1-10題每題3分

In questions 1–5 suppose P(x, y) is a predicate and the **universe for the variables x and y is {1, 2, 3}.**
Suppose P(1,3), P(2, 2), P(2, 3), P(3, 1), P(3, 2) are true, and P(x, y) is false otherwise. Determine whether the following statements are **True or False.**(請直接勾選答案)

1.　True　False　　$\forall x \exists y P(x, y)$.
2.　True　False　　$\exists x \forall y P(x, y)$.
3.　True　False　　$\neg \exists x \exists y (P(x, y) \wedge \neg P(y, x))$.
4.　True　False　　$\forall y \exists x (P(x, y) \rightarrow P(y, x))$
5.　True　False　　$\forall x \forall y (x \neq y \rightarrow (P(x, y) \vee P(y, x)))$.

In questions 6-10 determine whether the proposition is TRUE(T) or FALSE(F). (請直接勾選答案)

6.　True　False　　If $1 < 0$, then $3 = 4$.
7.　True　False　　If it is raining, then it is raining.
8.　True　False　　$1 + 1 = 2$ if and only if $2 + 2 = 3$.
9.　True　False　　If $2 + 1 = 3$, then $1 = 3 - 2$.
10.　True　False　　If $2 + 2 = 3$ or $2 + 2 = 4$, then $1 + 1 = 1$ and $1 + 1 = 2$.

11. (8%)How many satisfying assignments are there for the following system specification?
$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

12. (8%) $x \in Z^+$, if $1285^x - 1116^x = 637^x$. Find x.

13. (8%) Find a proposition using only p, q, $\neg$, and the connective $\vee$ that has the following truth table.

p	q	?
T	T	T
T	F	F
F	T	T
F	F	T

14. (8%) Find a compound proposition involving the propositional variables p, q, and r that is **true when exactly two of p, q, and r are true** and is **false otherwise.**

15. (8%) Prove or disprove that the following compound proposition is a **tautology.**

$((p \vee q) \wedge \neg p) \rightarrow q$

16. (12%) Suppose the variables x and y represent real numbers, and
    $L(x, y) : x < y$;
    $Q(x, y) : x = y$;
    $E(x) : x$ is even;
    $I(x) : x$ is an integer.
    Write each of the following statements using these predicates and any needed quantifiers.
    (a) Every integer even is an integer.
    (b) If x = y, then x is greater than or equal to y.
    (c) There is no smallest real number.

17. (8%) What are the **converse** and the **contrapositive** of the statement "If it rains, then I get wet"?

18. (8%) **Prove or disprove** that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are **equivalent.**

19. (8%) Given the lemma: if (p,q)=1, then (p,q²)=1. **Prove** that all the solutions to the equation $x^2 = x + 1$ are **irrational**(無理數). [註：(p,q)表p,q的最大公因數]

20. (8%) A staff member of a small hospital said, "The hospital staff consists of 13 doctors and nurses, including me. The following facts apply to the staff members; whether you include me or not does not make any difference."
    The staff consists of:
    - more nurses than doctors;
    - more male doctors than male nurses;
    - more male nurses than female nurses;
    - at least one female doctor.

    Give the sex and job of the speaker.

離散數學 第一次小考 2024/10/8　　　　　　姓名:_____　學號:_____

1-10題每題3分

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Suppose P(1,3), P(2, 2), P(2, 3), P(3, 1), P(3, 2) are true, and P(x, y) is false otherwise. Determine whether the following statements are **True or False**.(請直接勾選答案)

1. **(True)** False　　　$\forall x \exists y P(x, y)$.
2. True **(False)**　　　$\exists x \forall y P(x, y)$.
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11. (8%)How many **satisfying assignments** are there for the following system specification? **2**
$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

12. (8%) $x \in Z^+$,if $1285^x - 1116^x = 637^x$. Find x.

13. (8%) Find a proposition using only p, q, ¬, and the connective ∨ that has the following truth table. ¬p∨q

p	q	?
T	T	T
T	F	F
F	T	T
F	F	T

14. (8%) Find a compound proposition involving the propositional variables p, q, and r that is **true when exactly two of p, q, and r are true** and is **false otherwise.**

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16. (12%) Suppose the variables x and y represent real numbers, and
L(x, y) : $x < y$;
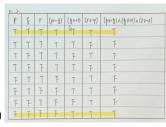Q(x, y) : $x = y$;
E(x) : x is even;
I(x) : x is an integer.
Write each of the following statements using these predicates and any needed quantifiers.
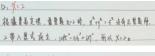(a) Every integer even is an integer.
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20. (8%) A staff member of a small hospital said, "The hospital staff consists of 13 doctors and **無解** nurses, including me. The following facts apply to the staff members; whether you include me or not does not make any difference."
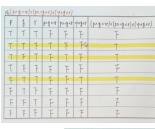The staff consists of:
- more nurses than doctors;
- more male doctors than male nurses;
- more male nurses than female nurses;
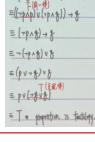- at least one female doctor.
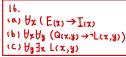
Give the sex and job of the speaker.