EVHS Algebra 2 Unit 7 Handout

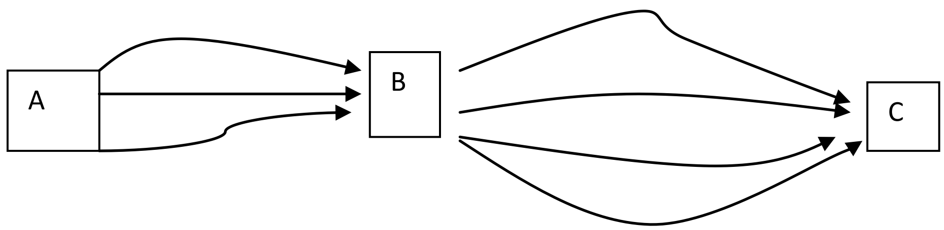
Contents

Topic 1 Fundamental Counting Principle

Objective: Student will explore the fundamental counting principles through different examples

The Lesson

Amanda who lives in city A wants to visit her friend Joy who lives in City C. From City A she has to first pass City B to City C. Following is the map from City A to City C



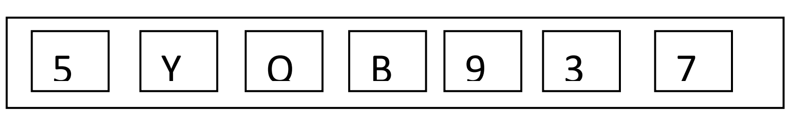
(1) How many possible routes for Amanda to take from A through B to C?

> Fundamental Counting Principle
>
> If there are m different ways to do one thing (from City A to City B, for example), and n different ways to do the other thing (from City B to City C, for example), then there are $(m \cdot n)$ ways to do both. (from A to B ***and*** from B to C)

(2) After the visit, she is going home many different choices will Amanda to return to City A from City C through city B?

(3) If Amanda decides to go back to City A on different routes than how she comes to visit Joy, how many choices does she have to return back to City A?

Example 2: A normal (not special made) California License plate has a Format of the following



With the first, 5^{th}, 6^{th} and 7^{th} number to be a digit from 0 – 9 and the second, 3^{rd} and the fourth places to be letters

(1) How many different combinations are possible for a license plate?

(2) if Tommy choose to have license plate with all different numbers/letters on his plates, how many possible combinations are there for him to choose?

(3) Chinese/Taiwanese people like numbers in the sequence of 888 or 168 on their license plates. If Mr. Chen (who is a Taiwanese) would like to get a new license plate for his new car with either 888 or 168 in the last 3 places, how many different license plates he may choose?

Example 3: At a pizza my heart, there is a soda machine that dispenses different drinks with different flavors. This particular one in San Jose downtown gives the customers choices soda/soft drinks of Coca Cola, Diet Coke, Coke Zero, Lemonade, Lemonade light, Dr. Pepper, diet Dr. Pepper, Root beer and diet Root beer. Every drink also offers different flavors of syrup, such as raspberry, strawberry, lime, and orange; and strawberry and lime syrup also provides sugar free versions for the customers to choose. When a person try to ask for a drink, he/she has to first select a drink and then select a syrup flavor. (one may also order a drink without added syrup by pressing on the button of a drink twice) Use the information to answer the following questions:

1. How many different soft drinks can this machine dispense?

2. Mr. Chen brings his son and daughter to the pizza my heart for dinner. After ordered the pizza for his children, he decides to order a drink for them to share; however, he does not like to give his kids sugary drinks. How many different drinks can he order for his kids?

3. Caffeine Free is very important for Mr. Jonah. How many different caffeine free drinks can this machine dispense?

4. Ms. Chen is on a sugar free and caffeine free diet. How many different drinks from the machine are caffeine free and sugar free?

Topic 2 Permutation

Objective: Students will learn what is permutation and how permutation works when there are repetitions.

The Lesson

Define: Permutation

An ordering of n objects is a permutation of the objects. For example :

There are 24 permutations of ABCD.

ABCD	ADBC	BCAD	CABD	CDAB	DBAC
ABDC	ADCB	BCDA	CADB	CDBA	DBCA
ACBD	BACD	BDAC	CBAD	DABC	DCAB
ACDB	BADC	BDCA	CBDA	DACB	DCBA

Example 1: 5 toy cars(A, B, C, D, E) race on a tournament. how many different outcomes can there be?

Example 2: 5 toy cars(A, B, C, D, E) race on a tournament. how many different outcomes can there be for the first place, the second place and the third place?

Using Permutation Notation:
(When there is no replacement and order matters)
The permutation of taken r objects from n distinctive objects (in our example, the first 3 places, 1^{st}, 2^{nd}, and 3^{rd} places from 5 toy car race) can be calculated by

$${}_nP_r = \frac{n!}{(n-r)!} \quad , \ P(\mathrm{n,r}) = {}_n\mathrm{P}_r$$

Practice Factorials:

define $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

$n! = n \times (n-1)!$ [This expression is true for all $n \geq 1$]

⇨ $0! = 1$

1. 5!	2. 10!	3. 6!

Practice Permutation notation:

4. ${}_6P_4$	5. ${}_3P_3$	6. ${}_5P_2$

Example 3:

A bank pin number is a 4 digit number. Each digit can be one of the numerals from 0 – 9.

(1) how many different pins a person may choose?

(2) if the bank set a restriction and require that no digit in a pin is the same. How many different pins can a customer choose?

Example 4:

(1) How many distinguishable permutations are in the word “TEA”?

(2) How many *distinguishable* permutations are in the word “PEET”?

(3) How many *distinguishable* permutations are in the word “ABBA”?

Numbers of distinguishable permutations with repetitions:

The total number of distinguishable permutations of K elements with the 1st element repeated n_1 times (if the element appears only 1 time, then $n_1 = 1$, if it appears two times, then $n_1 = 2$ …) and the 2nd element repeated n_2 times, … and the kth element repeated n_k times is

$$\frac{n!}{n_1!n_2!...n_k!}$$

and

$$n = n_1 + n_2 + ... + n_k$$

Example 5: How many distinguish permutations are in the word “ABRACADABRA”?

Topic 3 Combination

Objective: students will learn what combination is and how to use combination to apply to some practical experiences in life.

The Lesson

Definition:

Combination: An un-ordered collection of elements selected from some set.

Example: In a Set A={a, b, c, d}. List all permutations with 3 elements. List all combinations with 3 elements.

permutations/combinations of 3 elements of A

permutations (24)	combinations (4)
{a, b, c}, {a, c, b}, {b, a, c}, {b, c, a}, {c, a, b}, {c, b, a}	{a, b, c}
{a, b, d}, {a, d, b}, {b, a, d}, {b, d, a}, {d, a, b}, {d, b, a}	{a, b, d}
{a, c, d}, {a, d, c}, {c, a, d}, {c, d, a}, {d, a, c}, {d, c, a}	{a, c, d}
{b, c, d}, {b, d, c}, {c, b, d}, {c, d, b}, {d, b, c}, {d, c, b}	{b, c, d}

Use this example to derive the combination formula:

Given:

$$\text{Total permutations} = (\text{\# of unique groups}) \times (\text{\# of permutations for a unique group})$$

Now in our case:

1. Total permutations of 4 objects (a, b, c ,d) taken 3 at a time is: $_4P_3 = \dfrac{4!}{(4-3)!}$

2. Every 3 unique elements group can have $\underline{3!}$ permutations

3. Therefore, # of unique groups(which is what we called combination) = $\dfrac{_4P_3}{3!} = \dfrac{\frac{4!}{(4-3)!}}{3!} = \dfrac{4!}{3!(4-3)!}$

# of combinations of n objects taken r object at a time (or # of unique group for r objects in a n object pool) is $$_nC_r = \frac{n!}{r!(n-r)!}$$

It is true that ${}_{n}C_{0}=1$ (explain what does this mean?) ___________________________________

Example1: Jonah is preparing a fruit basket from his backyard. Jonah grows strawberries, apples, pears, and oranges in his backyard, how many different baskets can he prepare if he decides to put 3 different kinds of fruits in a basket?

Example2: Hannah has a 24-color pencil package, 16 of those colors are regular colors and the other 8 are off-regular colors. She wants to take some on a field trip. Mama says she is allowed to have 4 regular colors and 3 off-regular colors on her trip, in case she lost them. How many different ways can she bring her color pencils to the field trip?

Example 3: Texas Holdem is a Poker gram, a person deals a hand of 5 cards from a total of 52 (4 different suits: spades, hearts, diamonds and clubs with 13 faces, A, K, Q, J, 2, 3, 4, …, 9)

(a) how many different poker hands are possible?

(b) In how many hands does one have four-of-a-kind?

(c) In how many hands does one have three-of-a-kind?

(d) In how many hands does one have a full-house?

(e) In how many hands does one have a flush (including straight flush)?

(f) In how many hands does one have a flush but not straight flush?

Topic 4 Binomial Theorem and Combination

Objective: In the topic, student will connect the combination and the coefficient of binomial expansions

The Lesson

Binomial expansions: Following are some common binomial expansions,

power of a binomial	multiple form	end result
$(a+b)^2 =$	$(a+b)(a+b)=$	$a^2+2ab+b^2$
$(a+b)^3 =$	$(a+b)(a+b)(a+b)=$	$a^3+3a^2b+3ab^2+b^3$
$(a+b)^4 =$	$(a+b)(a+b)(a+b)(a+b)=$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$

Here are some interesting fact about a binomial expansion $(a+b)^n$:

1. The coefficient of each term of a binomial expansion is ${}_nC_r$ [therefore, the number of combination can also be called coefficient of binomial expansion]

[Video of the topic will explain why the coefficient of a term in a binomial expansion is exactly ${}_nC_r$]

2. The degrees of each term (adding the powers of both variables) = n.

3. Each term of a binomial expansion (in decending order of degrees of a) = ${}_nC_r a^{n-r} b^r$

Example 1: Expand the following expressions (or use binomial expansion principles to non-binomials)

1. $(x-2y)^4$	2. $(2a+3b)^4$	3. $(x-\frac{1}{y})^3$
4. $(a+b-\frac{1}{a}-\frac{1}{b})^3$	5. $(x^3-2y)^5$	6. $(x+y-z)^4$

Example 2:

(1) Find the coefficient of x^6 of $(2x+3)^{10}$

(2) Find the coefficient of x^2b^4 of $\left(\frac{2}{3}x-\frac{3}{4}b\right)^6$

Topic 5 Introduction to set operations

Objective: students will learn what set operations are and use them on the probabilities for the coming topics

The Lesson

This topic is not in the textbook, however, it is important that these notations and terminologies are introduced. The text book uses some of the notations and terminologies without explanations. (section 10.3, 10.4 and 10.5)

Definitions

- Set: a collection of objects, called elements, or members.
  - A={ a, b, c, d} is a set of four objects. S={ all students in the classroom}
    - elements of a set is normally in lower case,
    - the order of the elements does not matter
    - every element should be unique
- Some set relations:

Notation	Read	Meaning
A=B	A equals B	Set A and B have exactly same members
$x \in A$	x is in A (x belongs to A)	Element x is in set A
$x \notin A$	x is not in A	x is not an element of A
$A \subseteq B$	A is a subset of B	Every element of A is also an element of B
$\varnothing$	empty set	{}, a set with no members.

  - Universal set (U): a set of all possible elements.

Example 1: Given the universal set U={1, 2, 3, 4, 5, 6, 7, a, b, c, d, e, i, o, u, z}

A={x| x is a number when rolling a die}

B={a, e, i, o, u}

C={a, b, c, d, e, i, o, u, z}

D={3, 4, 5, 6, 1, 2}

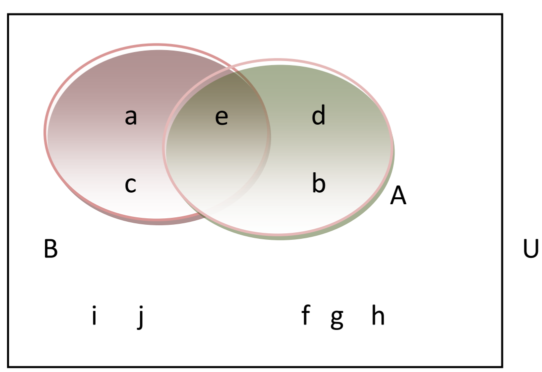
E={1, 3, 5, 7}

Then following are true: $B \subseteq C$, $3 \in A$, $e \in B$, D=A, $7 \notin A$

More relations regarding sets:

Notation	Reads	meaning.. and use the sets from example 1
$A \cup B$	A union B	A set of all elements in A OR B $A \cup B$ ={a, e, i, o, u, 1, 2, 3, 4, 5, 6}
$D \cap E$	D intersect E	A set of all elements in D AND E simultaneously. $D \cap E = \{1, 3, 5\}$
$\overline{C}$	complement of C (or simply C bar)	A set of all elements in universe except in C $\overline{C} = \{a, \mathrm{b, c, d, e, i, o, u, z}\}$
$A \cap B = \varnothing$	A disjoints B	A and B has no elements in common. We can also say they are mutually exculsive.
$\lvert A \rvert$	cardinality of A	total number of elements in set A, in example 1 $\lvert A \rvert$ =6

Example 2:

Let U={a, b, c, d, e, f, g, h, i, j} and S= {a, b, d, g, i} G={b, c, e, f, g} H={a, d, j} (1) $S \cup G$ (2) $S \cap G$ (3) $S \cap \overline{G}$ (4) $G \cap H$ (5) $(G \cup H) \cap S$ (6) $G \cup (H \cap S)$	For questions on the left, find its solution and the corresponding Venn-Diagram. Like this example here: A={b, e, d} B= {a, e, c}  $A \cup B = \{a, \mathrm{b, c, d, e}\}$

Topic 6 Theoretical probability

Objectives: Students will learn the definition of probability and apply the set operation to it.

The Lesson

Defintions:

- experiment: any repeatable activity that has well-observed outcomes
    - Ex: rolling a die.
    - Ex: pick a card from a deck.
- Sample Space: The set of all possible outcomes, normally denote as U, universal set.
    - Ex: sample space of rolling a die:
    - Ex: sample space of pick a card from a deck:
- Event: a subset of the sample space of an experiment.
    - Ex: an event of rolling a die:
    - Ex: an event of picking a card from a deck:

<u>Example 1</u>:

An experiment of flip a coin and then roll a die: Identify the Sample space, U.

Use {H (for head), T (for tail)} to represent the outcomes of flipping a coin

Use {1, 2, 3, 4, 5, 6} to represent the outcomes of rolling a die

<u>Example 2:</u> (continue from example 1)

Use set notation to represent the following events as subsets and count the cardinalities for each subset:

(a) the outcomes are all heads.

(b) John rolls no more than 3.

(c) James rolls a 7.

(d) Julie flips a tail and rolls a 2

<u>Example 3:</u> Let E be the events of rolling a 4 or higher. Let F be the event of flipping the coin in tails.

Find the result as well as translate the result in word for

(a) $E \cup F$ (b) $E \cap F$

Define 3 more terms:

- random experiment: the outcome of an event in the experiment cannot be predicted
- relative frequency: if an experiment is repeated n times, and of those trials, the event E occurs m times, (of course $m \le n$) then the relative frequency of the event E is $\frac{m}{n}$
- equal-likelihood: Two outcomes of a random experiment are said to be equally-likely if they tend to occur with the same relative frequency, when the experiment is repeated many times.

Ok, after all, we are at the place to define what is **the probability in a simple random experiment**:

If a random experiment has a finite sample space U (means of size U is a real number), containing only equally-likely outcomes, then the probability of an event E (which is a subset of U) is defined as

$$P(E) = \frac{|E|}{|U|} = \frac{\#\text{ of outcomes favorable to E}}{\#\text{ of all possible outcomes}}$$

Example4: In this example, you will use the experiment described from ex 1 – 3

(a) find $|U|$ of the experiment

(b) the probability of John rolling no more than 3 [refer to Ex2 (b)]

(c) the probability of James roll a 7 [refer to Ex2(c)]

(d)the probability of rolling a 4 or higher or flipped a tail [refer to Ex3 (a)]

Example 5: In a sack, James put in 5 blue marbles, 2 red marbles, and 4 green marbles. What are the probabilities of the following events if James draws …

(a) a blue marble	(b) a red marble	(c) a blue or a red marble	(d) a white marble

Example 6: James now rolls two dice in sequence. What are the following probabilities?

(a) He rolls a double

(b) He rolls a sum of 5

(c) He rolls at least one ace

(d) He rolls a double and rolls a sum of 7

Topic 7 The probability of OR events

Objective: student will learn what the probability of OR events

The Lesson

More from set operations: given any two sets E and F, the following properties are always true:

$|E \cup F| = |E| + |F| - |E \cap F|$ (this can be illustrated by ex1)

Example 1: Following is the result of a survey of the extra-curricular programs of 15 students in a certain class at EVHS

Name	YMCA youth club	Kumon Tutoring
Albert	yes	yes
Ben	yes	
Cathy		yes
Darren		
Erica	yes	
George	yes	yes
Helen		
Isabella	yes	yes
Joanna		
Ken	yes	
Lynn	yes	yes
Michael		
Nick		yes
Olivia	yes	yes
Peter		

(a) E is a set of students who join YMCA. Describe E by enumerating the names. How many students are in YMCA?

(b) F is a set of students who join Kumon tutoring. Describe F by enumerating the names. How many students are in Kumon turtoring?

(c) Draw a Venn-Diagram to describe the survey.

(d) find $|E|$, $|F|$, $|E \cap F|$ and $|E \cup F|$

(e) use the result of (d) to verify $|E \cup F| = |E| + |F| - |E \cap F|$

Example 2: Use data found from Example 1 to answer the questions in this example. San Jose Mercury New wants to learn what students are doing after school and decide to randomly pick students for interview:

(1) What is the probability for the reporter to interview a student that is a member of the YMCA club?

(2) What is the probability for the reporter to interview a student that goes to Kumon tutoring center?

(3) What is the probability for the reporter to interview a student that is a member of YMCA club AND goes to Kumon tutoring center?

(4) What is the probability for the reporter to interview a student that is a member of the YMCA club OR goes to Kumon tutoring center?

Example 3: In a bag, there are 4 yellow marbles marked with number 1, 2, 3, 4 and 5 blue marbles marked with numbers 1, 2, 3, 4, 5.

Event A: draw a marble with a number less than 3.

Event B: draw a blue marble

(1) Describe the event $A \cup B$

(2) Write event A, event B and event $A \cap B$ in set representation

(3) Find P(A), P(B) and P($A \cap B$)

(4) Find P($A \cup B$)

Example 4: Jonah is rolling a pair of dice.

(1) what is the probability that Jonah rolls a double ace?

(2) what is the probability that Jonah rolls a sum of 5?

(3) What is the probability that Jonah rolls a sum more than 10 or a double?

Example 5: In a box of crayons, there are 2 green crayons, 2 yellow crayons and 2 red crayons. Samuel gets 3 crayons out of the box. What is the probability for Samuel to take exactly one green, one yellow and one red if

(1) all 6 crayons in the box are distinguishable (like, 3 crayons (1R,1G,1Y) has Samuel's name on and the other 3 has Hannah's name on them).

(2) crayons with the same colors are not distinguishable.

Topic 8 The probability of NOT events

Objective: students will learn the probability of the complementary of an event

The Lesson

Two more set relations

1. $E \cap \overline{E} = \varnothing$ (any event intersects its complement is an empty set.)

2. $E \cup \overline{E} = U$ (any event unions its complement is the universe)

Example:

if E = { you roll a 3} then $\overline{E}$ ={you do not roll a 3}, it is easy to see that the intersection of these two sets has to be empty and the union of these two sets is all possible events in the universal set.

Now, use the property we learn from last topic

$$P(E \cup \overline{E}) = P(E) + P(\overline{E}) - \mathrm{P}(E \cap \overline{E})$$

From our example we learned:

$$E \cup \overline{E} = U \Rightarrow \text{P(U)=1;}$$

$$E \cap \overline{E} = \varnothing \Rightarrow P(\varnothing) = 0$$

therefore,

$$1 = P(E) + P(\overline{E}) - 0$$

The probability of a complement of any event E is…

The probability of a complementary event is $P(\overline{E}) = 1 - P(\mathrm{E})$ similarly, $P(E) = 1 - P(\overline{E})$

Example 1: In a tennis match, the probability for James to win is $\frac{2}{5}$. What is the probability for him to lose?

Example 2: Pascal rolls a pair of dice.

(1) What is the probability of NOT rolling a double?

(2) What is the probability of rolling a sum more than 4?

<u>Example3:</u> Two cards are randomly drawn from a full deck of 52, without replacement (you do not put it back)

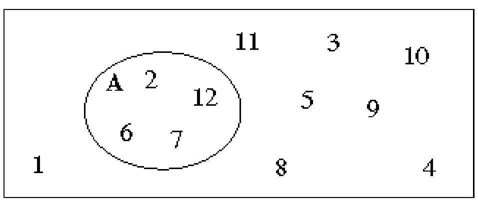
(1) what is the probability that they are NOT both aces?

(2) what is the probability that they are at least one ace?

<u>Example 4:</u> Hanson tosses a fair coin 10 times in a row. What is the probability of tossing…

(1) at least 1 head? (2) exactly 1 head? (3) at least 2 heads?

<u>Example 5:</u> A number is chosen randomly from the diagram. Find the probability if... (the letter A is the name of the set)

	(1) the number is from A (2) the number is neither from A nor less than 3 (3) the number is not more than 20 (4) the number is a multiple of 2 (5) the number is not a prime number? (6) the number is neither from A nor a perfect square?

<u>Example 6:</u> In a chess tournament, every match can have 3 different outcomes: win, lose, or draw. Peter and Alice are in the same chess club. In the past there were 30 matches between Peter and Alice. Of these 30 games, Peter won 7 matches and lost 20 matches when he played with Alice. Peter and Alice has a game with each other in a tournament on the coming Saturday

(1) what is the likely-hood that Peter and Alice draw in the game?

(2) what is the likely-hood that Alice does not win against Peter in the game?

Topic 9 The Odds

Objective: students will understand what the meaning of odds in a game.

The Lesson

In a game (sports or gambling), odds is used to describe the likely-hood of some event happens, it is a synonym of the probability. Or you may say that odds is the "Las Vegas" version (the gambling version) of probability.

For example, in a roulette [pronounce as: Roo-Let, i believe it is French, I research at least 20 min on how this game is played. that been said, i never played this game before…] game, you may hear a rumor around the table that the odds in favor of the certain bet is 1 to 20. You may wonder exactly what is the probability for winning the bet?

The answer is not $\frac{1}{20}$. Instead the probability of winning on that bet is $\frac{1}{1+20}=\frac{1}{21}$

In the textbook, the term odds is defined in a very odd manner. Here is a better version of that same definition

For any event E of a random experiment,
The odds **<u>in favor of</u>** E = $\frac{P(E)}{P(\overline{E})}$
The odds **<u>against</u>** E= $\frac{P(\overline{E})}{P(E)}$

Even though I put the definition of odds in fraction form, answer to an odds is **<u>never a fraction or decimal.</u>** You need to simplify an odds to the simplest form of a fraction, and use "numerator to denominator" or "numerator : denominator" in answers.

Example 1:

(a) the probability for John to win a tennis match is $\frac{2}{5}$. What is his winning odds?

(b) the probability to guess a right answer in a multiple choice question is 0.2. What is the odds in favor of guessing? What is the odds against guessing?

(c) the probability to guess a right answer in a True/False question is 0.5. What is the odds in favor of guessing? What is the odds against guessing?

Example 2: When a person renew a driver license, sometimes the DMV will require the applicant to take a written test again. The questions are in multiple choices form with 3 choices. One of those three options is correct. John was renewing his license and taking a test. In the test he decided to guess on one of the questions. What is the odds against him to guess the right answer?

When given an odd either in favor of or against an event, one can find the probability of the event

Given the odds in favor of an event E to be s:f (or s to f),

$$p(E) = \frac{s}{s+f}$$

$$P(\overline{E}) = \frac{f}{s+f}$$

Example 3: the odds in favor of getting a King in a standard deck is 1:12. What is the probability of not getting a King?

Example 4: Samantha tosses a fair coins 4 times in a row.

(1) What is her odds of getting all heads?

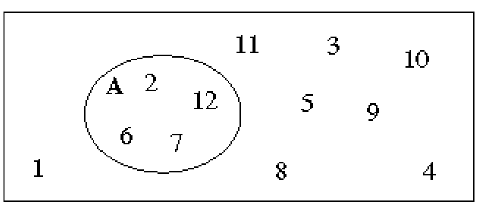
(2) What is odds against her to get no more than 3 tails?

Example 5: In a bag there are 3 quarters, 4 dimes, and 6 nickels. (all coins are different from each other even if they are the have the same value), When pick 3 coins at the same time

(1) what is the probability of getting exactly 1 quarter, 1 dime and 1 nickel?

(2) what is the odds against getting exactly 1 quarter, 1 dime and 1 nickel?

Example 6: A number is chosen randomly from the diagram (the letter A is the name of the set). What is the odds…



(1) in favor of a number from A?
(2) against a number neither from A nor less than 3?
(3) in favor of a number greater than 14?
(4) against of a prime number?
(5) in favor of a perfect square?

Topic 10 The probability of dependent events

Objective: In this topic students will learn how to find dependent event probability

The Lesson

Example: Jonah is rolling a pair of dice.

(1) the probability of Jonah rolling a 6 is P(roll a 6) = $\frac{5}{36}$

(2) Alice is guessing what is the outcome. If jonah said it is a double, what is the probability if Alice guesses it right that she said it is a 6? [see video a bout this]

Conditional Probability

Given two events E and F, the conditional probability of event E when F happens (<u>the sum of two dice is 6</u> (event E) when <u>it is a double</u>(event F))

P(E|F)= $\frac{P(E \cap F)}{P(F)}$ [the probability of both events happens / the probability of event F along]

Example1: Rolling a pair of dice. Find the following probabilities:

(a) P(rolling a double | rolled a 10)

(b) P(rolling a 7 | NOT rolling a double)

(c) P(rolling a 3 | one die lands on "1")

(d) P(rolling a 2 | rolling an odd number)

Example2: Let n be a randomly selected integer from 1 to 20

(a) P(n=2 | n is even)

(b) P(n=5 | n<8)

(c) P(n= prime number | n has 2 digit)

(d) P(n is odd | n is prime)

(e) P(n is prime | n is odd)

Example 3: You roll 2 dice. The first die shows 1 and the other die fell under the table that you can not see. What is the probability of

(1) both to be 1?

(2) the sum of two dice to be 8?

(3) the sum of two dice to be 4?

Example 4: The probability of drawing two cards of the same suite in a row? (if you do not replace the first one into the deck)

Example 5: In a box, there are 8 different novels, 8 different biographies, and 8 different war history books. If Jane picks two books at random, what is the probability that Jane will get two different kinds of books?

Example 6: What is probability of the sum of two die will be greater than 8, given that the first die is 6?

Topic 11 The probability of independent AND events

Objective: In this topic students will learn how to find probability for independent events.

The Lesson

Example: Jonah rolls a die twice.

(1) the probability of Jonah to roll a 6 for the first time is P(6) = 1/6

(2) the probability of Jonah to roll a 6 for the second time is P(6) = 1/6

(3) the probability of Jonah to roll two 6 in the two events is $P(6 \cap 6) =$

Probability of Independent events

Given two independent events E and F, $P(E \cap F) = P(E) \cdot P(F)$

Examle 1: You randomly select two cards from a deck of cards. After you select the first card, you record what you select and put it back to the deck. What is the probability of selecting …

(a) first time to be a king and the second time not a king.

(b) first time to be a spade and the second time to be a heart.

Example2: A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles.(marbles with the same colors can not be distinguished from one to the other.) A marble is chosen at random, place it back and the second marble is chosen in the same manner. What is the probability of choosing ..

(a) green marbles for both time?

(b) yellow marble first and the blue?

(c) blue marble first and then the yellow?

Example3: In the United States there are about 70% of the population declares they love pizza. If we pick 3 people at random and ask if they like pizza, what is the probability that none of the 3 says that they like pizza?

Example 4: continue from Ex3, what is the probability that at least one of the 3 says they like pizza?

Example 5: If from Ex 3 only two people are asked. What is the probability that at least one like pizza?

Example 6: In the United States a survey found 65% of children says that they do not like to eat vegetables. If two kids are asked by the survey, what is the probability that at least one of the children will say s/he does not like to eat vegetable