1. (15%) Determine which relationship, ⊆, =, or ⊇, is true for the pair of sets.(每小題僅需圈選一個最適合答案即可;
   (a) [⊆ = ⊇] A − (B − C) A ∪ B, A ∪ (B − A).
   (b) [⊆ = ⊇] A ∪ (B ∩ C), (A ∪ B) ∩ C.
   (c) [⊆ = ⊇] (A − B) ∪ (A − C), A − (B ∩ C).
   (d) [⊆ = ⊇] A − (B ∩ C) , (A − B) ∩ (A − C)
   (e) [⊆ = ⊇] (A − C) − (B − C), A − B.

2. (12%) Find
   (a) $\bigcup_{i=1}^{+\infty} [-\frac{1}{i}, \frac{1}{i}]$
   (b) $\bigcap_{i=1}^{+\infty} (1-\frac{1}{i}, 1)$
   (c) $\bigcap_{i=1}^{+\infty} [1-\frac{1}{i}, 1]$
   (d) $\bigcap_{i=1}^{+\infty} (i, \infty)$

3. (18%) Suppose A = {x, y} and B = {x, {x}}. Mark each of the following statements TRUE or FALSE. (每小題僅需填O或X即可)
   (a) [T F] x ⊆ B.
   (b) [T F] Ø ∈ *P*(B).
   (c) [T F] {x} ⊆ A − B.
   (d) [T F] |*P*(A)| = 4.
   (e) [T F] {{x}} ⊆ *P*(B).
   (f) [T F] {({x},{x})}⊆ A×A.

4. (8%)Suppose A = $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 1 \\ 0 & 6 \end{bmatrix}$. Find a matrix B such that AB = C^t or prove that no such matrix exists.

5. (10%)Suppose f: R → R and g: R → R where g(x) = 2x + 1 and g◦f(x) = 2x + 11.
   (a) Find f.
   (b) Find g^{-1}.

6. (16%) Find the solution to each of these recurrence relations and initial conditions.

(a) $a_n = a_{n-1} + 2$, $a_0 = 3$
(b) $a_n = 3a_{n-1} + 1$, $a_0 = 1$

7. (24%) (The Cantor-Bernstein-Schröeder Theorem)
If $|A| \le |B|$ and $|B| \le |A|$, then $|A| = |B|$. In other words, if there are injections $f : A \to B$ and $g : B \to A$, then there is a bijection $h : A \to B$.
(a) Show that (0, 1) has the same cardinality(基數) as (2, 5).
(b) show that (0,1) has the same cardinality as [0, 1).
(c) show that (0,1) has the same cardinality as $(0, \infty)$.

8. (16%)
(a) Show that the set of finite strings *S* over a finite alphabet(有限字元集) **Σ** is countably infinite.
(b) Show that the set of all Java programs J is countable.

離散數學 第二章小考　　　　　　　　　　　　姓名: ________________ 學號:______________

1. (15%) Determine which relationship, ⊆, =, or ⊇, is true for the pair of sets.(每小題僅需圈選一個最適合答案即可;
   (a) [⊆ = ⊇]　A − (B − C) A ∪ B,　　　A ∪ (B − A).
   (b) [⊆ = ⊇]　A ∪ (B ∩ C),　　　(A ∪ B) ∩ C.
   (c) [⊆ = ⊇]　(A − B) ∪ (A − C),　　　A − (B ∩ C).
   (d) [⊆ = ⊇]　A − (B ∩ C) ,　　　(A − B) ∩ (A − C)
   (e) [⊆ = ⊇]　(A − C) − (B − C),　　　A − B.

ans;= ⊇. =. ⊇, ⊆.

2. (12%) Find
   (a) $\bigcup_{i=1}^{+\infty} [-\frac{1}{i}, \frac{1}{i}]$
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   (c) $\bigcap_{i=1}^{+\infty} [1-\frac{1}{i}, 1]$
   (d) $\bigcap_{i=1}^{+\infty} (i, \infty)$

ans: (a)[-1,1](b)$\Phi(C)\{1\}(d)\Phi$

3. (18%) Suppose A = {x, y} and B = {x, {x}}. Mark each of the following statements TRUE or FALSE. (每小題僅需填O或X即可)
   (a) [T F]　x ⊆ B.
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   (c) [T F]　{x} ⊆ A − B.
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   (e) [T F]　{{x}} ⊆ *P*(B).
   (f) [T F]　{({x},{x})}⊆ A×A.
   Ans: FTFTTF

4. (8%)Suppose A = $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 1 \\ 0 & 6 \end{bmatrix}$. Find a matrix B such that AB = C^t or prove that no such matrix exists.
   Ans:

$$\begin{matrix} 3/2 & -15 \\ -1/2 & 9 \end{matrix}$$

5. (10%)Suppose f: R → R and g: R → R where g(x) = 2x + 1 and g∘f(x) = 2x + 11.
   (a) Find f.
   (b) Find g^{-1}.

Ans: (a) f(x)=x+5 (b) g^{-1}(x)= (x-1)/2

6. (16%) Find the solution to each of these recurrence relations and initial conditions.

(a) $a_n = a_{n-1} + 2$, $a_0 = 3$
(b) $a_n = 3a_{n-1} + 1$, $a_0 = 1$

Ans:(a) $a_n=2n+3$ (b) $a_n=(3^{n+1}-1)/2$

7. (24%) (The Cantor-Bernstein-Schröeder Theorem)
If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$. In other words, if there are injections $f : A \to B$ and $g : B \to A$, then there is a bijection $h : A \to B$.
(a) Show that (0, 1) has the same cardinality(基數) as (2, 5).
(b) show that (0,1) has the same cardinality as [0, 1).
(c) show that (0,1) has the same cardinality as $(0, \infty)$.

ans;
(a) The function f: $(0,1)\to(2,5)$, $f(x) = 3x+2$ is one-to-one and onto.
(b) f: $(0,1)\to[0,1)$, $f(x) = x$ is one-to-one; g: $[0,1)\to(0,1)$, $g(x) = 1/4+x/2$ is one-to-one
(c) f: $(0,1)\to(0,\infty)$, $f(x) = x$ is one-to-one; g: $(0,\infty)\to(0,1)$, $g(x) = x/(x+1)$ is one-to-one
另解: f: $(0,\infty)\to(0,1)$, $f(x) = x/(x+1)$ is one-to-one and onto

8. (16%)
(a) Show that the set of finite strings S over a finite alphabet(有限字元集) Σ is countably infinite.
(b) Show that the set of all Java programs J is countable.

ans:

(a)

S 中的字串可依下列順序列出:

1.λ (空字串; 長度為0)

2.依字母順序列出長度為 1的字串.

3.依lexicographic (as in a dictionary)順序列出長度為 2的字串

4.依lexicographic (as in a dictionary)順序列出長度為 3的字串

5.依此類推....

This implies a bijection from **N** to S and hence it is a countably infinite set.

(b) 承(a)令 Σ的元素為所有可出現在Java程式中的符號，則$J \subseteq S$, 又S 可數，故J可數

(J中的程式可依下列順序列出:

的將S中的字串依(a)的順序逐一取出並檢視是否為正確的Java程式，若不是則將其剔除)