Chapter 25

Jungle Bridge

25.1 Overview

We have crossed the Rainbow Road just to end up on a Jungle Island, thick with trees and forcing us to leave our Neatokarts behind. Our goal is to get to the other side unscathed, but beware! It isn't just bushwhacking – the island is split by a giant canyon, and we'll need to employ some clever engineering to cross. Introducing: the "Jungle Bridge Challenge."

Learning Objectives

By the end of this project, you should be able to:

- Perform linear regression on an n-dimensional dataset.
- Perform gradient descent for an n-dimensional dataset in either unconstrained or constrained settings.
- Validate a model for a catenary bridge empirically.
- Develop a technically rigorous lab report.

25.2 The Challenge

To cross the canyon, we'll need to rely on the materials we have on hand: boulders (nickel weights) and vines of various varieties (rubberbands and string). This compels us to construct a hanging bridge (AKA a simple suspension bridge AKA a catenary bridge) (see Fig. 25.1 for reference). But how do we go about making sure the bridge span and shape are suitable for crossing the canyon?

Your challenge is to predict the shape that your bridge will take, given a set of materials to construct it. We can think about this prediction problem through the lens of optimization: the final hanging configuration of our bridge will be an equilibrium of all forces and so it will be the *minimum potential energy* configuration of all the materials.

The potential energy function to describe our bridge is thus:

$$U = \sum_{i=1}^{n} g m_i y_i + \sum_{i=1}^{n} \frac{1}{2} k_i (\max(l_i - l_{0,i}, 0))^2 \tag{25.1}$$

$$l_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{25.2}$$

where

- $g = 9.81$ is acceleration due to gravity in m/s^2,
- m_i is the mass of the ith bridge weight,
- l_i is the ith stretched length of a rubberband or string member,

Figure 25.1: An illustration (dramatized) of the Jungle Bridge challenge and bridge components.

- $l_{0,i}$ is the natural length of the ith rubberband or string member,
- k_i is the elasticity of the ith rubberband or string member,
- and x and y indicate the cartesian coordinates of the locations of weights along the length of the bridge.

Over the next several classes, we will walk through a series of steps to characterize our bridge materials (estimate k_i and $l_{0,i}$) and then predict the shape of a given assembly of our materials (that is, finding all l_i (or, rather, x_i, y_i)) by minimizing our potential energy function.

25.2.1 Building Bridges while Building Bridges

To build a model of our bridge, we will tackle this project in stages during in-class activities, and learn the concepts that we need to perform our optimization "just-in-time" for each stage. To that end, we will be structuring classes around key mathematical concepts and building in specific dedicated project work time with incremental project deliverables, and homeworks will contain material to deepen your understanding of the topics introduced in-class. We will prompt each activity with a specific documentation guide, and all documentation will be due in the form of a report at the end of the module project.

25.2.2 A Pair of Bridge Builders

We're asking that this project be performed in groups of two (with groups of three in extenuating circumstances). Our expectation is that all team members will feel reasonably comfortable with all aspects of this project and will collaborate fully and equitably. You may want to consider taking a pair programming approach as you write MATLAB code, and a pair writing approach as you develop your report. Ask the teaching team, especially the CAs, about some best practices here.

As part of your written deliverable, you will be asked to write an attribution statement together to outline your teaming approach to the project. At the end of the project, we'll then ask you to complete a brief peer- and self-assessment survey.

25.2.3 Bridge Specifications

Bridges will be assembled using hand-crafted nickel weights (made in-class), rubberbands (measured in class) and/or string (measured in class). Sets of magnets and paperclips will be provided so that the bridges can be suspended against the whiteboards in the classroom. A minimal bridge should be composed of the following:

- At least 5 weights
- At least 6 rubberband/string members

Teams may make bridges larger than this, if so desired.

All supplies to make your bridge will be provided in-class. You will be responsible for keeping track of your bridge materials throughout the unit; labels and plastic storage bags will be provided to assist.

25.3 Crossing the Canyon: Project Deliverables

This module project is heavily integrated into the in-class and homework activities for the next few weeks. The main deliverable we are looking for is a comprehensive lab report of your work, complemented by your MATLAB implementation and any other supporting documentation you generate.

Exercise 25.1

Lab Report Your lab report will be composed of all the in-class documentation steps that will be prompted during in-class activities, as well as any additional written material or evidence to glue it all together into a comprehensive and complete report of your work. Please structure your report using the template below. We have included the questions you must address in each report section, and the tables, figures and images that you should include in the body of the report.

Section 1: Overview and Introduction: This section should answer the following questions at a very high level - about a paragraph or two total.

- What were the goals of this project?
- What did you do?
- What were the approaches or methods you used?

Section 2: Methodology and Results

- **A. Construction of the Jungle Bridge and Characterization of the Building Materials**
  - **For the rubber band bridge**
    - How did you characterize and then model the spring properties of the rubber bands, including any measurements or regression analysis you did?
    - What configuration did you choose (number of rubber bands, weights) for the rubber band bridge you built?
    - In this section, you should include and embed (and make explicit reference to) the following tables, figures and photos:
      - TABLE 1: A table with the measured data (weight(s) added, corresponding stretched length) for each of your rubber bands.
      - TABLE 2: A table showing the stiffness, k, and the natural length, lo, of each rubber band.
      - TABLE 3: A table of the x-y coordinates of each of the measured positions of the rubber band bridge
      - TABLE 4: A table of how much each of the individual weights used in the construction of the rubber band Jungle bridge weight weigh
      - FIGURE 1: For a single rubber band of your choice, a plot comparing the line of best fit to the measured data, including appropriately labeled axes, a legend, and caption.
      - FIGURE 2: For the same rubber band, a contour plot OR a 3D surface plot showing the cost function E(m, b), including appropriately labeled axes, a legend and caption.
      - FIGURE 3: Photo of your rubber band bridge
  - **For the string bridge**

        * How did you characterize the properties of the strings including any measurements you performed?
        * What configuration did you choose (number of rubber bands, weights) for the string bridge you built?
        * In this section, you should include and embed (and make explicit reference to) the following tables, figures and photos.
            · TABLE 5: A table of the measurements of the lengths of the strings used in the construction of the string Jungle Bridge
            · TABLE 6: A table of how much each of the individual weights used in the construction of the string Jungle bridge weight weigh
            · TABLE 7: A table of the x-y coordinates of each of the measured positions of the string bridge.
            · FIGURE 4: Photo of your string bridge

- **B. Using Gradient Descent Optimization for the Jungle Bridge - Unconstrained Optimization**
    - Describe your methodology: provide a description of how to compute a numerical gradient and how to perform gradient descent, including appropriate equations and/or snippets of code
    - Share your results: include and discuss appropriate plots comparing your measured jungle bridge to predicted jungle bridge configuration.
    - In this section, you should include and embed (and make explicit reference to) the following tables, figures and photos
        * FIGURE 5: A visualization of the progression of the gradient descent method starting from the initial guess on the appropriate potential function (similar to Figure 24.5 in the Day 18 activity).
        * FIGURE 6: Comparison plot between your measured (solid line) and predicted (dashed line) bridge geometry for the rubber band (unconstrained) case including appropriately labeled axes, a legend and caption.

- **C. Using Gradient Descent Optimization for the Jungle Bridge - Constrained Optimization**
    - Describe your methodology: Provide a description of the constrained optimization approach to gradient descent, including appropriate equations and/or snippets of code. Explain the difference between the constrained and unconstrained optimization approaches.
    - Share your results: include and discuss appropriate plots comparing your measured jungle bridge to predicted jungle bridge configuration.
    - In this section, you should include (and make explicit reference to) the following tables, figures and photos:
        * FIGURE 7: Comparison plot between your measured (solid line) and predicted (dashed line) bridge geometry for the string (constrained) case including appropriately labeled axes, a legend and caption.

Section 3: Interpretation of Results and Discussion: In this section, please address the following:

- How well does the gradient descent optimization predict the position of the rubber band Jungle Bridge? Of the string Jungle Bridge?
- What sources of error might contribute to any differences you see? What sources of error do you think contribute the most? How might you approach reducing the effects of these sources?

Section 4: Team Attribution: A brief attribution statement in your report, at the end, which lists the members of the team and aspects of the report they may have been primarily responsible for or generally contributed to.

Section 5: Code and Supplementary Documentation: Alongside your report, please submit all your code, and any supplementary artifacts/evidence/documents that you may have generated for your project. We will be checking for the following:

- The submitted MatLab files are all .m scripts; no .mlx files will be accepted
- Good annotation/commenting practices are used
  - Functions have "docstrings" or comments that describe their purpose, the inputs expected, and any outputs generated
  - Key methodological steps are documented with a brief comment

25.4 Summary of Tables, Plots and Figures Required for Lab Report

To check that you have created all the necessary visual elements of the lab report, we've put together this list of all the figures you'll need. These figures are compiled from the above report template.

- TABLE 1: A table with the measured data (weight(s) added, corresponding stretched length) for each of your rubber bands.
- TABLE 2: A table showing the stiffness, k, and the natural length, lo, of each rubber band.
- TABLE 3: A table of the x-y coordinates of each of the measured positions of the rubber band bridge
- TABLE 4: A table of how much each of the individual weights used in the construction of the rubber band Jungle bridge weight weigh
- TABLE 5: A table of the measurements of the lengths of the strings used in the construction of the string Jungle Bridge
- TABLE 6: A table of how much each of the individual weights used in the construction of the string Jungle bridge weight weigh
- TABLE 7: A table of the x-y coordinates of each of the measured positions of the string bridge.
- FIGURE 1: For a single rubber band of your choice, a plot comparing the line of best fit to the measured data, including appropriately labeled axes, a legend, and caption.
- FIGURE 2: For the same rubber band, a contour plot OR a 3D surface plot showing the cost function E(m, b), including appropriately labeled axes, a legend and caption.
- FIGURE 3: Photo of your rubber band bridge
- FIGURE 4: Photo of your string bridge
- FIGURE 5: A visualization of the progression of the gradient descent method starting from the initial guess on the appropriate potential function (similar to Figure 24.5 in the Day 18 activity). This is only for the rubber band bridge. You do not need to do this for the string bridge.
- FIGURE 6: Comparison plot between your measured (solid line) and predicted (dashed line) bridge geometry for the rubber band (unconstrained) case including appropriately labeled axes, a legend and caption.
- FIGURE 7: Comparison plot between your measured (solid line) and predicted (dashed line) bridge geometry for the string (constrained) case including appropriately labeled axes, a legend and caption.

25.5 Style Guide for the Lab Report

This section provides some guidance on *how* you write your report, as opposed to *what* you're writing about.

25.5.1 Headings

Headings are short-and-sweet signposts for your writing. Please use headings and subheadings to convey when you are transitioning into a different discussion. A skim of headings in a paper should give a reader a good sense of all the material that will be covered; headings should also be useful bookmarks for anyone looking for a particular piece of information to go back and re-read or review. Headings will generally not convey particular results or methods; rather, they will convey themes. Examples of some possible headings for a Jungle Bridge report include:

- Bridge Assembly
- Concluding Remarks
- Assumptions
- Results
- Project Description
- ...and so on...

Unacceptable headings would be examples like: "Material Properties of Each Rubber Band We Measured Using Different Weight Profiles" (too long, too specific); "Our Model and Experimental Bridge Data Were a Match" (results embedded into the heading); "It Worked" (results-oriented and too vague and colloquial to understand what this might be in reference to).

25.5.2 Text

The methods, analysis, results, and discussion of your work will all be written underneath your headings. 'Text' here refers to narrative writing, embedded symbolic equations, code snippets, and numerical tables. Narrative writing should be descriptive and explanatory, and make plain your thinking process while completing a project. It should not just re-hash the procedure for a project, but expand upon it – why is performing a particular analysis useful? how did you derive this equation? what is the interpretation of a particular result?

Equations

Some ideas are not necessarily well-described with narrative text. For instance, particular mathematical procedures (e.g., "taking the derivative of a function") are more precisely described symbolically through an equation or set of equations. When used, equations should be on their own lines (and ideally numbered/labeled), and the variables/values in the equations introduced in the text directly preceding or following the equation. Symbols should be used; rather than the word "theta" use θ. Since you define your symbols, and will describe them in your text, choose symbols that are clear shorthand: rather than x_{right} let the symbol simply be x_r. An example of how to render an equation follows, where x is a vector variable of real numbers, and $f(x)$ is a vector-valued function:

$$\frac{df(x)}{dx} = 2x - 5 \tag{25.3}$$

Code Snippets

As the projects we work on in this class have a significant implementation component in MATLAB, it can be helpful when describing your methods to selectively include code snippets. If you find that incorporating code into your explanations is helpful, we recommend the following best practices:

- Do not copy and paste all of your code in giant blocks without commentary. Include code comments or narrative text comments if you are copying over large sections of code.
- Try to limit code snippets to single lines or single function calls; you can incorporate snippets like this directly into a narrative paragraph.
- When adding code to your report, render the text in some other font or style, so it is clear to the reader that you are providing a code snippet.

Tables

You may have numerical data that you'd like to convey in the report, and writing it in a paragraph would be too cumbersome or unclear for a reader. Tables are a great organizational tool for numerical data (or even categorical data and lists!) that you are encouraged to use when appropriate. A table should have clear row and column labels, be given a table number and caption, and be cited somewhere directly in the text you write (see Tab. 25.1).

Example header	Example header 2
Some data!	Some data!

Table 25.1: This is an example table.

25.5.3 Figures and Images

Graphical representations of data, annotated diagrams, or images are an excellent supplement to written text, since they can convey complex information and analysis in a relatively compact format. You should feel empowered to include whatever graphical imagery is necessary to explain any key points you would like to make. When adding a figure to a report, there are some key practices you should follow:

- The figures should have an appropriate resolution to be rendered clearly in your report (figures at 300DPI and approximately 5 x 5 inches in scale may be a good rule of thumb here).
- For graphical figures, all axes should be labeled, the plot and subplots should be labeled, the legend should be complete, and any annotation added in post-processing should not block any portion of the data being visualized. Figure captions should include a brief description of what is in the plot, as well as brief interpretative remarks.
- For diagrams, all key components should be labeled clearly in words or symbols, which are then defined in the figure caption. The figure caption should summarize how to interpret the diagram, and any other key pieces of information.
- For images, annotations can be helpful in addition to descriptive figure captions that directly describe what is being rendered in the image and its interpretation. For instance, a caption like "The Bridge" is not very useful; instead consider "The bridge we assembled, centered in the image, with 5 weights and 6 rubberband members with properties recorded in Table X."
- When included, figures should be directly referenced in the text of the report. For instance, see Fig. 25.2.

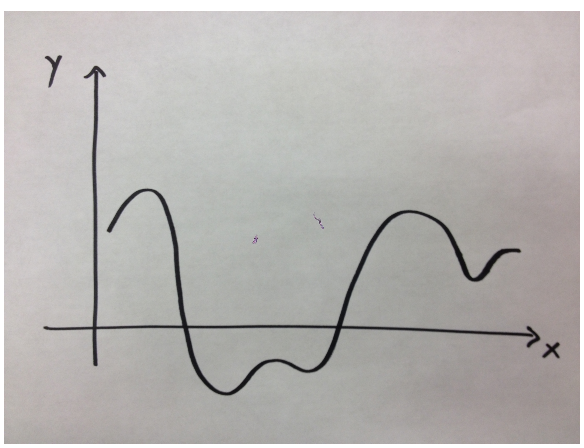

Figure 25.2: A placeholder figure in this style guide, to illustrate how to embed a figure in a report and reference it in the text.

25.5.4 References and Citations

In technical documentation you may write for other classes or in the workplace, citing any sources you used along the way is an essential practice. While we do not explicitly require you to use outside sources in this project, if you independently use outside sources to better understand a particular topic or implement a certain piece of code, please reference them in the text (e.g., with a style like "statement that needs cited (citation author, year)" or "statement that needs cited [Citation number]") and provide a full citation at the end of your report. You can choose the reference and citation format (e.g., MLA, APA, IEEE – this author recommends IEEE).

25.5.5 Resources

You can prepare your report in any text editing software you are most comfortable with. For collaborative report writing, cloud-based services like Google Docs or Overleaf are good options. In academia and industry, many reports or papers are written in a typesetting language called "LaTeX" which is how we write many of the documents we generate for this class. This may be a great opportunity to learn it! For generating figures, consider using a vector graphics editor like Inkscape, Adobe Illustrator, Gimp, and so on. You can also generate figures using cloud-based software like Google Slides or Google Drawing.