離散數學 第二章小考(兩面都有試題)　　姓名: _____ 學號:_____

1. (24%) Let A, B, C be finite sets. Find each of the following: Mark the following statement TRUE or FALSE

(每小題直接圈選T/F即可)

(a) T F　$\boldsymbol{P}(A \cup B)= \boldsymbol{P}(A) \cup \boldsymbol{P}(B)$

(b) T F　If $A \cup C= B \cup C$, then $A=B$

(c) T F　If $A \cap C= B \cap C$, then $A=B$

(d) T F　IF $A - C=B-C$, then $A=B$

(e) T F　$\emptyset \in \{\emptyset\}$

(f) T F　$\emptyset \subseteq \{\emptyset\}$

(g) T F　$\{\emptyset\} \in \{\emptyset,\{\{\emptyset\}\}\}$

(h) T F　$\{\{\emptyset\}\} \subset \{\{\emptyset\},\{\emptyset\}\}$

2. (18%) Determine which relationship, $\subseteq$, =, or $\supseteq$, is true for the pair of sets.(每小題圈選一個最適合答案即可)

(a) [$\subseteq$ = $\supseteq$]　$A \cap (B \cup C)$,　$(A \cap B) \cup C$.

(b) [$\subseteq$ = $\supseteq$]　$A - (B - C) \cup B$,　$A \cup (B - A)$.

(c) [$\subseteq$ = $\supseteq$]　$A - (B \cup C)$,　$(A - B) \cap (A - C)$.

(d) [$\subseteq$ = $\supseteq$]　$(A - B) - (C - B)$,　$A - C$.

(e) [$\subseteq$ = $\supseteq$]　{a,c,c,e,,e},　{e,c,a,a}.

(f) [$\subseteq$ = $\supseteq$]　$A - (B \cup C)$,　$(A - B) - C$).

3. (15%) Let f: B→C and g: A→B be functions. Mark the following statement TRUE or FALSE. (直接圈選T/F即可)

(a) T F　If $\boldsymbol{f} \circ \boldsymbol{g}$ is one-to-one, so is $\boldsymbol{f}$.

(b) T F　If $\boldsymbol{f} \circ \boldsymbol{g}$ is one-to-one, so is $\boldsymbol{g}$.

(c) T F　If $\boldsymbol{f} \circ \boldsymbol{g}$ is onto, so is $\boldsymbol{f}$.

(d) T F　If $\boldsymbol{f} \circ \boldsymbol{g}$ is onto, so is $\boldsymbol{g}$.

(e) T F　If $\boldsymbol{f} \circ \boldsymbol{g}$ is bijection, so are $\boldsymbol{g}$ and $\boldsymbol{f}$.

4. (21%) Determine whether each of these **sets** is **C**ountable(可數) or **U**ncountable(不可數). (圈選C/U即可，註Q:有理數；R:實數)

(a)　C U　**R**

(b)　C U　**Q**

(c)　C U　the set of **all Java programs**

(d)　C U　(2,5)

(e)　C U　the set $S=\{a+b\boldsymbol{\pi} \mid a,b \in \mathbf{Q}\}$

(f)　C U　the set of integers divisible by 3 but not by 7

(g)　C U　A−B, where A is an Uncountable set and B is a Countable set.

5. (10%) Let f, g: $\mathbf{R}\rightarrow\mathbf{R}$, where $g(x)=1-x+x^2$ and $f(x)=ax+b$. If $(\boldsymbol{g}\circ\boldsymbol{f})(x)=9x^2-9x+3$, determine a, b.

6. (10%)Suppose A = $\begin{bmatrix}3 & 5\\ 2 & 4\end{bmatrix}$ and C = $\begin{bmatrix}2 & 1\\ 0 & 6\end{bmatrix}$. Find a matrix B such that AB = C or prove that no such matrix exists.

7. (10%)Show that if A, B, and C are sets, then $A-(B\cup C)=(A-B)\cap(A-C)$ **using a membership table**.

離散數學第二次小考

1.

(a) F
(b) F
(c) F
(d) F
(e) T
(f) T
(g) F
(h) F

2.

(a) $\subseteq$
(b) $=$
(c) $=$
(d) $\subseteq$
(e) $=$
(f) $=$

3.

(a) F
(b) T
(c) T
(d) F
(e) F

4.

(a) U
(b) C
(c) C
(d) U
(e) C
(f) C
(g) U

5.

$g(f(x)) = 1 - (ax + b) + (ax + b)^2 = a^2x^2 + 2abx - ax + b^2 - b + 1 = 9x^2 - 9x + 3$

$a^2x^2 = 9x^2$
$a = \pm 3$

$b^2 - b + 1 = 3$
$(b - 2)(b + 1) = 0$
$b = 2, -1$

$2abx - ax = -9x$
if a = 3, b = -1 (5 分)
if a = -3, b = 2 (5 分)

6.

$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ 0 & 6 \end{bmatrix}$

B = $\frac{1}{4\times3-(-5)\times(-2)}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 6 \end{bmatrix}$

B = $\begin{bmatrix} 4 & -13 \\ -2 & 8 \end{bmatrix}$

7.

A	B	C	A-(B∪C)	(A-B)∩(A-C)
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	1	1
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0