EVHS Algebra II Unit 6 Handouts

Logarithm and Conic Sections (Sec 7.3-7.5, Ch 9)

Before your start…

- You should know that "Important Policy Change" is in this handout, please read (the calendar).
- **YOU** are strongly **encouraged** to have the following items when you come to class
  - **papers (to show your work, can be binder papers or any white papers, as long as they are clean)**
  - **graph papers**
  - **pencils (or color pencils)**
- **All class work has to meet the minimal requirement to be graded. Following bullets constitute the minimal requirement:**
  - **legible**
  - **on a clean paper**
  - **#2 pencils only (color pencils ok, only if graphing)**
  - **graphs need to be on a graph paper.**
- **Work/paper does not meet the minimal requirement will not be accepted until fixed.** +

+ if you have any questions about the minimal requirement due to financial hardship, please see Mr. Chen

Contents

EVHS Algebra II Unit 6 Calendar

Monday	Tuesday	Wednesday	Thursday	Friday
Feb 24 Get Ready for Unit 6	25 **Topic 1**	26	27 **Topic 2**	28
Mar 3 **Topic 3**	4	5 **Topic 4**	6	7 **Topic 5**
10 (Daylight Saving Starts) Review for logarithm	11 Quiz (U6 Topic 1 – 5) 30 points	12 **Topic 6**	13	14 **Topic 7**
17	18 **Topic 8**	19	20 **Topic 9**	21
24 **Topic 10**	25	26 **Topic 11**	27	28 (Minimum Day)
31 Cesar Chavez Day No School	1 Review for SA6 (1)	2 Review for SA6(2)	3 SA6 (part 1) 100 points	4 SA6 (part 2)

Important Policy Change:

- **Assignments and Assessments**: There will be no more **Homework Assignment** due in class. Your **everyday** assignment is to watch a new video before you come to class (whenever there is one). In class, you are going to **complete** the handout/textbook questions (assignment category: 2 pts) for the topic and take an **exit ticket quiz** (assessment category: 4 pts) for the day.
- All exit ticket quizzes need to be answered **in pencils only**, (color pencils ok for graphing). Exit ticket quizzes can be make-up **only if** the absence is excused.
- **Projects and Portfolios**: In a learning unit, you may be assigned Projects and/or Portfolios that may include (1) demonstrate what you know about what you learn, (2) research the history of a mathematical concept or (3) find an application for a mathematical concept. Points earned in a project will be based on a performance rubric and it will be in the participation category. Details about projects and portfolios will be disclosed when a project is assigned.
- **Off-task behaviors in class may disqualify you in demonstrating your projects or turning in your portfolios. Some** of the off-task behaviors include (1) leaving your seat without permission, (2) improper use of technology, or (3) distract/disrupt others to work in class.

For Ex 1-3 and practice, please show your work and answer in your own paper.

Topic 1 Definition and Evaluate the Simple Logarithm expressions

Definition:

$$\log_b x$$

(read as :logarithm of x with respect to the base of b. or "**Log x based b**") is a real number. This number is the exponent y of the following exponential equation,

$$x = b^y$$

Therefore exponent y is just a real number with a complicated form.

Ex1: What is the exponent of the following equations?

1. $4^y = 16$	2. $(\frac{1}{3})^y = 9$	3. $25^y = 125$	4. $343^x = 49$
5. $\left(\frac{1}{16}\right)^x = 512$	6. $8^a = \frac{1}{1024}$	7. $9^x = 243$	8. $\left(\frac{1}{25}\right)^x = 1$

Ex2: Use logarithm expressions to write every answer from example 1

Ex 3: Evaluate following expressions :

1. $\log_2 16$	2. $\log_{\frac{1}{3}} 27$	3. $\log_{64} 16$
4. $\log_{25} 5$	5. $\log_{\frac{1}{5}} 625$	6. $\log_7 343$

Properties of Logarithm: (Based on the observation of Ex 4-6 to conclude the properties)

(1) Logarithm of 1: $\log_b 1$ =_____

Ex 4:

1. $\log_7 1$	2. $\log_{\frac{1}{4}} 1$	3. $\log_{.2} 1$

For Ex 1-3 and practice, please show your work and answer in your own paper.

(2) Logarithm of base: $\log_b b =$ _____

Ex 5:

1. $\log_7 7$	2. $\log_{\frac{1}{4}} \frac{1}{4}$	3. $\log_{.2} .2$

(3) Logarithm of reciprocal: $\log_b \left(\frac{1}{b}\right)$ =_____

Ex 6:

1. $\log_3 \left(\frac{1}{3}\right)$	2. $\log_{\frac{1}{11}} 11$	3. $\log_{0.4} \left(\frac{5}{2}\right)$

Common Logarithm and Natural Logarithm

These two logarithms are logarithms with specific bases :

Common logarithm : base of 10. Mathematical convention allows the following abbreviation:

$\log_{10} 102 = \log 102$ (read as "log 102" not "log 102 based 10"), $\log_{10} 5 = \log 5$ (read as "log five")

Natural logarithm: base of e. (The euler number), mathematical convention allows (requires)

$\log_e 24 = \ln 24$ and instead of reading it as "log 24 based e", you **have to** read the logarithm as "natural log 24"

Practice: Evaluate the following expressions:

1. $\log_{27} 9$	2. $\log_8 32$	3. $8^{(\log_2 4)-(\log_3 27)}$
4. $(\log_4 16)^{\log_{\frac{1}{4}} 64}$	5. $\log_{12} \left(\frac{1}{144}\right)$	6. $(\log_3 3)+\left(\log_{\frac{1}{2}} 2\right)+\log_6 1$

For Ex1,2 and 3, please show your work and answer on your own paper.

Topic 2 Exploring logarithm properties 1

Objective: In this topic, you will learn 1. inverse property, 2. product property, 3. quotient property and 3. the power property.

Inverse properties

1. $\log_b\left(b^x\right)=x$

2. $b^{\log_b x}=x$

Ex1 Simplify the expressions

1.	2.	3.	4.
$7^{\log_7 x}$	$\log_3 81^x$	$30^{\log_{30} 4}$	$e^{\ln 2}$

Prove Inverse Properties:

1. $\log_b\left(b^x\right)=x$	2. $b^{\log_b x}=x$

Product Property:

3. $\log_b(xy)=\log_b x+\log_b y$

Quotient Property:

4. $\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

Power Property:

5. $\log_b\left(m^n\right)=n\log_b m$

Ex2. Expand the logarithm expressions:

For Ex1,2 and 3, please show your work and answer on your own paper.

1. $\log 105 =$	2. $\log_3 45$	3. $\log_5 25x$
4. $\ln(\frac{5x^2}{64})$	5. $\log\left(\frac{45}{y^2}\right)$	6. $\log_2 36$

Ex3. Condense the logarithm expressions (and simplify if possible):

1. $\ln 9+3\ln 2-\ln 3$	2. $\log 4+3\log 3-\log 12$	3. $\log_4 24-\log_4 3$
4. $5\log x - 4\log y$	5. $\ln 40 + 2\ln\frac{1}{2} + \ln x$	6. $e^{\ln x^2}+\ln(e^x)^2+e^{\ln 1}$

prove the product, quotient and power properties

Prove: $\log_b(xy)=\log_b x+\log_b y$
Prove: $\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$
Prove: $\log_b(m^n)=n\log_b m$

For ex1, please show your work and answers in your own paper.

Topic 3 Exploring the Logarithm Properties 2

Objective: You will learn 1. uniqueness property and 2. change of base property for logarithms

Uniqueness Property

6. Let $a, b, c > 0$

$a = b$ if and only if $\log_c a = \log_c b$

Uniqueness property may look trivial, however, it is one of the most important property you will use for solving logarithm equations and algebraic reasoning.

Prove:

if $a = b$ then $\log_c a = \log_c b$

if $\log_c a = \log_c b$ then $a = b$

Change of base formula

7. if $a, b, c > 0$ and $b \neq 1, c \neq 1$, $\log_c b = \dfrac{\log_a b}{\log_a c}$

Reciprocal Property

8. $\log_b a = \dfrac{1}{\log_a b}$ (you can also rewrite it as: $\left(\log_a b\right) \cdot \left(\log_b a\right) = 1$

For ex1, please show your work and answers in your own paper.

Ex 1: Given

$$\log 2 \approx 0.3 \text{ , } \log 3 \approx 0.48 \text{ , } \log 5 \approx 0.7 \text{ , } \log(e) \approx 0.43 \text{ , } \ln 7 \approx 1.94$$

Without using a scientific calculator evaluate:

1. $\log_2 3$	2. $\ln 32$	3. $\log_8 2 + \log_4 12$
4. $\log_5 8$	5. $3\log_4 6$	6. $\ln 10$
7. $\ln\left(\frac{4}{3}\right)$	8. $\log_9 12$	9. $\log_{\frac{1}{6}} 15$
10. $\log_2 4 + \log_6 5 - \log_8 10$	11. $\log 25 - \log 2.5$	12. $\log 7$
13 $\log_3 21$	14. $\log_{16} 5$	15. $\log_{0.4}\left(\frac{1}{10}\right)$

Prove: if $a, b, c > 0$ and $b \neq 1, c \neq 1$, $\log_c b = \dfrac{\log_a b}{\log_a c}$

Prove: if $a, b > 0$ and $a \neq 1, b \neq 1$, $\log_b a = \dfrac{1}{\log_a b}$

For questions in Practice, please show your work and answer on your own paper.

Topic 4 Logarithm as an inverse function

Objective: Students will learn that logarithm is the inverse operation of exponential functions.

The Lesson

Based on the definition of $y = \log_b x$, y is the power to raise, with the respect of the base b, in order to equal to x. If x>0, this definition $y = \log_b x$ can be a function with domain x>0.

$g(x) = \log_b x$, $x > 0$ And g(x) is actually an inverse function of $f(x) = b^x$.

Verify with chained operations:

$f \circ g =$ $g \circ f =$

What is the behavior of the inverse function? Let us use the following example to explore:

Ex 1: Graph $f(x) = 2^x$ and $g(x) = \log_2 x$

$f(x) = 2^x$

x	-1	0	1	2
f(x)				

Domain:

Range:

Horizontal Asymptote:

$g(x) = \log_2 x$

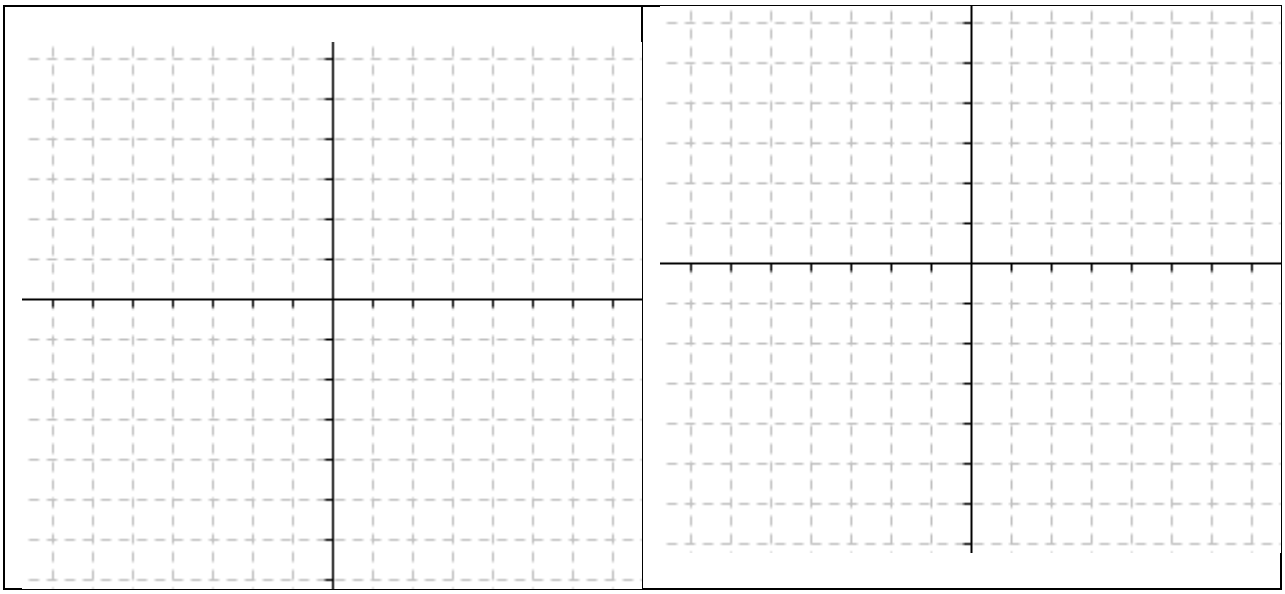
x				
g(x)	-1	0	1	2

Domain:

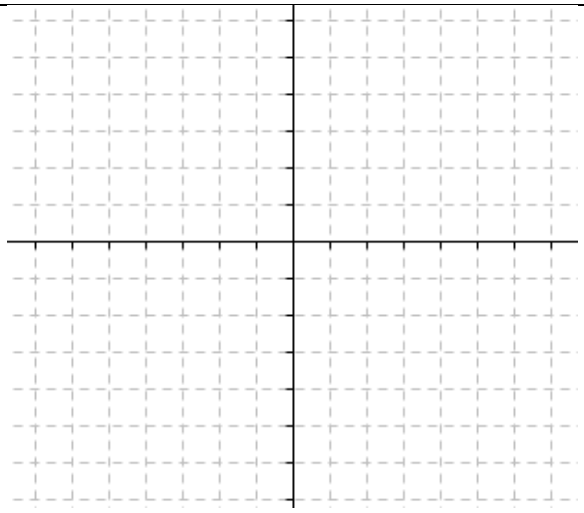
Range:

Vertical Asymptote:

For questions in Practice, please show your work and answer on your own paper.



Now, Combine the graphs onto the same coordinate plane



Observation and conclusions:

1. about domain and range

2. about asymptotes

3. about axis of symmetry

Practice:

Find the inverse function of a given function, and graph both functions on the same coordinate plane

For questions in Practice, please show your work and answer on your own paper.

For each function identify the domain, range and asymptotes

1. $y = \log_3 x$	2. $y = \log x$
3. $y = \log_{\frac{1}{4}} x$	4. $y = \ln x$
5. $y = \left[\log_{\frac{1}{3}} (x-3) \right] - 1$	6. $y = \left[\log_2 (x+2) \right] - 1$
7. $y = [\log(x+1)] + 2$	8. $y = \log_2 (x^2 + 4x + 4)$
9. $y = \ln x^2$	10. $y = 2^x - 3$

For Ex 1-5, please work on your own paper.

Topic 5 Logarithm Applications in the Real World

Objectives: You are going to expose to several types of word problems

Ex1 The altitude h (in meters) above the sea level is related to the atmospheric pressure at the level. It can be modeled by

$$h = -8000 \ln\left(\frac{P}{P_0}\right)$$

Where P_0 is the reference pressure (in Kpa). Given that the pressure is 65Kpa at the altitude of 3,500 m, What is the height if the pressure is 57Kpa?

Ex2 pH value is defined as

$$pH = \log\left[H^+\right],$$

$\left[H^+\right]$ is the concentration of hydrogen ions in a solution in (mole/L).

Now, if a solution has a pH= 5.6, what is the hydrongen ion concentration in the solution?

Ex3 (Continue from Ex 2) All Aqueous solution has an interesting constant called the product of water, which states

$$\left[H^+\right]\left[OH^-\right] = 10^{-14}$$

where $\left[H^+\right]$ is the concentration of hydrongen ions, and $\left[OH^-\right]$ is the concentration of hydroxide.

If the Lipton ice tea Mrs. Chen made Sunday night has an initial pH= 7.8 what was the concentration of hydroxide in the tea?

Ex4 (Continue Ex 3) Mrs. Chen decide to put some Lemonade flavored Crystal Lite into the tea and after the Crystal Lite was totally dissolved, the concentration of $\left[OH^-\right] = 5\times10^{-8}$, what is the pH of the tea?

Ex 5 In music, the governing equation of the frequencies between two notes can be modeled as follow:

$\log_{\sqrt[12]{2}}\left(\frac{f_n}{f_0}\right) = n$ where

n is steps between a note and a reference note

f_n is the frequency (the pitch) of the note n steps away from the reference note

f_0 is the frequency of the reference note

For example, Tone (note) A has a frequency of 440 Hz, we can calculate the frequency of B flat which is 1 step away from A using this model. What is the frequency of B flat?

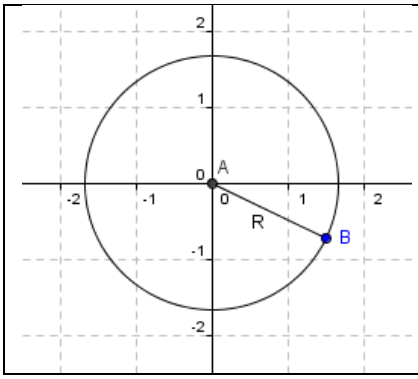
Topic 6 Circles

Objective: Students will learn how the equation of a circle come about, students will also identify a circle when given an equation or a graph.

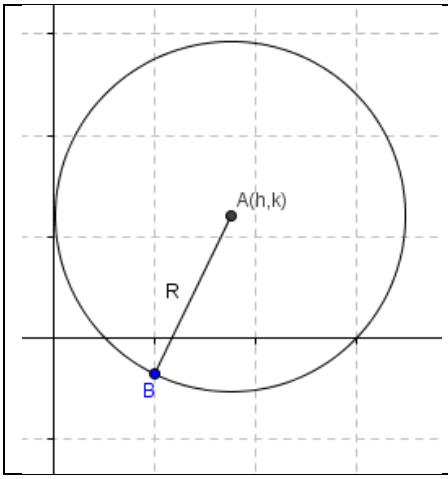
Definition: Circle is a set of all points (x, y) in a plane that are equidistant to a fixed point, (the center).

Use the definition a circle to form a relation between x and y so that all solutions of (x, y) will describe the circle.

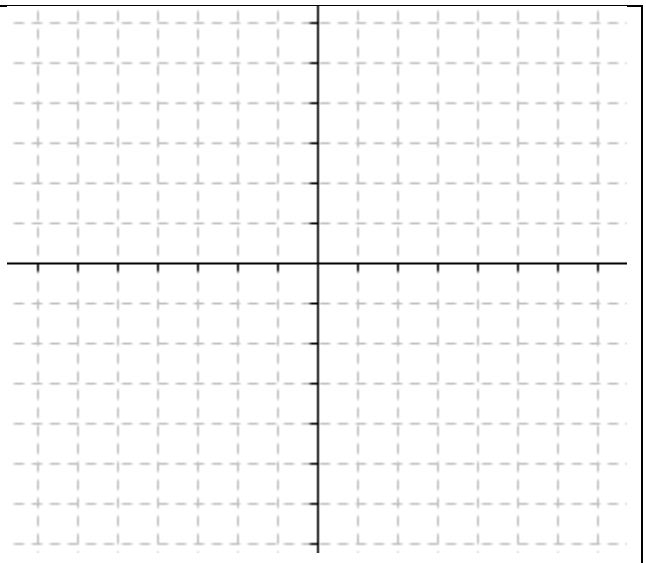
Ex 0: Center (0, 0), and radius r.



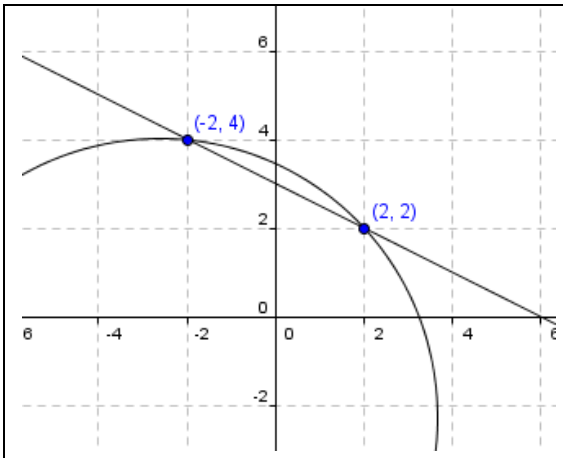
Ex1: Center (h, k) and radius r.



Ex 2: Let H(2, 3) and K(3, -5) be the end points of a diameter on a circle.
(a) Write a relation of x and y so that it describe the circle.
(b)Identify the coordinates of the center and the radius;
(c) graph the circle.

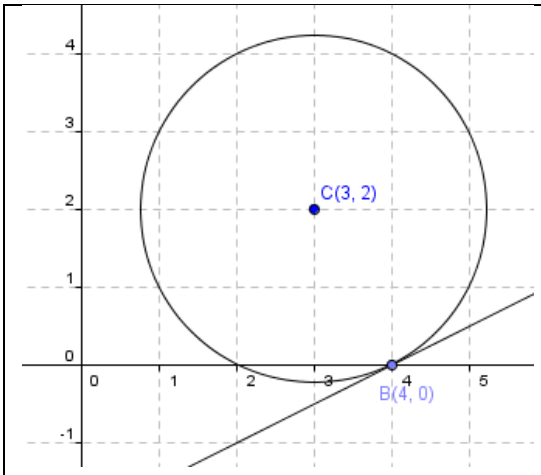


Ex 3: Line A passes (-2, 4) and (2, 2) as shown in the diagram. If a circle T also passes through the two points with radius of 5. Find all possible centers of circle T? What is the equation of circle T?

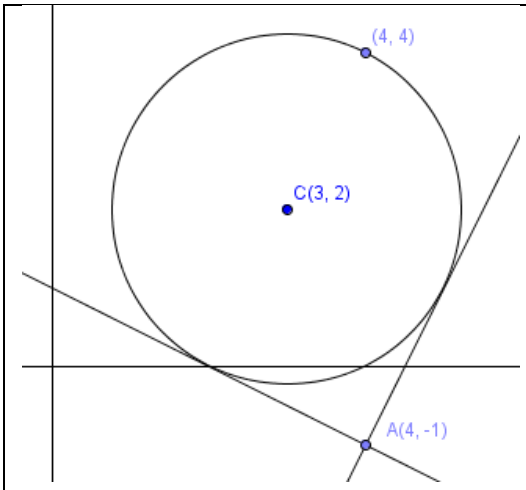


Objective 2: Tangents of a circle

Ex 4: As shown in the diagram, circle C centered at (3, 2) and passes B(4,0), if a Line T is the tangent line shown in the diagram. Write the equation of T?



Ex5 As shown in the diagram, circle C centered at (3, 2) and passes (4,4), if a Line Q and R are the tangent lines passes through A, what are the equations of the tangent lines?

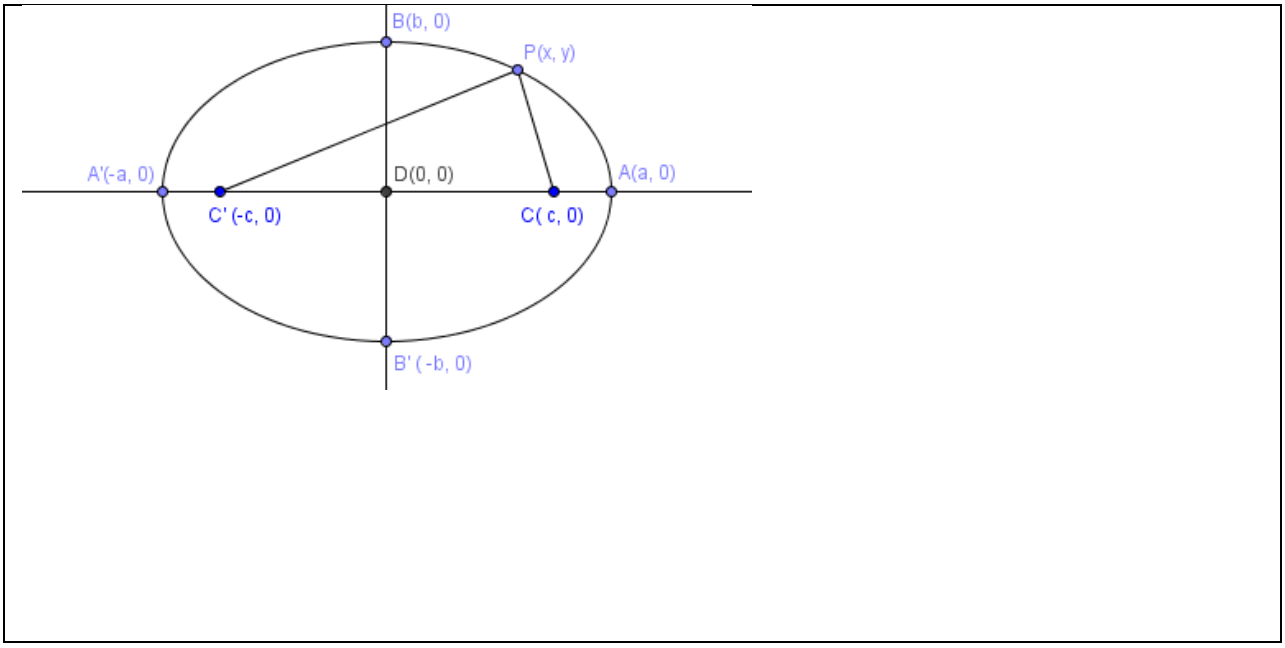


For example 1-4, please show your work on your own paper.

Topic 7 Ellipses

Definition: An ellipse is the set of all points P(x, y) in a plane such that the sum of distances between P and two fixed points are constant.

Ex: Ellipse E is shown below. Use the definition of an ellipse to derive the standard form of an ellipse centered at origin D(0, 0).[That is find an equation for x and y so that all solutions for P(x,y) is the set of the ellipse]



Terms for ellipse and their definitions: (as illustrated in the diagram above)

Foci: Point C and C'	The two fixed points on the plane. CC'=2c
Center: Point D	The midpoint of C and C'
Major axis	The line passes through two foci , $\overleftrightarrow{CC'}$
Minor axis	The perpendicular line of the major axis that passes through the Center, $\overleftrightarrow{BB'}$
Vertices: Point A and A'	The two points on the major axis on the opposite sides of the center, AA' = 2a and a > c
Co-vertices: Point B and B'	The two points on the minor axis that are on the opposite sides of the center. BB' = 2b and $a^2 = b^2 + c^2$

Based on the definition of vertices and co-vertices, the vertices always lie on the longer slant side.

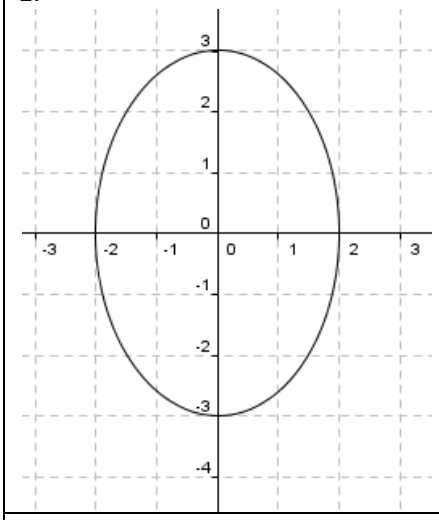
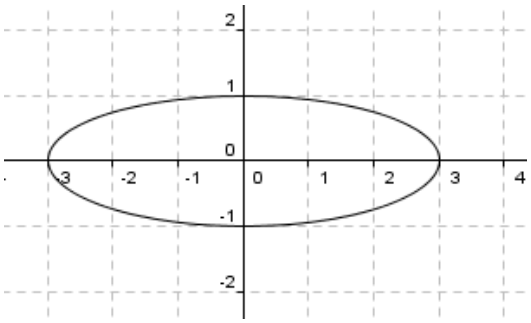
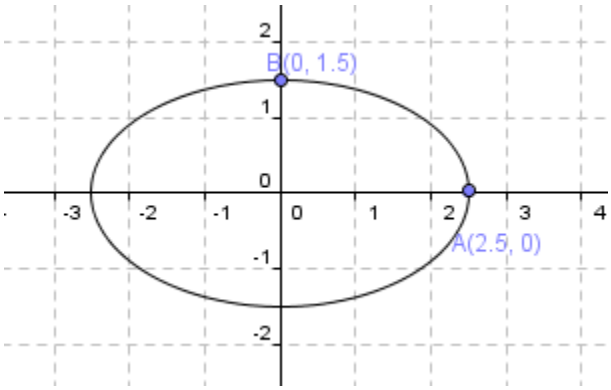
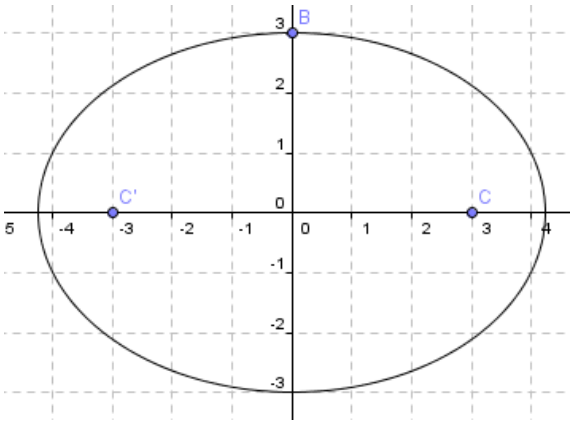
Why?

Ex1: Graph the ellipses, identify the vertices, co-vetices, major axis, minor axis and foci.

For example 1-4, please show your work on your own paper.

1. $\frac{x^2}{16}+\frac{y^2}{4}=1$	2. $4x^2+y^2=36$
3. $16x^2+25y^2=1600$	4. $72x^2+8y^2=648$

Ex2: Find the equation of the ellipse from the graphs below, find the coordinates of the foci or vertices (depends on which pair is missing):

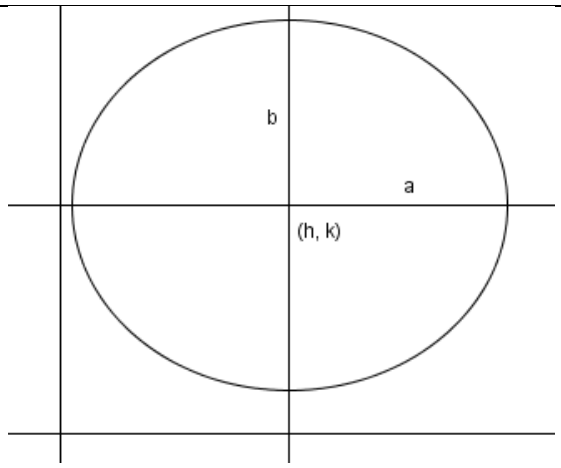
1. 	2. 
3. 	4 

Ex3 An ellipse has foci at (-2, 0) and (2, 0). If the same ellipse also passes through (3, 3), find the equation of the ellipse, the coordinates of the vertices and the co-vertices.

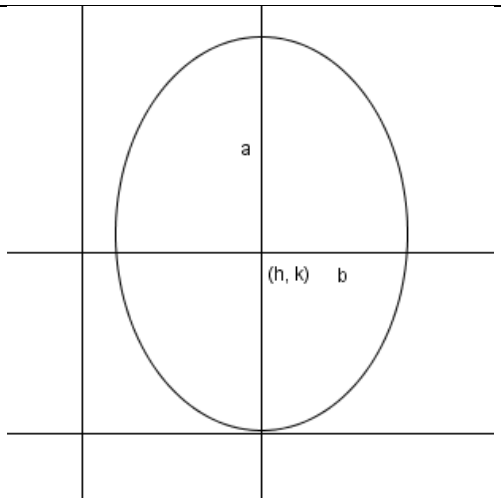
Objective 2: Translations

When the center of an ellipse moves to (h, k), the equation of the ellipse changes to

For example 1-4, please show your work on your own paper.



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



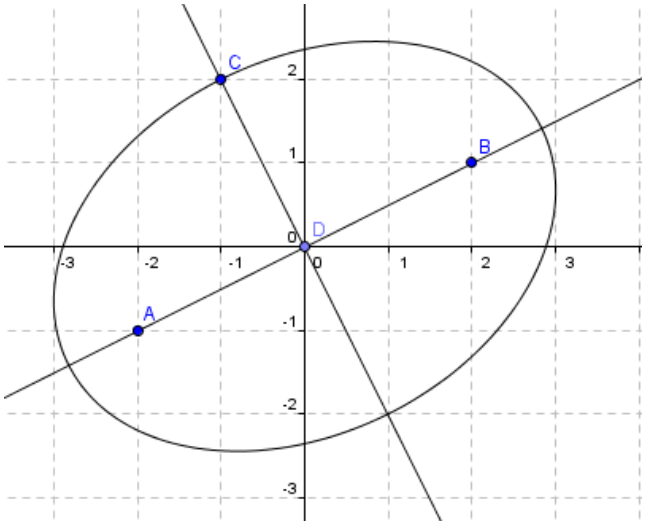
$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Ex4 Graph the ellipses

1. $6(x-4)^2+3(y+2)^2=48$	2. $\frac{(x-3)^2}{4}+\frac{(y-4)^2}{9}=1$
3. $9(x+1)^2+(y+2)^2=4$	4. $\frac{y^2}{16}+\frac{4(x-1)^2}{9}=1$

Ex5:

Ellipes D is titled. Use the information in the graph to find the equation of ellipse D. A and B are foci of the foci.



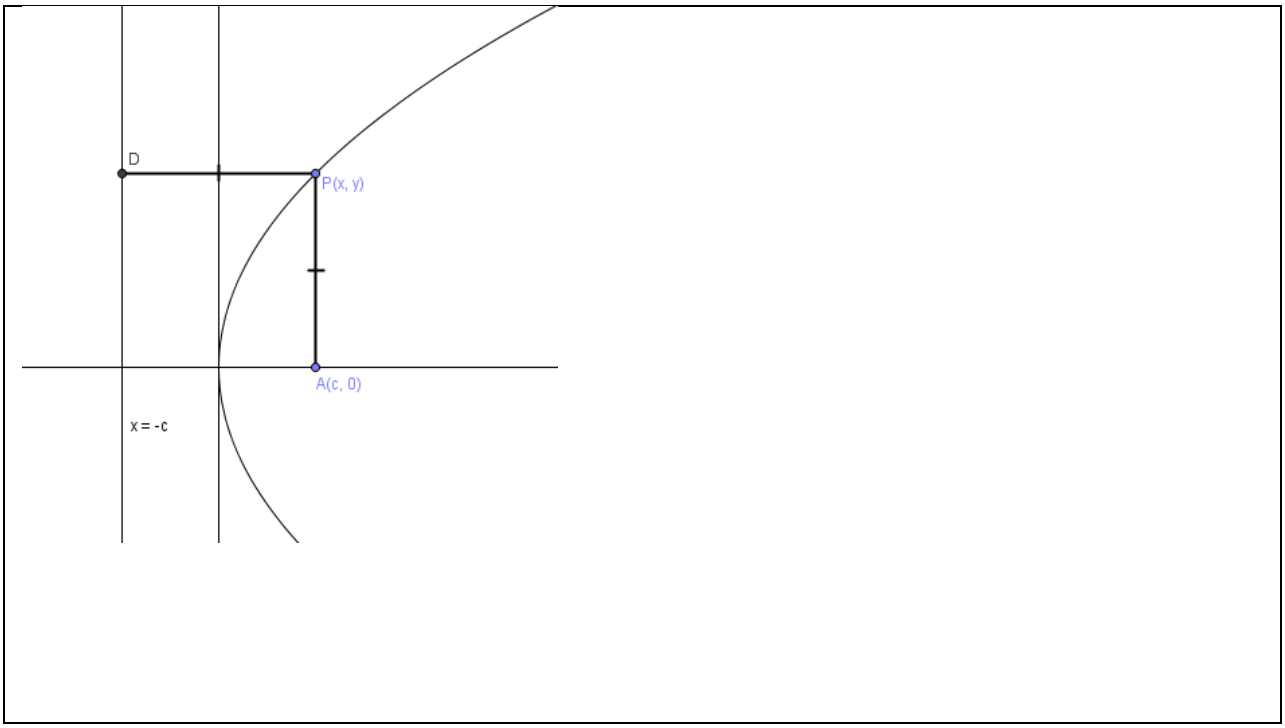
Ex 2, 3, 5 please show your work and answer on your own paper.

Topic 8 Parabolas

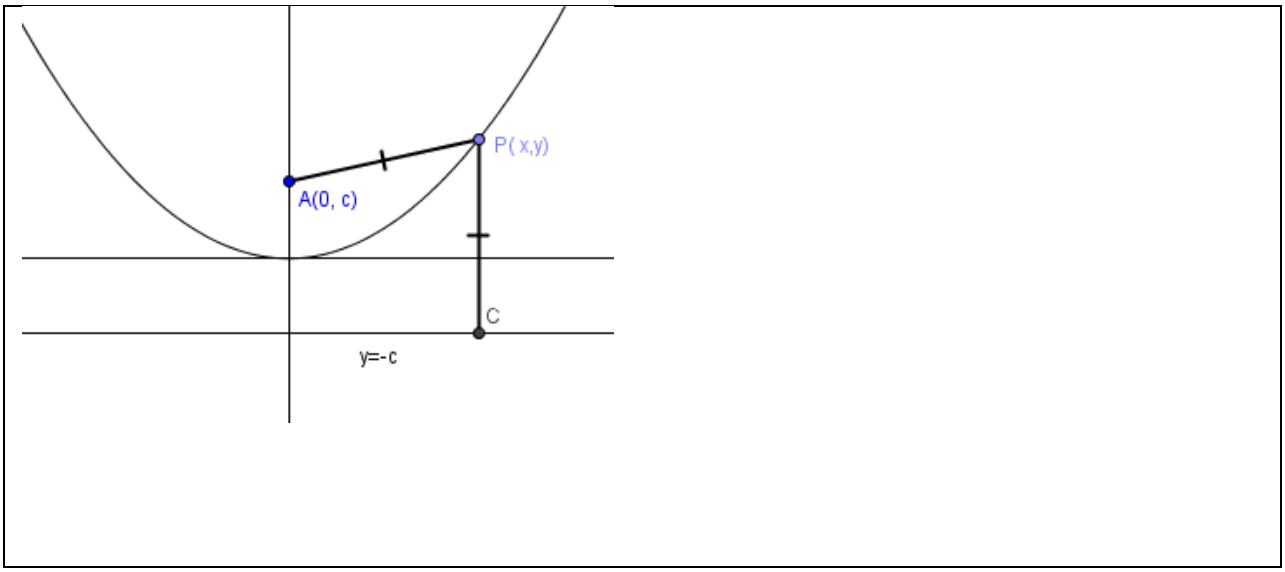
Objectives: Learn the parabolas from the perspectives of a focus and a directrix

Definition: A parabola is a set of points that is equidistant from a line, the directrix and a point, the focus.

Ex0: Use the definition of a parabola to derive a parabola with focus at (c, 0) where c > 0 and the directrix x=-c.



Ex1: Use the similar idea to derive a parabola with focus at (0,c) where c > 0 and the directrix y = -c.

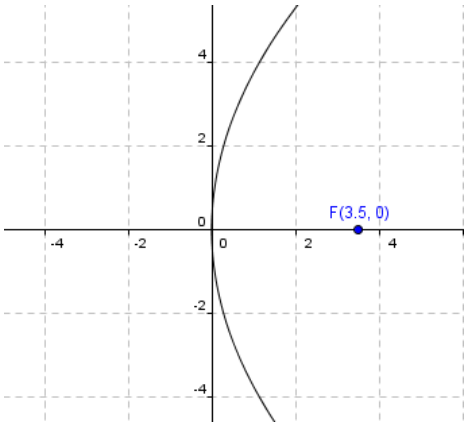
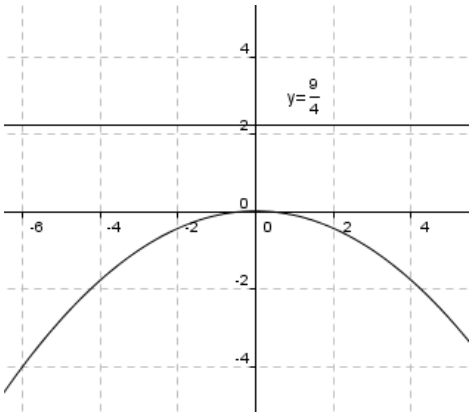


Ex 2, 3, 5 please show your work and answer on your own paper.

Ex2: Graph and identify the coordinates of the focus and directrix of the parabola

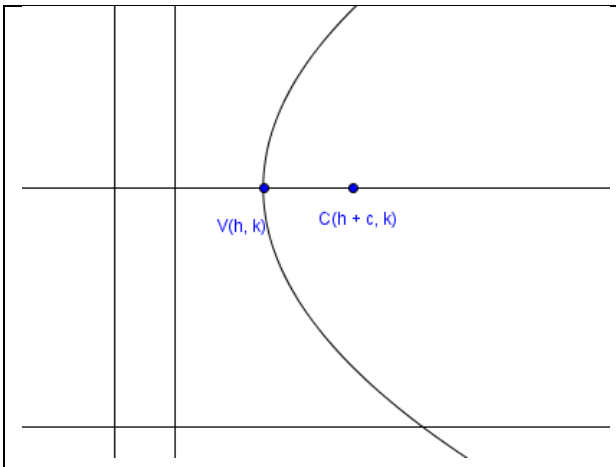
1. $8x = y^2$	2. $6x = -y^2$
3. $4y = x^2$	4. $10y = -x^2$

Ex3 Write the parabola equations for each graph, when the focus or directrix is given

1.  Point F is the focus.	2.  $y = \frac{9}{4}$ is the directrix

Objective 2: With translation of vertex to (h, k), focus distance c >0

Ex 4. The parabola in the graph has a vertex at (h, k), with focus at (h + c, k), where c > 0. What is the equation?



Ex 2, 3, 5 please show your work and answer on your own paper.

Ex 5 Write the equation of the parabola with the given information, and then graph the parabola

1. vertex at (-4, -3) and focus at (1, -3)
2. vertex at (5, 3) and directrix y = 6
3. focus at (-5, 2) and directrix x=-3

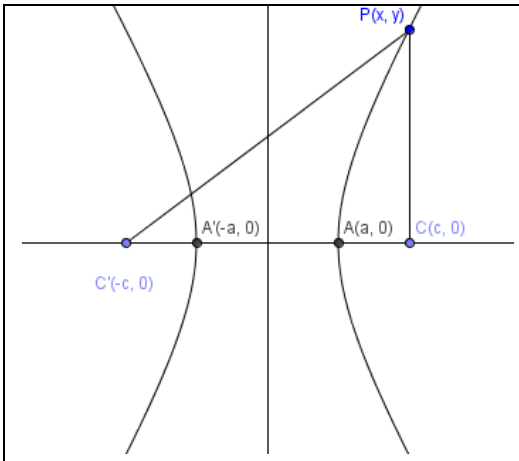
Ex 1, 2, 4, 5 please show your work and answer on your own paper.

Topic 9 Hyperbolas

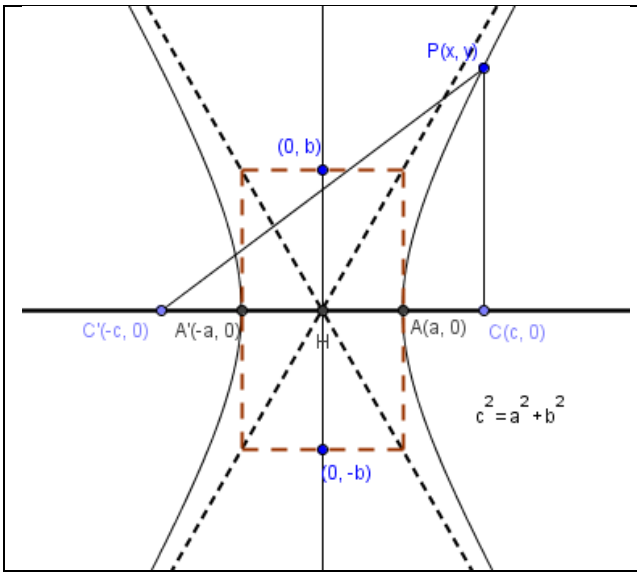
Definition: A hyperbola is a set of the points, P(x, y) that the difference of the distances between P and two fixed points, the foci is a constant.

For Example 1,2,4,5 please provide your own paper to finish.

Ex0: Use the definition to derive the equation of a hyperbola if the foci are at (c, 0) and (-c,0) and the difference of the distances is 2a.



Terms for hyperbola and their definitions: (as illustrated in the diagram below



Foci: the two points used to define the hyperbola
transverse axis: the line passes through the foci
Vertices: The two points are on the transverse axis. The distance between the vertices is the distance in which the difference between the distances between points on the hyperbola and two foci.
constant b: this constant is defined by a and c using $c^2 = a^2 + b^2$
Asymptotes: the diagonal defined by the rectangles between the two branches of the hyperbolas.

Ex 1, 2, 4, 5 please show your work and answer on your own paper.

Ex1: Graph the hyperbolas, identify asymptotes and coordinates of vertices and foci.

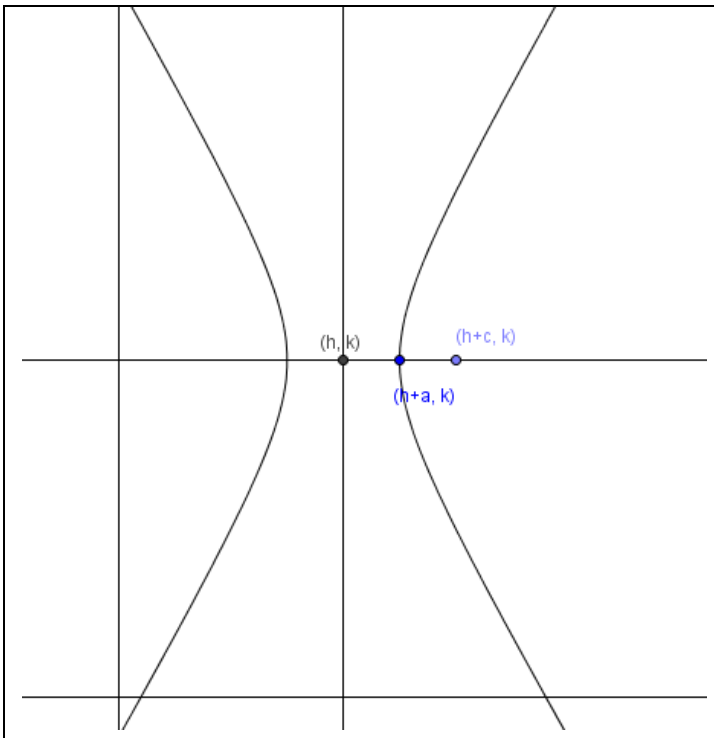
1. $25y^2 - 4x^2 = 100$	2. $\frac{x^2}{36} - \frac{y^2}{4} = 1$
3. $\frac{y^2}{25} - \frac{x^2}{16} = 1$	4. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Ex2: Write the equation of the hyperbolas, identify the asymptotes.

1. foci: (2, 0), (-2, 0), vertices: (1, 0), (-1, 0)	2. foci: (0, 5), (0, -5); vertices: (0,3), (0,-3)

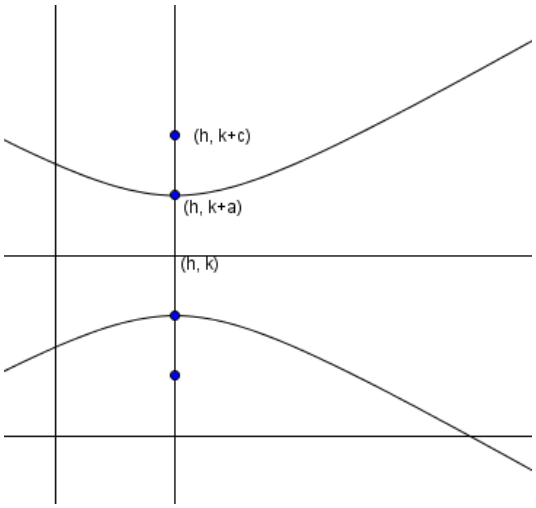
Objective 2: translate the center from (0, 0) to (h, k)

Ex3: Find the equation of the hyperbola if the foci are (h+c, k), (h-c, k) and the vertices are (h+a, k) and (h-a, k) where c > a.



Ex 1, 2, 4, 5 please show your work and answer on your own paper.

Ex $3\frac{1}{2}$: Find the equation of the hyperbola if the foci are (h, k+c), (h, k-c) and the vertices are (h, k+a) and (h, k-a) where c > a.



Ex4: Graph the equations, identify the vertices, asymptotes, and foci

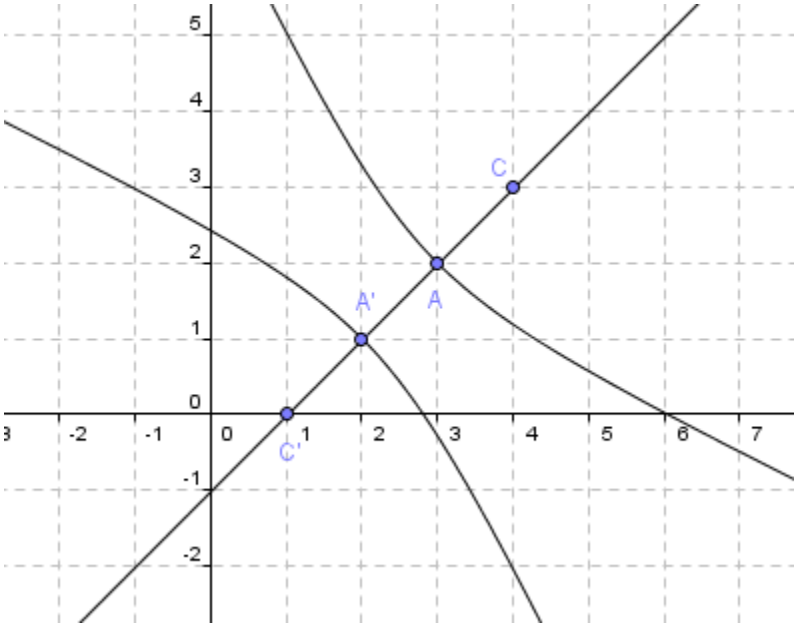
1. $\frac{(x-6)^2}{25} - (y+1)^2 = 1$	2. $\frac{(x+3)^2}{9} - \frac{(y-4)^2}{16} = 1$
3. $\frac{x^2}{25} - \frac{(y-2)^2}{4} = 1$	4. $-\frac{(x+2)^2}{9} + \frac{(y-2)^2}{4} = 1$

Ex5: Write the equations of the hyperbolas based on the descriptions, identify the center, the asymptotes and graph the hyperbola.

1. Foci: (1, 3), (5, 3); vertices: $(\frac{7}{2}, 3)$, $\left(\frac{5}{2}, 3\right)$	2. Foci: (-3, 2), (-3, -4); vertices: (-3, 0), (-3, -2)

Ex 1, 2, 4, 5 please show your work and answer on your own paper.

Ex 6 Find the hyperbola as shown in the diagram. Where A and A' are the vertices; C and C' are the foci



Please provide your own paper to work on each question.

Topic 10 Classify the Conics

Objective: In this topic, students will learn how to identify and graph the conic sections.

For each conic section:

1. classify it to either a circle, ellipse, parabola or hyperbola

2. graph the conic.

1. $6x^2-2y^2+24x+2y-1=0$	6. $x^2-16x-8y+80=0$
2. $y^2-10y-5x+57=0$	7. $9y^2-x^2-54y+8x+56=0$
3. $x^2+y^2-6x+8y-24=0$	8. $x^2-18x+6y+99=0$
4. $9x^2+4y^2-36x-24y+36=0$	9. $y^2+14y+16x+33=0$
5. $8x^2-9y^2-40x+4y+145=0$	10. $4x^2+y^2+32x-10y+85=0$

Please provide your own paper to work on each question.

Topic 11 Solving Quadratic Systems

For each system,

(1) graph the equations and

(2) use algebraic procedures to find the exact solutions

1. $\begin{cases} y = 2x \\ x^2 + y^2 = 45 \end{cases}$	
2. $\begin{cases} x^2 + 2y^2 - 3y = 0 \\ x^2 + y^2 = 2 \end{cases}$	
3. $\begin{cases} x^2 - 4x + 4y^2 - 8y = 8 \\ 2x + y = 7 \end{cases}$	
4. $\begin{cases} 2x^2 + 3y^2 = 0 \\ x^2 + y^2 + 4 = 0 \end{cases}$	
5. $\begin{cases} x - 2x^2 - 3y = 1 \\ 9x^2 + 4y^2 = 36 \end{cases}$	
6. $\begin{cases} 2x^2 - y^2 - x - 4 = 0 \\ -x^2 + y^2 + 3x - 4 = 0 \end{cases}$	