VD: Unit 9 Topic 3 Part 3

Rotate the axes of x and y so that the xy term in the general form below will be eliminated. Write the general form into the standard form in the rotated coordinate, and classify the conic.

conic in xy coordinate	conic in x'y' coordinate
1. $6x^2+12xy+y^2-10=0$	$x=\frac{3}{\sqrt{13}}x'-\frac{2}{\sqrt{13}}y'$, $y=\frac{2}{\sqrt{13}}x'+\frac{3}{\sqrt{13}}y'$ Hyperbola, $\frac{135}{130}x'^2-\frac{24}{130}y'^2=1$
2. $x^2-4xy+4y^2+10x-30=0$	$x=\frac{2}{\sqrt{5}}x'-\frac{1}{\sqrt{5}}y'$ $y=\frac{1}{\sqrt{5}}x'+\frac{2}{\sqrt{5}}y'$ Parabola, $-\frac{4}{\sqrt{5}}\left(x'-\frac{31\sqrt{5}}{20}\right)=\left(y'-\frac{1}{\sqrt{5}}\right)^2$
3. $8x^2-12xy+3y^2-\sqrt{13}x+2\sqrt{13}y-5=0$	$x=\frac{2}{\sqrt{13}}x'-\frac{3}{\sqrt{13}}y'$ $y=\frac{3}{\sqrt{13}}x'+\frac{2}{\sqrt{13}}y'$ Hyperbola, $\frac{\left(y'+\frac{7}{24}\right)}{\frac{97}{576}}-\frac{(x'-2)^2}{\frac{97}{48}}=1$
4. $x^2-40xy+10y^2+2\sqrt{41}x-4\sqrt{41}y+4=0$	$x=\frac{5}{\sqrt{41}}x'-\frac{4}{\sqrt{41}}y'$ $y=\frac{4}{\sqrt{41}}x'+\frac{5}{\sqrt{41}}y'$ hyperbola, $\frac{\left(y'-\frac{7}{13}\right)^2}{\frac{399}{338}}-\frac{\left(x'-\frac{1}{5}\right)^2}{\frac{133}{65}}=1$
5. $5x^2-6xy+5y^2-7\sqrt{2}x+6\sqrt{2}y+4=0$	$x=\frac{1}{\sqrt{2}}x'-\frac{1}{\sqrt{2}}y'$ $y=\frac{1}{\sqrt{2}}x'+\frac{1}{\sqrt{2}}y'$

	ellipse, $\frac{\left(x'-\frac{1}{4}\right)^2}{\frac{45}{32}}+\frac{\left(y'-\frac{13}{16}\right)^2}{\frac{45}{256}}=1$
6. $2x^2-24xy-5y^2+5x-20y+3=0$	$x=\frac{3}{5}x'-\frac{4}{5}y'$ $y=\frac{4}{5}x'+\frac{3}{5}y'$ hyperbola, $\frac{\left(x'-\frac{13}{28}\right)^2}{\frac{123}{44}}-\frac{\left(y'-\frac{8}{11}\right)^2}{\frac{123}{56}}=1$
7. $3x^2-2\sqrt{3}xy+y^2+2x+2\sqrt{3}y=0$	$x=\frac{1}{2}x'-\frac{\sqrt{3}}{2}y'$ $y=\frac{\sqrt{3}}{2}x'+\frac{1}{2}y'$ parabola $y'^2=-x$
8. $-2x^2+3xy+2y^2+3=0$	$x=\frac{1}{\sqrt{10}}x'-\frac{3}{\sqrt{10}}y'$ $y=\frac{3}{\sqrt{10}}x'+\frac{1}{\sqrt{10}}y'$ hyperbola, $\frac{x'^2}{\frac{10}{3}}-\frac{y'^2}{\frac{10}{3}}=1$