Class Discussion

Unit 5 Topic 4 Sum and different Angle formula

Objective: This is a set of formula must-memorized. Ask the students to memorize

Sum and difference of angle formulas

1. $\sin(a+b) = \sin a \cos b + \cos a \sin b$

2. $\sin(a-b) = \sin a \cos b - \cos a \sin b$

3. $\cos(a+b) = \cos a \cos b - \sin a \sin b$

4. $\cos(a-b) = \cos a \cos b + \sin a \sin b$

Ex 1:

Use the following diagram to prove $\cos(a-b)$. [The setup order is very important, the proof will not make sense to the students if the steps are not followed. However, on the other hand, if the steps are followed precisely, most students understand and the proof comes to plain sight of itself]

1. Make circle O be a unit circle.

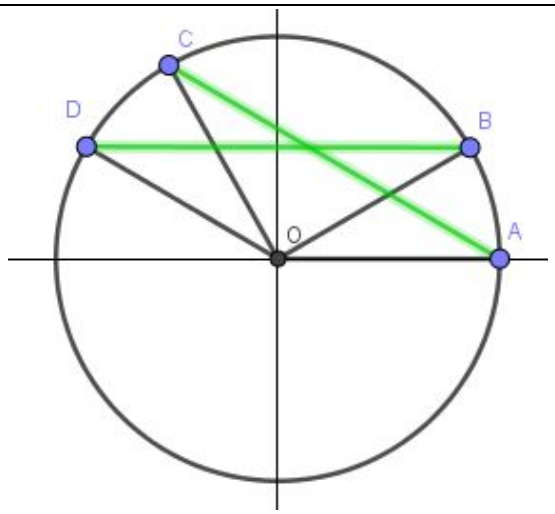
2. Set up points $A(1,0)$, $B(x_1, y_1)$, $D(x_3, y_3)$ on the circle so the $m\angle AB = b$, $m\angle AD = a$ and $m\angle BD = a - b$.

3. Choose a point $C(x_2, y_2)$ on $\overset{\frown}{BD}$ so that $m\angle ABC = a - b$.

4. Construct $\overline{AC}$ and $\overline{BD}$.

5. Based the setup from step 1 – 4, $\Delta AOC \cong \Delta BOD$ (SAS)

6. $\because$ A, B, C, and D are on the unit circle.

	$x_2 = \cos(a-b)$, $x_3 = \cos a$, $y_3 = \sin a$ $x_1 = \cos b$, $y_1 = \sin b$

7. Based on step 5, $\overline{AC} \cong \overline{BD}$, apply distance formula and simplify, eventually,

$$x_2 = x_1 x_3 + y_1 y_3$$

8. and thus prove the formula

Ex 2: if $\sin x = \frac{4}{5}$, $\frac{\pi}{2} < x < \pi$; $\cos y = -\frac{5}{13}$, $\pi < y < \frac{3\pi}{2}$, k and m are real numbers

$$\begin{cases} k\sin(x+y) - m\cos(x-y) = \frac{74}{65} \\ k\sin(x-y) + m\cos(x+y) = \frac{98}{65} \end{cases}$$, find m and K?