離散數學 Quiz 5　　姓名:_____ 學號:_____

1. (32%) For each of the following relations decide whether it is **reflexive**, whether it is **symmetric**, whether it is **antisymmetric**, and whether it is **transitive**? (請直接於表格中填O或X)
   (a) The relation $R_1$ on {w, x, y, z} where $R_1$ = {(w,w), (w, x), (x,w), (x, x), (x, z), (y, y), (z, y), (z, z)}.
   (b) The relation $R_2$ on Z where a $R_2$ b means $|a - b| \le 1$.
   (c) The relation $R_3$ on Z where a$R_3$ b means $a^2 = b^2$.
   (d) The relation $R_4$ on {a, b, c} where $R_4$ = {(a, a), (b, b), (a, b), (a, c), (c, b)}.

Ans:

| | reflexive | symmetric | antisymmetric | transitive |
|---|---|---|---|---|
| $R_1$ | O | X 沒有{z,x} | X找到{a,b},{b,a} | X 沒有{w,z} |
| $R_2$ | O | O | X {2,1},{1,2} | X {1,2},{2,3},沒有{1,3} |
| $R_3$ | O | O | X {2,-2},{-2,2} | O |
| $R_4$ | X 沒有{c,c} | X 沒有{b,a} | O | O |

2. (16%) For each of the following relations decide whether it is a **partial ordering** and whether it is an **equivalence relation**? (請直接於表格中填O或X)
   (a) The relation $R_1$ on {1, 2, 3, . . .} where a $R_1$ b means a|b.
   (b) The relation $R_2$ on {w, x, y, z} where $R_2$ = {(w,w), (x, x), (y, y), (z, z)}.
   (c) $R_3$ is the symmetry closure of a partial ordering ⇒對稱閉包.即滿足 symmetry
   (d) The relation $R_4$ on the set of all functions $f$: N⟶$\mathbf{R}^+$ where $R_4$={($f_1$, $f_2$) | $f_1(n) \le f_2(n)$, $\forall n \in \mathbf{N}$ }.

Ans:

| | partial ordering | equivalence relation |
|---|---|---|
| $R_1$ | O | X |
| $R_2$ | O | O |
| $R_3$ | X | O |
| $R_4$ | O | X |

3. **(8%) {{1,2,3}, {4}}is a partition of A={1, 2, 3, 4}. Find the equivalence relation R on A such that R's different equivalence classes form the same partition of A.**
   **Ans: R = {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3, 2),(3,3), (4,4)}**

4. Let R = {(1, 2), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1)} be the relation on the set {1,2,3,4}.
   (a) (4%) Find the **reflexive closure** of R. 少{1,1},{4,4}
   (b) (4%) Find the **symmetric closure** of R. 少{2,1},{3,2},{1,3},{1,4}
   (c) (5%) Find the **matrix representation** (矩陣表示法)for the **transitive closure** of R

Ans: (a) {(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 4)}
(b) {(1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 1), (1, 3), (3, 3), (4, 1),(1, 4)}
(c) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, $M_R^{[2]} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $M_R^{[3]} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$, $M_R^{[4]} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $M_R^{[5]} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

$M_R^* = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee M_R^{[4]} \vee \cdots \vee M_R^{[n]}$

5. Let A={1,2,3,4,5,6,7,8,9,10}.
   (a) (4%) How many **reflexive** relations on A are there?
   (b) (4%) how many **symmetric** relations on A are there?
   (c) (4%) how many **anti-symmetric** relations on A are there?

Ans: (a) $2^{90}$ (b) $2^{55}$ (c) $2^{10} \times 3^{45}$

6. (8%) Draw the **Hasse diagram** for the relation R on A = {1, 2, 3, 4, 6, 8, 12} where a R b means a|b.

Ans:

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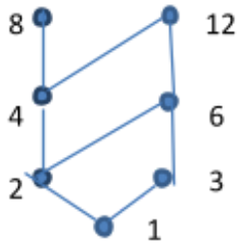
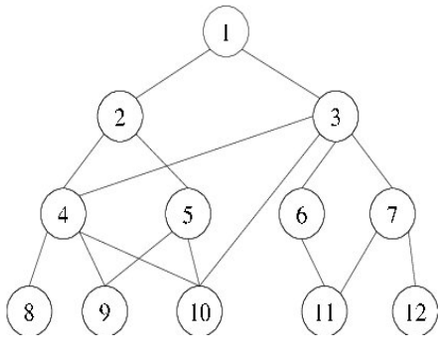



7. For the partial order represented by the Hasse diagram shown right.

(a) (18%) Answer these questions (請直接於表格中作答, 若該題答案不存在請填X).

| **Question** | **Answer** |
|---|---|
| Find the maximal element(s). | *1* |
| Find the greatest element. | *1* |
| Find the minimal elements(s). | *8,9,10,11,12* |
| Find the least element. | *X* |
| Find all upper bounds of {5,10}. | *5,2,1* |
| Find the least upper bound of {5,10}. | *5* |
| Find all lower bounds of {2, 4, 5}. | *9, 10* |
| Find the greatest lower bound of {b, d, e}. | *X* |
| Is it a lattice? (Yes or No) | *No* |




(b) (4%) Give a total order of the elements that are compatible with the partial order.

Ans: (b) 8,9,10,11,12,4,5,6,7,2,3,1