

Please read this instruction before you start answering questions:

1. Your goal is to collect 65 points.
2. Different questions have different point values.
3. Your Point Value = sum of [(questions point) * (point factors)]
4. Questions with (*), the point factor = 1, questions without (*), the point factor = 0.2
5. Write your answers in the answer documents. Please make your writing legible. It is your responsibility to make the grader understand your reasoning process.

1. Solving the following systems of equations:

$$\text{a. (5 pts) } \begin{cases} 2x + y - 3z = -5 \\ x - y + z = 4 \\ 3x + 2y + z = 3 \end{cases}$$

$$\text{b. (5 pts) } \begin{cases} \frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 1 \\ \frac{5}{6}x + \frac{3}{8}y - \frac{5}{48}z = \frac{1}{8} \\ \frac{3}{4}x - \frac{1}{2}y + \frac{1}{8}z = \frac{7}{2} \end{cases}$$

2. Solve the following equations by factoring.

$$\text{a. (5 pts) } 32x^2 - 42x - 9 = 0$$

$$\text{b. (5 pts) } \frac{39}{7}x = 5 - 2x^2$$

$$\text{c. (5 pts) } x^2 = x - 552$$

$$\text{d. (5 pts) } 26x^2 - 173x + 26 = 0$$

3. Use complete the square to solve the following equations. (Some of the equations might have complex solutions)

$$\text{a. (5 pts) } x^2 = x + 1$$

$$\text{b. (5 pts) } 2x^2 - 8x + 17 = 0$$

$$\text{c. (5 pts) } \frac{1}{3}x^2 = 5x - 7$$

$$\text{d. (5 pts) } x^2 + 7x - 3 = 0$$

4. Rationalize the expressions:

$$\text{a. (5 pts) } \frac{\sqrt{10} + \sqrt{3}}{\sqrt{5}}$$

$$\text{b. (5 pts) } \frac{2}{\sqrt{5} - 1}$$

$$\text{c. (5 pts) } \frac{\sqrt{8} - \sqrt{12}}{\sqrt{3} - \sqrt{2}}$$

$$\text{d. (5 pts) } \sqrt{6} \frac{\sqrt{18}}{\sqrt{3} - \sqrt{7}}$$

5. Graph the quadratic functions and identify (i) the opening, (ii) the x-intercepts and y-intercept, (iii) the vertex and the axis of symmetry.

$$\text{a. (10 pts) } y = -\frac{1}{4}(x+2)(x-3)$$

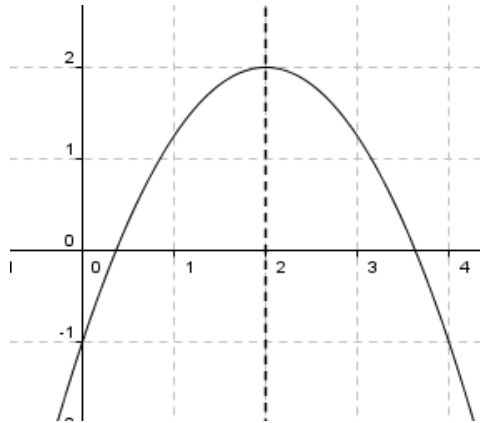
$$\text{b. (10 pts) } y = 4x^2 - 16x + 17$$

$$\text{c. (10 pts) } y = x^2 - x - \frac{15}{4}$$

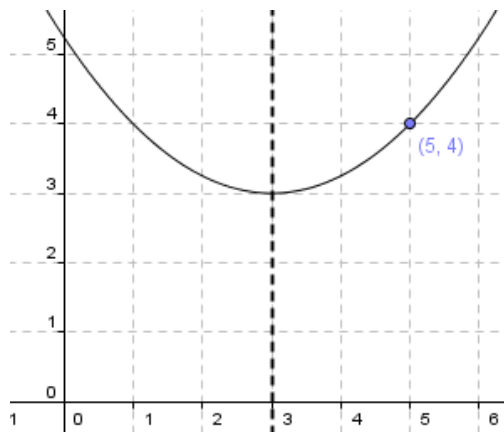
$$\text{d. (10 pts) } y = x^2 - 4x + 1$$

6. Write the function in standard forms that represents the graph

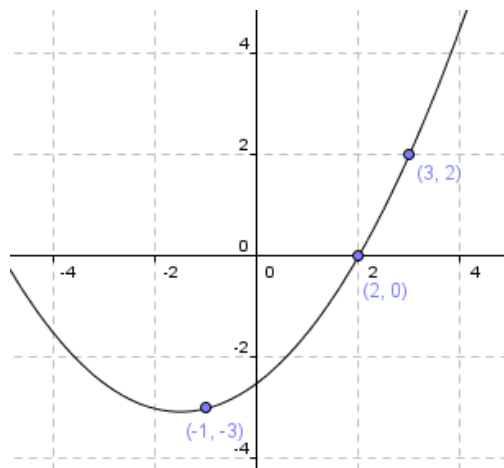
a. (10 pts)



b. (10 pts)



c. (10 pts)



7. Let $i = \sqrt{-1}$, if $z_1 = 1 - i$, $z_2 = i - \sqrt{3}$,
 $z_3 = -1 - i$

Evaluate

a. (5 pts) $z_4 = z_1 + z_2$

b. (5 pts) $z_5 = z_2^2$

c. (5 pts) $z_6 = z_1 z_2$

d. (5 pts) $z_7 = \frac{z_3}{z_1}$

8. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

Evaluate

a. (5 pts) AB

b. (5 pts) BA

c. (5 pts) ABA

d. (5 pts) BAB

9. (10 pts) The surface area of a right cylinder is 72π square inches. If the height of the cylinder is 5 inches, find the radius of the cylinder?

10. In art, a golden rectangle (as shown, ABCD) is considered to be the most aesthetically pleasing (the most beautiful) to human eyes.

It follows the rule: “ the ratio of the lengths of long side (AB) to short side(BC) and the ratio of the length of the short side(BC) to the difference of the lengths of the long side and short side (AB – BC) are the same.” In another words, $ABCD \sim BCFE$.

Now, let $BE = x$, $CB = y$; also that, AEFD is a square.

(a) (10 pts) Use the definition of golden rectangle to show that $y^2 - x^2 = xy$

(b) (10 pts) Define the golden ratio to be

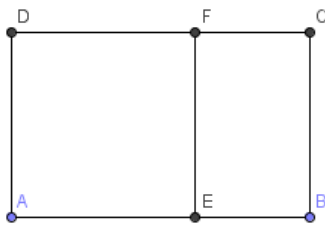
$r = \frac{y}{x}$. From the conclusion of question (a),

prove that $r = \frac{1+\sqrt{5}}{2}$.

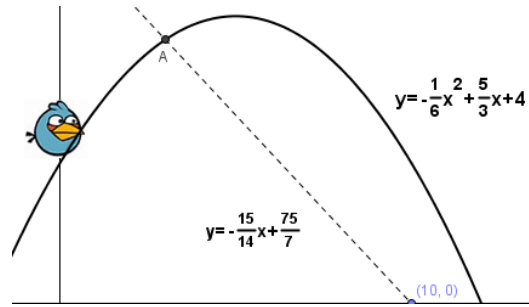
(c) (10 pts) Use the definition of golden ratio defined in (b) and show that the area of ABCD is:

$$r(r+1)x^2$$

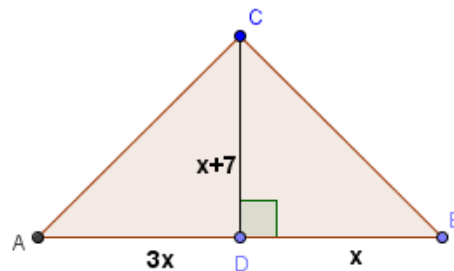
(d) (5 pts) What is the area of the golden rectangle ABCD, if $BE = \sqrt{2}$?



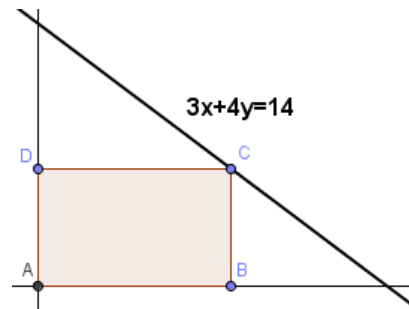
11. (10 pts) You shot an angry bird from (10,0) as shown in the diagram. If the trajectory of the angry bird was $y = -\frac{1}{6}x^2 + \frac{5}{3}x + 4$, and the trajectory of the bullet was $y = -\frac{15}{14}x + \frac{75}{7}$, what were the coordinates where the angry bird was shot (point A)?



12. (10 pts) As shown below, the area of $\triangle ABC$ is 16 square units, find the perimeter of $\triangle ABC$.



13. (10 pts) Herald has a triangular backyard. (the triangle bounded by x-axis, y-axis, and $3x + 4y = 14$) He plants tomatoes in the rectangular area ABCD.



(a) Find the coordinates of C so that he can maximize the rectangle.

(b) What is the area of ABCD when it reaches its maximum?