Class Discussion

Unit 2 Topic 2 Behaviors of a polynomial

Ex0:

Identify polynomials from the following functions

A	$f(x)=\frac{1}{5}$
B	$f(x)=x^2+x^{-2}$
C	$f(x)=x^2+x^3+x^4+x^5$
D	$f(x)=\frac{1}{x+1}-\frac{x}{x^2+1}$
E	$f(x)=\sqrt{x}+\sqrt[3]{x}$

Algebraic descriptions of a polynomial: polynomials only have terms with variables to an exponent that a whole number(exponents n = $\{n \mid n \in \mathbb{N}+0\}$)

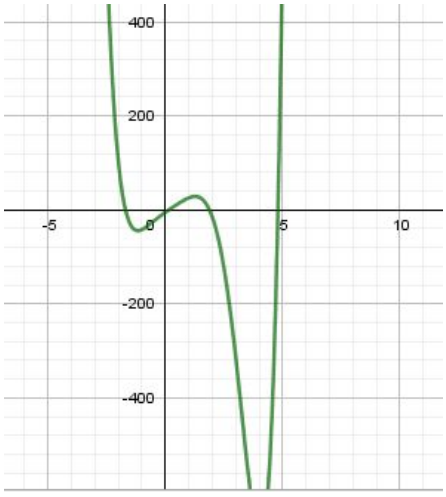
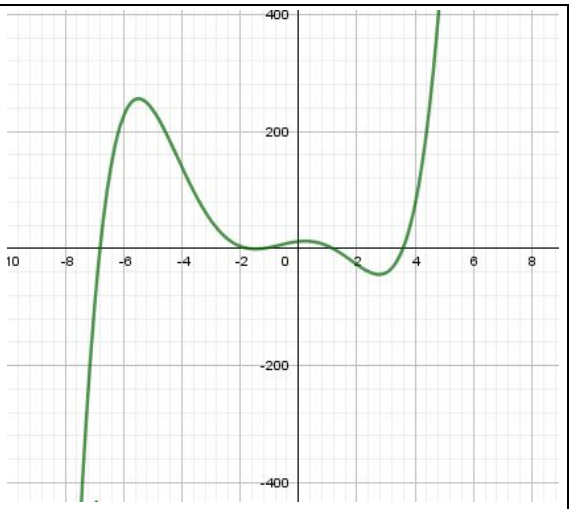
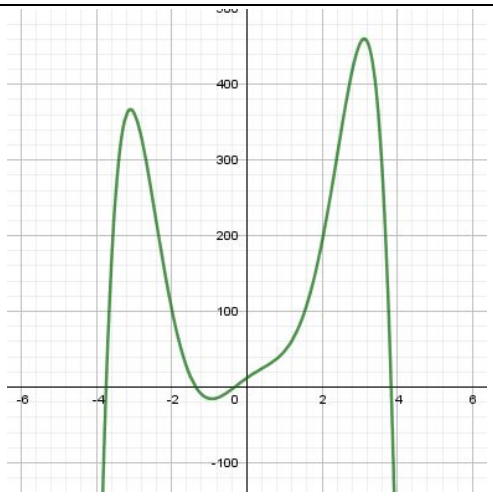
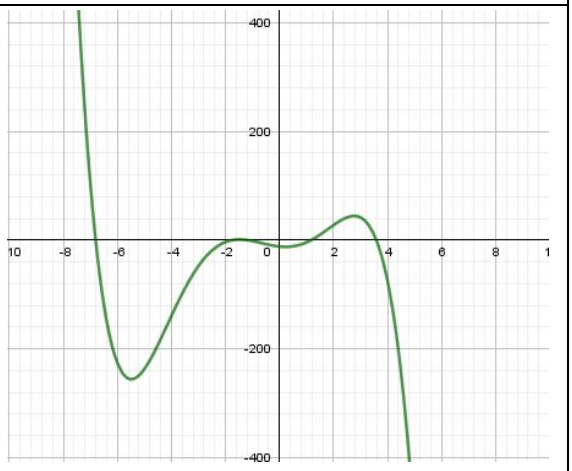
General description of a polynomial: smooth and continuous

Besides the general description of polynomials, we describe a polynomial with the following two characteristics:

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_3x^3+a_2x^2+a_1x+a_0$$

1. Ending Behaviors ($f \to ? \text{ if } x \to \pm\infty$)

a_nx^n	n is even	n is odd

$a_n > 0$	 $x \to \infty, f \to \infty$ $x \to -\infty, f \to \infty$	 $x \to \infty, f \to \infty$ $x \to -\infty, f \to -\infty$
$a_n < 0$	 $x \to \infty, f \to -\infty$ $x \to -\infty, f \to -\infty$	 $x \to \infty, f \to -\infty$ $x \to -\infty, f \to \infty$

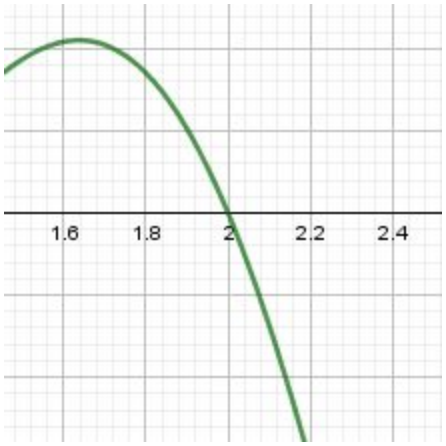
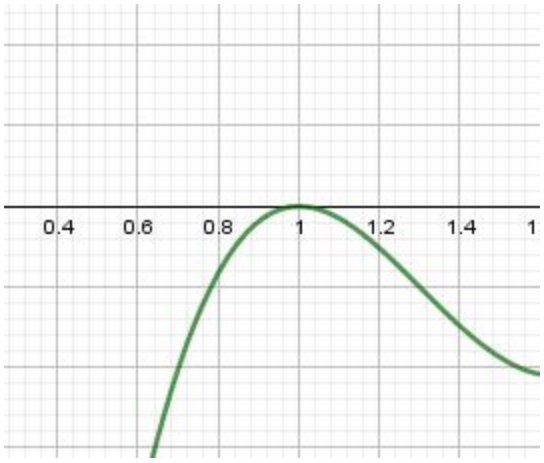
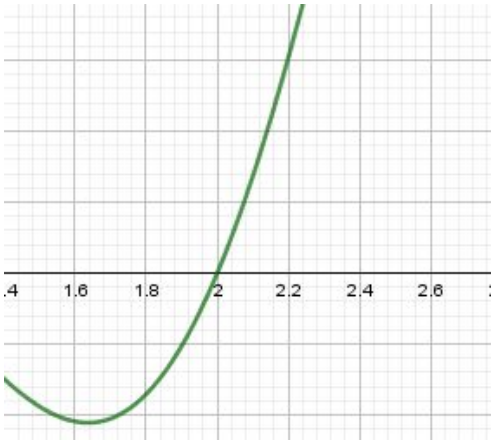
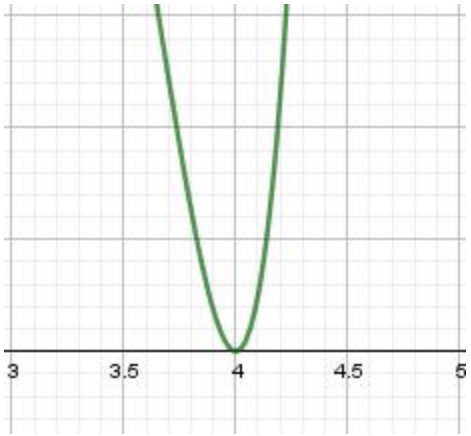
2. Behaviors near zeros(multiplicity)

If $f(x) = a(x-\alpha_1)^{k_1}(x-\alpha_2)^{k_2}\cdots(x-\alpha_m)^{k_m}$ then f has m distinctive zeros, however, the degree of f is $k_1 + k_2 + \cdots + k_m$

Multiplicity:

k_i is the multiplicity of a zero of f @ $x = \alpha_i$

k_i	Odd (Cross)	Even (Touch)

	from + to – 	from – to – 
	from – to + 	from + to + 

Example: Let $f(x) = (x-1)(x^2-5x+4)+(x+1)^2$

(a) describe the Ending behavior

(b) describe the behavior near a real zero

(C) use (a) and (b) to sketch $f(x)$