離散數學 第一次小考 2023/10/03　　　　　　　　姓名:______________ 學號:_______________

1-14題每題4分

In questions 1–5 suppose **P(x, y)** is a predicate and the **universe for the variables x and y is {1, 2, 3}**.
Suppose P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2) are true, and P(x, y) is false otherwise. Determine whether the following statements are **True or False**.(請直接勾選答案)

1. ○True ○False $\forall x \exists y P(x, y)$.
2. ○True ○False $\exists x \forall y P(x, y)$.
3. ○True ○False $\neg \exists x \exists y\, (P(x, y) \land \neg P(y, x))$.
4. ○True ○False $\forall y \exists x\, (P(x, y) \to P(y, x))$
5. ○True ○False $\forall x \forall y\, (x \neq y \to (P(x, y) \lor P(y, x))$.

In questions 6-9, determine whether the following statements are **True or False**.(請直接勾選答案)

6. ○True ○False $\forall x\, [P(x) \land Q(x)] \to \forall x\, P(x) \land \forall x\, Q(x)$
7. ○True ○False $\forall x\, [P(x) \lor Q(x)] \to \forall x\, P(x) \lor \forall x\, Q(x)$
8. ○True ○False $\exists x\, [P(x) \land Q(x)] \to \exists x\, P(x) \land \exists x\, Q(x)$
9. ○True ○False $\exists x\, [P(x) \lor Q(x)] \to \exists x\, P(x) \lor \exists x\, Q(x)$

In questions 10-14, Determine whether the proposition is a **tautology(恒真)**, **contradiction(矛盾)**, or **contingency(偶發)**.
(請直接勾選答案)

10. ○Tautology ○Contradiction ○Contingency $p \to p \lor q$
11. ○Tautology ○Contradiction ○Contingency $(p \to q) \to (p \land q)$
12. ○Tautology ○Contradiction ○Contingency $[(p \to r) \land (q \to r)] \to [(p \land q) \to r]$
13. ○Tautology ○Contradiction ○Contingency $[q \land (p \to \neg q)] \to \neg p$
14. ○Tautology ○Contradiction ○Contingency $(p \to q) \land (p \land \neg q)$

15. (8%) Find a proposition using only p, q, ¬, and the connective $\land$ that has the following truth table.

p	q	?
T	T	F
T	F	T
F	T	F
F	F	F

16. Show that $p \to (\neg q \land r)$ and $\neg p \lor \neg (r \to q)$ are **equivalent**
    (a) (8%) using a truth table.
    (b) (8%) using logical equivalences

17. Let
    C(x) denote “x is in this class,”
    B(x) denote “ x has read the book,” and
    P(x) denote “x passed the first exam.”
    The **premises:**
    “A student in this class has not read the book.”
    “Everyone in this class passed the first exam.”
    The **conclusion:**
    “Someone who passed the first exam has not read the book.”
    (a) (8%) translate the premises and conclusion into symbolic form.
    (b) (8%) Use the **rules of inference** to **construct a valid argument(有效論證)** for the conclusion.

18. (8%) **Prove** that there are no solutions in positive integers to the equation $x^4 + y^4 = 100$.

19. (8%) Show that there exist **irrational numbers** x and y such that x^y is **rational**. (hint: a nonconstructive existence proof)

離散數學第一次小考

1 – 5
True
True
False
True
True

6 – 9
True
False
True
True

10 – 14
Tautology
Contingency
Tautology
Tautology
Contradiction

15.

p	q	?
T	T	F
T	F	**T**
F	T	F
F	F	F

Ans: $p \land \neg q$

當 $p = T$，$q = F$ 時，? 為 T，因此 $? = p \land \neg q$

16.

(a)

p	q	r	$p \to (\neg q \wedge r)$	$\neg p \vee \neg(r \to q)$
T	T	T	F	F
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

(b)

$p \to (\neg q \wedge r)$

$\equiv$ **$\neg p \vee (\neg q \wedge r)$**

$\equiv$ **$\neg p \vee \neg(q \vee \neg r)$**

$\equiv \neg p \vee \neg(\neg(\neg q) \vee \neg r)$

$\equiv \neg p \vee \neg(\neg q \to \neg r)$

$\equiv$ **$\neg p \vee \neg(r \to q)$**

(只寫紅色部分也可)

17.

投影片 P87, 88

(a)

$$\begin{array}{l} \exists x(C(x) \wedge \neg B(x)) \\ \forall x(C(x) \to P(x)) \\ \hline \therefore\ \exists x(P(x) \wedge \neg B(x)) \end{array}$$

(b)

Step	**Reason**
1. $\exists x(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	EI from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x(C(x) \to P(x))$	Premise
5. $C(a) \to P(a)$	UI from (4)
6. $P(a)$	MP from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$	Conj from (6) and (7)
9. $\exists x(P(x) \wedge \neg B(x))$	EG from (8)

18.

case	$x^4 + y^4$
1. $x = 1, y = 1$	2
2. $x = 1, y = 2$	17
3. $x = 1, y = 3$	82
4. $x = 2, y = 1$	17
5. $x = 2, y = 2$	32
6. $x = 2, y = 3$	97
7. $x = 3, y = 1$	82
8. $x = 3, y = 2$	97
9. $x = 3, y = 3$	162
10. $x > 3$ or $y > 3$	>256

19.

$\sqrt{2}$ is irrational

if $\sqrt{2}^{\sqrt{2}}$ is rational, $x = \sqrt{2}$, $y = \sqrt{2}$, $x^y = \sqrt{2}^{\sqrt{2}}$ (4 分)

if $\sqrt{2}^{\sqrt{2}}$ is irrational, $x = \sqrt{2}$, $y = \sqrt{2}^{\sqrt{2}}$, $x^y = 2$ (4 分)