MATH ANALYSIS
PRACTICE QUESTION BANK

For end of the year summative test

Before You Start…

1. Show your work in detail (including graphs, reasoning process, tables…) on separate papers.
2. Compile your report in unit sequence, and do not skip units.

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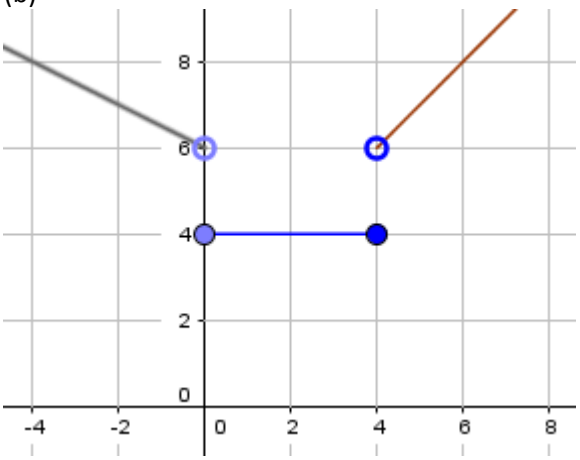
2018 – 2019 Math Analysis Practice Question Bank (Chen)

[Unit 1]

1. Given a graph below, write a piecewise defined function (assumed each segment can be represented by a linear function), the entire real number line is the domain. Indicate the range of the function.

(a)

(b)



2. Use transformation to graph each function and its parent function below. Identify the transformations between the given function and its parent function.

a) $f(x)=2-\sqrt{4x+3}$

b) $f(x)=\dfrac{1}{x-2}+1$

3. Find the intersection of $f(x)$ and $g(x)$ algebraically and verify your result graphically.

a) $f(x)=\dfrac{1}{x-2}$, $g(x)=x-\dfrac{1}{2}$

b) $f(x)=\dfrac{6x}{3x+2}, x>-\dfrac{2}{3}$ and $g(x)=\dfrac{-2x}{3x-6}, x<2$

4. Given $f(x)=|x-1|$, $g(x)=\dfrac{x-2}{x^2}$, graph

(a) $g(x-1)$

(b) $f(x^2)$

5. Given $f(x)=1-\dfrac{1}{1+\dfrac{1}{x}}, g(x)=\dfrac{2-x}{x}$, find $(g\circ f)(x)$, $(f\circ g)(x)$

[Unit 2]

$f(x)=6x^2-7x-3$

6. Given $g(x)=x-\frac{3}{2}$

(A) Find the intersection of f(x) and g(x) algebraically if possible.

(B) verify your solution graphically.

(C) if $h(x)=f(x)-g(x)$, What are the range and x-intercepts of h(x)?

7. Find the rational zeros of

(a) $f(x)=2x^4-5x^3+10x^2-20x+8$ (b) $f(x)=2x^5+3x^4-4x^3-6x^2+2x+3$

8. Graph the rational function $f(x)$, identify the holes of the function if it exists.

(a) $f(x)=\frac{3x-1}{3x^2-7x+2}$ (b) $f(x)=\frac{x^2-4x+4}{(x-6)(x^2+3x-10)}$

(c) $f(x)=\frac{x^2+3x-4}{-x^3+27}$

9. Find f(x), if f(x) is a 4th degree polynomial with real coefficients, and f(x) has zeros of

(a) f(x) has zeros of $2-i$, $3-2i$, and $f(0)=13$

(b) f(x) has zeros of $3-i$, 2 , $\frac{3}{2}$, and $f(0)=-6$

10. Consider the following polynomials,

a) $f(x)=(x-2)(x^2-4)-(x^2-4)$ b) $f(x)=x^3+6x^2-9x-54$

1) Describe the ending behaviors for each polynomial

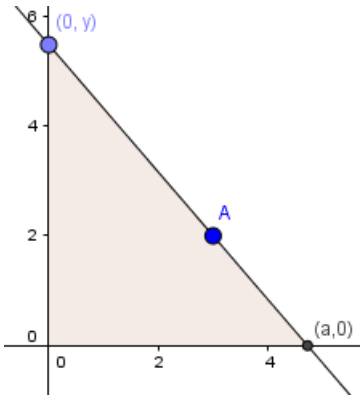
2) Describe the behaviors of each polynomial around its zero.

3) Sketch each $f(x)$ on its own graph, based on the results from 1) and 2)

11. Graph the rational function $f(x)$, identify the holes of the function if it exists.

a) $f(x)=\frac{x^3}{x^2-4}$ b) $f(x)=\frac{-x^3+x^2}{x^2-x-2}$

12. A right triangle is formed in the first quadrant by the x-axis, y-axis and the a line segment through the point A(3, 2)



a) Show that equation of the hypotenuse is $y = \frac{2(a-x)}{a-3}, 0 \le x \le a$

b) Show that the area of the triangle is given by $A(a) = \frac{a^2}{a-3}$

c) Graph the function of triangle's area on a coordinate plane. Estimate the minimal area of the triangle.

[Unit 3]

13. Graph f(x), describe the transformations between f(x) and its parent function and find x, y intercepts if possible.

a) $f(x)=3^{x-2}+1$

b) $f(x)=-\left(\frac{1}{2}\right)^{x-1}+2$

c) $f(x)=2+\log_{\frac{1}{4}}(x+4)$

d) $f(x)=2+\log_3(x-1)$

14. A credit card company charges 22% APR and compounding the balance daily. Jonah carries a balance of $2,000 on this credit card on the beginning of a billing cycle 10/5/2016. If he decides to pay the balance $300 on the beginning of every month, (and not have any purchase any more on the card). When will he pay off his balance?

15. Evaluate following expressions:

a) $\left(\log_{\frac{1}{\sqrt{5}}} 625\right)+\log_{32} 2$

b) $\left(\log_{\sqrt{2}} \frac{1}{2}\right)-\log_{729} 3$

16. Solve the exponential or logarithmic equations (or systems of equations)

a) $\begin{cases} 2^{x+y}=\frac{1}{4} \\ \frac{9^x}{3^y}=27 \end{cases}$

b) $\begin{cases} \frac{5^{x+y}}{25}=25^z \\ 9^x=9(27)^y \\ 8^x=(16)^{z+1} \end{cases}$

c) $2\log_6(2x+1)=\log_{36} 4+\log_6(6x-1)$

d) $\begin{cases} \log\frac{(x-2)^2}{y-1}=\frac{5}{2} \\ \log(x-2)^3(y-1)\sqrt{y-1}=\frac{27}{4} \end{cases}$

e) $\log_4(2x-3)=\log_2 5$

f) $\begin{cases} 12^x=\frac{1}{144} \\ 2\cdot 4^{x-y}=128 \end{cases}$

g) $x\ln x+x=0$

17. **(Richter scale)** The Richter scale is used for measuring the magnitude of an earth quake. The Richter scale is given by

$$R = \frac{2}{3}\log(0.37E) + 1.46$$

Where E is the energy in (Kwh). If two successive quakes are recorded with 0.1 difference on the scale, what is the ratio of the energy released between these two quakes?

18. **(Yeast Growth)** Following model represents a yeast population when a sour dough bread was rising:

$$Y(t) = \frac{80}{1 + 20e^{-.75t}}, 0 \le t \le 10$$

Where t represents the time (in hours), Y(t) represent the numbers of yeast in millions.

(a) What is the initial population when the yeast was just added into the dough?

(b) Make a graph of Y(t) over the whole domain [0, 10] by evaluating the yeast population at whole hours.

(c) Interpret this function $Y'(t) = \frac{Y(t+1) - Y(t)}{(t+1) - t}$ in terms of yeast growth in a sour dough bread.

(d) Compare Y'(1), Y'(5) and Y'(9). Make a conjecture based on the values of Y' you observe.

(e) If this model still apply after the 10th hour, Evaluate Y(12), Y'(12) and Y(15), Y'(15). Interpret your finding.

(f) When the population of the yeast reaches 97% of the final population, we said the dough is fully risen. In this case, when will the dough be fully risen? (Round to tenth hour)

19. Evaluate $2 + \cfrac{\sqrt{e}}{1 + \cfrac{\sqrt{e}}{2 + \cfrac{\sqrt{e}}{1 + \cfrac{\sqrt{e}}{\ddots}}}}$

20. Assume that x is a real number, Solve for x if $\frac{2}{x+e} = \frac{e}{x} - \frac{2}{x-e}$

21. Condense the following expression into a single logarithmic expression to the assigned base. Your condensed logarithmic expression need to be reduced to the simplest form with radical removed whenever the argument is an expression with radical sign.

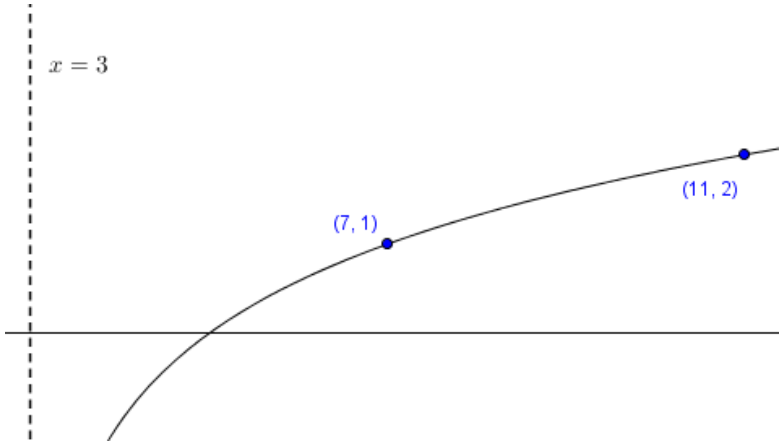
(a) (change to base 3) $\log_{\frac{1}{\sqrt{3}}}\left(\sqrt{x-3}\right)+\log_9\left(x^2+3x-18\right)$

(b) (change to base 4) $\log_2\sqrt{(x+2)^3}+\log_{\frac{1}{4}}\left(x^3+2x^2-4x-8\right)$

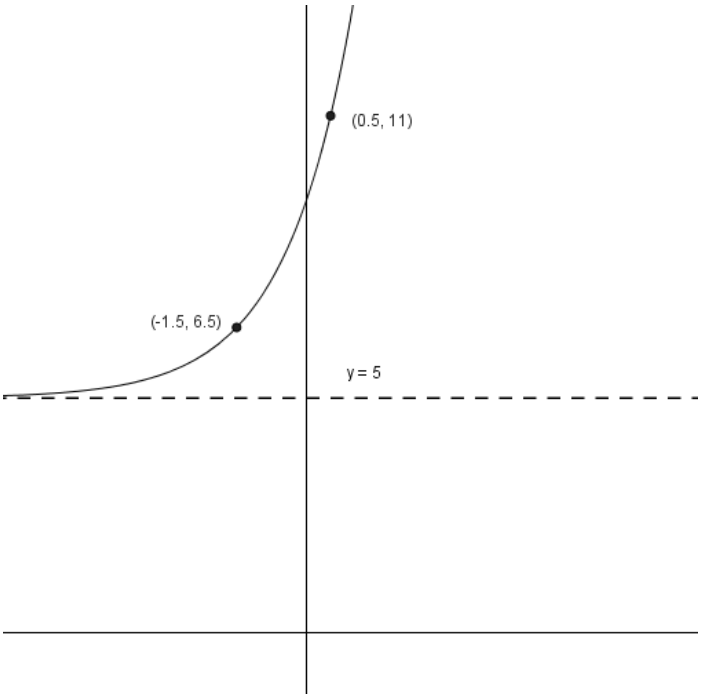
(c) (change to natural logarithm) $\log_3\left(2x^2+3x-20\right)-\log_9\left(x^2+8x+16\right)$

22. Use the given graph to find the function

(a) The graph shown is below a logarithm function. x=3 is the vertical asymptote. Two given points that the function passes through are (7, 1) and (11, 2)



(b) The graph shown below is an exponential function. y = 5 is the horizontal asymptote. Two given points that the function passes through are $(\frac{-3}{2},\frac{13}{2})$ and $(\frac{1}{2},11)$



23. **(Transparency)** Ms. King uses transparencies in her art class to show students how to draw a painting in different layers. However, light intensity will decrease as the transparency overlaid. The intensity decrease follows the model below:

$$I_n = I_0(1-x)^n$$

Where

n: number of transparency overlaid

x: intensity reduction per sheet of transparency, in percent

I_0: intensity of light before going through the transparency, in lumens

I_n: intensity of light after going through the transparency, in lumens

(a) Ms. King has a Panasonic Projector produces 1600 lumens of light when it is in fully operation. If Ms. King overlaid 20 transparencies on the projector, the light intensity dropped to 1250 lumens. What is the light intensity reduction (in percent, round to the hundredth percent) if only one transparency is on the projector?

(b) Assuming Ms. King is using the same projector, she now needs to overlay at most 25 transparencies in a project. She knows that if light intensity cannot be lower than 1400 lumens, her current transparencies will not be adequate. Which of the following brands at Office Depot can be recommended for her, if you were a sales assistant? If her budget to get 25 transparencies cannot be more than $25 dollars, which brands can she consider?

Brands of the transparency	price per package	sheets per package	Light intensity reduction
ClearPix	$5.00	10	0.57%
As Air	$6.00	8	0.51%
Ultra	$7.00	15	0.55%
No Lost	$9.00	5	0.52%
Last Long	$10.00	10	0.48%

[Unit 4]

24. Assume the Sunrise today in Austin, Texas (30 16'1" N,97 44'35"W) was 7:34 AM. If your friend who lives 400 miles west from you, called you when he saw the sun rose. When did you get the phone call this morning? What is the longitude where your friend lives?

25. Given that

(a) $\sec\theta = 7$ and $\tan\theta > 0$ find $\sin\theta$

(b) $\cos\theta = \frac{3}{4}$ and $\sin\theta < 0$, find $\tan(\pi - \theta)$

(c) $\tan\theta = \frac{9}{4}$ and $\sin\theta < 0$, find $\sin(\theta + \frac{\pi}{4})$

26. Graph the following trigonometric functions

(a) $f(x) = 4\cos(\frac{x}{2} + \frac{\pi}{4}) - 2$

(b) $f(x) = -4\sin(2x - 3\pi) + 1$

(c) $f(x) = -\frac{\pi}{2}\cot(\frac{3}{4}x + \pi)$

(d) $f(x) = 2\sec\left(\frac{3}{2}x - 3\pi\right) + 1$

(e) $f(x) = -\frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$

27. Verify the following identities:

(a) $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

(b) $\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

(c) $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

(d) $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

(e) $\frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

(f) $1 - \frac{\sin^2 x}{1 - \cos x} = -\cos x$

28. Evaluate (Do not use a calculator)

(a) $\csc(\arcsin\left(-\frac{\sqrt{6}}{3}\right))$

b) $\cot\left[\arctan\left(-\frac{15}{8}\right)\right]$

c) $\arccos\left(-\frac{1}{\sqrt{2}}\right)$

d) $\arccos\left(\csc\frac{7\pi}{6}\right)$

29. Write an algebraic expression that is equivalent to the trigonometric expression

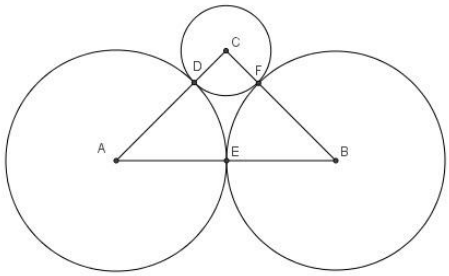
a) $\tan(\arcsin\frac{x}{5})$

b) $\sin(\arctan(\sqrt{x^2-1}))$

c) $\cos(\arcsin(\frac{2}{\sqrt{4+x}}))$

d) $\csc(\arccos(\frac{\sqrt{x+6}}{x}))$

30. Given



$\bigcirc A \cong \bigcirc B$. $\triangle ABC$ is an isosceles right triangle with $\overline{BC} \perp \overline{AC}$. The radius of $\bigcirc A$ is r. If $\bigcirc A$, $\bigcirc B$ and $\bigcirc C$ are tangent to each other as shown in the diagram above

(1) Show that $\frac{AD}{DC} = \sqrt{2}+1$

(2) Find the area bounded by DE, EF, and FD.

(3) Construct $\overline{DF}$ and $\overline{CE}$, show that $\overline{DF} \perp \overline{CE}$

(4) Construct $\triangle DEF$, show that area of $\triangle ABC = 2(\sqrt{2}+1)$ area of $\triangle DEF$

31.Find the indicated trigonometric ratios

(a) $\sin\theta$ if $9 = 6\sin^2\theta + 11\cos\theta$

(b) $\cot\theta$ if $\frac{3}{2\left(1-\sin^2\theta\right)} = 2+\tan\theta$

[Unit 5]

32. Rewrite the following expressions so that they are not in the fractional form

a) $\dfrac{\sin x}{\tan x}+\dfrac{\cos x}{\cot x}$

b) $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}$

33. Solve the following trigonometric equations if $x\in[0,2\pi)$

a) $\sin^2 3x-\sin^2 x=0$

b) $\sin\dfrac{x}{2}+\cos x=0$

c) $\sin 6x+\sin 2x=0$

d) $\sin x\cos x=-\dfrac{\sqrt{2}}{4}$

e) $\sin 4x+2\sin 2x=0$

f) $4\cos^3 x-8\cos^2 x-3\cos x+6=0$

g) $\csc^2 x-4\cot x+2=0$

h) $3\sin x=2\cos x+3$

34. Evaluate the following expressions

a) $\sin\left(\cos^{-1}\left(-\dfrac{3}{5}\right)-\sin^{-1}\left(\dfrac{40}{41}\right)\right)$

b) $\cos\left(\sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)-\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\right)$

c) $\tan\left(\dfrac{2}{3}\pi+\arccos\left(-\dfrac{1}{5}\right)\right)$

35. Find indicated trigonometric ratios

(a) if $\sin x=\dfrac{3}{5}$, $0<x<\dfrac{\pi}{2}$, find $\cos 2x$

(b) if $\sec x=-\dfrac{7}{2}$, $\dfrac{\pi}{2}<x<\pi$, find $\sin\dfrac{x}{2}$

36. Rewrite the following expressions in terms of the first power of cosine (if the result is a rational expression, the numerator and denominator need to be in the first power of cosine)

a) $\sin^6 x$

b) $\cos^4\dfrac{x}{2}$

c) $\sin^4 x\cos^2 x$

37. Solve the following trigonometric equations

a) $\cot x + \csc x = 0$

b) $\sec x \csc x = 2\csc x$

38. Given $\sin x = -\frac{4}{5}, \pi < x < \frac{3\pi}{2}$, $\tan y = -\frac{3}{4}, \frac{3\pi}{2} < y < 2\pi$, and $\begin{cases} k\sin(x-y) - m\cos(x-y) = 2 \\ k\sin(x+y) + m\cos(x+y) = -\frac{2}{5} \end{cases}$.

Answer the following questions

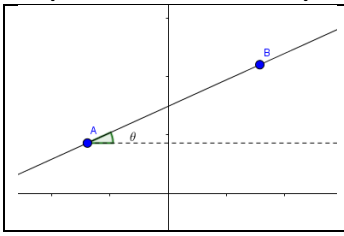
a) Find $\frac{\cot y}{\tan x}$

b) Find $\csc\left(\frac{x}{2}\right)$

c) Find $\cot(x-y)$

d) Find m and k.

39. **(Intersection of Lines)**



Define θ is the angle with vertex at the point of intersection between a horizontal ray (pointed to the positive x direction), the horizontal ray as the initial side and the line as the terminal side.

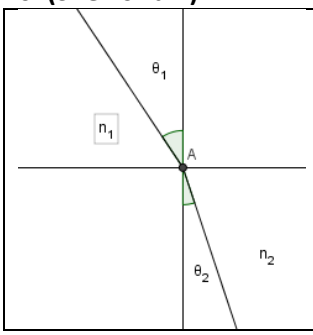
a) Show that $m = \tan\theta$

b) Line M and Line N are non-vertical and non-perpendicular. If the slopes of Line M and Line N are m_1 and m_2 respectively, if the acute angle between these two lines is α, show that $\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

c) if M: $3x - y = 5$, N: $x + y = 1$, what is the angle?

d) if $\alpha = 37^\circ$ and N passes through (1,3), find the equation of N (assume M is $y = 3x - 5$)

40. **(Snell's Law)**



Snell's law of refraction says that the angle of incident θ_1 and the angle of refraction θ_2 has the following relationship

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n are the refractive index of the medium where the light travel.
in vacuum (air) n = 1
in water, n = 1.3

A person 5 feet 5 inches tall standing in a river with water reached his waist (about 3 feet deep) saw a trout appeared to be 4 feet away on the riverbed from where he stood. Assume his eye line is 2 inches below when measured from the top of his head. Use the Snell's law to estimate actually how far away was the trout from where he stood?

41. **(Cannon ball)** A cannon ball was fired at an angle θ (measured from the horizon) with an initial velocity of v_0. We know that the trajectory of the flying cannon ball is a parabola. Let the location where the cannon ball was fired be the origin, the x and y coordinates of the cannon ball can be modeled as $\left(v_0 t\cos\theta, v_0 t\sin\theta - \frac{1}{2}gt^2\right)$ where t is the time in second when the cannon was flying and g is the acceleration caused by gravity. Use the model to

a) Show that the flying time for a cannon ball is T, and $T = \frac{2v_0\sin\theta}{g}$

b) Show that for a given initial velocity, the maximum horizontal distance a cannon ball can travel may occur when the cannon ball was fired at 45°

c) Write the function of the trajectory of the cannon ball, $y = f(x)$. Show that $f(x) = (\tan\theta)x - \left(\frac{g\sec^2\theta}{2v_0^2}\right)x^2$

[Unit 6]

42. Given $\vec{u} = 2\vec{i} + 3\vec{j}$ and $\vec{v} = 2\vec{i} + 2\vec{j}$, find

a) $\vec{w} = \vec{u} + \vec{v}$

b) $\vec{w} = \vec{u} + \vec{v}$, $\vec{a} = \vec{u} - \vec{v}$ and $\vec{w} \cdot \vec{a}$

c) θ between $\vec{w}$ and $\vec{a}$ (degree)

d) $\text{Proj}_{\vec{a}} \vec{w}$

e) If $\vec{w} = \text{Proj}_{\vec{a}} \vec{w} + \vec{n}$, find $\vec{n}$

43. Find $z_1 z_2$ and $\frac{z_1}{z_2}$ if

a) $z_1 = -4i$, $z_2 = 12 - 5i$

b) $z_1 = 3 + i$, $z_2 = 12 - 5i$

44. Use DeMoivre Theorem to find the n-th root of the complex numbers

a) $-64i$, 6th root

b) 16, 4th root

45. If $\vec{v} = \langle -2, 2 \rangle$, $\vec{w}$ is defined as $\|\vec{w}\| = 3$ and $\theta_{\vec{w}} = 240^\circ$, $\vec{u} = 2\vec{v} + 3\vec{w}$, Find

a) $\|\vec{u}\|$

b) $\theta_{\vec{u}}$

46. Solve the following triangles, assuming the triangle is ΔABC, a is the opposite side of $\angle A$, b is the opposite side of $\angle B$ and c is the opposite side of $\angle C$.

a) $\angle A = 65^\circ$, $a = 7, b = 10$

b) $\angle A = 35^\circ$, $a = 10, b = 9$

c) $\angle C = 10^\circ$, $a = 8, b = 15$

d) $\angle A = 65^\circ$, $a = 9, c = 10$

e) $a = 5, b = 9, c = 10$

f) $\angle C = 108^\circ$, $a = 10, b = 8$

47. Given $\vec{u} = \langle 4, -1 \rangle$, $\vec{v} = 3\vec{j}$, the magnitude of vector $\vec{a}$ is 5 and the directional angle of vector $\vec{a}$ is $\pi + \tan^{-1}\left(\frac{1}{2}\right)$, $\vec{b} = \langle 3, 0 \rangle$. If $\vec{w} = -4\vec{u} + 3\vec{v} = x\vec{a} + y\vec{b}$, find the exact value of x and y.

48. **Forest Fire** Two watch towers spotted the same forest fire with bearings N 52° E (from tower A) and N 30° W (from tower B). Further, two watch towers are 10 miles apart, and the bearing of tower A from tower B is N 65° W. If the rescue center C is 15 miles away from tower B and the bearing of center C from tower B is S 45° W
(a) Find the bearing a helicopter pilot should set from center C to the fire. If the average speed of the helicopter is 40 mph, (b) how long in time would it take the helicopter to reach the fire?

49. **Height of a Tree** A tree is on a hillside of slope 13° (from horizontal). 60 feet downhill from where the tree is, the angle of elevation at the top of the tree is 30°. Find the height of the tree.

50. Let z_i ($i = 1, 2, 3, 4, 5, 6$) be the 6th roots of 1. And define $\overrightarrow{a_i} = \mathrm{Re}(z_i)\vec{i} + \mathrm{Im}(z_i)\vec{j}$ where $\mathrm{Re}(z)$ is the real part of the complex number z and $\mathrm{Im}(z)$ is the imaginary part of the complex number z.

(A) Locate z_i on the complex plane.

(B) Find the angle between $\overrightarrow{a_2}$ and $\overrightarrow{a_5}$

(C) Let vector $\overrightarrow{m} = \overrightarrow{a_3} - \overrightarrow{a_1}$ and $\vec{n} = \overrightarrow{a_4} - \overrightarrow{a_2}$, find the angle between vector $\overrightarrow{m}$ and $\vec{n}$

(D) Find $\left\|\overrightarrow{m}\right\|$ and $\left\|\vec{n}\right\|$

[Unit 7]

51. Solve the following systems of equations and verifies it graphically

a) $\begin{cases} y=|x| \\ y^2=x+2 \end{cases}$

b) $\begin{cases} (x+1)^2+y^2=4 \\ x^2+y^2=1 \end{cases}$

c) $\begin{cases} y=x^2 \\ x^2+y^2=8 \end{cases}$

d) $\begin{cases} y=2\sqrt{x-2} \\ y=-\frac{1}{3}x+3 \end{cases}$

52. Solve the following systems

a) $\begin{cases} 3x-2y-6z=-1 \\ -3x+2y+6z=1 \\ 8x+3y+2z=3 \end{cases}$

b) $\begin{cases} 2x+4y+z=-4 \\ 2x-4y+6z=13 \\ 4x-2y+z=6 \end{cases}$

c) $\begin{cases} \sqrt{x+1}+2\sqrt{y+2}=4 \\ 3\sqrt{x+1}-7\sqrt{y+2}=-\frac{15}{2} \end{cases}$

d) $\begin{cases} \frac{2x+1}{x+1}+\frac{2y-1}{y-2}=\frac{3}{2} \\ \frac{3}{x+1}+\frac{y}{y-2}=3 \end{cases}$

e) $\begin{cases} e^x+2e^y=2 \\ 2e^x-e^y=\frac{1}{2} \end{cases}$

g) $\begin{cases} x-3y=-7 \\ -3x+10y+z=23 \\ 4x-10y+2z=-24 \end{cases}$

h) $\begin{cases} x-y-z=1 \\ 5x-4y+z=8 \\ -6x+8y+18z=0 \end{cases}$

53. Given $f(x)=2-x^2$, if $g(x)=-x+b$ where $b>0$ has exactly one intersection.

a) Find b.

b) Find the point of intersection.

54. Let $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 5 \\ 6 & 4 & 3 \end{pmatrix}$ and $I=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. If λ is a real number and $\det(A-\lambda I)=0$, (a) Find λ .For every solution of λ , there exists at least one non-trivial (means, no all elements are zeros) matrix $v=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, such that $(A-\lambda I)v=0$.

(b) Find 3 matrices v_1, v_2, v_3 corresponding to 3 different solutions $\lambda_1, \lambda_2, \lambda_3$.

(c) Find A^{-1}

(d) Find $\left(A^{-1}\right)^2$

(e) Find $\det(A)$

55. Find the partial fraction decomposition of each rational expression

a) $\frac{3}{x^2+x-2}$

b) $\frac{2x^3-x^2+x+5}{x^2+3x+2}$

c) $\dfrac{x-1}{x^3+x^2}$

d) $\dfrac{x^3-7x+8}{\left(x^2-x\right)\left(x^2-4x+4\right)}$

56. Let $A=\begin{pmatrix} 4 & -1 & -4 \\ 0 & 1 & 0 \\ -2 & 3 & 0 \end{pmatrix}$ Find $A^{-1}+A^2$.

[Unit 9]

57. Given a circle $C_1:(x-2)^2+(y-3)^2=13$, and a point (4,6), find all possible equations of another circle C_2 with a radius of 3. Graph both C_1 and all possible C_2 on the same coordinate plane.

58. Rewrite following conics in their standard forms and classify.

a) $8x^2+18y^2+24x-108y+90=0$

b) $x^2+y^2+6x+4=0$

c) $y^2-4x-10y+33=0$

d) $-4x^2+5y^2+24x-16=0$

59. Following conic is an ellipse. Choose an angle θ to rotate the coordinate system (x', y') so that the ellipse can be written in the standard form.

$$4x^2+\sqrt{3}xy+3y^2+6\sqrt{3}x-4y+10=0$$

(a) Find the angle θ (Exact value)

(b) Find the standard form of the ellipse in the rotated coordinate system (x', y') .

(c) Find the coordinates of the center in the original coordinate system (x, y)

60. Find all possible tangent lines of the following conics T that passes through the given point P

(a) $T:(x-1)^2+(y+2)^2=20$, $P(1,3)$

(b) $T:-2(x-6)=(y-1)^2$, $P(4,3)$

(c) $T:4(y-3)=(x-2)^2$, $P(1,2)$

(d) $T:(x+1)^2+(y+2)^2=29$, $P(1,3)$

61. Find all possible standard forms of the conics below with given characteristics

(a) a hyperbola: focus @ $\left(-2,-5-\sqrt{26}\right)$ with asymptotes $y=\frac{1}{2}x+\frac{3}{2}, y=-\frac{1}{2}x+\frac{5}{2}$

(b) a parabola: focus @ (1,0) with directrix y = -4

(c) a circle: passes though $(0,5),(3,0),(-3,1)$

(d) an ellipse: eccentricity is 0.2 and foci @ $(5,1),(9,1)$

62. Graph the following polar equations

(a) $r = 2\sin\theta$

(b) $r = 3 - 2\cos\theta$

(c) $r = 2 + 2\sin\theta$

63. Identify the following conics, and find the distance between the pole and its directrix.

(a) $r = \dfrac{12}{2 - \cos\theta}$

(b) $r = \dfrac{4}{3 + 3\sin\theta}$

(c) $r = \dfrac{2}{1 + 2\cos\theta}$

[Unit 8]

64. Sequence a_n is defined recursively, find $a_n = f(n)$ based on the definition of the recursive expression and prove your finding using mathematical induction.

a) $a_1 = -1, a_k = 2a_{k-1} + 1$ b) $a_1 = -2, a_k = a_{k-1} + 2$ c) $a_1 = -1, a_k = -\dfrac{2}{5}a_{k-1}$

65. Assume that a_n is an arithmetic sequence, and $a_{k_1} = p, a_{k_2} = q$, where k_1 and k_2 are integers greater than 1 and $k_1 \neq k_2$, show that the partial sum of the first k_3 terms of the sequence can be written as

$$S_{k_3} = \frac{k_3}{k_2 - k_1}\left[pk_2 - qk_1 + (q - p)\left(\frac{k_3 + 1}{2}\right)\right]$$

66. A beach ball after it was thrown in a gym and it started bouncing and advancing. Assume that the height of each bounce of the ball can reach forms a geometric sequence, and the distance measured from every touch down to its initial thrown location forms an arithmetic sequence. Following table gives some observation of the beach ball after it starts bouncing:

	2nd	4th
height of the n-th bounce	16 feet	$\frac{32}{3}$ feet
distance of the n-th touch down	9 feet	17 feet

a) What would be the height of the ball at the 8th bounce?

(b) What would be the distance of the ball at the 8th touch down?

(c) What is the initial height of the ball right before it was thrown?

67. Find sums.

a) $\sum_{k=0}^{\infty} 2\left(\frac{1}{3}\right)^k$ b) $\sum_{k=0}^{\infty} 2\left(-\frac{1}{3}\right)^{2k+1}$ c) $\sum_{k=0}^{10} \left(\frac{2}{5}\right)^{3k-1}$

68. The Hacked Lock

A lock with a 4-digit pin with no repeats (made up by Arabic numerals) is partially hacked. If the first digit of the combination is 2 and none of the rest of the digits are greater than 6.

a) What is the probability if the combination is a multiple of 5?

b) What is the probability if the combination is a multiple of 3?

c) What is the probability if the combination is a multiple of 5 or a multiple of 3?

69. Make Your Own Sub Bistro

Fiona often visits "Make Your Own Sub" on Wednesdays. Server Pete noticed that the probability that Fiona orders a sandwich with provolone cheese on Wednesdays is about 75%. Further, Pete also noticed that whenever she orders a sub with provolone cheese, 80% of the time she would ask him to toast the bread; the same probability (toasting the bread) drops to 60% when she does not order a sub with provolone cheese. It's a Wednesday again. Pete sees Fiona at the store with her colleagues Susan and Jerry. How likely will Fiona reply with "OMG! You read my mind!" to Pete, if he offers her to toast her bread before she asks today? (Answer "Very likely" if you think he guesses it right with probability of 75% or above, "Maybe" if the probability is more than 50% but less than 75%, "Not really" if the probability is 50% or below)

70. The Pluto's Restaurant

At the Santana Row (San Jose) offers fresh salads and other dishes. When a customer orders a salad, s/he goes through the following steps: (Copied from the store menu)

Step 1: Choose Your Lettuce (Choose only ONE)

Farmer's Greens	**Romaine**	**Baby Spinach**

Step 2: Choose Your Size (Choose only ONE)

Side Salad	**Main Salad**

Step 3: Choose Your Seven (7) Salad Fixings

Raw Fixings(10 options)	Cooked Fixings(8 options)	Other Fixings(7 options)
Broccoli	grilled fennel	Sunflower Seeds
Cucumbers	sweet walnuts	Dried Cranberry
Red Onions	Green Beans	Pine nuts
Beets	Sweet corn	Crunchy Croutons
Jicama	Garbanzo Beans	Parmesan Cheese
Peas	Sautéed Mushrooms	California Raisins
Carrots	Caramelized Balsamic Onions	Crumbled Blue Cheese
Granny Smith Apples	Roasted Red Bell Peppers	
Plum Tomatoes		
Navel Oranges		

Step 4: Choose A Fresh Baked Bread(Choose only ONE)

Sourdough	**Wheat**	**Foccacia**

Step 5: (Optional) Add Meat or Vegetarian Option (Add only ONE)

Grilled Herbed Chicken Breast	**Herb Roasted Sonoma Turkey**	**Grilled Certified Angus Beef**
Aidells Poultry Sausage	**Crispy Chicken Asteroids**	**Grilled Portobello Mushroom**

Step 6: (Optional) Dress Your Salad with One of Our Homemade Dressings (Choose only ONE)

Balsamic Vinaigrette	**Cilantro Lime Vinaigrette**	**Gorgonzola Vinaigrrete**
Caesar Dressing	**L.F. Yogurt Based Honey Mustard**	**House Special Dressing**

So, after all, Fiona did not marry Pete. It was Pete's buddy, Sal, who eventually became her husband. Last weekend was the 8th anniversary for Fiona and Sal. Use the menu above from the Pluto's restaurant to answer the following questions regarding Fiona and Sal's visit to the restaurant for their anniversary.

1. At step 3, how many different ways can a customer fix his salad?

2. At step 3, if a customer picks 2 fixings from Raw Fixings, and 3 fixings from Cooked Fixings, and 2 fixings from Other Fixings, how many different ways can s/he fix her/is salad?

3. Going through all 6 steps, how many different salads can the Pluto's restaurant serve?

4. Fiona loves to have beets, cranberries and crumbled blue cheese whenever she fixes up her salad. At step #3, if she decided to have the rest of her fixings from the Raw Fixings category, what is the probability she got peas in her fixings?