離散數學 Quiz 3 姓名:____________學號:______________

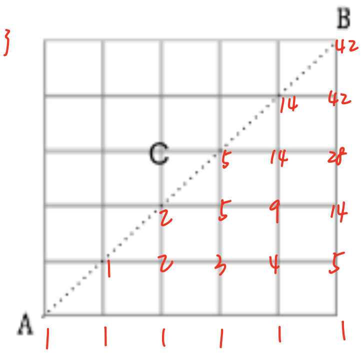
1. (20%) Let A be the set of all bit strings of length 10.
   (a) How many bit strings of length 10 are there?
   (b) How many bit strings of length 10 begin with 1101?
   (c) How many bit strings of length 10 have exactly six 0's?
   (d) How many bit strings of length 10 have equal numbers of 0's and 1's?
   (e) How many bit strings of length 10 begin with 10 and end with 110 ?

(a) 2^{10} (b) 2^6 (c) C^{10}_6 (d) C^{10}_5 (e) 2^5

2. (10%)
   (a) Find the **next 5** (緊接著連續5個) permutations in lexicographic order after 6715243.
   (b) Find the **next 5** 4-combinations of the set {1, 2, 3, 4, 5, 6, 7} after {1, 2, 5, 7}.

(a)
6715324 → 6715342 → 6715423 → 6715432 → 6721345

(b)
{1, 2, 6, 7}. {1, 3, 4, 5}. {1, 3, 4, 6}. {1, 3, 4, 7}. {1, 3, 5, 6}



3. The figure at the right shows a 5-block by 5-block grid of streets. Suppose that starting at the point labeled A you can go **one step up** or **one step to the right** in each move. This is continued until the point labeled B is reached.
   (a) (8%)How many different paths go from A to B ?
   (b) (5%)How many different paths go from A to B by way of C?
   (c) (5%)How many different paths go from A to B without passing above the diagonal?

(a) $C^{10}_5 = \frac{10!}{5!5!}$ (b) $C^5_2 \times C^5_2 = \frac{5!}{2!3!} \times \frac{5!}{2!3!}$ (c) 42

4. A group contains m boys and n girls. How many ways are there to arrange these people in a row so that no two boys are seated next to each other if
   (a) (8%) m=n=5?
   (b) (5%) m=6 and n=8?

(a) $5! \times C^6_5 \times 5!$ (先排女生，男生插空位)

(b) $8! \times C^9_6 \times 6!$

5. $(a+b)^n = \sum_{n=0}^{k} C(n,k)\, a^{n-k} b^k$
   (a) (8%) Find the coefficient of x^6 in the expansion of $(3 - 2x)^{10}$.
   (b) (5%) Find the coefficient of x^6 in the expansion of $(x - 2)^2(3 - 2x)^{10}$.

(a) $C^{10}_6 \times 3^4 \times (-2)^6$ (b) $C^2_0\, 1^2 \times (-2)^0 \times C^{10}_4 \times 3^6 \times (-2)^4 + C^2_1\, 1 \times (-2) \times C^{10}_5 \times 3^5 \times (-2)^5 + C^2_2\, 1^0 \times (-2)^2 \times C^{10}_6 \times 3^4 \times (-2)^6$

6. (10%) Stirling number, $S(n,k) = k\,S(n-1,k) + S(n-1,k-1)$
   (a) How many ways are there to distribute 7 **distinguishable(可區別的) objects** into 4 **indistinguishable(不可區別的) boxes?**
   (b) as (a) but each of the boxes contains at least one object?

(一)

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	3	1				
4	1	7	6	1			
5	1	15	25	10	1		
6	1	31	90	65	15	1	
7	1	63	301	350	140	21	1

(a) $S(7,1) + S(7,2) + S(7,3) + S(7,4)$
$= 715$

(b) $S(7,4) = 350$

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(二) 討論箱子內數量的可能

① 一個盒子: 1

② 二個盒子: $2^7-2=126$ (扣掉所有物品在同一盒的情況)

$126\div 2=63$ ({A,B},{C}) 和 ({C},{A,B})為相同情況

③ 三個盒子: $\{5,1,1\}=\frac{C^7_5\times C^2_1\times C^1_1}{2}=21$

$\{4,2,1\}=C^7_4\times C^3_2\times C^1_1=105$

$\{3,3,1\}=\frac{C^7_3\times C^4_3\times C^1_1}{2}=70$

$\{3,2,2\}=\frac{C^7_3\times C^4_2\times C^2_2}{2}=105$

$\fallingdotseq 310$

④ 四個盒子

$\{4,1,1,1\}=\frac{C^7_4\times C^3_1\times C^2_1\times C^1_1}{3!}=35$

$\{3,2,1,1\}=\frac{C^7_3\times C^4_2\times C^2_1\times C^1_1}{2!}=210$

$\{2,2,2,1\}=\frac{C^7_2\times C^5_2\times C^3_2\times C^1_1}{3!}=105$

$\fallingdotseq 350$

7.

(a) (8%)Find the number of **nonnegative integer** solutions to x + y + z = 20.
(b) (5%)Answer part (a), but assume that x≥1, y≥2, and z≥3.
(c) (4%)Find the number of nonnegative integer solutions to x + y + z ≤ 20.
(d) (3%)Find the number of nonnegative integer solutions to 15 ≤ x + y + z ≤ 20.

(a) C^{22}_2

(b) C^{16}_2

(c) 當作 $x+y+z+t=20$
C^{23}_3

(4) 全部 − $x+y+z<15$
$C^{23}_3-C^{17}_3$

8. (10%) Prove that given 7 distinct integers, there must exist two integers such that the difference is divisible by 6.[hint: **pigeonhole principle**]

Ans: 將整數n除以6的餘數分成以下6類:
[I] if n=6k+1 for some k∈Z; (即餘數為1)
[II] if n=6k+2 for some k∈Z; (即餘數為2)
[III] if n=6k+3 for some k∈Z; (即餘數為3)
[IV] if n=6k+4 for some k∈Z. (即餘數為4)
[V] if n=6k+5 for some k∈Z. (即餘數為5)
[VI] if n=6k for some k∈Z; (即餘數為0)
根據鴿籠原理，7個整數中必有2個整數x,y, (x>y) 同屬i類，即x＝6k+i, y=6k’+i, 故x-y=6(k-k’) 為6的倍數