離散數學 第二章小考　　　　　　　姓名: ____________ 學號: ____________

1. (30%) Let A, B, C be finite sets. Find each of the following: Mark the following statement TRUE or FALSE
Note: $A \oplus B$, is the set containing those elements in either A or B, but not in both A an B. (每小題直接圈選 T/F 即可)

(a) T F $P(A \cap B) = P(A) \cap P(B)$
(b) T F $A - (B \cup C) = (A - B) - C$
(c) T F IF $A - C = B - C$, then $A = B$
(d) T F If $A \oplus C = B \oplus C$, then $A = B$
(e) T F $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$
(f) T F $\emptyset \in \{\emptyset\}$
(g) T F $\emptyset \subseteq \{\emptyset\}$
(h) T F $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
(i) T F $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
(j) T F If $|A|=2$, then $|A^3|=8$.

2. (15%) Let f is a function from Z^+ to Z^+. For each of the following, determine if f is (1) 1-to-1 but not onto, (2) onto but not 1-to-1, (3) both 1-to-1 and onto, or (4)neither 1-to-1 nor onto. (請直接圈選答案 1,2,3,或 4)

(a) 1 2 3 4 $f(n) = 2n - 1$
(b) 1 2 3 4 $f(n) = \lceil n/2 \rceil$
(c) 1 2 3 4 $f(n) = n+1$
(d) 1 2 3 4 $f(n)=1$
(e) 1 2 3 4 $f(n)=\varphi(n)$, where the Euler phi function $\varphi(n)$ computes the number of integers between 1 and n that are relatively prime (also known as coprime) to n.

3. (10%) Determine whether each of these sets is Countable(可數) or Uncountable(不可數). (圈選 C/U 即可)

(a) T F **R**
(b) T F **Q**
(c) T F all bit strings not containing the bit 1
(d) T F the set of **all C programs**
(e) T F all real numbers containing only 1s in their decimal representation

4. (10%)Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3\}$, $C = \{2, 3, 8\}$, and g and f are defined by $g = \{(1, 2), (2, 1), (3, 3), (4, 2)\}$ and $f = \{(1, 8), (2, 3), (3, 2)\}$.

(a) Find $f \circ g$.
(b) Find f^{-1}.

5. (10%) Suppose $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$. Find A^{-1} or prove that no such matrix exists.

6. (10%) Determine a recurrence relation and its initial condition(s) for the sequence 1,3,4,8,15,27,50,92, ...

7. (10%) Find the solution to the recurrence relation $a_n=3a_{n-1}+1$ with the initial condition $a_0=1$.

8. (10%) Let A=C={1,2,3} and f: A→B and g: B→C be functions. Provide **an example** that includes set B, and functions f and g such that $g \circ f$ is one-to-one, but g is not. (答案不是唯一)

9. (10%) Let A, B, and C be sets. Show that (A-B)-C = (A-C)-(B-C) using **a membership table.**

10. (10%) The Schröeder-Bernstein Theorem:If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$. In other words, if there are injections $f : A \rightarrow B$ and $g : B \rightarrow A$, then there is a bijection $h : A \rightarrow B$.

Use the given Schröeder-Bernstein Theorem to show that $(0, 1)$ and $(0,1]$ have the same cardinality.

離散數學 第二章小考 姓名: ____________ 學號:____________

1. (30%) Let A, B, C be finite sets. Find each of the following: Mark the following statement TRUE or FALSE
Note: A⊕B, is the set containing those elements in either A or B, but not in both A an B. (每小題直接圈選 T/F 即可)

(a) T F $\boldsymbol{P}(A \cap B) = \boldsymbol{P}(A) \cap \boldsymbol{P}(B)$
(b) T F $A - (B \cup C) = (A - B) - C)$
(c) T F IF A – C=B–C, then A=B
(d) T F If A⊕C= B⊕C, then A=B
(e) T F A∩(B⊕C)= (A∩B)⊕(A∩C)

(c) A: {1,2,3} B: {1,2,3,4,5} C: {4,5} A−C={1,2,3} B−C={1,2,3} 但 A≠B

(f) T F $\emptyset \in \{\emptyset\}$
(g) T F $\emptyset \subseteq \{\emptyset\}$
(h) T F $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
(i) T F $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
(j) T F If |A|=2, then $|A^3|=8$.

2. (15%) Let *f* is a function from $\mathbf{Z}^+$ to $\mathbf{Z}^+$. For each of the following, determine if *f* is (1) 1-to-1 but not onto, (2) onto but not 1-to-1, (3) both 1-to-1 and onto, or (4)neither 1-to-1 nor onto. (請直接圈選答案 1,2,3,或 4)

(a) 1 2 3 4 f(n) = 2n -1
(b) 1 2 3 4 f(n) = ⌈n/2⌉
(c) 1 2 3 4 f(n) = n+1
(d) 1 2 3 4 $f(n)=1$
(e) 1 2 3 4 f(n)=φ(n), where the Euler phi function φ(n) computes the number of integers between 1 and n that are relatively prime (also known as coprime) to n.

假設 f: A → B
判斷 function 是否為 one-to-one：若 f(x)=f(y)，則 x=y
判斷 function 是否為 onto：考慮任何 element y ∈ B，找出 element x ∈ A，使 f(x)=y.

(e) φ(6) = φ(3) = 2 ⟶ not one to one

suppose φ(n) = 3 , can't find n ⟶ not onto

3. (10%) Determine whether each of these **sets** is **C**ountable(可數) or **U**ncountable(不可數). (圈選 C/U 即可)

ppt97 (a) T F **R**
ppt93 (b) T F **Q**
(c) T F all bit strings not containing the bit 1
(d) T F the set of **all C programs**
(e) T F all real numbers containing only 1s in their decimal representation

判斷可數集還是不可數集主要是看有無對應性

4. (10%)Suppose *g*: A → B and *f*: B → C where A = {1, 2, 3, 4}, B = {1, 2, 3}, C = {2, 3, 8}, and g and f are defined by g = {(1, 2), (2, 1), (3, 3), (4, 2)} and f = {(1, 8), (2, 3), (3, 2)}.
(a) Find f ∘ g.
(b) Find f^{-1}.

(a): {(1,3), (2,8), (3,2), (4,3)}

(b): {(2,3), (3,2), (8,1)}

5. (10%) Suppose A = $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$. Find A^{-1} or prove that no such matrix exists.

$$A^{-1} = \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$$

6. (10%) Determine a recurrence relation and its initial condition(s) for the sequence 1,3,4,8,15,27,50,92, ...

$$\begin{cases} f_n = f_{n-1} + f_{n-2} + f_{n-3} \\ f_1 = 1,\ f_2 = 3,\ f_3 = 4 \end{cases}$$

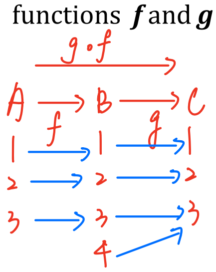
7. (10%) Find the solution to the recurrence relation $a_n=3a_{n-1}+1$ with the initial condition $a_0=1$.

$a_1 = 3a_0 + 1 = 4$
$a_2 = 3a_1 + 1 = 13$
$a_3 = 3a_2 + 1 = 40$
⋮
$a_{n-2} = 3a_{n-3} + 1$
$a_{n-1} = 3a_{n-2} + 1$
$a_n = 3a_{n-1} + 1$
將每個式子帶入,

$a_n = 3(3a_{n-2}+1)+1$
$= 3^2 a_{n-2} + 3 + 1$
$= 3^3 a_{n-3} + 3^2 + 3 + 1$
$= 3^n \underset{1}{\underline{a_0}} + \underbrace{(3^{n-1} + 3^{n-2} + \cdots + 3^1 + 3^0)}_{\text{等比數列}}$

$= 3^n + \frac{3^n - 1}{2} = \frac{2 \cdot 3^n}{2} + \frac{3^n - 1}{2} = \boxed{\frac{3^{n+1} - 1}{2}}$ #

8. (10%) Let A=C={1,2,3} and f: A→B and g: B→C be functions. Provide **an example** that includes set B, and functions ***f*** and ***g*** such that $g \circ f$ is one-to-one, but ***g*** is not. (答案不是唯一)



$B: \{1,2,3,4\}$, $f(1)=1$, $f(2)=2$, $f(3)=3$, $g(1)=1$, $g(2)=2$, $g(3)=3$, $g(4)=3$

9. (10%) Let A, B, and C be sets. Show that (A-B)-C = (A-C)-(B-C) using **a membership table**.

A	B	C	A-B	(A-B)-C	A-C	B-C	(A-C)-(B-C)
1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0
1	0	1	1	0	0	0	0
1	0	0	1	1	1	0	1
0	1	1	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

10. (10%) The Schröeder-Bernstein Theorem:If |A| ≤ |B| and |B| ≤ |A|, then |A| = |B|. In other words, if there are injections f : A → B and g : B → A, then there is a bijection h : A → B.
Use the given Schröeder-Bernstein Theorem to show that (0, 1) and **(0,1]** have the same cardinality.

$f: (0,1) \longrightarrow (0,1]$，即 $f(x)=x$，是 one-to-one

$g: (0,1] \longrightarrow (0,1)$，即 $g(x)=\frac{x}{2}$，是 one-to-one

根據 Schröeder-Bernstein Theorem，$|(0,1)| = |(0,1]|$ #