離散數學　第6章小考　　　　　　　　　　　　　　　　姓名:____________學號:_______________

1. (10%) How many different <u>license plates</u> (車牌) can be made if each license plate consists of three **letters (A-Z)** followed by three **digits (0-9)** or four digits followed by two letters? (答案不用乘開)
Ans:

2. (10%) How many students must be in a class to guarantee(保證) that **at least 5** were born in the same **month**?

Ans:

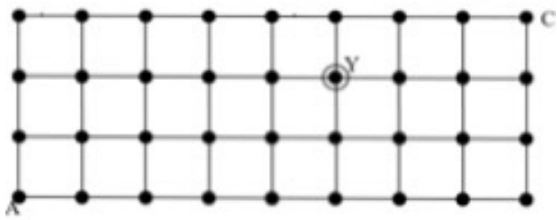
3. (12%) IF $|A| = 2^5$ and $|B| = 2^4$
(a) Find the number of functions $f:A\rightarrow B$.
(b) Find the number of **one-to-one** functions $g:A\rightarrow B$.

Ans:
(a)
(b)

4. (12%)
(a) Find the **next 5** (緊接著5個) permutation in lexicographic order after 143256.
(b) Find the **next 5** 4-combinations of the set {1, 2, 3, 4, 5, 6, 7} after {1, 3, 4, 6}.

Ans:
(a)
(b)

5. (10%) The figure at the right shows a 3-block by 8-block grid of streets. Suppose that starting at the point labeled A you can go one step up or one step to the right in each move. This is continued until the point labeled C is reached. How many different paths go from A to C which **DO NOT** go through the circled point Y.



Ans:

6. (12%)
(a) Find the **coefficient of x^3y^{12}** in the expansion of $(2x - 2y)^{15}$.
(b) The **sum of all coefficients (係數總和)** in expansion of $(2x - 2y)^{15}$.

Ans: (a)

(b)

7. (10%)Find the number of solutions to $x + y + z < 7$, where x, y, and z are non-negative integers.

Ans:

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8. (10%))Find the number of **bit strings** of length 8 either start with "00" or end with "111"?

Ans:

9. (10%) In how many ways can we distribute 7 red balls and 9 blue balls among 5 children so that **each child receives at least one red ball and one blue ball**?

Ans:

10. (12%) How many ways are there to put 5 temporary employees into 4 **identical**(相同的) offices,
    (a) if each office contains at least 1 employee?
    (b) if offices are allowed to be empty?

Ans:

(a)
(b)

11. (10%) Prove by **pigeonhole principle** that among any 6 integers there will always be a pair whose **sum** or **difference** is divisible by 9. [**hint**: 依除以9的餘數設法分成5類]

Ans:
[I]
[II]
[III]
[IV]
[V]

(1)
(2)

離散數學　第6章小考　　　　　　　　　　　　姓名:____________學號:______________

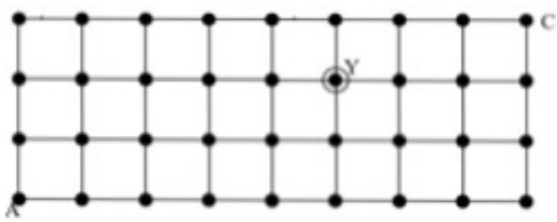
1. (10%) How many different <u>license plates</u> (車牌) can be made if each license plate consists of three **letters (A-Z)** followed by three **digits (0-9)** or four digits followed by two letters? (答案不用乘開)
Ans: $26^3 10^3 + 10^4 26^2$

2. (10%) How many students must be in a class to guarantee(保證) that **at least 5** were born in the same **month**?
Ans: $\lceil \frac{N}{12} \rceil = 5$ => N=49

3. (12%) IF $|A| = 2^5$ and $|B| = 2^4$
(a) Find the number of functions f:A→B.
(b) Find the number of **one-to-one** functions g:A→B.
Ans:
(a) 2^9
(b) 0

4. (12%)
(a) Find the **next 5** (緊接著5個) permutation in lexicographic order after 143256.
(b) Find the **next 5** 4-combinations of the set {1, 2, 3, 4, 5, 6, 7} after {1, 3, 4, 6}.
Ans:
(a) 143265, 143526, 143562,143625, 143652
(b) {1,3,4,7}, {1,3,5,6}, {1,3,5,7}, {1,3,6,7}, {1,4,5,6}

5. (10%) The figure at the right shows a 3-block by 8-block grid of streets. Suppose that starting at the point labeled A you can go one step up or one step to the right in each move. This is continued until the point labeled C is reached. How many different paths go from A to C which **DO NOT** go through the circled point Y.



Ans: C(11,3)-C(7,2)*C(4,1)

6. (12%)
(a) Find the **coefficient of x^3y^{12}** in the expansion of $(2x - 2y)^{15}$.
(b) The **sum of all coefficients (係數總和)** in expansion of $(2x - 2y)^{15}$.

Ans: (a) $C(15,3) \cdot 2^3 \cdot (-2)^{12}$

(b) $(2x - 2y)^{15} = \sum_{i=0}^{15} c(15,i) \times 2^i \times (-2)^{15-i} \cdot x^i y^{15-i}$

代入x=1, y=1可得係數總和=0

7. (10%)Find the number of solutions to $x + y + z < 7$, where x, y, and z are non-negative integers.
Ans:

x + y + z =0	c(2,2)=1
x + y + z =1	C(3,2)=3
x + y + z =2	C(4,2)=6
x + y + z =3	C(5,2)=10

x + y + z =4	C(6,2)=15
x + y + z =5	C(7,2)=21
x + y + z =6	C(8.2)=28
	共84組解

8. (10%))Find the number of **bit strings** of length 8 either start with "00" or end with "111"?
Ans: $2^6+2^5-2^3$

9. (10%) In how many ways can we distribute 7 red balls and 9 blue balls among 5 children so that **each child receives at least one red ball and one blue ball**?
Ans:C(6,4)▪C(8,4)

10. (12%) How many ways are there to put 5 temporary employees into 4 **identical**(相同的) offices,
(a) if each office contains at least 1 employee?
(b) if offices are allowed to be empty?

Ans: (stirling number)
S(n,j)=j*S(n-1,j)+S(n-1,j-1)

n \ j	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	3	1				
4	1	7	6	1			
5	1	15	25	10	1		
6	1	31	90	65	15	1	
7	1	63	301	350	140	21	1

(a) S(5,4)=10
(b) S(5,1)+S(5,2)+S(5,3)+S(5,4)=51

11. (10%) Prove by **pigeonhole principle** that among any 6 integers there will always be a pair whose **sum** or **difference** is divisible by 9. [**hint**: 依除以9的餘數設法分成5類]

Ans: 將整數n除以9的餘數分成以下5類:
[I]　if n=9k±1 for some k∈Z; (即餘數為1或餘數為8)
[II]　if n=9k±2 for some k∈Z; (即餘數為2或餘數為7)
[III]　if n=9k±3 for some k∈Z; (即餘數為3或餘數為6)
[IV]　if n=9k±4 for some k∈Z. (即餘數為4或餘數為5)
[V]　if n=9k for some k∈Z; (即餘數為0)
根據鴿籠原理，6個整數中必有2個整數m,n屬於同一類，
(1)若m，n餘數相同，則m-n可被9整除
(2)若m，n餘數不同，則m+n可被9整除