EVHS 2013-14 Algebra II Unit 2 Handouts

Matrices and Quadratic Relations

Contents

c_{ij}

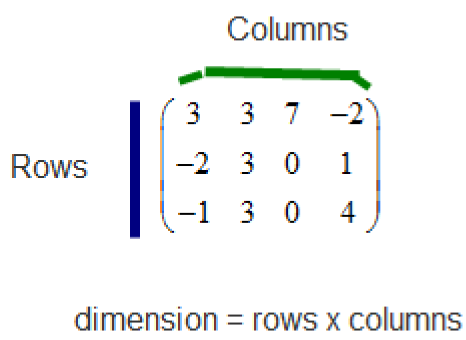
Topic 1 Matrix 101

Objective:

Introduce the object, matrix, and what is the constant scaling property, commutative property of addition in matrices

The Lesson

Matrix anatomy:



A matrix M is a mathematic entity of _________dimensional arrays of numbers. If it has m rows and n columns, it has a dimension of ____________________, and each number in the matrix is called an ___________of the matrix. It is a mathematical convention to name an element of a matrix with two indices like m_{24}=________, and the location for the element 7 in the matrix is ____________

Matrices have the following properties:

1. Commutative properties of Addition

A + B = B + A

Addition

To add two matrices with same dimensions, follow the following rules

Let C = A + B (A, B and C are matrices with the same dimensions)

$$c_{ij} = a_{ij} + b_{ij}$$

Constant Scaling

A matrix can be scaled by a constant, $C = kA$, where C and A are matrices and k is a constant.

$$c_{ij} = ka_{ij}$$

Let $A = \begin{pmatrix} \frac{1}{2} & 1 & 2 \\ 2 & -1 & \frac{3}{5} \\ -8 & 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -\frac{1}{2} & -2 \\ -\frac{2}{5} & 2 & 3 \\ -1 & \frac{2}{3} & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{2} \\ \frac{-3}{4} & \frac{1}{6} & -2 \\ 2 & \frac{1}{2} & 3 \end{pmatrix}$

Example 1:

Find the followings

1. A+2B	2. C-3A
3. –A + 0.5B	4. A + B + C

Example 2:

If A+B = C + D, find Matrix D?

Example 3:

$$\begin{pmatrix} x & 2x-1 & 11 \\ -2x+3 & -4 & -3 \end{pmatrix} = 3\begin{pmatrix} x & x & 5 \\ 1 & 0 & -1 \end{pmatrix} - 2\begin{pmatrix} x & 4 & 2 \\ x & 2 & 0 \end{pmatrix}$$

Exit Tickets:

Solve x and y

1.

$$4x\begin{bmatrix} -1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ -24 & 3y \end{bmatrix}$$

2.

$$\begin{bmatrix} -2x & 6 \\ 1 & -8 \end{bmatrix} + 2\begin{bmatrix} 5 & -1 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -9 & 4 \\ -13 & y \end{bmatrix}$$

Topic 2 Multiplying Matrices

Objective:

In this topic you will learn what matrices can have product and how to calculate them.

The Lesson

Not any two matrices can have product. The product of two matrices can only be defined, if both are of certain dimensions.

Let the dimension of A be m x n

Let the dimension of B be n x p

Then the dimension of the product matrix C=AB will be m x p

If $n \neq p$, then there is no product defined for BA

Example 1

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 3 & 2 \\ -1 & -2 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 4 & 1 & 2 \\ 3 & 2 & 2 \\ 2 & 1 & -3 \end{pmatrix}$$

1. AB	2. BA

Conclusion:____

Ex2 Find the product

1.

$$\begin{bmatrix} 2 & 5 \\ -1 & 4 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ -3 & 10 & -4 \end{bmatrix}$$

2.

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix} \begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix}$$

Ex3: Use distribution to find the result

$$A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix}, E = \begin{bmatrix} -3 & 1 & 4 \\ 7 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

3.
A(B+C)

4.
0.5(D+E)D

Exit Tickets:

1. Find x and y

$$\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} y & -4 \\ -13 & 8 \end{bmatrix}$$

2. Use Matrix D in the Example 3, and find the results of (a) and (b)

(a) $(D+E)^2$

(b) $D^2 + 2DE + E^2$

Topic 3 Calculate Determinants of 3 x 3 matrices

Objective:

In this topic, you will learn how to calculate the determinant of a 3x3 matrix

The Lesson

Given a 3 x 3 matrix $M = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$, the determinant of M is defined as

$$\det M = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3$$

Objective 1: Use the rule to calculate the determinant of each 3x3 matrix

1.	2.
$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & \frac{2}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{1}{2} & -1 \\ \frac{1}{3} & -1 & \frac{1}{4} \end{pmatrix}$

3. $\begin{pmatrix} -3 & 8 & 5 \\ -2 & 1 & -1 \\ -4 & -7 & 2 \end{pmatrix}$	4. $\begin{pmatrix} 1 & 0 & -1 \\ -2.1 & 0.2 & -1.3 \\ 4.1 & -0.3 & 0.2 \end{pmatrix}$
5. $\begin{pmatrix} -1 & -2 & -8 \\ 2 & -2 & -1 \\ 8 & 1 & 1 \end{pmatrix}$	6. $\begin{pmatrix} 0.01 & 6 & 1.25 \\ 7.21 & 2 & 0.01 \\ 0.1 & 0 & 0 \end{pmatrix}$

Objective 2:

Given 3 points in a coordinate plane $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$: the area of the triangle formed by $P_1P_2P_3 = \frac{1}{2}\left|\det A\right|$, where A is a matrix $A = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$

Example: Find the area of ΔABC

7. A(2, -3), B(4, -1), C(1, -1)	8. A(0, 2), B(0, 5), C(15, 2)
9. A(4, 2), B(9, 0), C(0, -3)	10. A(1, 1), B(-1, 1), C(6, 1)

Exit Ticket:

If the determinant of $\begin{pmatrix} 1 & 0 & 3 \\ -3 & 2x & 4 \\ 1 & x-2 & 1 \end{pmatrix}$ is -8, find the value of x.

Topic 4 Cramer's Rule

Objective:

In this topic you will use Cramer's rule to solve a system with 3 variables.

The Lesson

In the history of mathematics, it was Gabriel Cramer who proposed his rule in 1750 to solve a system of equations with n-variables. Although his rule can be used to solve a system of any variables, we will see in this topic how it works in a system of 3 variables.

Consider a system

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

One can construct four 3 x 3 matrices

$$\Delta = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix},\ \Delta_x = \begin{pmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{pmatrix},\ \Delta_y = \begin{pmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{pmatrix},\ \Delta_z = \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{pmatrix}$$

and the solution of the system will be (if exist)

$$(x, y, z) = \left(\frac{\det \Delta_x}{\det \Delta}, \frac{\det \Delta_y}{\det \Delta}, \frac{\det \Delta_z}{\det \Delta}\right)$$

Ex: Solve

$$\begin{cases} 4x - y + 3z = 13 \\ x + y + z = 2 \\ x + 3y - 2z = -17 \end{cases}$$

To use Cramer's rule, you need to first construct all delta matrices, in this case,

$$\Delta = \begin{pmatrix} 4 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -2 \end{pmatrix},\ \Delta_x = \begin{pmatrix} 13 & -1 & 3 \\ 2 & 1 & 1 \\ -17 & 3 & -2 \end{pmatrix}, \Delta_y = \begin{pmatrix} 4 & 13 & 3 \\ 1 & 2 & 1 \\ 1 & -17 & -2 \end{pmatrix}, \Delta_z = \begin{pmatrix} 4 & -1 & 13 \\ 1 & 1 & 2 \\ 1 & 3 & -17 \end{pmatrix}$$

Then you will need to find the determinant of each matrix, in this case,

$\det\Delta = -17$, $\det\Delta_x = 17$, $\det\Delta_y = 34$, $\det\Delta_z = -85$

Therefore,

$$x = \frac{\det\Delta_x}{\det\Delta} = \frac{17}{-17} = -1,$$
$$y = \frac{\det\Delta_y}{\det\Delta} = \frac{34}{-17} = -2,$$
$$z = \frac{\det\Delta_z}{\det\Delta} = \frac{-85}{-17} = 5$$

Practice:

Use Cramer's rules to solve the following systems

1.	2.
$\begin{cases} 4x + y + 3z = 7 \\ 2x - 5y + 4z = -19 \\ x - y + 2z = -2 \end{cases}$	$\begin{cases} 3x - y + z = 25 \\ -x + 2y - 3z = -17 \\ x + y - 2z = 0 \end{cases}$

3.	4.
$\begin{cases} 5x - y - 2z = -6 \\ 2x + 3y + 4z = 16 \\ 2x - 4y + z = -15 \end{cases}$	$\begin{cases} x + 2y - 2z = 3 \\ 2x - y - z = 2 \\ 2x + 4y + 2z = 3 \end{cases}$

No exit tickets for topic 4

Topic 5 Factor and Solve $x^2+bx+c=0$

Objective:

Today I will learn how to use the small diamond method to find solutions for $x^2+bx+c=0$.

The Lesson

In the equation: $x^2+2x-3=0$ can be rewritten into the factored form $(x-a)(x-b)=0$ with a puzzle diagram that I called "Small diamond".

-3 2	To form the puzzle, you always **put the constant at the top of the diamond**, and **the linear coefficient at the bottom of the diamond**. Once you found the solution for the puzzle, you can write out your equations as the factored form.

The Small Diamond Method

The factored form of the trinomial is

Solve the following equations:

1. $x^2+4x+3=0$	2. $x^2-17x+30=0$
3. $x^2-5x-104=0$	4. $x^2-64=0$

5. $x^2 = 4x + 5$	6. $7x = x^2 - 3x + 24$
7. $-2x^2 + 8x + 90 = 0$	8. $12x = -45 + x^2$

Exit Tickets:

a. $x^2 + 10x - 75 = 0$	b. $x^2 = -7x + 60$
c. $x^2 - 49 = 0$	d. $x^2 + 4x + 1 = 13$

Topic 6 Factor and Solve $ax^2+bx+c=0$

Objective:

Today I will learn the 'big diamond' method to solve quadratic equations like $ax^2+bx+c=0$

The Lesson

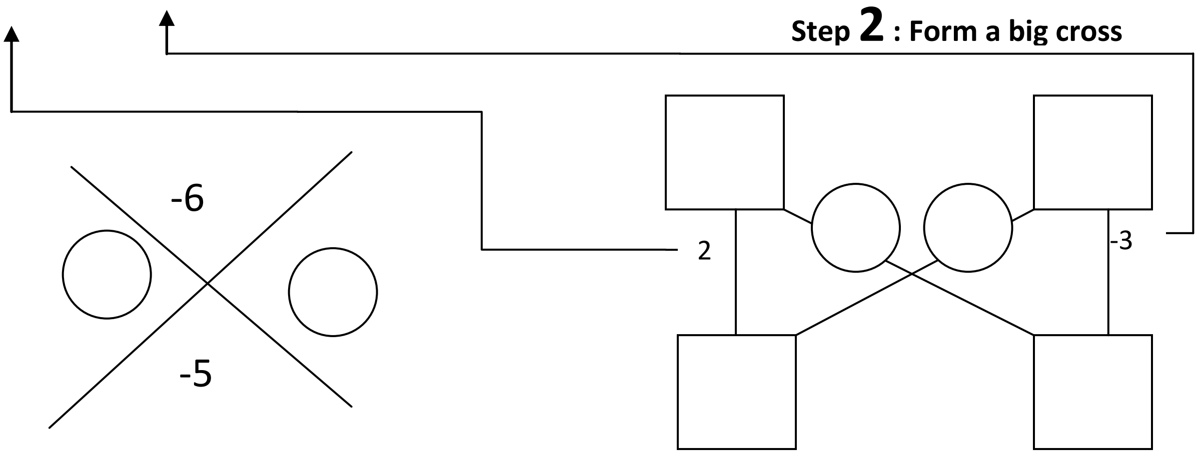
Although the **small diamond method** in Topic 1 can solve quadratic equation like $x^2+7x-18=0$ **. It falls short when encountering a question like:** $2x^2-5x-3=0$**. Big diamond method is to alleviate the situation**. The following diagram use $2x^2-5x-3=0$ as an example to introduce the method.

The Method

The Big Diamond Method

$2x^2-5x-3=0$

Step 2 : Form a big cross



The solution of the puzzle is: find 4 numbers in the box that will make 4 products that satisfy all connections in the big cross at the same time.

Step 1: Form a small diamond.

On the top of the diamond is the product of coef. of x^2 and constant: $2\cdot(-3)=-6$

At the bottom of the diamond is still the middle term -5

Step 3: Write the equation in factor form:

( ) ( ) = 0

Solve the following equations:

1. $30x^2 + x - 1 = 0$	2. $2x^2 + 7x + 3 = 0$
3. $4x^2 - 9 = 0$	4. $2x^2 + 21x + 49 = 0$
5. $-45x^2 + 20 = 0$	6. $12x^2 + 5x = 2$
7. $6x^2 - 11x - 112 = 0$	8. $12x^2 - 85x - 125 = 0$

Exit tickets

a. $8x^2 + 2x - 1 = 0$	b. $12x^2 + 5x - 3 = 0$

Topic 7 Rationalize Radicals

Objective:

Today I will learn how to simplify radicals like: $\sqrt{28}$, $\frac{3}{\sqrt{12}}$ *and* $\frac{2}{\sqrt{6}-\sqrt{2}}$.

The Lesson:

There are 3 parts for today's lesson:

1. What is the conjugate of a radical expression?
2. What is the purpose of the conjugate?
3. How does the conjugate of a radical expression help to rationalize a radical?

Define the conjugate of $\sqrt{a}+\sqrt{b}$ to be $\sqrt{a}-\sqrt{b}$

Example: Write the conjugates of the following:

$\sqrt{2}+\sqrt{5}$	$\sqrt{11}+\sqrt{7}$	$2+\sqrt{3}$	$\sqrt{6}$
$\sqrt{6}-\sqrt{3}$	$\sqrt{7}-\sqrt{5}$	$\sqrt{13}-4$	$\sqrt{7}$

Purpose of the conjugate:

The product of a pair of conjugates of an irrational number is rational.

Ex: $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) =$

Rationalize the denominators of following radical expressions:

1. $\dfrac{2}{2-\sqrt{2}}$	2. $\dfrac{\sqrt{6}}{\sqrt{5}-\sqrt{11}}$	3. $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{3}}$
4. $\dfrac{1}{\sqrt{3}}$	5. $\dfrac{\sqrt{6}}{3\sqrt{2}}$	6. $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}$
7. $\dfrac{\sqrt{8}-\sqrt{6}}{2-\sqrt{3}}$	8. $\dfrac{4}{\sqrt{6}-\sqrt{2}}$	9. $(\sqrt{3}-\sqrt{2})\dfrac{\sqrt{2}}{\sqrt{6}-2}$

No exit tickets for this topic.

Topic 8 Solving $ax^2 + c = 0$

Objective:

Today I will learn how to solve a quadratic equation in a form of $ax^2 + c = 0$

The Lesson

To solve the equation $ax^2 + c = 0$, first you isolate the quadratic term

$$x^2 = -\frac{c}{a}$$

and solve for x^2

Practice: Solve the following equations

1. $\frac{2}{3}x^2 - 4 = 14$	2. $3(x-3)^2 + 2 = 26$
3. $\frac{2}{3}(x+8)^2 - 66 = 0$	4. $7 - 3x^2 = 1$
5. $20(x-5)^2 + 14 = 0$	6. $-2(x+2)^2 + 72 = 0$

No exit tickets.

Topic 9 Completing the square

Objective:

Today I will learn how to complete the square for a quadratic expression like x^2+bx+c.

The Lesson

The term "completing the square", in general means to rewrite $x^2+bx+c=(x+h)^2+k$. The process follows a certain steps (3 steps actually) to make it done. Although the process is very straight forward and easy to follow; yet, to most students, completing the square seem to be a strange and difficult concept. The mostly likely problem may be:

1. What is the purpose? To my best knowledge, the purpose is historical: in the history, when people encountered equations like $x^2+6x+4=0$ that cannot be factored, they asked this question: can it be rewritten to some perfect square form? If the answer is yes; then square root the perfect square may yield the solutions.

2. When rewriting the expression, why do you divide *b* by 2 and and square it, $\left(\frac{b}{2}\right)^2$? Hopefully the powerpoint animation you are about to watch will answer the question…

<u>Watch the power point (What the $^&# do you divide the middle term and square it?)</u>

Steps to completing the square: for x^2+6x+4

Step 1 : Separate the x terms and the constant	$x^2+6x+4=(x^2+6x)+4$
Step 2: Divide the middle term by 2:	$x^2+6x+4=(x^2+6x)+4$ $=\left[(x+3)^2-9\right]+4$
Step 3: Combine the constants,	$x^2+6x+4=(x^2+6x)+4$ $=(x+3)^2-5$

Examples: Complete the square for following trinomials

1. $x^2+8x+13$	2. $x^2-18x+21$
3. x^2-7x+5	4. $x^2+\frac{3}{2}x-\frac{7}{16}$
5. $x^2+\frac{8}{3}x+\frac{16}{9}$	6. $x^2-\frac{4}{5}x+\frac{29}{25}$

Exit Tickets:

1. $x^2+\frac{1}{2}x+\frac{5}{16}$	2. $x^2+\frac{1}{3}x-\frac{1}{12}$
3. $x^2+2x+\frac{5}{2}$	4. $x^2+\frac{6}{7}x+\frac{9}{49}$

Topic 10 Completing the square II

Objective:

Today I will learn how to complete the square for a quadratic expression like ax^2+bx+c.

The Lesson

Today we are going to use what we learned from yesterday and apply it to a more general form of a quadratic expression like $2x^2+4x-7$**. How can we complete a square like this? You will see me demonstrate through the examples**

Example:

1. $2x^2+4x-7$	2. $3x^2-6x+19$
3. $-3x^2+12x-8$	4. $2x^2+5x-2$

Conclusion: If coefficient $a\neq 0$**, factor out** a **from the expression and then you can complete the square like you did yesterday.**

Exit Tickets:

1. $6x^2+6x-4$	2. $7x^2+28x-9$
3. $2x^2-5x+10$	4. $4x^2+4x-1$
5. $2x^2+7x-\frac{9}{16}$	6. $-2x^2-3x+\frac{5}{16}$

Topic 11 Use Completing the Square to Solve $ax^2+bx+c=0$

Objective:

Today I will learn how to apply the method of completing the square for an equation like $ax^2+bx+c=0$.

The Lesson

If you still remember, when we are going through the topic of completing the square, I said, the reason that we use this method (or study this method) is partly historical. Meaning, once in a time, mathematicians really use this method (although seems lengthy) to solve problems. Today's topic will bring you back to that time…

Example: Solve $3x^2+8x-2=0$

This is an equation, you cannot factor, even the big-diamond method falls short on this problem. Complete the square will be the method to consider when big-diamond is not good enough.

Step 1: Move the constant to the other side: add 2 to both sides. $3x^2+8x=2$
Step 2: Complete the square for the left-hand side of the equation. (I will show you how to get there) $3x^2+8x=2$ $3(x+\frac{4}{3})^2-\frac{16}{3}=2$
Step 3: Now solve the equation like $ax^2+c=0$

More examples: Solve the equations:

1. $x^2 - 5x + 2 = 0$	2. $2x^2 + 8x + 3 = 0$

Practice:

1. $x^2 + 6x + 4 = 0$	2. $-x^2 + 5x - 3 = 0$
3. $4x^2 + 4x - 1 = 0$	4. $2x^2 - 8x - 1 = 0$
5. $3x^2 - 6x - 5 = 0$	6. $2x^2 + 7x = \frac{23}{4}$

Topic 12 Graph $y = a(x-h)^2 + k$

Objective:

Today I will learn how to graph a quadratic equation in its vertex form.

The Lesson

Although intercept form of a quadratic function seems to be a good tool to know the characteristics of the graph of a parabola, the biggest problem of the intercept form is: "What if a parabola cannot be factored?" Make it worse, "What if a parabola does not have x-intercepts?"

The answer to the questions above is simple: "Don't use the intercept form." Use **the vertex form** of the quadratic function instead.

The vertex form is the based on the fact that "every quadratic function $y = ax^2 + bx + c$ can be rewritten into $y = a(x-h)^2 + k$ by completing the square."

Example: Graph $y = 2x^2 + 4x + 1$

Compare the function with $y = ax^2 + bx + c$,

$a =$	
$b =$	
$c =$	

Now, use complete the square to rewrite the expression into vertex form you will get

$$y = 2(x+1)^2 - 1$$

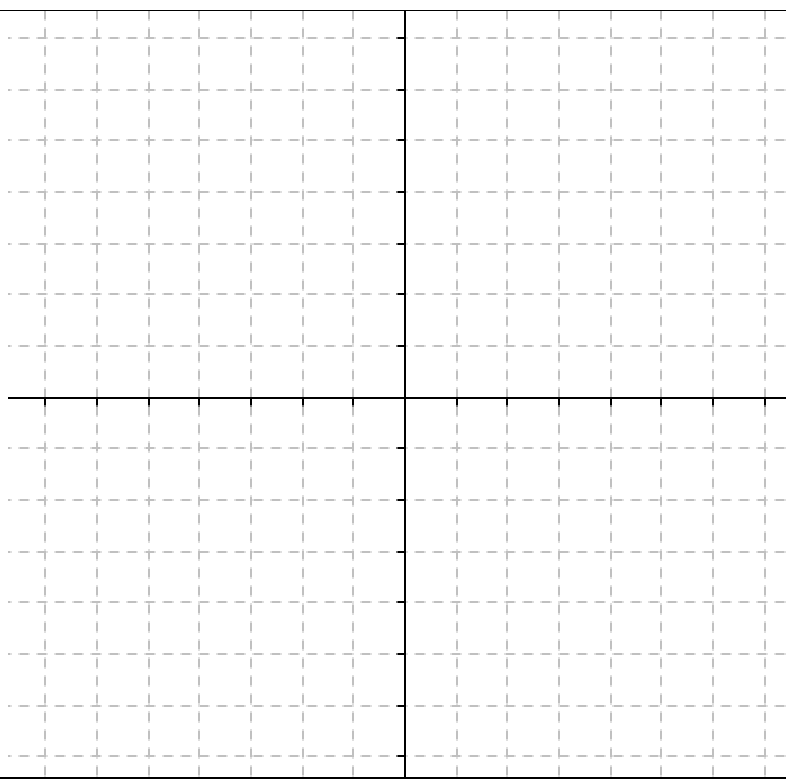
Compare the function with $y = a(x-h)^2 + k$,

$h =$	
$k =$	

Now, let us analyze the function as we did before,

Opening	When $a>0$, parabola opens up When $a<0$, parabola opens down	In our case, the function
Axis of symmetry	$x-h=0$, (that is $x=h$)	In our case, the axis of symmetry is
Coordinates of the vertex (Min or Max of the range)	(h,k)	In our case, the coordinates of the vertex are
y-intercept	$(0,c)$	In our case, the y-intercept is
Possible x-intercepts	Solve $a(x-h)^2+k=0$ $(h+\sqrt{\frac{-k}{a}},0)$, and $(h-\sqrt{\frac{-k}{a}},0)$	In our case, the x-intercepts are
Supplemental points	At least one supplemental point can be found (for all quadratic functions): $\left(-\frac{b}{a},c\right)$	In our case, the coordinates of the supplemental point are

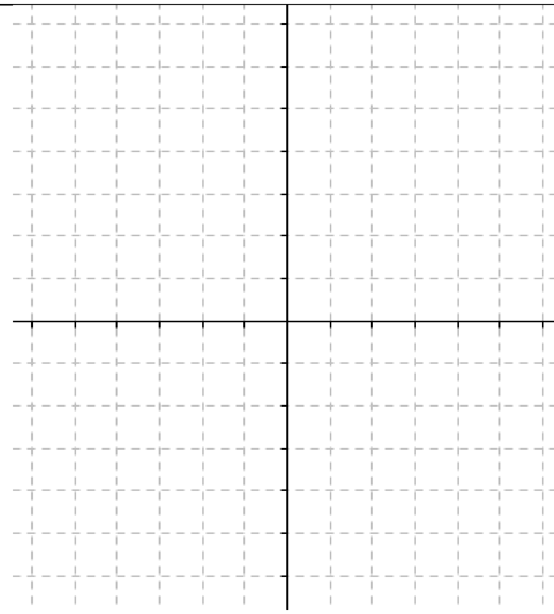
Now, graph the function



Practice:

1. $y = -x^2 + 6x + 2$

$a =$	
$b =$	
$c =$	
$h =$	
$k =$	

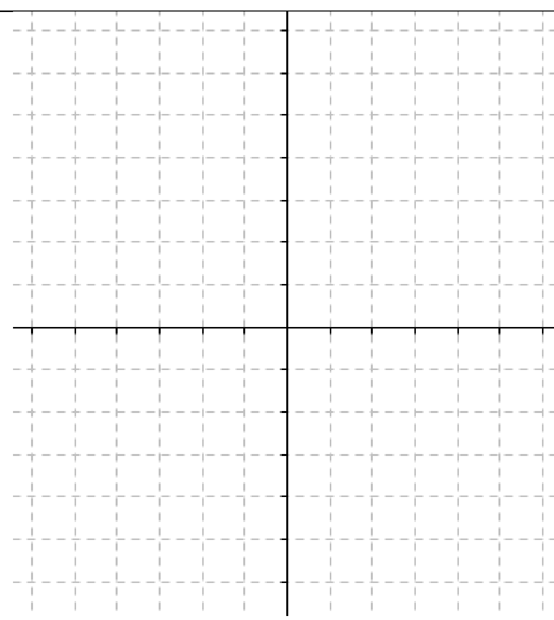


Opening	
Axis of symmetry	
Coordinates of vertex	

y-intercept	
x-intercepts	
Supplemental point	

2. $y = x^2 + 3x + 5$

$a =$	
$b =$	
$c =$	
$h =$	
$k =$	



Opening	

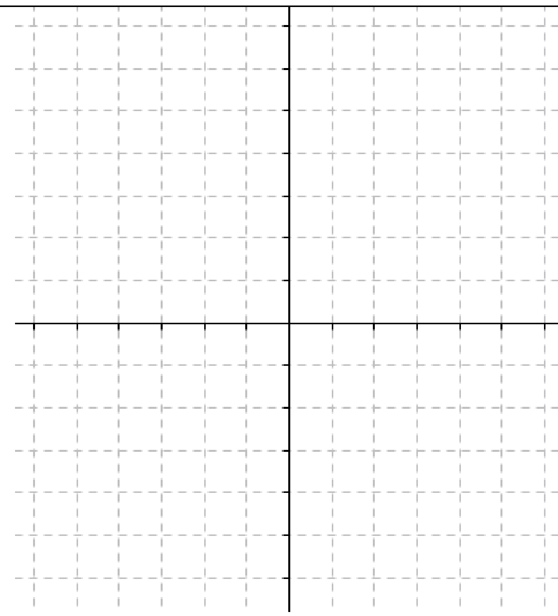
y-intercept	

Axis of symmetry	
Coordinates of vertex	

x-intercepts	
Supplemental point	

3. $y = -2x^2 + 8x + 5$

$a =$	
$b =$	
$c =$	
$h =$	
$k =$	



Opening	
Axis of symmetry	
Coordinates of vertex	

y-intercept	
x-intercepts	
Supplemental point	

Topic 13 Graph $y=a(x-\alpha)(x-\beta)$

Objective:

Today I will learn how to graph a quadratic equation in its intercept form.

The Lesson

The graph of a quadratic function $y=ax^2+bx+c$ is a parabola. Similar to linear functions, quadratic equations can be written into different forms that help us to visualize/analyze them.

One of these forms is the intercept form $y=a(x-\alpha)(x-\beta)$.

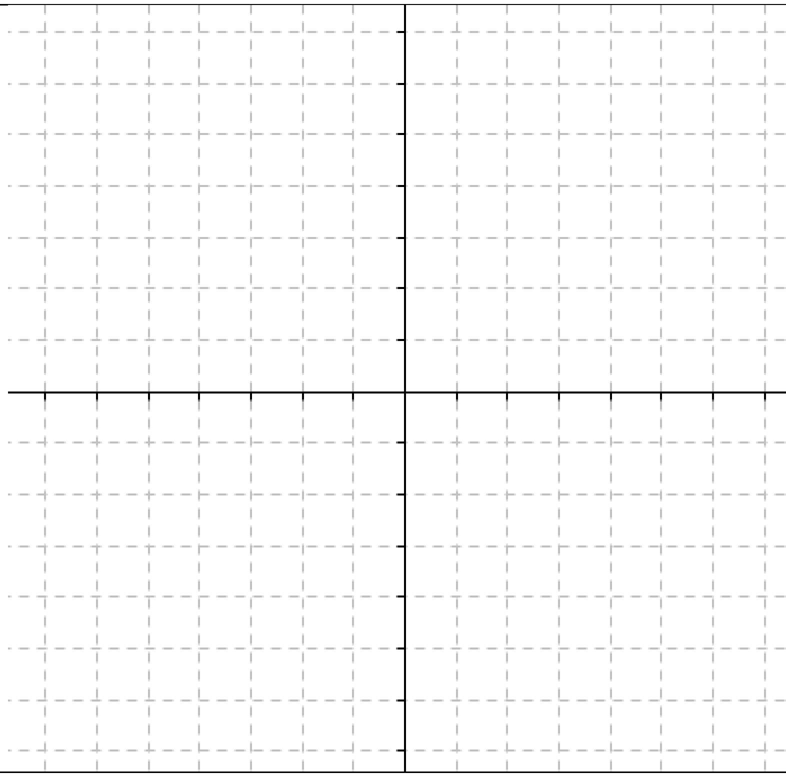
Consider $y=-x^2-2x+8$, if you use small diamond method, you can easily factor the function into $y=-(x-2)(x+4)$. Compare it with the form $y=a(x-\alpha)(x-\beta)$, you get

$a=$
$\alpha=$
$\beta=$

When you rewrite a quadratic function into the intercept form, it reveals the following characteristics of the function:

Opening	When $a>0$, parabola opens up When $a<0$, parabola opens down	In our case, the function
x-intercepts	The x-intercepts are at $x=\alpha, x=\beta$	In our case, the x-intercepts are $x=$, $x=$
y-intercept	The y-intercept is at $y=a\alpha\beta$	In our case, the y-intercept is $y=$
Axis of symmetry	The axis of symmetry is $x=\dfrac{\alpha+\beta}{2}$	In our case, the axis of symmetry is $x=$
Vertex (Min. or Max. of the range)	The coordinates of the vertex are $\left(\dfrac{\alpha+\beta}{2},-a\left(\dfrac{\alpha-\beta}{2}\right)^2\right)$	In our case, the coordinates of the vertex are

With these pieces of information about the function, you can graph the function with ease

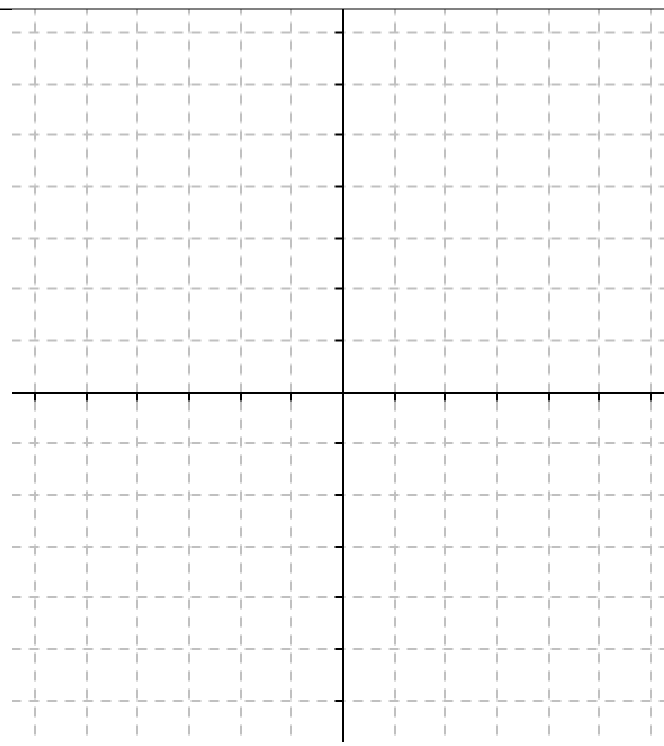


Graph the following quadratic function using the intercept form.

1. $y = 2(x-2)(x+2)$

$a =$	
$\alpha =$	
$\beta =$	

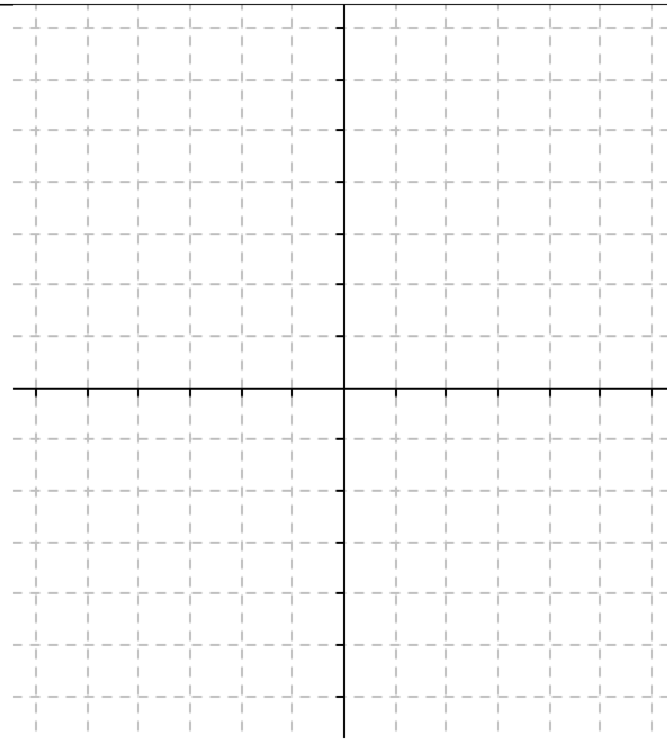
Opening	
x-intercepts	
y-intercept	
Axis of symmetry	
Coordinates of vertex	



2. $y=-2x^2-4x+6$

$a=$	
$\alpha=$	
$\beta=$	

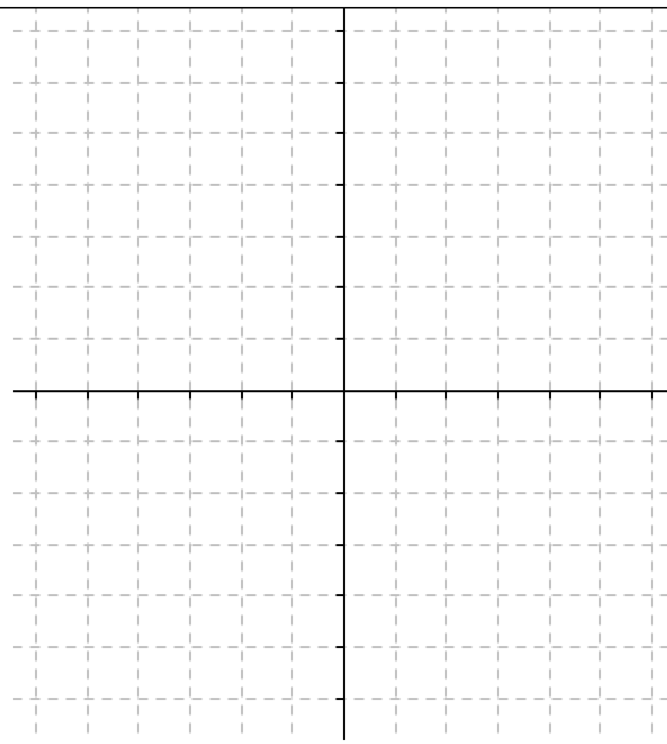
Opening	
x-intercepts	
y-intercept	
Axis of symmetry	
Coordinates of vertex	



3. $y=-\frac{1}{2}x^2+x+\frac{15}{2}$

$a=$	
$\alpha=$	
$\beta=$	

Opening	
x-intercepts	
y-intercept	
Axis of symmetry	
Coordinates of vertex	



Topic 14 $i = \sqrt{-1}$, The Imaginary Number

Objective:

Today I will learn the basics of the complex system.

The Lesson

In the history, the imaginary number was not documented until the late 16th century. It was the works of Leonhard Euler and Carl F. Gauss (in the late 18th century) that showed the significance and usefulness of imaginary numbers to the modern world. Believe it or not, many modern technologies including, computer science, 3D graphics, and your cell phone, hinge their existences on this concept ($\sqrt{-1}$) that does not even have a place on the real number line. So, exactly, what is $\sqrt{-1}$?

Definition of the imaginary number

Define the imaginary number $i = \sqrt{-1}$ as one of the solution of $x^2 = -1$. $x = \pm i$.

Investigating $i = \sqrt{-1}$:

Example 1: Simplify the expressions below:

1. $i^2 + i^4$	2. $2i^2 + i^3 - 1$	3. $i^3 - 6i + 2i^6$	4. $i^4 + i^3 + i^2 + i$

Complex Number and the Z-Plane

Expanding the idea of the imaginary number on a plane, use two real numbers a, b, and combine it as

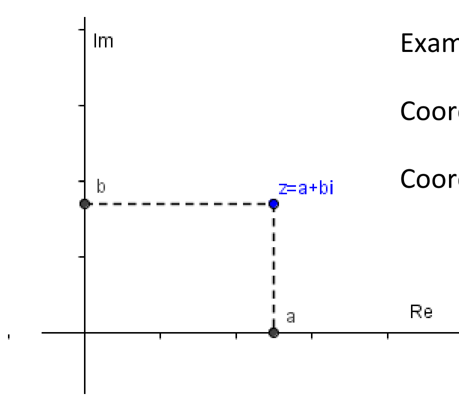
$$z = a + bi$$

z is a **complex number**, which can be located on a z-plane (like a xy-plane)

<u>z-plane: a plane forms by two perpendicular number lines (axes).</u>

Real axis (Re): the x-axis

Imaginary axis (Im): the y-axis



Example: if $z = 3 + 2i$, that means:

Coordinate on the real axis = 3

Coordinate on the imaginary axis = 2

Adding two complex numbers:

Let

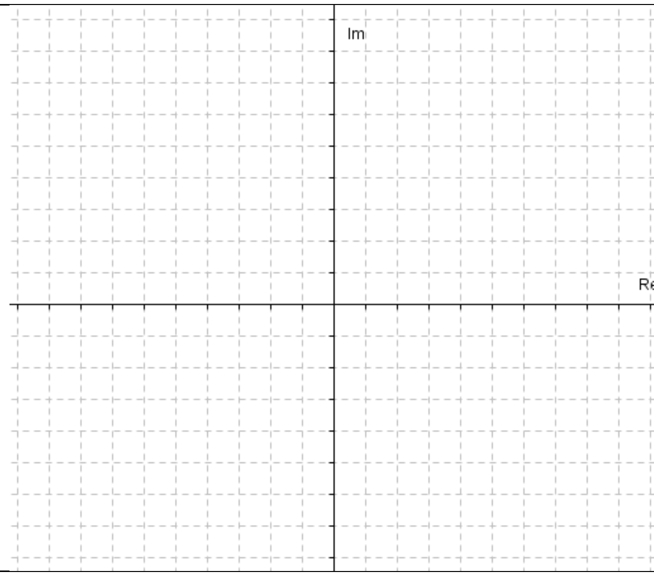
$$z_1 = a_1 + b_1 i, \qquad z_2 = a_2 + b_2 i$$

Then

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$

Ex 2: Locate each complex number below on the z-plane

$z_1 = 1 + i$	$z_6 = z_1 + z_2$
$z_2 = -2 + 4i$	$z_7 = 3i + z_3$
$z_3 = 2i$	$z_8 = 10 - 2i + z_5 + z_2$
$z_4 = 2 - 3i$	$z_9 = z_7 + z_5$
$z_5 = -4$	$z_{10} = 3 + z_1$



Multiplying two complex numbers:

Complex numbers possess the distributive property, when multiplying, you are actually distributing the real and imaginary part of the number:

$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$

$$= (ac - bd) + (ad + bc)i$$

Ex 3 : Find the product of the following complex numbers

1. $(3+i)(4+2i)$ =

2. $(-4+5i)(\frac{1}{2} - \frac{3}{4}i)$

Dividing two complex numbers:

Like radical numbers, mathematical convention demands that when computing $\frac{z_1}{z_2}$, no imaginary part of a complex number is allowed in the denominator. Therefore, the fraction needs to be "rationalized" by using a complex conjugate to make the denominator real.

A complex conjugate of a complex number $z = a + bi$ is defined as $\bar{z} = a - bi$

Example 4 : Find a complex conjugate of z=4+2i ,and find the product of $z \cdot \bar{z}$

Find quotient of the followings

1. $\frac{1+2i}{1-3i}$	2. $\frac{5}{1+i}$	3. $\frac{5+2i}{3-2i}$

The Norm of a complex number:

Let $z = a + bi$, and we define the norm of the complex number z to be

$$|z| = \sqrt{z \cdot \bar{z}} = \sqrt{a^2 + b^2}$$

Example 5: Find the norms of the following:

1. $3+4i$	2. $\frac{-1}{2i}$	3. $\frac{3+i}{2}$	4. $\frac{-i+2}{(i-4)(5+i)}$

Exit Tickets: Given 3 complex numbers, rewrite each expression in the grid below in standard form and compute the norm of each complex number.

$z_1 = 1 + i$,

$z_2 = -2 + 4i$

$z_3 = 2i$

1. $z_4 = (z_1 + z_2)(z_2 - z_3)$	2. $\dfrac{2z_1 + z_4}{z_2}$	3. $\dfrac{z_1}{z_3 - \frac{1}{2} z_2}$
4. $z_1^2 + 2z_1 + 1$	5. $z_2^{-1} + z_1^{-2}$	6. $\dfrac{z_1 + z_2}{z_3 + z_4}$

Topic 15 Derive and Use the Quadratic Formula for $ax^2 + bx + c = 0$

Objective:

Today I will learn how to derive and use the quadratic formula to solve an equation like $ax^2 + bx + c = 0$.

The Lesson

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **is the general solution for a quadratic equation like** $ax^2 + bx + c = 0$. This means if you can identify the a, b, and c in an equation like $3x^2 - 2x - 1 = 0$, then you can use the formula and find the solutions.

You will watch an power point animation to find out how does this formula come about…

(Derive quadratic formula using complete the square)

Identify a, b, and c and use the quadratic formula to solve the equations.

1. $6 = x^2 + 3x$	2. $4x + 7 = x^2$
3. $4x = x^2 - 2$	4. $5x = 3x^2$

Practice: Identify a, b, and c and use the quadratic formula to find solutions for the following equations.

1. $x^2 - 4x - 3 = 0$	2. $-x - x^2 = 4$
3. $-6x^2 + 3x + 2 = 3$	4. $-4x + 5x^2 + 1 = 3x$
5. $x + x^2 + 1 = 0$	6. $2 = x + 4x^2$

Topic 16 Quadratic Inequalities

Objective:

In this topic, you will learn how to graph the solution of the quadratic inequalities and solve quadratic inequalities of 1 variable.

The Lesson:

Objective 1:

We will apply the idea of graphing an inequality in two variables for linear relation into the quadratic relations.

1. Graph $y \le x^2 - 4x - 5$	2. Graph $\begin{cases} y \ge x+1 \\ y \le -x^2 + 3x + 4 \end{cases}$

3. Graph $y > x + 2x^2 - 3$	4. Graph $\begin{cases} y < -x^2 + 2x - 4 \\ y > x^2 - 4x + 4 \end{cases}$

Objective2:

Solve the quadratic inequalities with one variable

1. $2x^2 - 3x + 1 \le 0$	2. $2x^2 + 6x - 3 < 0$

3. $x^2+4x+8\geq 0$	4. $x^2-6x+12<0$
5. $x^2+x+\frac{1}{4}\leq 0$	6. $-x^2+3x-2\leq\frac{1}{4}$

Topic 17 Use $y = a(x-\alpha)(x-\beta)$ and $y = a(x-h)^2 + k$

Objective:

Today I will learn how to write a quadratic equation in its intercept and vertex form.

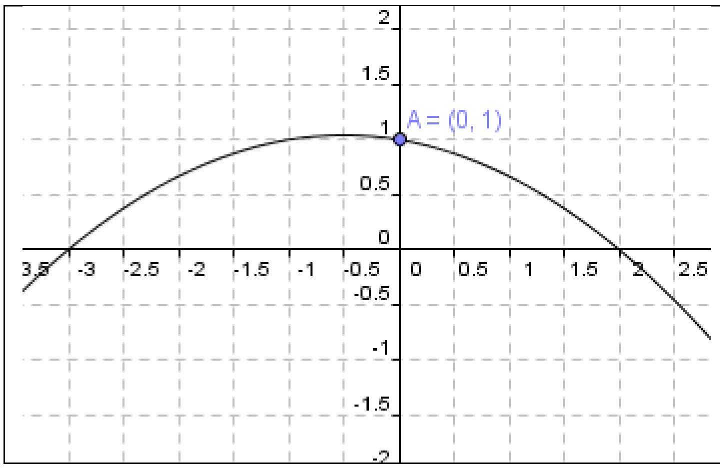
The Lesson

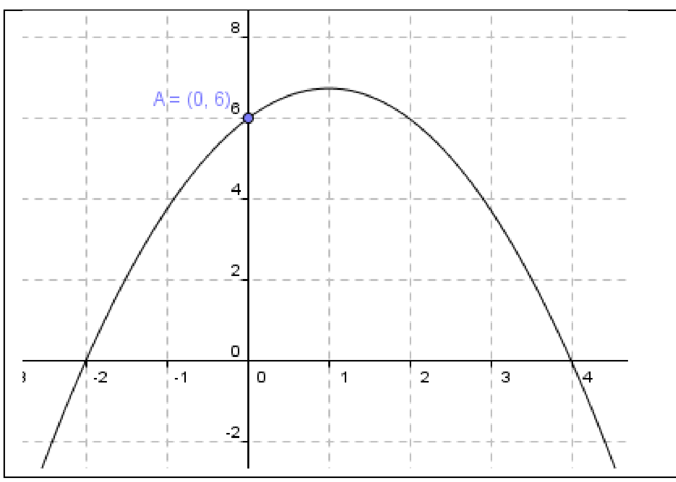
We will **apply the intercept and vertex form** of a quadratic relation **to applications.**

Objective 1: Interception form application

Use the graphs to find

(1) The quadratic equation.
(2) The coordinates of the vertex
(3) The axis of symmetry





Objective 2: The vertex form

Find Maximum or Minimum

Every quadratic function has a vertex. Depending on the value of a, the y-coordinate of the vertex is the minimum or the maximum of the function.

If $a > 0$, then the function has a minimum $y = k$.

If $a < 0$, then the function has a maximum $y = k$.

Find the maximum or minimum of the following quadratic functions:

1. $y = -2x^2 + 6x$	2. $y = x^2 - 6x + 7$

Find the Axis of Symmetry

Every quadratic function has an axis of symmetry, that is $x - h = 0$.

Axis of symmetry can also be found by $h = \frac{-b}{2a}$

Find the axis of symmetry for the following functions

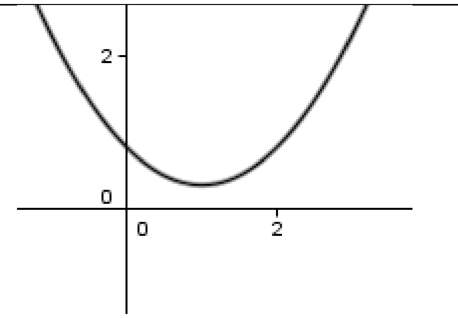
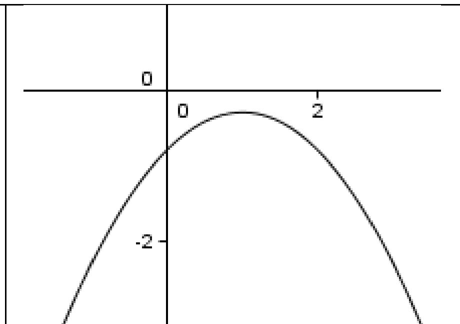
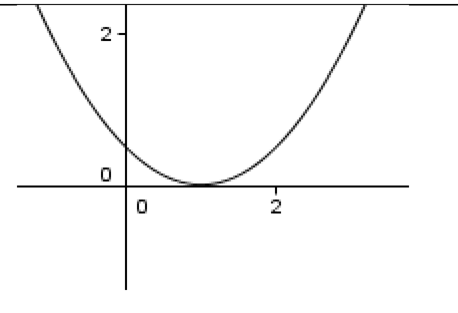
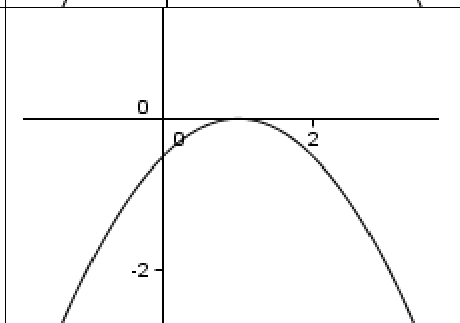
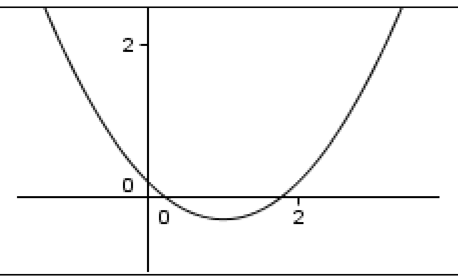
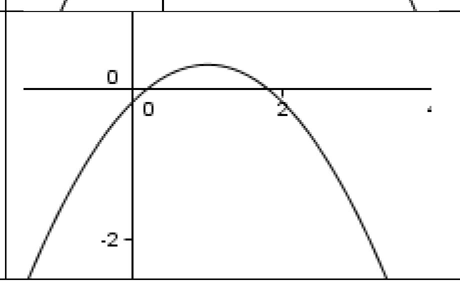
3. $y = \frac{1}{2}x^2 - x + 3$	4. $y = -\frac{2}{3}x^2 + 18x - 7$

Identify numbers of real zeros(x-intercepts)

When a quadratic function is rewritten into the vertex form

$$y = a(x-h)^2 + k$$

The product of a and k provides a very interesting fact, illustrated below:

$ak > 0$	No x-intercept		
$ak = 0$	1 x-intercept		
$ak < 0$	2 x-intercepts		

Use the vertex form to identify if the functions has x-intercepts? If it has x-intercepts, where are they?

5. $y = \frac{1}{2}x^2 + 2x$	6. $y = \frac{1}{3}x^2 - 4x + 16$

For the following quadratic functions

(1) Rewrite the function in the vertex form
(2) Find its minimum or maximum
(3) Find the axis of symmetry
(4) Identify the numbers of x-intercepts

1. $y = -2x^2 + 4x - 6$	2. $y = -\frac{1}{2}x(x+3)$

Topic 18 Write a quadratic relation

Objective:

In this topic, you will learn how to write a quadratic relation through 3 points.

The Lesson

Find a quadratic relation that passes through (-1, -3) , (0, -4) , (2, 6). Uses the relation you found to

1. Find the vertex

2. The x and y intercepts (if exists)

3. Axis of symmetry

4. Graph the quadratic relation.

5. Find the intersection of the quadratic function and y = 60-3x

Practice:

Write a quadratic functions of the following:

1. Vertex: (-2, 4) and passes (0, 3)	2. Intercepts of (4, 0), $\left(\frac{5}{2}, 0\right)$ with a vertex in the first quadrant and distance between the vertex and the x-axis is $\frac{3}{4}$.
3. passes through (-2, 6), (2, -1), (10, -2)	4. vertex of (-2, 1), passes (3, -9)