: In this topic, students will learn how to multiply and divide rational expressions

Prerequisite: factoring polynomials

The Lesson

Multiply and divide rational expressions are extensions from multiplying/dividing the fractions, even though the direct applications of these skills are not obvious, these skills are essential when dealing with expressions.

Just like multiplying two fractions

$$\frac{21}{45} \times \frac{5}{7}$$

You would like to find common factors among the fractions, like this

$$\frac{3 \cdot 7}{3^2 \cdot 5} \times \frac{5}{7}$$

, so that whenever the reduction is possible, reduce! And the result of the multiplication becomes,

$$\frac{1}{3}$$

This procedure can be extended into rational expressions like this

$$\frac{x^2+3x+2}{x^2-x} \cdot \frac{x^2-2x+1}{3x^2+2x-1}$$

This complicated rational expression can be factored as

$$\frac{(x+1)(x+2)}{x(x-1)} \cdot \frac{(x-1)^2}{(3x-1)(x+1)}$$

Now, assuming that none of the denominators is zero, you can reduce the same factors as you did for fractions, and your expressions become

$$\frac{(x+2)(x-1)}{x(3x-1)}$$

And this is the goal for multiplying two rational expressions.

Similar process can be applied to dividing two rational expressions as well, you will practice in the following examples

Example 1: Find products or quotients of rational expressions

1. $\frac{3x^2-16x-64}{3x^2-4x-32}\cdot\frac{2x^2+6x-56}{x^2-x-56}$	2. $\frac{x-2}{6x^2-13x-5}\cdot\frac{(3x+1)^2}{x^2-2x}$
3. $\frac{x^2-3x-28}{x^2-49}\cdot\frac{x^2-16}{x^2+3x-28}$	4. $\frac{3x+5}{8x^2+6x+1}\cdot\frac{2x-1}{6x^2+7x-5}$
5. $(2x+1)\cdot\frac{14x^2-9x+1}{18x^2+7x-1}$	6. $\frac{2x^2-3x-2}{2x^2+5x+2}\cdot(x+2)$

7. $\dfrac{1}{3x-9x^2}\cdot\dfrac{12x^2+5x-3}{4x+3}$	8. $\dfrac{1}{(4x-1)^2}\cdot\dfrac{8x^2+34x-9}{2x^2+23x+63}$
9. $\dfrac{3x+7}{2x+9}\div\dfrac{3x^2+25x+42}{2x^2+x-36}$	10. $\dfrac{4x^2+13x+3}{2x+1}\div\dfrac{2x^2+3x-9}{4x^2-4x-3}$
11. $\dfrac{4x^2-4x-35}{2x-3}\div\dfrac{2x+5}{4x^2-16x+15}$	12. $\dfrac{4x+8}{x+2}\div\dfrac{3x-6}{x^2-4x+4}$

13. $\frac{2x^2-17x+30}{4x-3} \div \frac{2x^2-7x-30}{4x+3}$	14. $\frac{5x-4}{(2x+1)^2} \div \frac{10x^2-3x-4}{4x-3}$
15. $\frac{(3x+1)^2}{3x+2} \div \frac{12x^2-11x-5}{21x^2+8x-4}$	16. $\frac{1}{6x^2+7x-5} \div \frac{1}{12x^2-4x-40}$

Topic 2 Add and Subtract Rational Expressions

Topic Objective: In this topic students will learn how to add and subtract rational expressions

Prerequisite: factoring polynomials

The Lesson

Just like multiplying and dividing rational expressions are extensions from multiplying and diving fractions, adding and subtracting rational expressions are the extensions from adding and subtracting two fractions. You will learn this skill in two steps (objectives).

Objective 1: Find the LCM (least common multiples) in two polynomials or LCD (least common denominators) in two rational expressions.

LCM and LCD are similar concepts. We will talk about the LCM and use the same skill to find LCD. See the following example:

What is the LCM for 24 and 28?

There are different ways of finding the LCM of 24 and 28, here is one of the effective approach:

since

$$24 = 2^3 \times 3^1$$

and

$$28 = 2^2 \times 7^1,$$

the LCM for

$$[24,28] = 2^3 \times 3^1 \times 7^1 = 168$$

What is the LCM for $6x^2 + 7x - 5$ and $2x^2 - 9x + 4$?

Using the same approach, we found

$$6x^2 + 7x - 5 = (2x-1)(3x+5)$$

and

$$2x^2 - 9x + 4 = (2x-1)(x-4)$$

Therefore, the LCM for $[6x^2 + 7x - 5, 2x^2 - 9x + 4] = (2x-1)(3x+5)(x-4)$

Ex1: Use the same skills to find the following LCMs

1. x^2+5x+6 and x^2-4	2. $x+6$ and $2x+12$
3. $10x^2-x-21$ and $35x^2+24x-35$	4. $(2x-5)(3x+2)$ and $3x^2+5x-12$

Ex2: Find the LCD for the following expressions and rewrite the rational expressions to have the same denominators as the LCD.

5. $\dfrac{x+4}{x-4}$ and $\dfrac{2x-3}{x+3}$	6. $\dfrac{x-2}{x+3}$ and $\dfrac{x}{3x-1}$

7. $\dfrac{1}{2x^2-7x-72}$ and $\dfrac{3x+7}{x^2-2x-48}$	8. $\dfrac{x-1}{x^2-6x+8}$ and $\dfrac{3x+6}{2x^2+x-10}$
9. $\dfrac{3x^2+7x-6}{x-4}$ and $\dfrac{1}{x^2-4x}$	10. $\dfrac{3x-8}{x-7}$ and $\dfrac{2}{x^2+4x-77}$
11. $\dfrac{1}{x+2}$, $\dfrac{2}{x+4}$ and $\dfrac{3x-1}{x^2+6x+8}$	12. $\dfrac{1}{x-3}$, $\dfrac{x}{x+2}$ and $\dfrac{x-3}{x^2-2x-8}$

Objective 2: Adding and subtracting rational expressions

When adding (or subtracting) two fractions like these:

$$\frac{5}{18}+\frac{3}{16}$$

We first find the LCD (which is 144) and rewrite both fractions so that they both have the same denominator,

$$\frac{5\times 8}{144}+\frac{3\times 9}{144},$$

and then you can add the numerators together to find the sum $\frac{67}{144}$

Now, we can apply the same principles to find the addition (or subtraction) of rational expressions

1. Check if all rational expressions involved have same denominators
2. Rewrite them so that every rational expressions have same denominators
3. Find the sum or difference.

Example 3:

1. $\frac{2}{x+3}+\frac{x}{x-4}$	2. $\frac{x}{3x-1}-\frac{4}{x+4}$

3. $\dfrac{x}{x^2-2x-3}+\dfrac{x-1}{2x^2+x-1}$	4. $\dfrac{x-4}{6x^2-x-1}+\dfrac{x}{2x^2+5x-3}$
5. $\dfrac{x}{x-4}-\dfrac{54}{x^2-10x+24}$	6. $\dfrac{2x+1}{x^2+x-42}-\dfrac{1}{x-8}$

7. $\dfrac{1}{2-x}+\dfrac{2}{x-\dfrac{2}{x-1}}$	8. $\dfrac{x-1}{x}+\dfrac{2}{1-\dfrac{1}{x}}$
9. $1-\dfrac{2}{x-x^2}$	10. $\dfrac{8x}{x^2-4}-2$

Topic 3 Solve Rational Equations

Topic Objective: In this topic, students will learn how to find solutions for rational equations.

Prerequisite: Add/Subtract/Multiply/Divide Rational expressions

The Lesson

This lesson is a review of solving the proportions which most of you have learned from Algebra 1. In this topic, you may simplify the equation on both sides before you solve it.

Examples: Solve

1. $\dfrac{-2x+7}{x^2-9}=\dfrac{2}{x-3}-\dfrac{3}{x+3}$

2. $\dfrac{x}{x-3}-\dfrac{1}{x+3}=1+\dfrac{x+9}{x^2-9}$

Practice:

1. $\dfrac{2x}{x^2-2}=\dfrac{-1}{x-1}$	2. $\dfrac{3}{x+4}=\dfrac{5x}{2x^2-5x-42}$

3. $\frac{1}{x+2}+\frac{2}{x-2}=5$	4. $\frac{6x}{x+4}+4=\frac{2x+2}{x-1}$
5. $\frac{18}{x^2-3x}-\frac{6}{x-3}=\frac{5}{x}$	6. $\frac{x+3}{x-3}+\frac{x}{x-5}=\frac{x+5}{x-5}$
7. $\frac{1}{x+6}+\frac{2}{x+5}=\frac{3}{x-4}$	8. $\frac{1}{x-2}-\frac{2}{x+5}=\frac{-1}{x+9}$

Topic 4 Graph Simple Rational Functions

Topic Objective: In this topic, students will learn how to graph some simple rational functions.

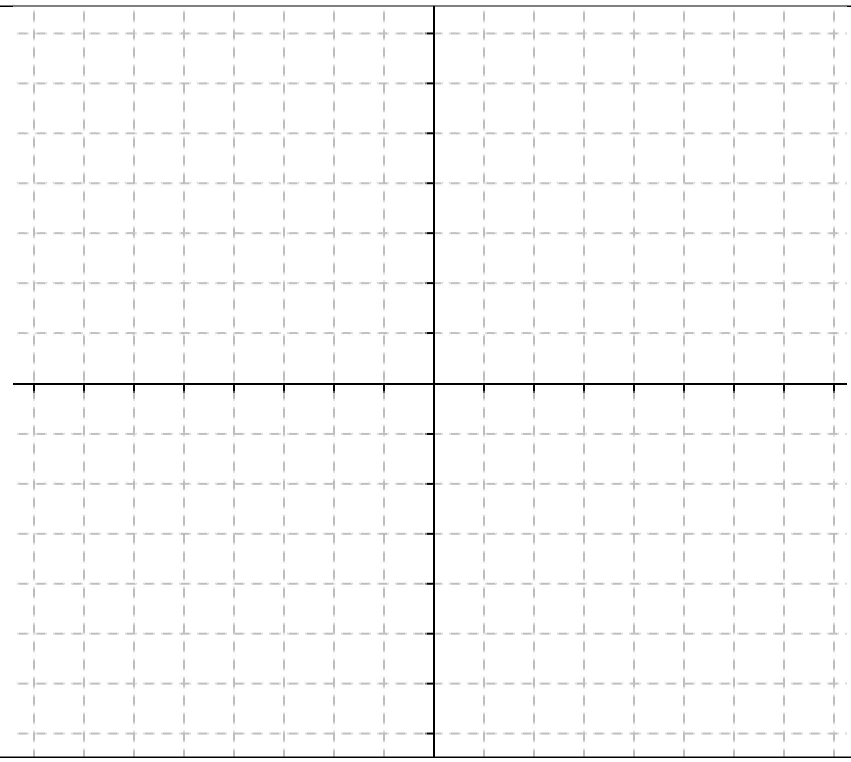
Prerequisite: Function translation and scaling.

The Lesson

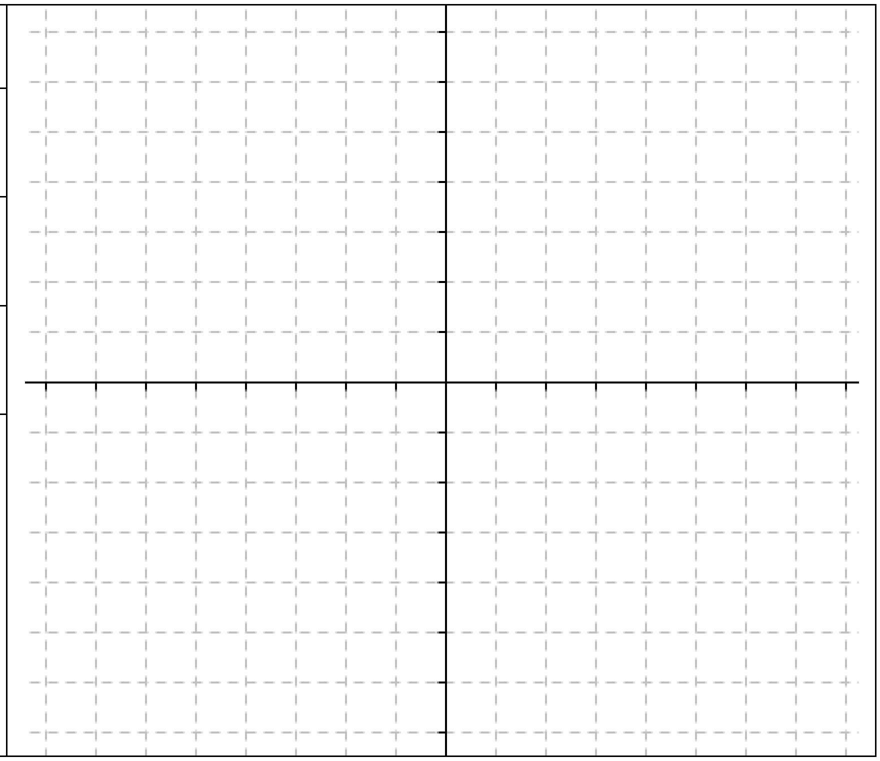
In your prior algebra classes, every function can be defined on the real domain. (that means, every x value gets you a y value) However, starting from rational functions, you will find out that not every real x will give you a real y. These exceptions are one of many characteristics that we will discuss about rational functions in order for us to understand their behaviors. First of all we will look at the parent function of all rational functions $y = \frac{1}{x}$. In this topic, you will provide 4 characteristics for each rational function before you sketch it.

Objective 1: Parent function and translation only

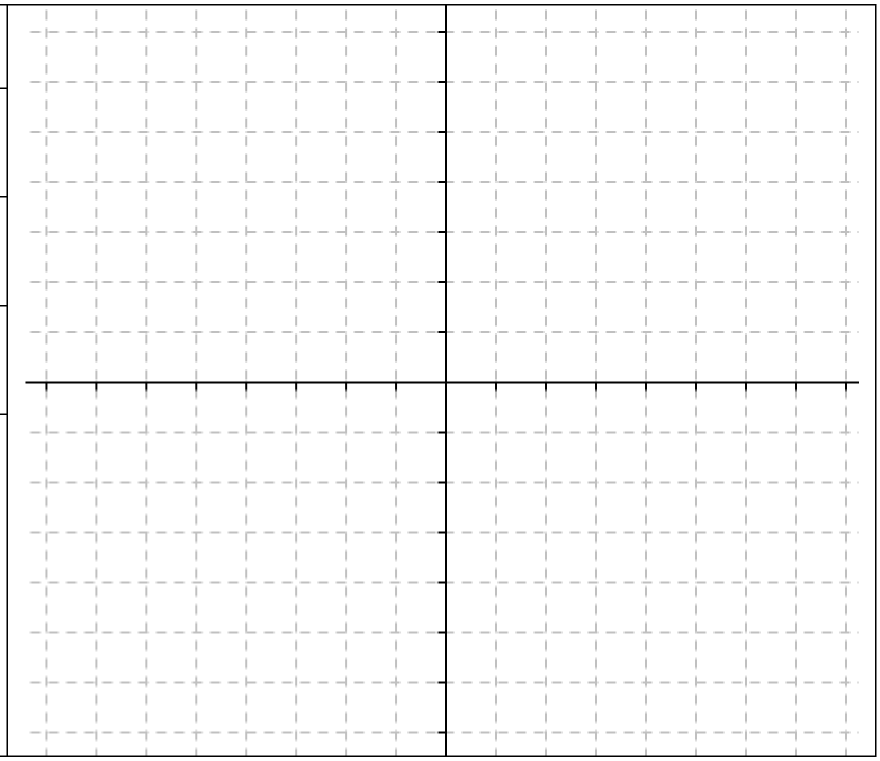
Function: $y = \frac{1}{x}$
Domain:
Range:
V. Asym.:
H. Asym.:



Function translation (Translate horizontally) $y = \dfrac{1}{x-h}$

Function: $y = \dfrac{1}{x-2}$	
Domain:	
Range:	
V. Asym.:	
H. Asym.:	

Function translation (Translate vertically) $y=\dfrac{1}{x}+k$

Function: $y=\dfrac{1}{x}+1$	
Domain:	
Range:	
V. Asym.:	
H. Asym.:	

Function translation (Translate both horizontally and vertically) $y = \dfrac{1}{x-h} + k$

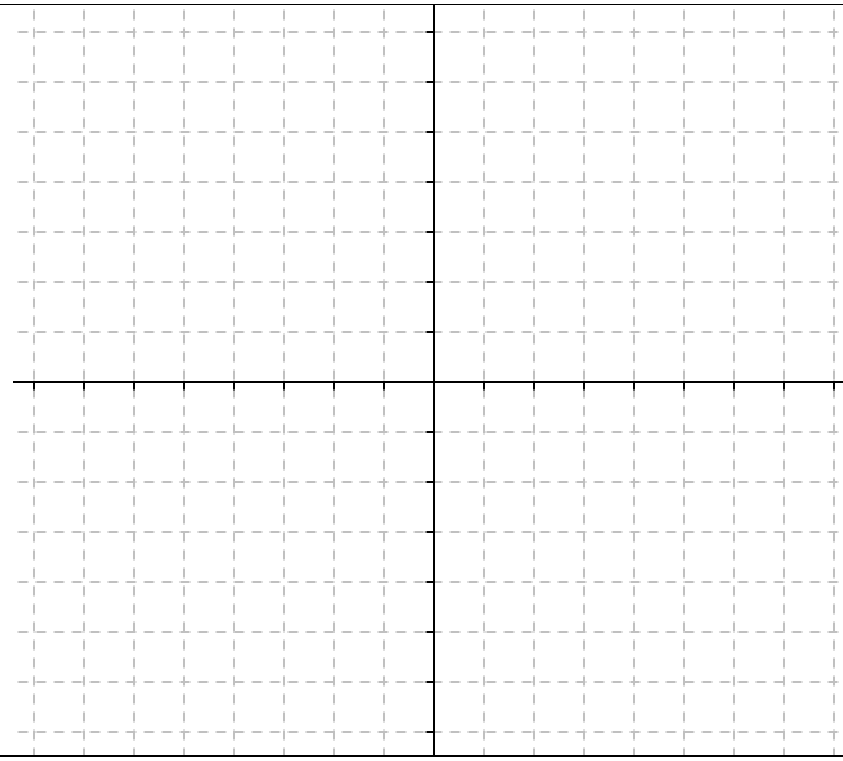
Function: $y = \dfrac{1}{x-3} + 2$

Domain:

Range:

V. Asym.:

H. Asym.:



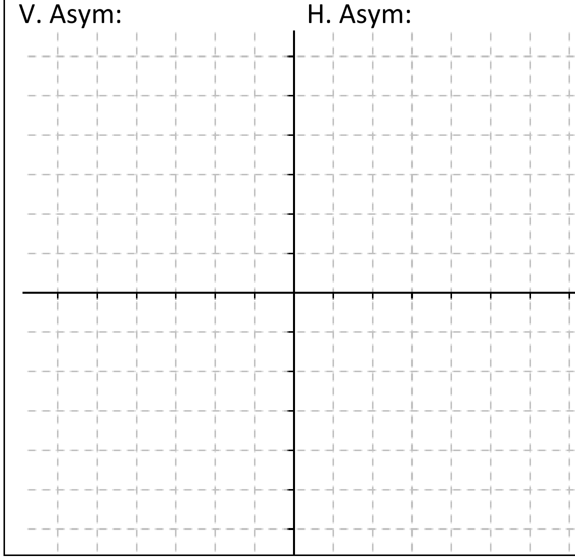
Practice: Sketch the following functions and identify the domain, range, and asymptotes.

1. $y = \dfrac{1}{x+2} - 3$

Domain:

Range:

V. Asym: H. Asym:

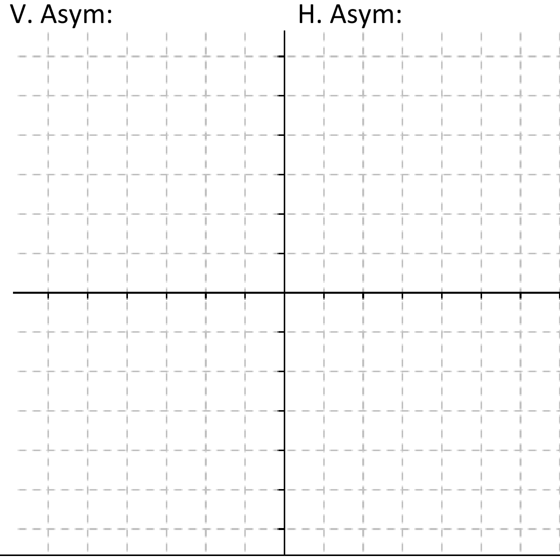


2. $y = \dfrac{1}{x+1} - 2$

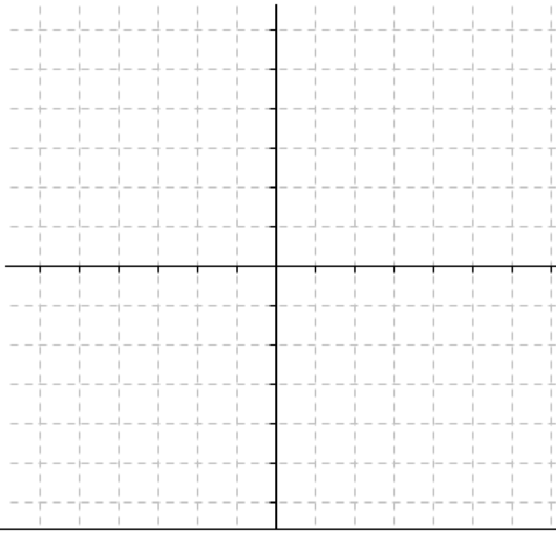
Domain:

Range:

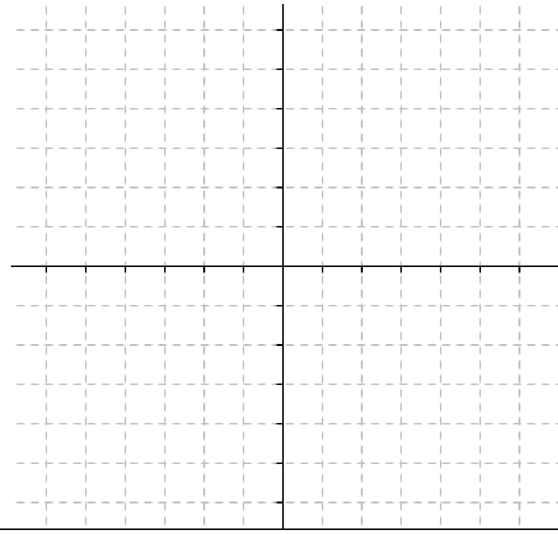
V. Asym: H. Asym:



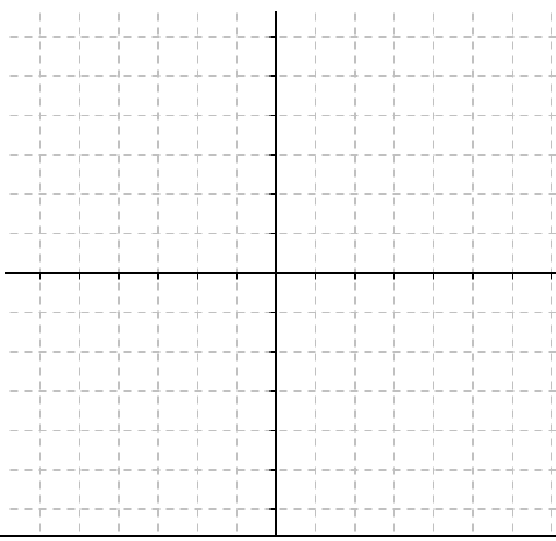
3. $y = \dfrac{-x-1}{x+2}$



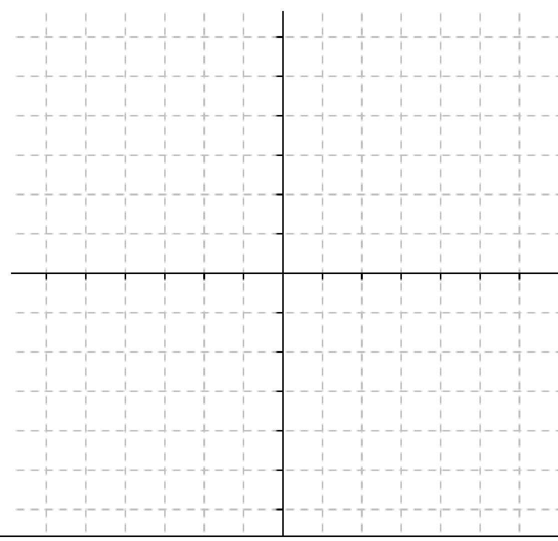
4. $y = \dfrac{2x+9}{x+4}$



5. $y = \dfrac{-3x+7}{x-2}$

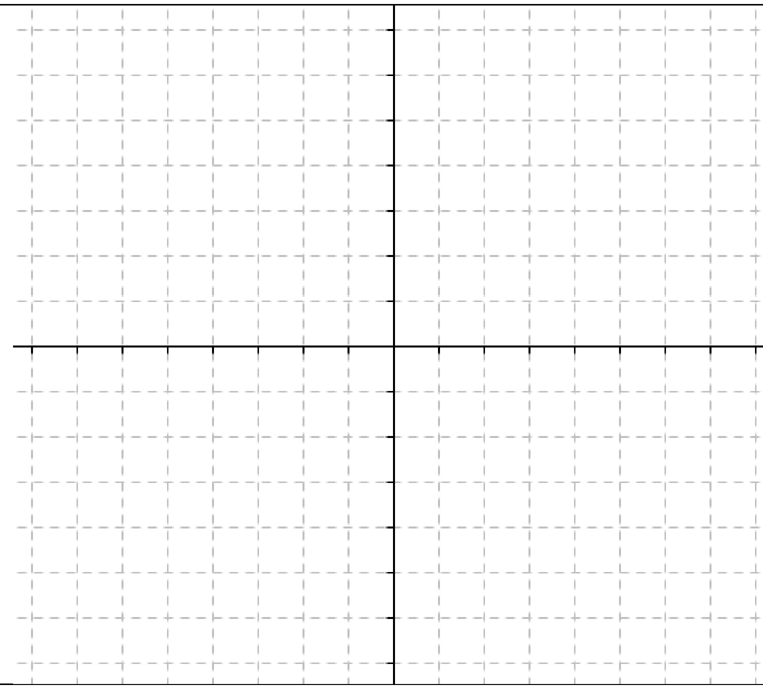


6. $y = \dfrac{4-x}{x-3}$

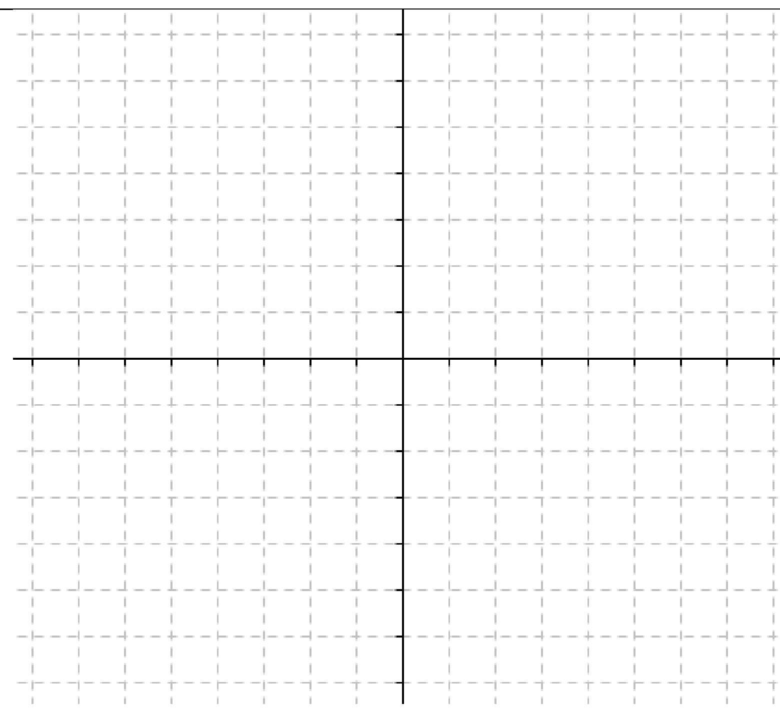


Objective 2: Function reflection and scaling

Function reflection: $y=-\frac{1}{x}$

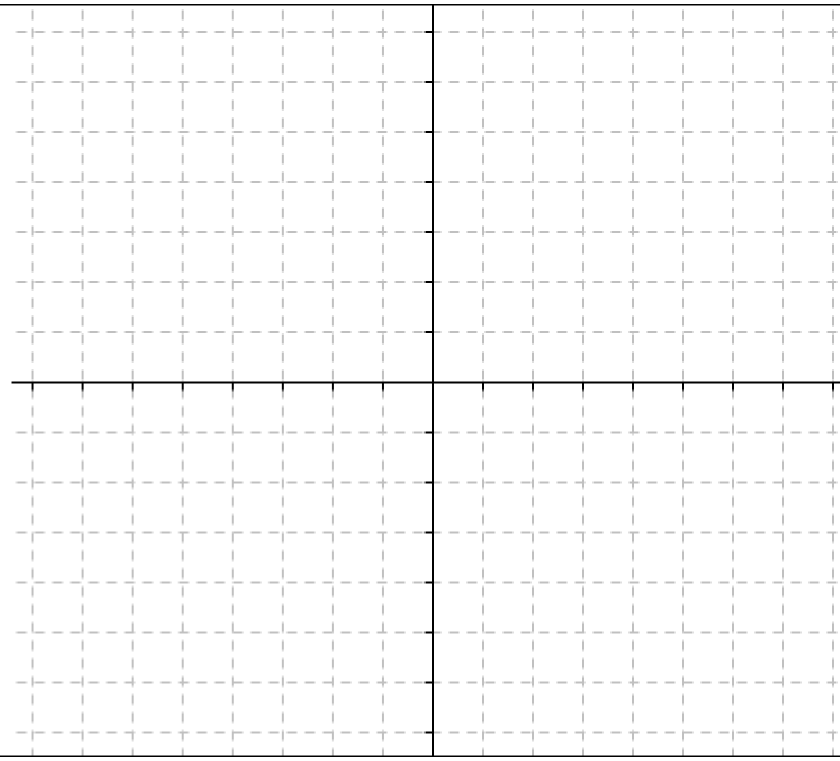
Function: $y=-\frac{1}{x}$	
Domain:	
Range:	
V. Asym.:	
H. Asym.:	

Function reflection: $y=\frac{a}{x}$

Function: $y=\frac{2}{x}$	
Domain:	
Range:	
V. Asym.:	
H. Asym.:	

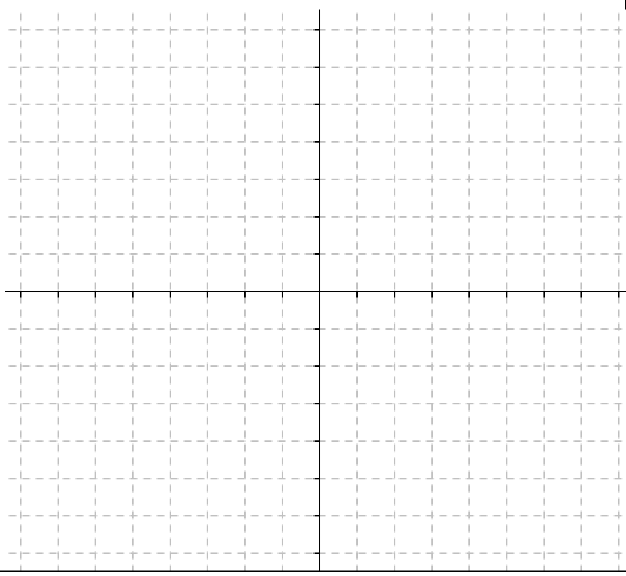
Mixed it up $y = \dfrac{-a}{x-h} + k$

Function: $y = \dfrac{-2}{x-2} + 1$
Domain:
Range:
V. Asym.:
H. Asym.:

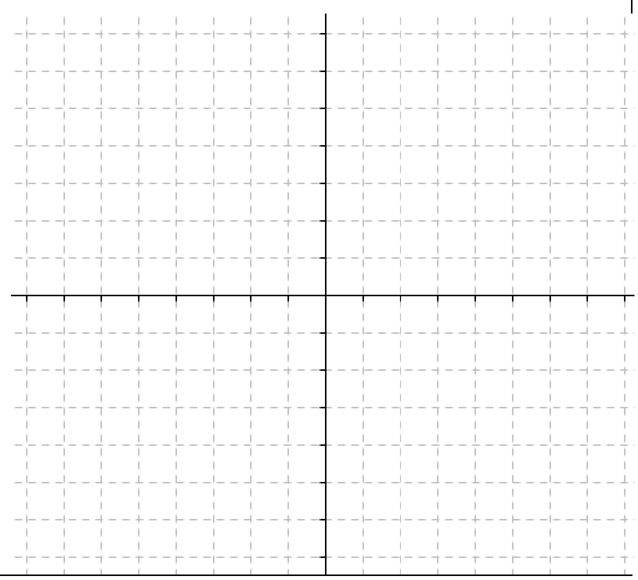


Practice:

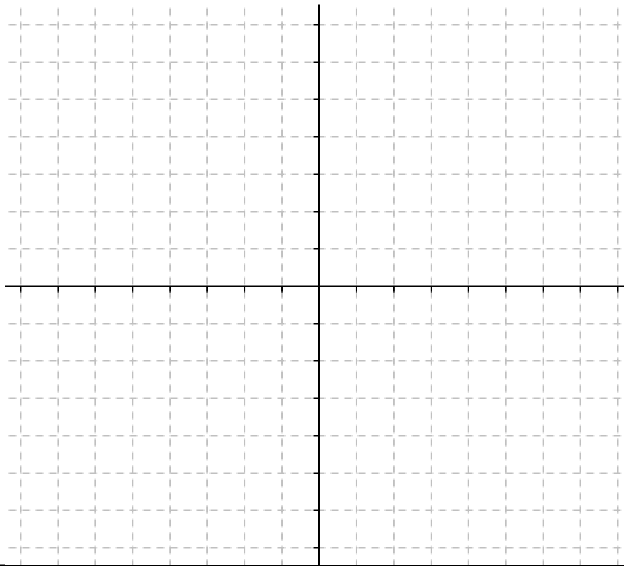
7. $y = \dfrac{-x}{x+3}$



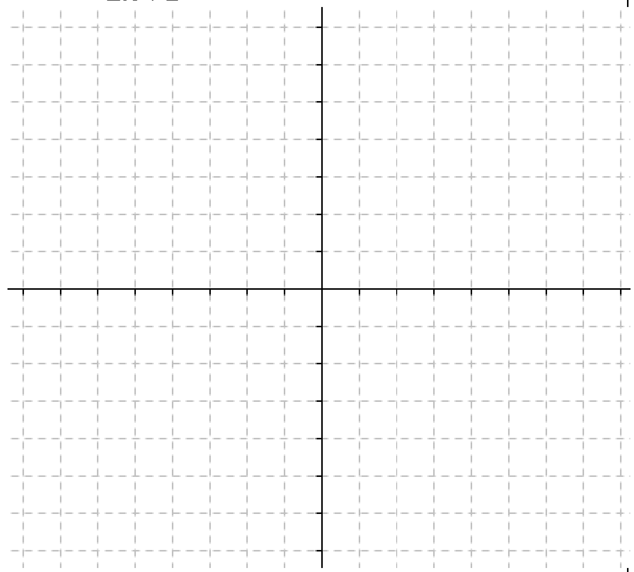
8. $y = \dfrac{3-2x}{x+2}$



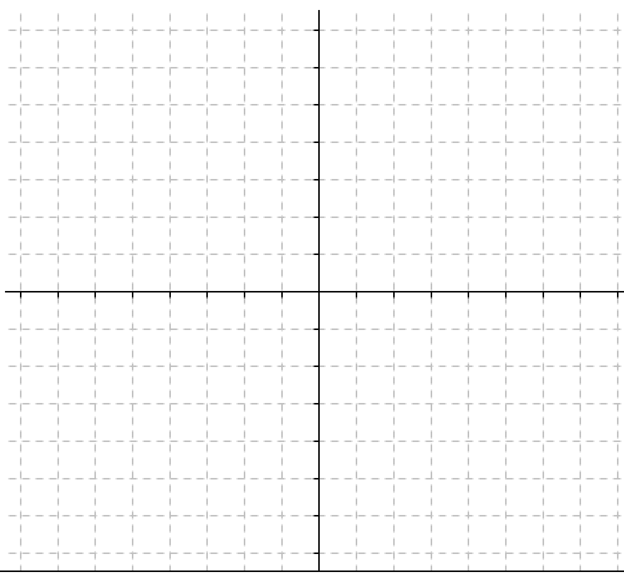
9. $y = \dfrac{1-2x}{x+1}$



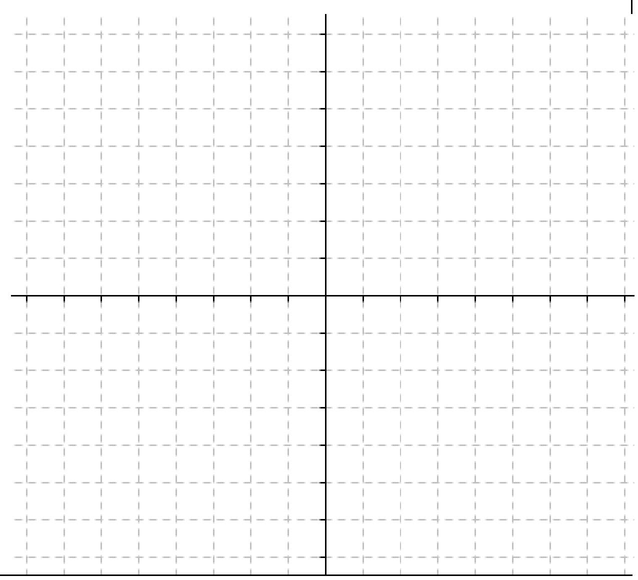
10. $y = \dfrac{-2x}{2x+1}$



11. $y = \dfrac{6x-5}{4x-6}$



12. $y = \dfrac{x+1}{x-3}$



Topic 5 Function operation: composition of functions

Topic Objective: In this topic, students will learn how to chain two functions into one.

Prerequisite: general algebraic operations

The Lesson

One of the operations between two functions does not have its arithmetic counterpart is composition. Before we jump into composition, here is a short review of the other operations:

Review Examples:

Given $f(x) = x + 3$ and $g(x) = x^2 - x$, (find the domain for each function)

Find (1) $(f + g)(x)$

(2) $(f - g)(x)$

(3) $(f \cdot g)(x)$

(4) $(\frac{f}{g})(x)$

Now, to compose two functions, we substitute one function into the other

$$f \circ g(x) = f(g(x))$$

and

$$g \circ f(x) = g(f(x))$$

In general,

$$f \circ g \neq g \circ f$$

If we use $f(x)$ and $g(x)$ defined in the review example, you will find:

$f \circ g(x) =$

$g \circ f(x) =$

Now, we will practice the chain (Composition) operation, using m(x), n(x), and k(x):

$m(x) = x + 3$, $n(x) = \sqrt{x}$, $k(x) = x + x^{-1}$

1. $n \circ k(x)$	2. $k \circ m(x)$
3. $m \circ n(x)$	4. $n \circ m \circ k(x)$
5. $k \circ n(4)$	6. $m \circ k(-3)$

7. $m \circ n(5)$	8. $n \circ k(1)$
9. $m \circ \left(\dfrac{m}{n}\right)(x)$	10. $m \circ \left(\dfrac{m}{n}\right)(4)$

Topic 6 Evaluating Nth Root

Topic objective: In this topic, students will learn how to evaluate the nth root of a real number.

Prerequisite: evaluating square root, properties of exponents

The lesson

Objective 1: Understand the relationship between radical symbol and rational exponent

In your previous algebra classes, you come to understand the radical symbol

$$\sqrt{}$$

Which means to find the square root of some number, for example

$$\sqrt{25}$$

you are asked to find a number that multiply itself twice will come to 25. And you know the answer is 5.

This operation can be interpreted two different ways. The first one is the traditional way:

since 5x5 = 25 therefore $\sqrt{25}$ =5.

Or the second perspective:

Seeing $\sqrt{25} = (25)^{\frac{1}{2}}$, so that

$$(25)^{\frac{1}{2}} \cdot (25)^{\frac{1}{2}} = (25)^{(\frac{1}{2}+\frac{1}{2})} = 25^1 = 25$$

If we see the radical from the second perspective, then the radical can be redefined as finding "**Half-power of a real number"**

$$\sqrt{} = (\quad)^{\frac{1}{2}}.$$

This concept can be extended to a higher order root, like

$\sqrt[3]{} = (\quad)^{\frac{1}{3}}$ [cubic root], $\sqrt[4]{} = (\quad)^{\frac{1}{4}}$ (4^{th} root)….

Ex1: Evaluate the following expressions

1. $\sqrt[3]{64}$	2. $\sqrt[4]{625}$	3. $\sqrt[4]{1296}$	4. $\sqrt[3]{343}$
5. $\sqrt[3]{1331}$	6. $\sqrt[5]{-1024}$	7. $\sqrt[3]{-729}$	8. $\sqrt[8]{512}$

Objective 2: apply properties of rational exponents

Properties of exponents can be applied to numbers with rational (or fractional) exponents, following table is a review of the properties of exponents

1. $a^m \cdot a^n = a^{m+n}$	2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$	4. $(ab)^m = a^m \cdot b^m$
5. $a^{-m} = \dfrac{1}{a^m}$	6. $(\dfrac{a}{b})^m = \dfrac{a^m}{b^m}$

Also, for radicals, rule 4 and rule 6 can look like this:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Further, when a radical is in its **simplest form**, the radical is rationalized, with radicant's power less than the index.

$$\sqrt[3]{48} = \sqrt[3]{2^4 \cdot 3} = \sqrt[3]{2^3} \cdot \sqrt[3]{2 \cdot 3} = 2\sqrt[3]{6}$$

the simplest form for $\sqrt[3]{48}$ is $2\sqrt[3]{6}$

Ex2: Evaluate (or rewrite) the following expressions and write your answer to the simplest form.

1. $\sqrt[4]{162} \cdot \sqrt[4]{8}$	2. $\sqrt[3]{3} \cdot \sqrt[6]{81}$
3. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}}$	4. $\dfrac{\sqrt[3]{72}}{\sqrt[4]{4}}$
5. $\sqrt[3]{48} - \dfrac{\sqrt[3]{4}}{\sqrt[3]{108}}$	6. $\sqrt[5]{125} - \dfrac{2}{\sqrt[5]{25}}$

7. $\sqrt[4]{1250} - 8\sqrt[4]{32} + \sqrt[4]{2}$	8. $5\sqrt[3]{48} - \sqrt[3]{750}$
9. $\sqrt[3]{\dfrac{x}{y^2}}$	10. $\dfrac{\sqrt[3]{x}}{\sqrt[3]{x^2 y}}$

Topic 7 Inverse function

Topic Objective: In this topic, students will learn how to and find its inverse function.

Prerequisite: The skill of “Solve for”—(Isolating variables)

The Lesson

Observation:

Let $f(x)=2x+1$, $g(x)=\frac{1}{2}x-1$.

(1) Find $f\circ g(x)$ and $g\circ f(x)$.

(2) $f(g(-2))$

(3) $g(f(-200))$

When two functions $f(x)$ and $g(x)$ are inverse functions, then the following conditions must be true

1. (1 to 1)

____

2. (sym. y=x)

____

3. (int. chain)

____

The symbol of the inverse function of any $f(x)$ is $f^{-1}(x)$. (read as “f inverse of x”)

Ex: Find the inverse of each function below, verify the domain and range of the original function and its inverse.

1. $f(x) = 3x + 1$
2. $f(x) = x^3 + 1$
3. $f(x) = x^2 + 1, x \geq 0$
4. $f(x) = \sqrt{2x - 3}, x \geq \frac{3}{2}$

Topic 8 Solve radical equations

Topic Objective: In this topic, students will learn how to solve radical equations

Prerequisite: solve linear and quadratic equations

The Lesson

The first step to solve a radical equation is to remove the radical, and then solve the equations after. This process may introduce some extraneous solutions. One should always test the solution in the original equations to see if it is an extraneous solution.

1. $\sqrt{-2x+3}-2=10$	2. $\sqrt[3]{12x}-13=-7$
3. $-5\sqrt[3]{8x}+12=-8$	4. $\sqrt[3]{4x+2}-6=-10$

5. $\sqrt{2x+5}=\sqrt{x+2}+1$	6. $\sqrt{5x+6}+3=\sqrt{3x+3}+4$
7. $\begin{aligned} 5\sqrt{x}-2\sqrt{y}&=4\sqrt{2} \\ 2\sqrt{x}+3\sqrt{y}&=13\sqrt{2} \end{aligned}$	8. $\begin{aligned} 3\sqrt{x}+5\sqrt{y}&=31 \\ 5\sqrt{x}-5\sqrt{y}&=-15 \end{aligned}$
9. $\sqrt{x+2}-2=\sqrt{x}$	10. $\sqrt{3x+8}+1=\sqrt{x+5}$

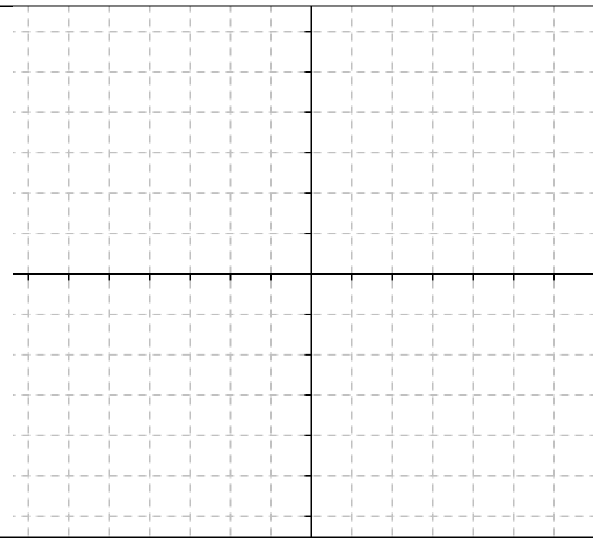
Topic 9 Sketch the square root and cubic root functions

Topic Objective: students will learn how to sketch the parent function of square root and cubic root. They will use the translation, reflection and scaling transformation to sketch the derived square root or cubic root relations.

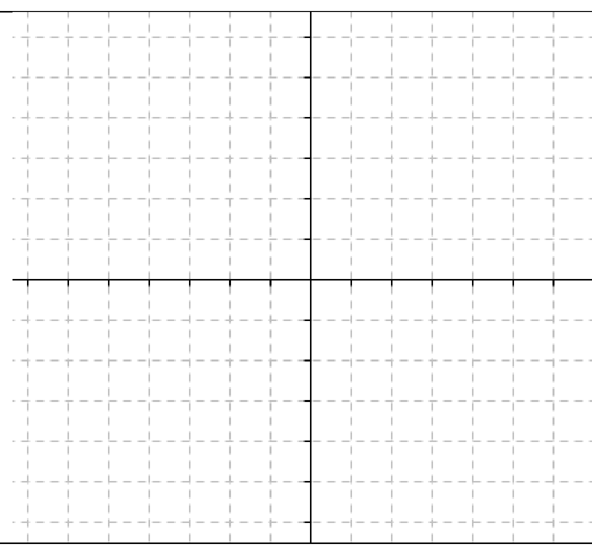
Prerequisite: translation, reflection and scaling transformations on functions.

The Lesson

In order to sketch common radical functions, you need to know first how their parent functions look like. In the example below, graph the radical parent functions.

Function : $y=\sqrt{x}, x\geq 0$ Domain: Range: when $x\to\infty$, $y\to$	

For cubic functions

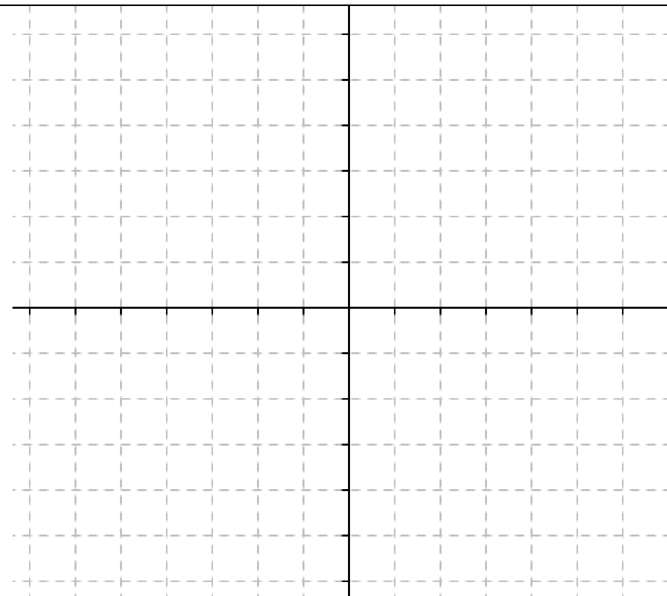
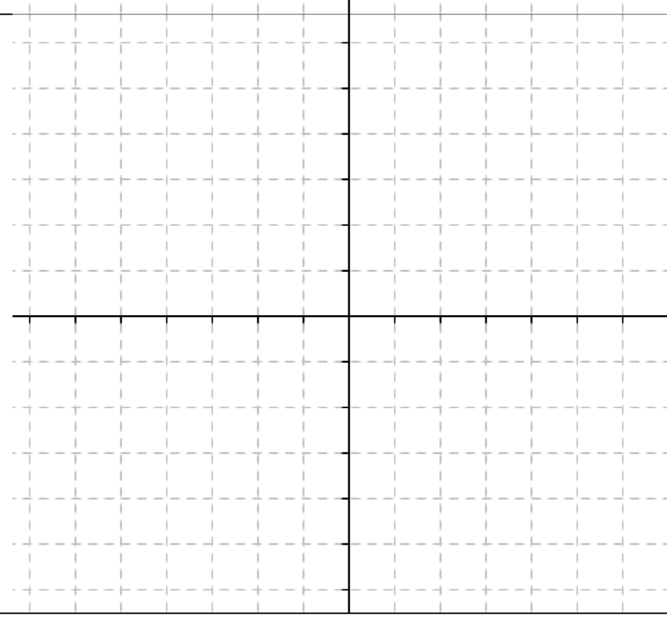
Function : $y=\sqrt[3]{x}$ Domain: Range: when $x\to\infty$, $y\to$	

Similar operation of translations, reflection and scaling can be applied to these parent functions as well…

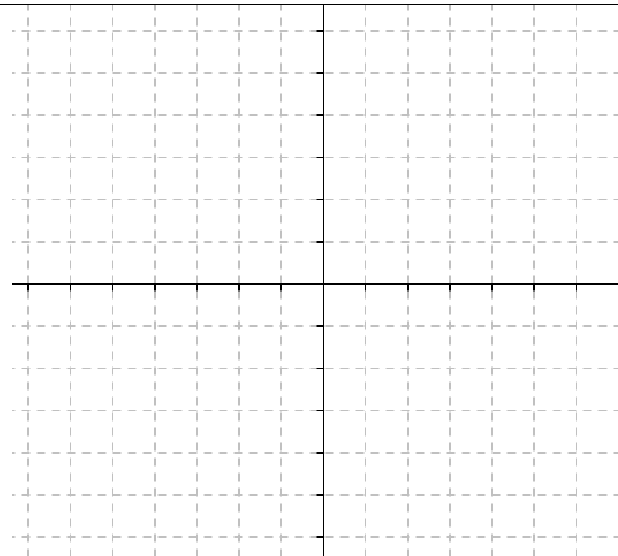
$f(x) = a\sqrt{x-h} + k$: the square root function is first scaled by a constant a (if a<0, then it reflects about the x-axis)and then translated h units horizontally and k units vertically.

$f(x) = a\sqrt[3]{x-h} + k$: The cubic root function is first scaled by a constant a (if a<0 then it reflects about the x-axis first) and then translated h units horizontally and k units vertically.

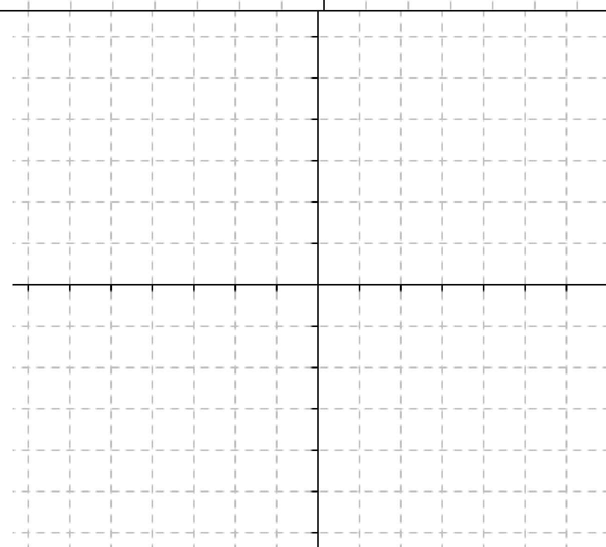
Graph the following radical functions. (Identify the domain and range)

1. $f(x) = -2\sqrt{x}$	
2. $f(x) = \frac{2}{3}\sqrt{x}$	

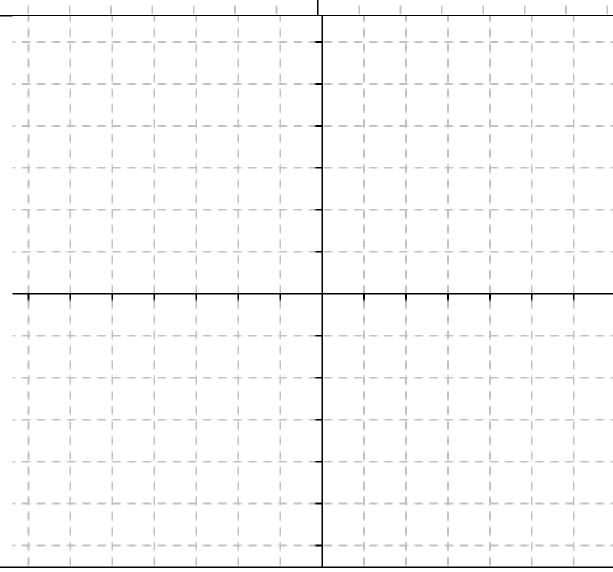
3. $f(x)=\sqrt{x-4}$

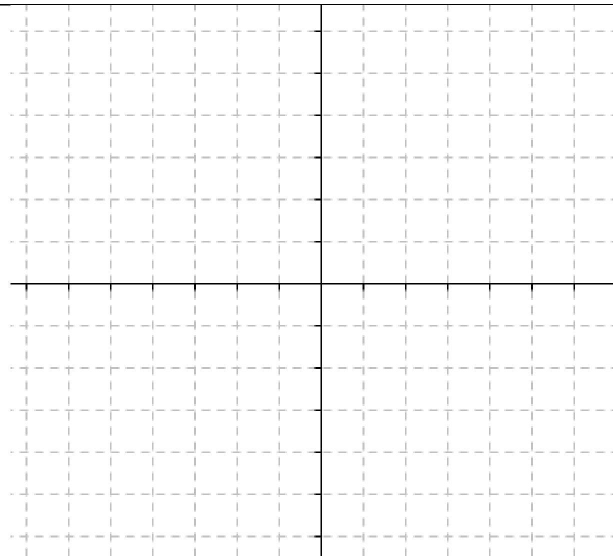
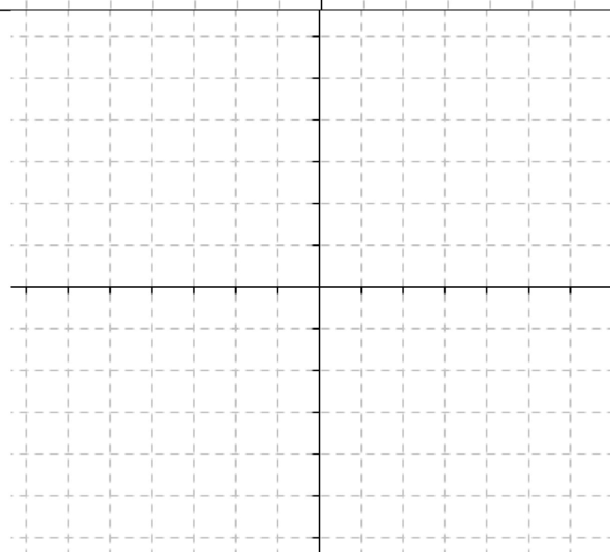
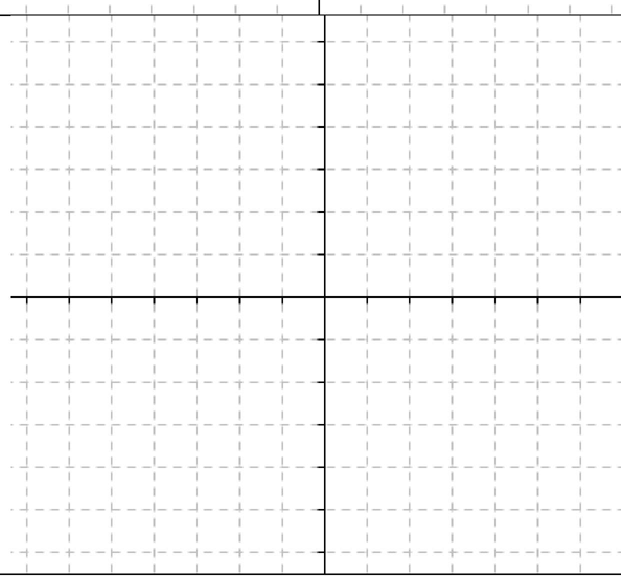


4. $f(x)=\sqrt{4x-2}$

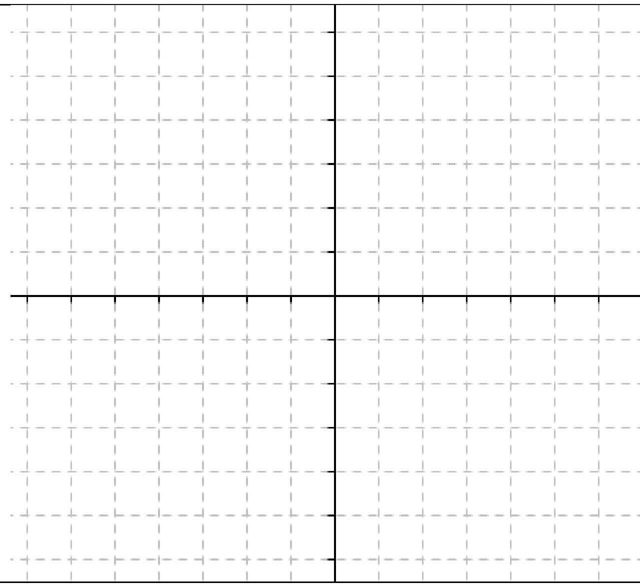


5. $f(x)=\sqrt{9x+18}-2$



6. $f(x)=\sqrt[3]{x+4}$	
7. $f(x)=-2\sqrt[3]{x-1}+2$	
8. $f(x)=-\sqrt[3]{x+3}-2$	

9. $f(x)=\frac{1}{2}\sqrt[3]{x-2}+4$



Topic 10 Graph Exponential Growth Functions

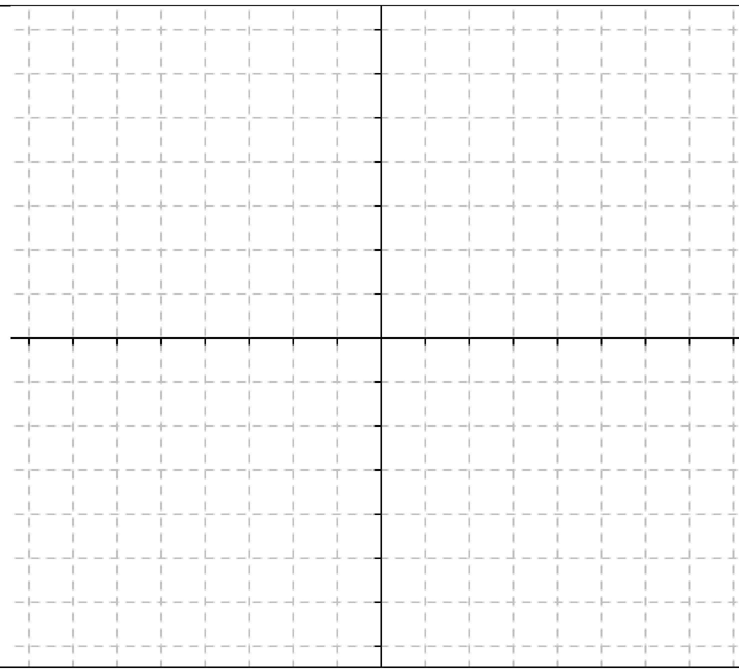
Topic Objective: in this topic, students will understand exponential growth relations. Students will also learn how to graph exponential growth functions with translation, reflection and scaling.

Prerequisite: function transformations

The Lesson

Exponential growth can be observed both in the lab (for small creatures like bacteria) as well as in the countries (like population). In this lesson, you will learn how to graph parent functions and use translations, scaling and reflections to move functions.

Example: $y = b^x$, b>1

$y = 2^x$ domain: range: H. asymptote: y-int: x-int:	

Scaling and reflection : $y = ab^x$, b>1 (if a<0, then reflect in the x-axis)

$y = \frac{1}{3} \cdot 2^x$

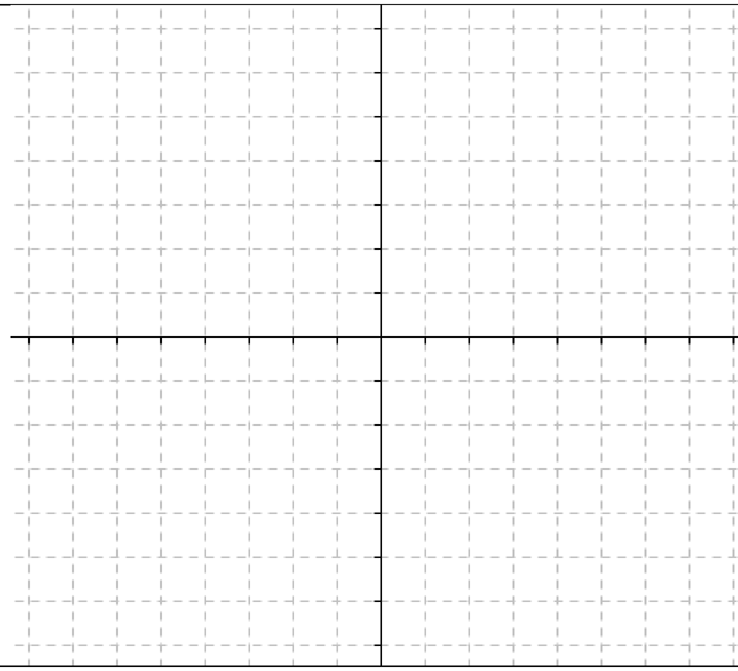
domain:

range:

H. asymptote:

y-int:

x-int:



Translations: $y = ab^{x-h} + k$,b>1 (move h units horizontally and k units vertically)

$y = -\frac{1}{4} \cdot 2^{x-3} + 1$

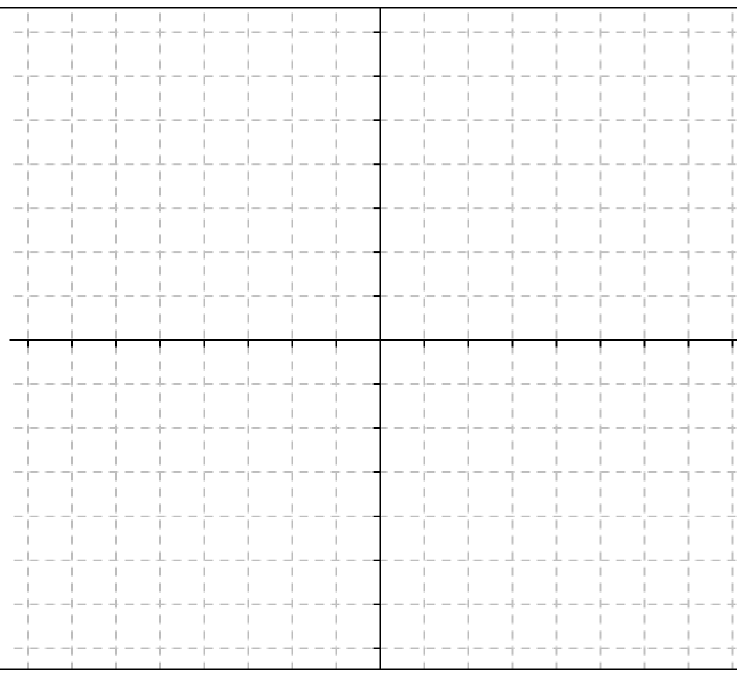
domain:

range:

H. asymptote:

y-int:

x-int:



Practice:

1.

$y = 3^x$

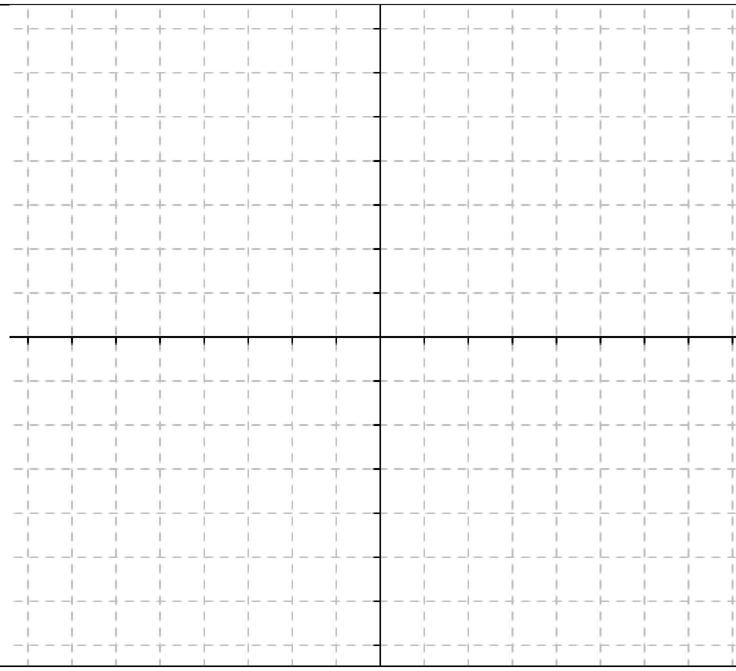
domain:

range:

H. asymptote:

y-int:

x-int:



2.

$y = \frac{3}{2} \cdot 3^x$

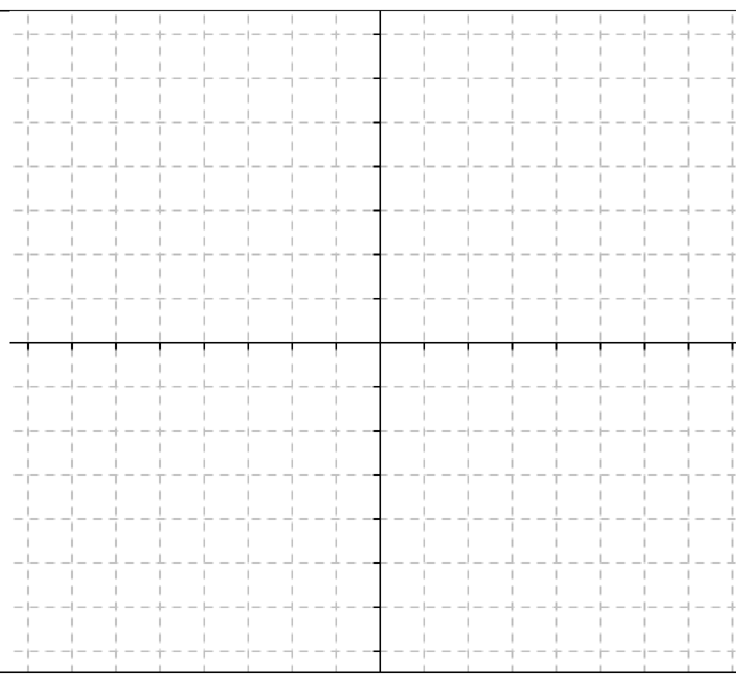
domain:

range:

H. asymptote:

y-int:

x-int:



3.

$y = -\frac{1}{2} \cdot 3^{x}$

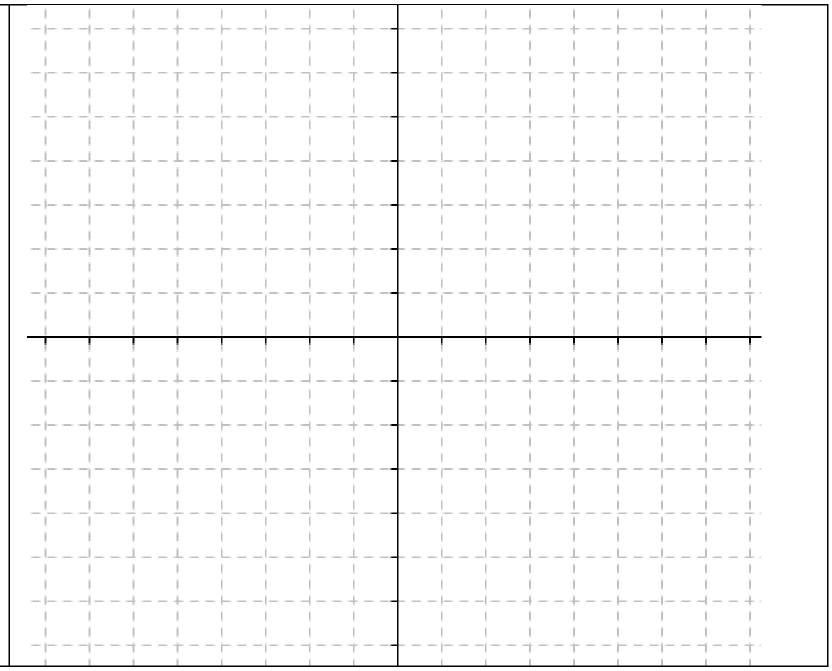
domain:

range:

H. asymptote:

y-int:

x-int:



4.

$y = -\frac{1}{4} \cdot 3^{x+2}$

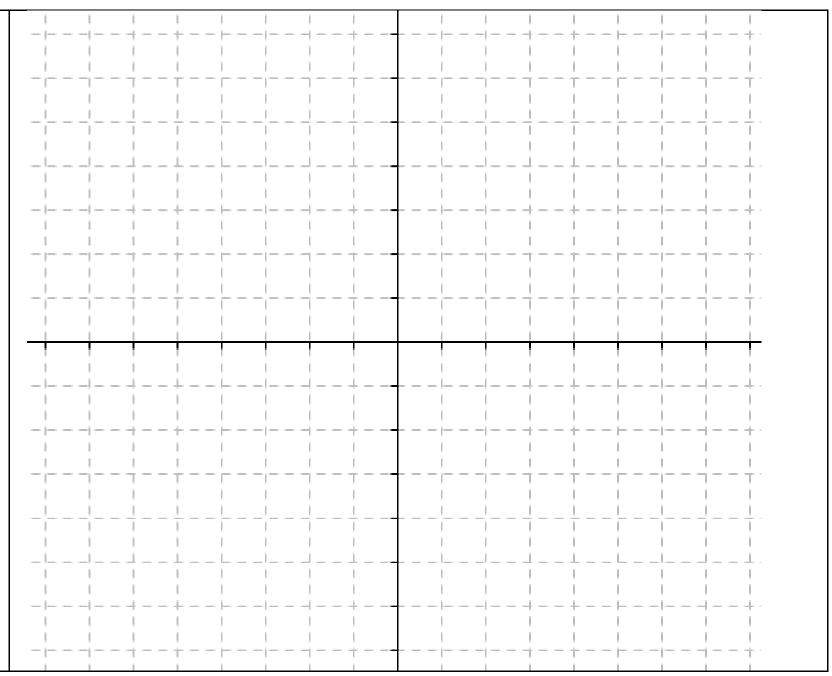
domain:

range:

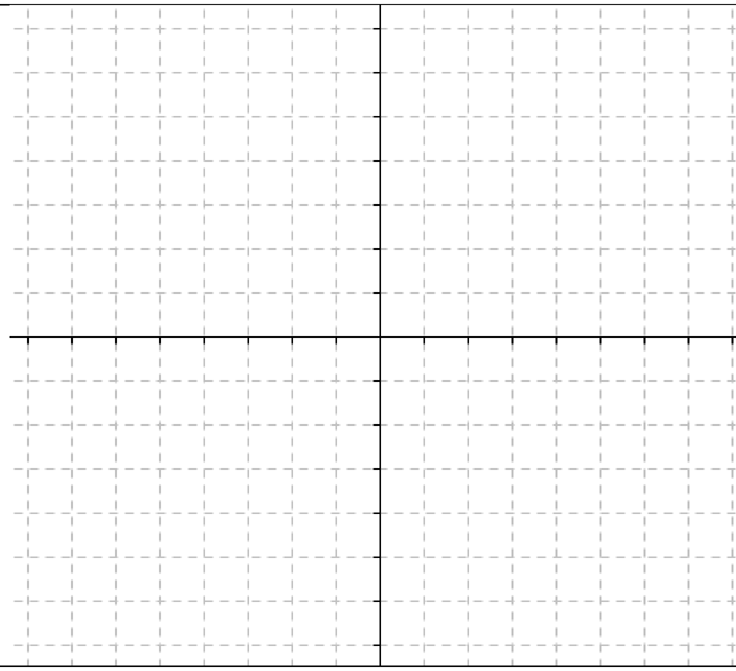
H. asymptote:

y-int:

x-int:



5.

$y = \frac{1}{6} \cdot 3^{x+2} - 3$ domain: range: H. asymptote: y-int: x-int:	

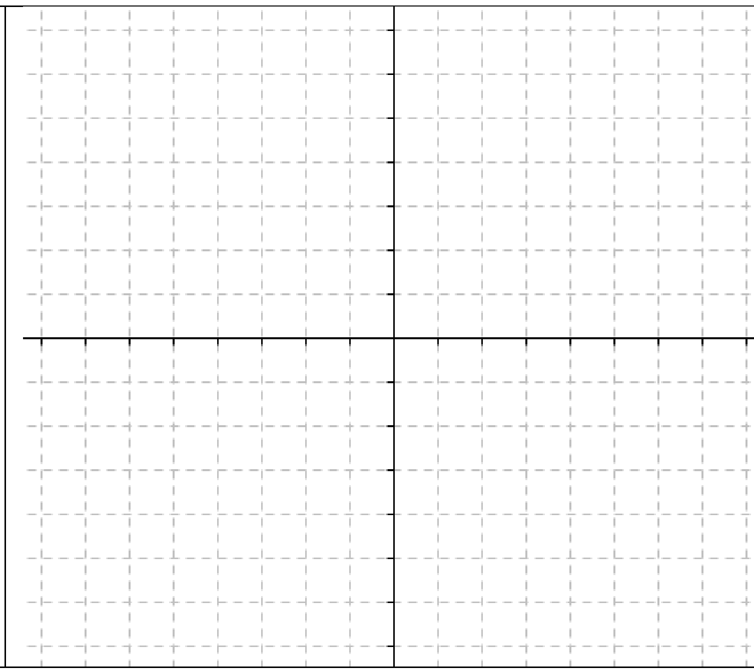
Topic 11 Graph Exponential Decay Functions

Topic Objective: in this topic, students will understand exponential decay relations. Students will also learn how to graph exponential decay functions with translation, reflection and scaling.

Prerequisite: function transformations

The Lesson

In topic 10, you learn how does an exponential growth function behave, now, in this lesson, you are learning the exact parallel of the similar types of the function in topic 10, however, instead of increasing to infinity as x approaching infinity, it decreases and approaching zero as x approaching infinity. In this lesson, you will also use the similar skill to move these functions by translating, scaling and reflecting. Example: $y = b^x$, 0<b<1

$y = (\frac{1}{2})^x$ domain: range: H. asymptote: y-int: x-int:	

Scaling and reflection : $y = ab^x$, 0<b<1 (if a<0, then reflect in the x-axis)

$y = \frac{1}{3} \cdot (\frac{1}{2})^x$

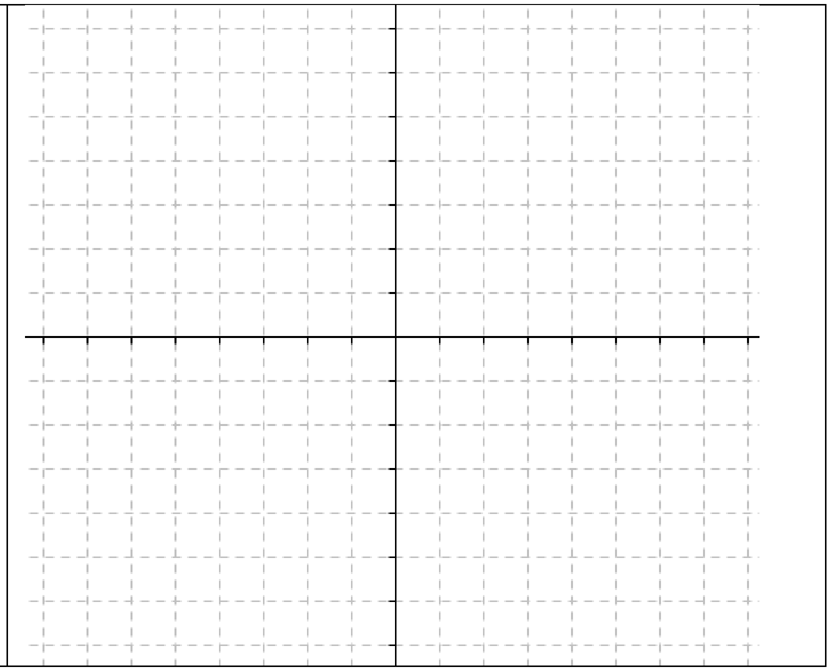
domain:

range:

H. asymptote:

y-int:

x-int:



Translations: $y = ab^{x-h} + k$, 0<b<1 (move h units horizontally and k units vertically)

$y = -\frac{1}{4} \cdot (\frac{1}{2})^{x-3} + 1$

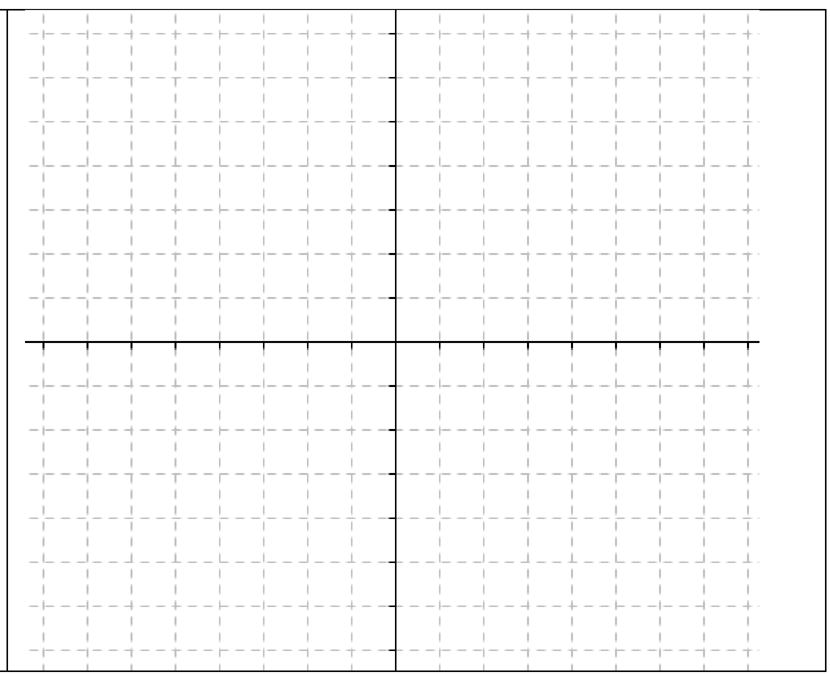
domain:

range:

H. asymptote:

y-int:

x-int:



Practice:

1.

$y = \left(\frac{1}{3}\right)^x$

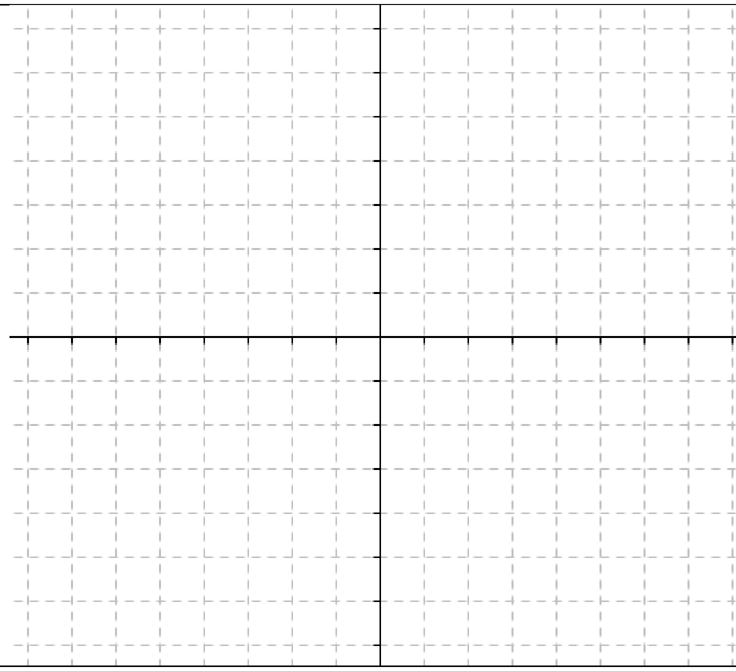
domain:

range:

H. asymptote:

y-int:

x-int:



2.

$y = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^x$

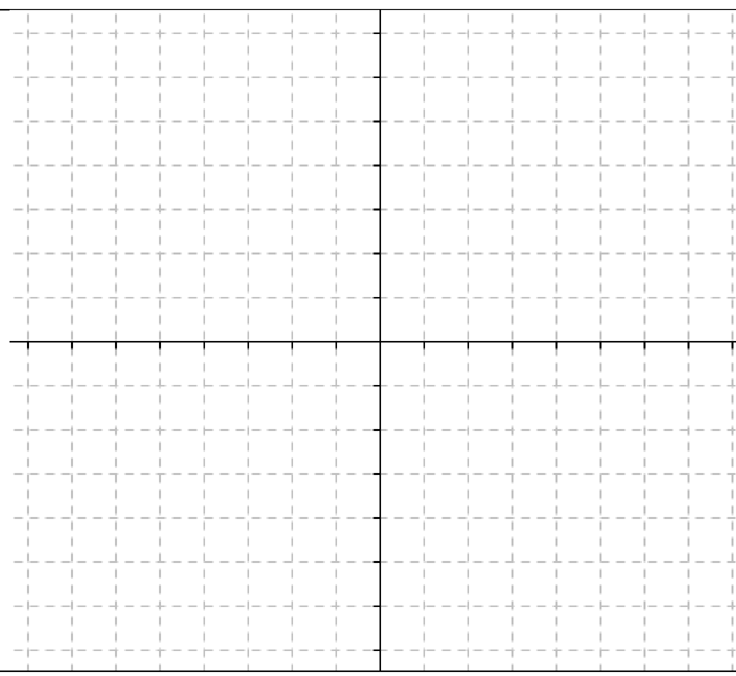
domain:

range:

H. asymptote:

y-int:

x-int:



3.

$y = -\left(\frac{1}{3}\right)^x$

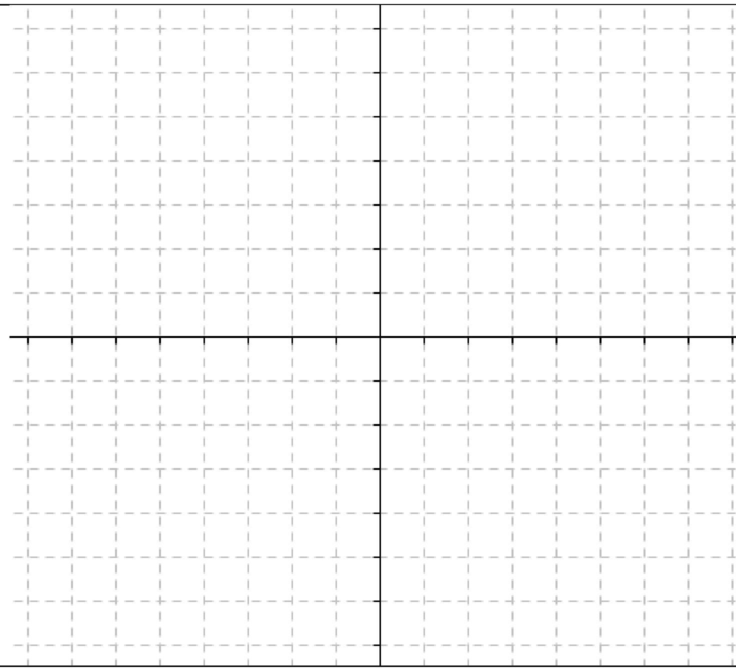
domain:

range:

H. asymptote:

y-int:

x-int:



4.

$y = -2 \cdot \left(\frac{1}{3}\right)^{x+2}$

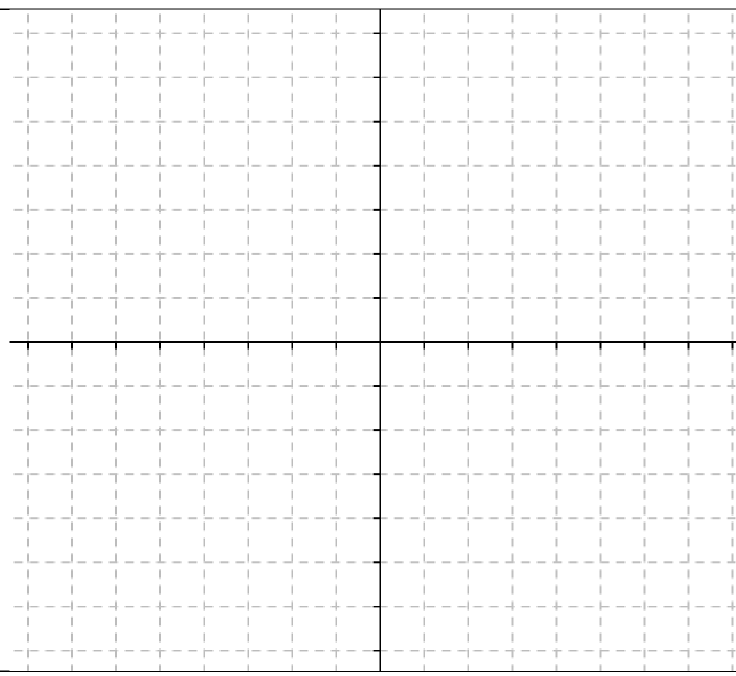
domain:

range:

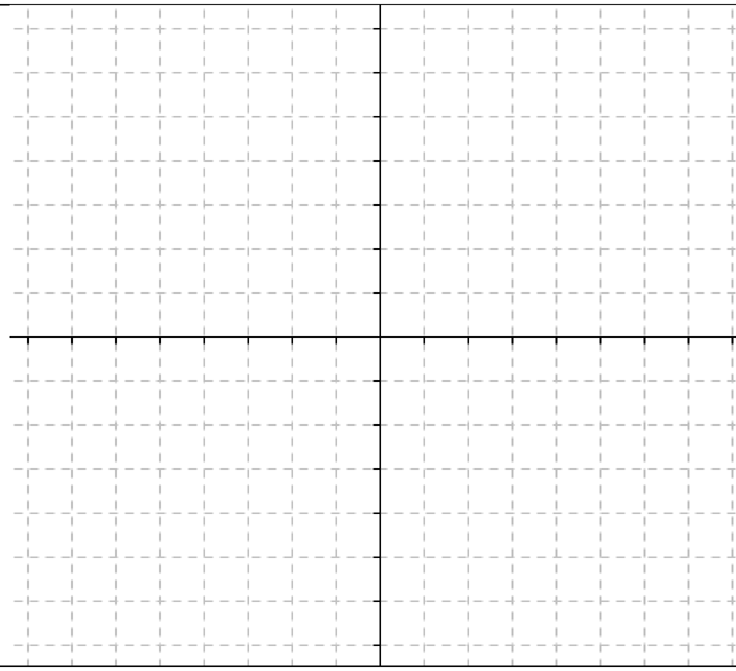
H. asymptote:

y-int:

x-int:



5.

$y = 6 \cdot \left(\frac{1}{3}\right)^{x+3} - 2$ domain: range: H. asymptote: y-int: x-int:	

Topic 12 The Natural Base e

Topic Objective: In this topic students will observe a very familiar model, compound interest. And learn the natural base discovered by Euler.

Prerequisite: Graphing exponential growth functions

The Lesson

Compound Interest:

When borrowing money (or saving money to a saving's account), the bank may calculate your principal in two different models: simple interest or compound interest. The major difference between these two models is the compound interest model count your earned interest as part of your principal. This might sound a little bit complicated, let us figure out by using the table below:

Say, you have $1,000 and would like to save into a saving's account. The bank agrees to pay you a 6% APR (annual percentage rate) for your account, and the bank also agrees to calculate your interest monthly. What is the balance (after 12 months)you will have if the account accrues interest using (a) simple interest model? (b) compound interest model?

Before we go too far, let us understand how a bank calculate your interest rate:

When a bank says it will pay 5% APR that means, in a year, you will get 6% or how much you invest as a reward, so, if we calculate this interest rate monthly, every month your interest rate should be $6\% \div 12 = 0.5\%$. Now compare to different models

Principal (P) = $1,000

Simple Interest model

Month (T)	Interest earned up to this cycle I = PxRxT R=0.5%	Balance after adding interest A=P+I
0	0	1000
1	1000x.5%x1=5	1005
2	1000x.5%x2=10	1010
3	1000x0.5%x3=15	1015

at 12^{th} cycle (1 year) I=1000x0.5%x12 = 60

Your balance after a year = $1,060.00

Principal (P) = $1,000

Compound Interest model

Month (T)	Balance after adding interest $A = P(1+R)^T$
0	1000
1	$1005 = 1000(1+0.5\%)^1$
2	$1010.03 = 1000(1+0.5\%)^2$
3	$1015.08 = 1000(1+0.5\%)^3$
4	$1020.15 = 1000(1+0.5\%)^4$

at the 12^{th} cycle (1 year) =

$1000(1+0.5\%)^{12} = 1061.68$

Using this calculation, we can easily find out how much you will have for the $1,000 in 5 years

years	1	2	3	4	5
cycles	12	24	36	48	60
Simple	1060.00	1120.00	1180.00	1240.00	1300.00
Compound	1061.68	1127.60	1196.68	1270.48	1348.85

Historically, a similar question was studied by Jocab Bernoulli:

An account starts with $1.00. with APR=100%, What is the Balance after a year if this account compound (1) yearly? (2) monthly? (3) daily? (4) hourly? (5) continuously..

Using compound interest model,

$$A = P(1+\frac{r}{n})^{nt} \ ,$$

the P = 1, r=1, so the return after a year will be

$$A = (1+\frac{1}{n})^{n}$$

n is how often the account will compound:

n	1	12	365	365x24=8760	∞
A	2	2.613	2.715	2.718	?

Leonard Euler found this number $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = 2.718281828...$

Mathematically, Euler Number e is very important, since it is related to a lot of many scientific models The exponential function using e as the base is called natural base functions. At algebra 2 level , students need to know how to graph the functions (in aide of calculators)Example 1: Manipulating e: (rewrite each expression so that final answer become e to the x-th power)

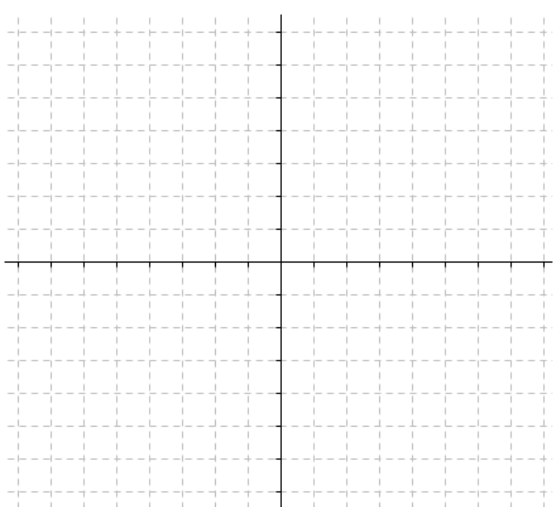
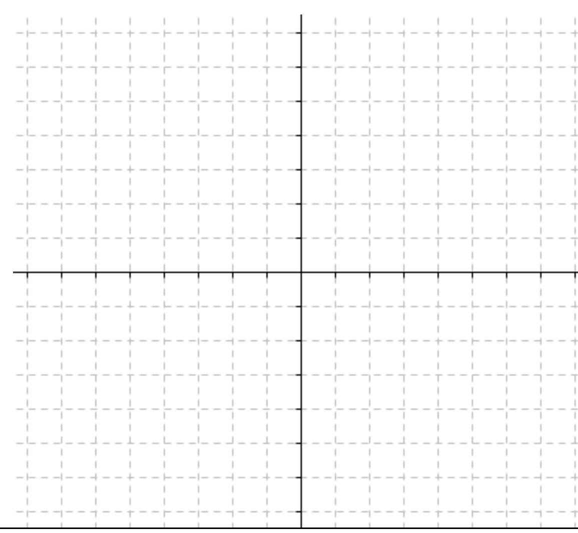
1. $e^2 \cdot e^x$	2. $\frac{\sqrt{e}}{e^2}$	3. $\left(e^2\right)^3 \cdot \left(2x\sqrt{e^3}\right)^2$

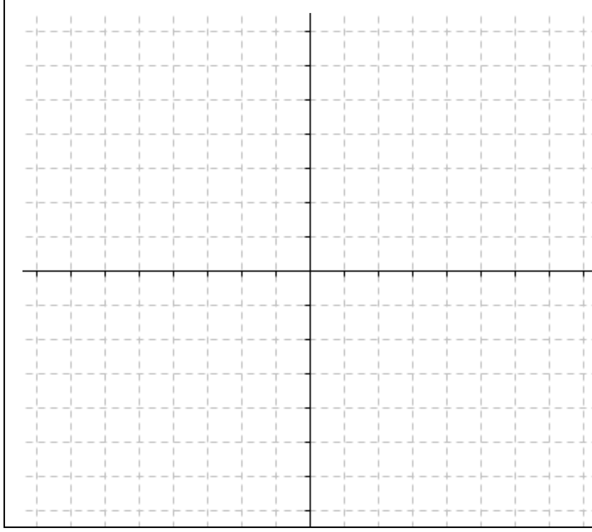
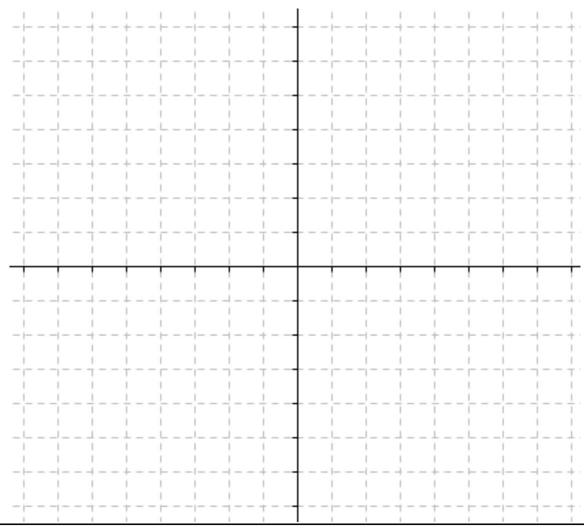
Example 2:

If a credit card charged 26% APR, and Jeffery carries a balance of $2,000 at the beginning of the year. Compute the balance at the end of the year if Jeffery did not pay the balance in a year and the interest of the credit compound monthly.

Example 3:

Graph the following functions

1. $y = e^{x}$ 	2. $y = -e^{x-1}$ 

3. $y = e^{-x}$ 	4. $y = \dfrac{1}{2}e^{x-2} + 1$ 

Topic 13 Exponential Models

Topic Objective: In this topic, students will learn how to find an exponential model based on the given information or graph.

Prerequisite: solving exponential equations

The Lesson

In this lesson, you will use the exponential model $y = ab^x$, to find the correct exponential relations in different situation..

Example 1:

An exponential model passes through (1, 12) and (3, 108), Find the exponential function?

Example 2:

The height of a bouncing ball can be model by an exponential model $h = ab^n$, where h is the height (ft) of the ball after the nth bounce. At the second bounce, the height was recorded at 5.14 ft and at the fourth bounce the height was recorded at 3.78 ft.

(a) what is the original height?

(b) what is the height at the 5^{th} bounce?