VD unit 2 topic 3 part 1

Given two conditions of $f(x)$,

(1) Find the polynomial $f(x)$

(2) Find the remainder of $f(x) \div (x-3)$

(3) Find $f(x) \div (x^2+1)$. Write your answer in the form of $Q(x) + \frac{R(x)}{x^2+1}$, where $Q(x)$ is the quotient and $R(x)$ is the remainder.

(a and b are real constants.)

$f(x)$	condition 1	condition 2	$f(x)$	R for $f(x) \div (x-3)$	$f(x) \div (x^2+1)$
$\frac{4}{3}x^3 + ax^2 + bx - 2$	$(x-2)$ is a factor	$-\frac{7}{3}$ is the remainder of $f(x) \div (x-1)$	$\frac{4}{3}x^3 - \frac{8}{3}x^2 + x - 2$	13	$\frac{4}{3}x - \frac{8}{3} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2+1}$
$3ax^3 - bx^2 - x + 1$	$f(-1) = 1$	3 is the remainder of $f(x) \div (x+2)$	$-2x^3 - 3x^2 - x + 1$	-83	$-2x - 2 + \frac{x-3}{x^2+1}$
$4ax^3 + bx^2 - x + 5$	$f(1) = 9$	-9 is the remainder of $f(x) \div (x-2)$	$-8x^3 + 13x^2 - x + 5$	-97	$-8x + 13 + \frac{7x-8}{x^2+1}$
$-\frac{3}{2}x^3 + ax^2 - 2bx - 1$	$f(1) = -\frac{9}{2}$	55 is the remainder of $f(x) \div (x+2)$	$-\frac{3}{2}x^3 + \frac{20}{3}x^2 - \frac{26}{3}x - 1$	$-\frac{15}{2}$	$-\frac{3}{2}x + \frac{20}{3} + \frac{-\frac{43}{6}x - \frac{23}{3}}{x^2+1}$
$ax^3 + bx^2 - x + \frac{1}{3}$	$f(1) = -\frac{1}{6}$	$\frac{7}{3}$ is the remainder of $f(x) \div (x-2)$	$\frac{1}{2}x^3 - x + \frac{1}{3}$	$\frac{65}{6}$	$\frac{1}{2}x + \frac{-\frac{3}{2}x + \frac{1}{3}}{x^2+1}$
$ax^3 - x^2 + bx + 6$	$f(1) = \frac{9}{2}$	-2 is the remainder of $f(x) \div (x-2)$	$-\frac{1}{2}x^3 - x^2 + 6$	$-\frac{33}{2}$	$-\frac{1}{2}x - 1 + \frac{\frac{1}{2}x + 7}{x^2+1}$
$ax^3 - 3x^2 + 4bx + 6$	$f(\frac{1}{2}) = -\frac{7}{2}$	22 is the remainder of $f(x) \div (x+1)$	$-2x^3 - 3x^2 - 17x + 6$	-126	$-2x - 3 + \frac{-15x+9}{x^2+1}$
$3ax^3 + 4x^2 - 5bx + 5$	$f(1) = 5$	17 is the remainder of $f(x) \div (x+2)$	$2x^3 + 4x^2 - 6x + 5$	77	$2x + 4 + \frac{-8x+1}{x^2+1}$