[Collect 40 points for 100%, total possible 57 points for 105%]

[Basic function]

1. (3 points) Given $f(x)=4+\frac{1}{2}x^2$, if $g(x)=-x+b$ where $b>0$ has exactly one intersection. Find b.

2. (3 points) Find the point of intersection in question 1 algebraically.

3. (3 points) Graph both functions on the same coordinate plane. Verify your solution from question 2 with functions the graph.

[Matrices]

4. (3 pts) Let $A=\begin{pmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 1 & 1 & 3 \end{pmatrix}$ and

$I=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

If λ is a real number and $\det(A-\lambda I)=0$, Find λ.

5. (6 pts) For every solution of λ, there exists at least one non-trivial (means, no all elements are zeros) matrix $v=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, such that

$(A-\lambda I)v=0$.

Find 3 matrices v_1, v_2, v_3 corresponding to 3 different solutions $\lambda_1, \lambda_2, \lambda_3$ from question 4.

6. (9 pts) Find A^{-1}

[DeMoivre Theorem]

7. (12 pts) Given c_1, c_2, c_3 and c_4 are the unique fourth roots of 16. If $\theta_1, \theta_2, \theta_3$ and θ_4 are the arguments for each roots when written in the trigonometric form and

$\theta_1<\theta_2<\theta_3<\theta_4$. Evaluate $\frac{c_1+c_2}{c_3-c_4}$

[Partial Fraction Decomposition]

8. (9 pts) Let $f(x)=\frac{4x^2-14x+4}{\left(x^2-x\right)\left(x^2-4\right)}$. If $f(x)$ can be uniquely written into the form of $f(x)=\frac{a}{x}+\frac{b}{x-1}+\frac{c}{x-2}+\frac{d}{x+2}$ where a,b,c and d are real numbers. Evaluate $\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

9. Let $g(x)=ax^3+bx^2+cx+d$ where a,b,c and d are from question 8.

(a) (3 pts) Find the possible x-intercepts

(b) (1 pt) Find the y-intercept

(c) (2 pts) Describe the ending behaviors of g(x)

(d) (2 pts) Evaluate the points in the following table

x	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2
g(x)				

(e) (1 pt) Sketch $g(x)$ on a coordinate plane with information from (a)-(d)