Practice Test, Unit 3 Assessment

1. Solve the exponential equations:

$$\frac{3^{x^2}}{27} = 9^x$$

2. In this question, e is the Euler number, (the base of the natural logarithm)

(a) Evaluate $\sqrt{e+\sqrt{e+\sqrt{e+\sqrt{\ldots}}}}$

(b) Solve for x if $\dfrac{x}{x+e} = \dfrac{2e}{x}$

3. Solve the following logarithmic equations (x is the variable)

(a) $2\log_3 x = \log_9 x$

(b) $\log_{\sqrt{2}}(x-5) = \log_2(2x-7)$

4. For the following question assume

$\log 2 = a$, $\log 3 = b$, $\log 7 = c$ and $\ln 10 = d$

Rewrite the logarithmic expression with a, b, c and/or d

(a) $\log_4 7$

(b) $\log_5 245$

5. Condense the following expression into a single logarithmic expression to the assigned base. Your condensed logarithmic expression need to be reduced to the simplest form with radical removed, if the argument is rational expression with a radical

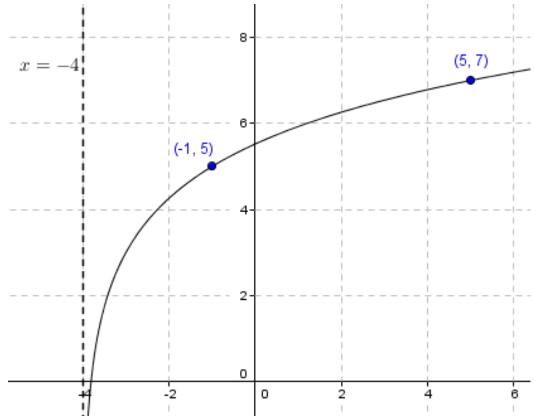
(change to natural logarithm)

$$\log_3\left(x^2-16\right) - \log_9\left(x^2+8x+16\right)$$

6.

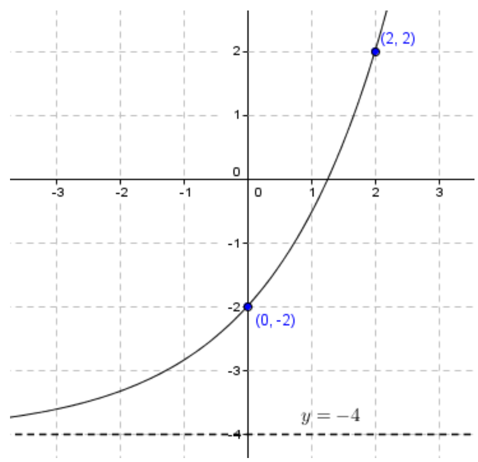
Use the graph to find the indicated function

(a) The graph shown is a logarithm function. x=-4 is the vertical asymptote.



(b) The graph shown is an exponential function

y=-4 is the horizontal asymptote of the function.



7. **(Cooling)** Newton's Law of Cooling describes the way the temperature of an object adjusts to the ambient temperature over time. If the following function,

$$L(t) = 78(0.93)^t + 68$$

describes the temperature ($^{\circ}F$) of a Starbucks grande latte over time (in minutes) after the barista called out the drink, answer the following questions:

(a) Graph the cooling function (temperature (y axis) – Time (x-axis)) in half an hour. [evaluate at every 10 minutes interval]

(b) Find the y-intercept of the function. Explain what is the meaning of the y-intercept in this context.

(c) Based on the cooling function of the latte, determines what could be the thermostat setting of the Starbucks store. Give an explanation why do you think your answer make sense.

(d) If the person who is going to enjoy the latte would not like the beverage to burn his tongue, as a barista how long would you suggest your customer to wait (round to minutes) before enjoying the latte. (Assume a person will feel his tongue burned when the temperature of the food is 15 $^\circ F$ higher than his normal body temperature. Assume further that an adult's normal body temperature is 98.5$^\circ F$)

(e) Assume that the "best enjoying" temperature for a latte is between 125$^\circ F$ and 95$^\circ F$. How long is this time period for any person to best enjoy a latte?

(f) Mary and Peter both bought a latte from the same Starbucks. Mary bought a tall latte (12 oz) and Peter bought a venti latte(20 oz). Mary argues that "Assume the room temperature keeps the same, my latte will cool off faster than yours, because mine is 12 oz and yours is 20 oz." However, Peter did not agree. He said, "As long as the barista heats up our lattes to the same temperature, the cooling off speed should be exactly the same for either latte." Whose argument is correct? Why? (Use Newton's Law of cooling and the given function to support your reasoning)

8. **(Yeast Growth)** Following model represents a yeast population when a sour dough bread was rising:

$$Y(t) = \frac{80}{1+20e^{-.75t}}, t \geq 0$$

Where t represents the time (in hours), Y(t) represent the numbers of yeast in millions.

(a) What is the initial population when the yeast was just added into the dough?

(b) Make a graph of Y(t) over the whole domain [0, 10] by evaluating the yeast population at whole hours.

(c) Final population of the yeast in the dough is the population of the yeast after it has been risen for a long time. And the dough is considered fully risen if the population of the yeast reaches 97% of the final population. Based on the model, how long will it take for a sourdough bread to be fully risen?