離散數學 第一次小考 2022/10/11 姓名:____________ 學號:______________

1. (10%) Find a proposition using only p, q, ¬, and the connective ∨ that has the following truth table.

p	q	?
T	T	F
T	F	F
F	T	T
F	F	F

2. (12%) Determine whether the following argument is valid: (請圈選T或F)

(a) T F

$$\begin{array}{l} p \to q \\ \underline{\neg p} \\ \therefore \neg q \end{array}$$

(b) T F

$$\begin{array}{l} p \to r \\ q \to r \\ \underline{\neg(p \vee q)} \\ \therefore \neg r \end{array}$$

(c) T F

$$\begin{array}{l} p \to r \\ q \to r \\ \underline{q \vee \neg r} \\ \therefore \neg p \end{array}$$

3. (12%) Show that ¬(p˅q) and ¬q˄¬p are **equivalent**
   (a) using a truth table.
   (b) using logical equivalences

4. (10%) Determine whether this proposition is a **tautology**: ((p → ¬q) ∧ q) → ¬p.

5. (16%) Let F(A) be the predicate "A is a finite set" and S(A, B) be the predicate "A is contained in B". Suppose the universe of discourse consists of all sets. Translate the statement these predicates and any needed quantifiers..
   (a) Not all sets are finite.
   (b) Every subset of a finite set is finite.
   (c) No infinite set is contained in a finite set.
   (d) The empty set is a subset of every finite set.

6. (10%) Prove that the equation $2x^2 + y^2 = 14$ has no positive integer solutions.

7. (10%) Show that there exist **irrational numbers** x and y such that x^y is **rational**. (hint: a nonconstructive existence proof)

8. (16%) P(x, y) is a predicate and the universe for the variables x and y is {1, 2, 3}.
Suppose P(1, 3), P(2, 2), P(2, 3), P(3, 1), P(3,2) are true, and P(x, y) is false otherwise. Determine whether the following statements are TRUE(**T**) or FALSE(**F**). (請圈選T或F)
(a) T F $\exists x \forall y P(x, y)$.
(b) T F $\neg \exists x \exists y\, (P(x, y) \wedge \neg P(y, x))$.
(c) T F $\forall y \exists x\, (P(x, y) \rightarrow P(y, x))$.
(d) T F $\forall x \forall y\, (x \neq y \rightarrow (P(x, y) \vee P(y, x))$.

9. (10%) Use **rules of inference** to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\neg R(a)$, where a is in the domain, are true, then the conclusion $\neg P(a)$ is true.

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1. (10%) Find a proposition using only p, q, ¬, and the connective ∨ that has the following truth table.

p	q	?
T	T	F
T	F	F
F	T	T
F	F	F

Ans: ¬(p ∨ ¬q).

2. (12%) Determine whether the following argument is valid: (請圈選T或F)

(a) T F

$$\begin{array}{l} p \rightarrow q \\ \underline{\neg p} \\ \therefore \neg q \end{array}$$

(b) T F

$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \underline{\neg(p \vee q)} \\ \therefore \neg r \end{array}$$

(c) T F

$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \underline{q \vee \neg r} \\ \therefore \neg p \end{array}$$

Ans: (a)F (b)F (c)F

3. (12%) Show that ¬(p˅q) and ¬q˄¬p are **equivalent**
   (a) using a truth table.
   (b) using logical equivalences

Ans: (a)

p	q	¬(p˅q)	¬q˄¬p
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Ans: (b)

¬(p˅q)

≡¬p∧¬q (by the 2nd De Morgan law)

≡¬q∧ ¬p (by the Commutative law)

4. (10%) Determine whether this proposition is a **tautology**: ((p → ¬q) ∧ q) → ¬p.

Ans: Yes, 用truth table或logical euuivalence推導出此命題為恆真即可 (略)

5. (16%) Let F(A) be the predicate “A is a finite set” and S(A, B) be the predicate “A is contained in B”. Suppose the universe of discourse consists of all sets. Translate the statement these predicates and any needed quantifiers..
   (a) Not all sets are finite.
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Ans: (a) ∃A ¬F(A). (b) ∀A ∀B [(F(B) ∧ S(A, B))→F(A)]. (c) ¬∃A ∃B (¬F(A) ∧ F(B) ∧ S(A, B)). (d) ∀A (F(A)→S(Ø, A)).

6. (10%) Prove that the equation $2x^2 + y^2 = 14$ has no positive integer solutions.

Ans: Give a proof by cases. There are only six cases for (x, y) that need to be considered : (1,1), (1,2),(1,3), (2,1), (2,2),(2,3)

7. (10%) Show that there exist **irrational numbers** x and y such that x^y is **rational**. (hint: a nonconstructive existence proof)

   Ans: (略，如上課講義)

8. (16%) P(x, y) is a predicate and the universe for the variables x and y is {1, 2, 3}.

   Suppose P(1, 3), P(2, 2), P(2, 3), P(3, 1), P(3,2) are true, and P(x, y) is false otherwise. Determine whether the following statements are TRUE(**T**) or FALSE(**F**). (請圈選T或F)

   (a) T F $\exists x \forall y P(x, y)$.

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   (c) T F $\forall y \exists x (P(x, y) \rightarrow P(y, x))$.

   (d) T F $\forall x \forall y (x \neq y \rightarrow (P(x, y) \vee P(y, x))$.

   Ans: (a)F(b)T(c)T(d)F

9. (10%) Use **rules of inference** to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\neg R(a)$, where a is in the domain, are true, then the conclusion $\neg P(a)$ is true.

   Ans:

   1. $\forall x(P(x) \rightarrow Q(x))$ Premise
   2. $P(a) \rightarrow Q(a)$ Universal Instantiation
   3. $\forall x(Q(x) \rightarrow R(x))$ Premise
   4. $Q(a) \rightarrow R(a)$ Universal Instantiation
   5. $\neg R(a)$ Premise
   6. $\neg Q(a)$ Modus tollens from steps 4 and 5
   7. $\neg P(a)$ Modus tollens from steps 2 and 6