Practice Test, Unit 7 Assessment

[Basic function, #1-3, pick 2]

1. Given $f(x)=4-x^2$, if $g(x)=-x+b$ where $b>0$ has exactly one intersection. Find b.

2. Find the point of intersection in question 1 algebraically.

3. Graph both functions on the same coordinate plane. Verify your solution from question 2 with functions the graph.

[Matrices, do #1 and pick 2 from #2 - #5]

1. Let $A=\begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ and $I=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

If λ is a real number and $\det(A-\lambda I)=0$

Find λ.

2. For every solution of λ, there exists at least one non-trivial (means, no all elements are zeros) matrix $v=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, such that $(A-\lambda I)v=0$.

Find 3 matrices v_1, v_2, v_3 corresponding to 3 different solutions $\lambda_1, \lambda_2, \lambda_3$ from question 1.

3. Find A^{-1}

4. Find $\left(A^{-1}\right)^2$

5. Find $\left(A^2\right)^{-1}$

[System of Equations and vectors, pick 2]

1. Solve system of equations

$$\begin{cases} \dfrac{1}{m+1}+\dfrac{1}{n-2}-\dfrac{1}{k+3}=1 \\ \dfrac{-m}{m+1}-\dfrac{n-1}{n-2}+\dfrac{2}{k+3}=5 \\ \dfrac{m+2}{m+1}+\dfrac{n-3}{n-2}+\dfrac{k+1}{k+3}=-6 \end{cases}$$

2. Assume vector $\vec{u}=m\vec{i}+n\vec{j}$ and $\vec{v}=k\vec{i}+\vec{j}$, m, n and k are from questions 6. Find the angle between vector $\vec{u}$ and $\vec{v}$

3. What is the area of the triangle formed by the origin, end point of vector $\left(\vec{u}+\vec{v}\right)$ and $\left(\vec{u}-\vec{v}\right)$

(u and v are from question 2)

[De Moivre Theorem, pick 1]

1. If the cubic root of a complex number $z=\sqrt{2}i$ are c_1, c_2, c_3, find $\dfrac{c_1+c_2}{c_3}$ if c_3 is the cubic root that does not have the real part.

2. Given $f(x)=x^5+x^4+3x^2-x+2$, if $x=i$ is a zero of $f(x)$,

(A) find all other zeros for $f(x)$

(B) Graph all zeros on a complex plane.

(C) Let z_1, z_2, z_3, z_4, z_5 be all the zeros of f(x) in its trigonometric forms. $\theta_i, i=1,2,3,4,5$ are the arguments for each zero, if $\theta_1<\theta_2<..<\theta_5$, evaluate the exact value of $z_3^{\,4}+z_1^{\,6}+z_4^{\,8}+z_2^{\,10}+z_5^{\,12}$

[Trigonometric equation, pick 1]

1. $x \in [0, 2\pi)$,

Solve $\sin\left(\frac{x}{2}\right) = \cos x - 1$

2. Given $x \in [0, \pi)$,

Solve $\cos 2x = \sin x$ (exact value)

[Partial Fractional Decomposition]

1. Let $f(x) = \frac{x^3 - 7x + 8}{(x^2 - x)(x^2 - 4x + 4)}$. If $f(x)$ can be uniquely written into the form of $f(x) = -\frac{a}{x} + \frac{b}{x-1} + \frac{c}{x-2} + \frac{d}{(x-2)^2}$ where a, b, c and d are real numbers. Evaluate $\sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$

2. Let $g(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are from question 1.

(a) Find the possible x-intercepts

(b) Find the y-intercept

(c) Describe the ending behaviors of g(x)

(d) Evaluate the points in the following table

x	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1
g(x)				

(e) Sketch $g(x)$ on a coordinate plane with information from (a)-(d)

[Applications]

<u>Forest Fire</u>

Two watch owers spotted the same forest fire with bearings N 42° E (from tower A) and N 45° W (from tower B). If two watch towers are 12 miles apart, and the bearing of tower A from tower B is S 75° W. If the rescue center C is 7miles away from tower B and the bearing of center C from tower B is S 38° W

Find the bearing a helicopter pilot should set from center C to the fire. If the average speed of the helicopter is 40 mph, how long in time would it take the helicopter to reach the fire?

<u>Height of a Tree</u>

A tree is on a hillside of slope 28° (from horizontal). 75 feet downhill from where the tree is, the angle of elevation at the top of the tree is 45° . Find the height of the tree.

(Yeast Growth)

Following model represents a yeast population when a sour dough bread was rising:

$$Y(t) = \frac{180}{1 + 35e^{-.85t}}, t \geq 0$$

Where t represents the time (in hours), Y(t) represent the numbers of yeast in millions. And the domain is called the rising period.
(a) What is the initial population when the yeast was just added into the dough?
(b) Make a graph of Y(t) over the domain $t \in [0, 10]$ by evaluating the yeast population at whole hours.
(c) What is the final population of the yeast in the dough? (when $t \to \infty$)