Unit 3 Assessment

1. (4 points) Solve the exponential equations:

$$\begin{cases} 2^{x+y} = \dfrac{1}{2} \\ \dfrac{9^x}{3^y} = \dfrac{1}{81} \end{cases}$$

2. (4 points, pick 1) In this question, e is the Euler number, (the base of the natural logarithm)

(a) Evaluate $1+\cfrac{\sqrt{e}}{1+\cfrac{\sqrt{e}}{1+\cfrac{\sqrt{e}}{1+\cfrac{\sqrt{e}}{\cdots}}}}$

(b) Assume that x is a real number, solve for x if

$$6e^{\frac{x}{2}} = 5 + 4e^{-(\frac{x}{2})}$$

3. (4 points, pick 1) Solve the following logarithmic equations (x is the variable)

(a) $\log_{\frac{1}{2}}(x-4) = \log_{\frac{1}{4}}(7x-50)$

(b) $2\log_6(2x+1) = \log_{36} 4 + \log_6(6x-1)$

4. (4 points, pick 1) For the following question assume

$\log 2 = a$, $\log 3 = b$, $\log 7 = c$ and $\ln 10 = d$

Rewrite the logarithmic expression with a, b, c and/or d

(a) $\log_2 \dfrac{e}{\sqrt{35}}$

(b) $\log_{\frac{e}{\sqrt{2}}}(6.25)$

5. (8 points) Condense the following expression into a single logarithmic expression to the assigned base. Your condensed logarithmic expression need to be reduced to the simplest form with radical removed, if the argument is rational expression with a radical

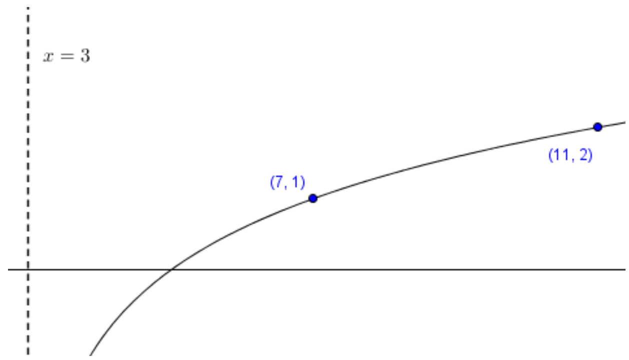
(change to base 4)

$$\log_2 \sqrt{(x+2)^3} + \log_{\frac{1}{4}}\left(x^3 + 2x^2 - 4x - 8\right)$$

6. (4 points, pick 1)

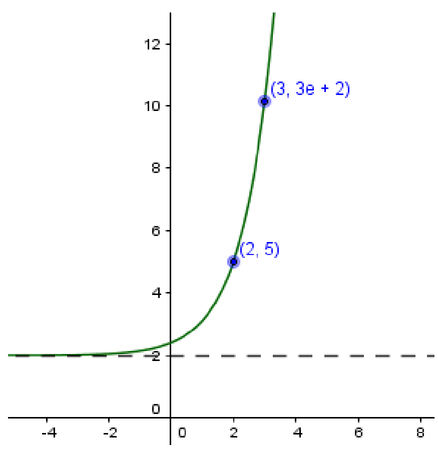
Use the graph to find the indicated function

(a) The graph shown is a logarithm function. x=3 is the vertical asymptote. Two given points that the function passes through are (7, 1) and (11, 2)



(b) The graph shown is an exponential function

y = 2 is the horizontal asymptote. Two given points that the function passes through are $(3, 3e+2)$ and (2, 5)



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7.(8 points, a = 4, b = 2, c = 1, d = 1) **(Cooling)** Newton's Law of Cooling describes the way the temperature of an object adjusts to the ambient temperature over time. If the following relationship,

$$t = -8.58\ln\left(\frac{G-67}{65}\right)$$

describes the temperature ($^{\circ}F$) of a Starbucks grande latte, G, over time, t, (in minutes) after the barista called out the drink, answer the following questions:

(a) Rewrite G from the given relationship, so that it is a function of t. ($y = G(t)$)

(b) Graph the cooling function (Temperature, G (y axis) – Time, t (x-axis)) for the first 20 minutes. [evaluate at every 5-minute interval]

(c) Use $G(t)$ to find the temperature of the latte when the barista called the drink and also find the Starbucks store's thermostat setting for the store.

(d) Johnny always think that Starbuck's latte is too hot for him. He likes to wait for a little while until the temperature of the latte is below $125^{\circ}F$ to enjoy it. If you are the barista, how long in time (round to whole minute) will you suggest Johnny to **at least** wait?

8. (6 points, a = 1, b, 2, c = 3) **(Yeast Growth)** Following model represents a yeast population when a sour dough bread was rising:

$$Y(t) = \frac{90}{1+23e^{-0.95t}}, t \geq 0$$

Where t represents the time (in hours), Y(t) represent the numbers of yeast in millions.

(a) What is the initial population when the yeast was just added into the dough? (Round to the tenth million)

(b) Make a graph of Y(t) over the whole domain [0, 10] by evaluating the yeast population at **every two hours**.

(c) Now, final population of the yeast in the dough is the population of the yeast after it has been risen for a long time and if the dough is considered fully risen when the population of the yeast reaches 95% of the final population, based on the model, **at the latest** when (provide your reasoning and state your answer in terms of time, round your answer to the whole minute) should a baker start to raise the sourdough bread if the store opens at 9:00 am very morning and it takes about 1 hour and fifteen minutes to bake the bread, and about 30 minutes to cool off before it can be served? (Assume the baker serves the bread when its door opens)