Chapter 25

Jungle Bridge

25.1 Overview

We have crossed the Rainbow Road just to end up on a Jungle Island, thick with trees and forcing us to leave our Neatokarts behind. Our goal is to get to the other side unscathed, but beware! It isn't just bushwhacking – the island is split by a giant canyon, and we'll need to employ some clever engineering to cross. Introducing: the "Jungle Bridge Challenge."

Learning Objectives

By the end of this project, you should be able to:

- Perform linear regression on an n-dimensional dataset.
- Perform gradient descent for an n-dimensional dataset in either unconstrained or constrained settings.
- Validate a model for a catenary bridge empirically.
- Develop a technically rigorous lab report.

25.2 The Challenge

To cross the canyon, we'll need to rely on the materials we have on hand: boulders (nickel weights) and vines of various varieties (rubberbands and string). This compels us to construct a hanging bridge (AKA a simple suspension bridge AKA a catenary bridge) (see Fig. 25.1 for reference). But how do we go about making sure the bridge span and shape are suitable for crossing the canyon?

Your challenge is to predict the shape that your bridge will take, given a set of materials to construct it. We can think about this prediction problem through the lens of optimization: the final hanging configuration of our bridge will be an equilibrium of all forces and so it will be the *minimum potential energy* configuration of all the materials.

The potential energy function to describe our bridge is thus:

$$U = \sum_{i=1}^{n} g m_i y_i + \sum_{i=1}^{n} \frac{1}{2} k_i (\max(l_i - l_{0,i}, 0))^2 \tag{25.1}$$

$$l_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{25.2}$$

where

- $g = 9.81$ is acceleration due to gravity in m/s^2,
- m_i is the mass of the ith bridge weight,
- l_i is the ith stretched length of a rubberband or string member,

Figure 25.1: An illustration (dramatized) of the Jungle Bridge challenge and bridge components.

- $l_{0,i}$ is the natural length of the ith rubberband or string member,
- k_i is the elasticity of the ith rubberband or string member,
- and x and y indicate the cartesian coordinates of the locations of weights along the length of the bridge.

Over the next several classes, we will walk through a series of steps to characterize our bridge materials (estimate k_i and $l_{0,i}$) and then predict the shape of a given assembly of our materials (that is, finding all l_i (or, rather, x_i, y_i)) by minimizing our potential energy function.

25.2.1 Building Bridges while Building Bridges

To build a model of our bridge, we will tackle this project in stages during in-class activities, and learn the concepts that we need to perform our optimization "just-in-time" for each stage. To that end, we will be structuring classes around key mathematical concepts and building in specific dedicated project work time with incremental project deliverables, and homeworks will contain material to deepen your understanding of the topics introduced in-class. We will prompt each activity with a specific documentation guide, and all documentation will be due in the form of a report at the end of the module project.

25.2.2 A Pair of Bridge Builders

We're asking that this project be performed in groups of two (with groups of three in extenuating circumstances). Our expectation is that all team members will feel reasonably comfortable with all aspects of this project and will collaborate fully and equitably. You may want to consider taking a pair programming approach as you write MATLAB code, and a pair writing approach as you develop your report. Ask the teaching team, especially the CAs, about some best practices here.

As part of your written deliverable, you will be asked to write an attribution statement together to outline your teaming approach to the project. At the end of the project, we'll then ask you to complete a brief peer- and self-assessment survey.

25.2.3 Bridge Specifications

Bridges will be assembled using hand-crafted nickel weights (made in-class), rubberbands (measured in class) and/or string (measured in class). Sets of magnets and paperclips will be provided so that the bridges can be suspended against the whiteboards in the classroom. A minimal bridge should be composed of the following:

- At least 5 weights
- At least 6 rubberband/string members

Teams may make bridges larger than this, if so desired.

All supplies to make your bridge will be provided in-class. You will be responsible for keeping track of your bridge materials throughout the unit; labels and plastic storage bags will be provided to assist.

25.3 Crossing the Canyon: Project Deliverables

This module project is heavily integrated into the in-class and homework activities for the next few weeks. The main deliverable we are looking for is a comprehensive lab report of your work, complemented by your MATLAB implementation and any other supporting documentation you generate.

Exercise 25.1

Lab Report Your lab report will be composed of all the in-class documentation steps that will be prompted during in-class activities, as well as any additional written material or evidence to glue it all together into a comprehensive and complete report of your work. We are expecting that your report will, at a minimum, contain the following elements:

- All prompted in-class documentation, including:
  - Day 17 – Regression – a table of all the rubberband measurements, a visualization of the rubberband data and line of fit, image evidence of the measurement activity, a description of the regression methodology
  - Day 18/19 – Gradient Descent – a description of how to compute a numerical gradient and how to perform gradient descent, a visualization of the results of a gradient descent method
  - Day 20/21 – Constrained Optimization – an explanation of the difference between constrained and unconstrained optimization, a description of the constrained optimization approach to gradient descent, a table of all the string measurements, a visualization of the string data
- Images of your rubberband and string bridges
- Comparison plots between your measured and predicted bridge geometries
- A description of all mathematical methods not otherwise captured in the in-class prompts
- Explanatory diagrams or code snippets to support written explanations

Your report *will not* simply be all your in-class documentation "stapled together" – it should be organized as a single cohesive report and consist of the following broad sections (please use this as a guide; you may have different section headers or add additional sections per your organizational preferences):

- Overview
- Introduction
- Methods
- Results and Interpretation
- Discussion (or Conclusion)
- Team Attribution

Your report should follow the Style Guide for this project, and will be scored based on technical writing quality, organization, and clarity. Please see the rubric on Canvas for more details on grading.

Exercise 25.2

Code and Supplementary Documentation Alongside your report, please submit all your code, and any supplementary artifacts/evidence/documents that you may have generated for your project. We will be checking for the following:

- The submitted MatLab files are all .m scripts; no .mlx files will be accepted
- Good annotation/commenting practices are used
    - Functions have "docstrings" or comments that describe their purpose, the inputs expected, and any outputs generated
    - Key methodological steps are documented with a brief comment