Experimental Characterization of an Oscillator

2.1 Introduction

In this project you will work with a partner to build and characterize a single degree-of-freedom (DOF) harmonic oscillator. This will involve using differential equations with real measurements. We talked in class about a mass-spring-damper model system, pictured in Figure 2.1.

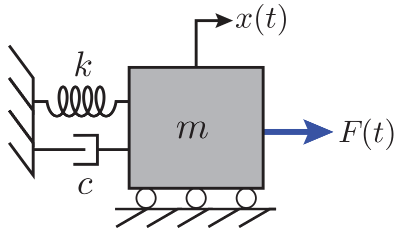

Figure 2.1: The mass-spring-damper model, leading to a second-order system.

In this project, you will set up a physical experiment (Fig. 2.2) that mimics this system, collect data, and validate the degree to which the simple mass-spring-damper model matches a real system.

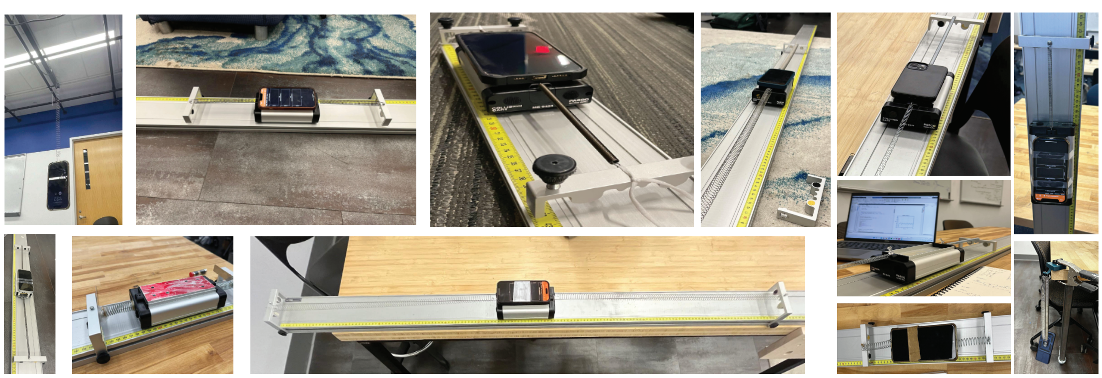

Figure 2.2: Example experimental setups.

2.2 Project Steps

1. Select a spring(s) from our spring library
2. Characterize the spring(s) (fit a natural length, l_0, and spring constant k) using the instructor-provided force measurement device and a meter stick.
   - We recommend that you use the same method you used to measure the rubber band natural length and stiffness in the QEA2 Jungle Bridge project.
   - Please use at least five (length, force) measurements in your characterization.
3. Set up a harmonic oscillator using either the Vertical or Horizontal option (see below)
4. Measure the mass (m) of your moving mass, defined as the box + phone for the Vertical option, or the cart + phone for the Horizontal option
5. Start your oscillator from a nonzero initial displacement with zero initial velocity.
6. Use a phone to collect acceleration data from the moving mass. We recommend the phyphox app for data collection—it's free! Alternatively, the MATLAB phone app is an option.
7. Use the acceleration data that you collected to estimate the natural frequency, ω_n, and damping ratio, ζ. You will need to solve the ODE for the damped harmonic oscillator to derive an analytic solution, manipulate the solution to represent the acceleration data, then use your data to fit the unknown coefficients in your analytic solution.
8. Use the measured/estimated values of m, k, ω_n, and ζ to estimate the damping constant, c.
9. Write a lab report that documents your methodology and results.

Notes:

- You will need to use curve fitting tools with an expression for the *acceleration*. We previously showed that, for an unforced linear system, the acceleration, $a = \ddot{x}$, obeys the same ODE as the displacement:

$$\frac{d^2}{dt^2}\left(m\ddot{x} + c\dot{x} + kx = 0\right) \quad \rightarrow \quad \boxed{m\ddot{a} + c\dot{a} + ka = 0} \tag{2.1}$$

  This means that the solution to the acceleration will have the same form as the solution to the displacement. Furthermore, the natural frequency and damping ratio for both ODEs are the same.

- You also do not need to show any quantities we're not explicitly asking for in the Deliverables section; for instance, you don't need to report a phase shift.
- You may want to use `fit_oscillator_acceleration` to help check your analysis.

2.2.1 Two Experimental Options

Vertical option (build a box)

Hang your phone (e.g., from a table) using a spring to collect data. If you use more than one spring, make sure to calculate an effective spring constant (series or parallel). Design and manufacture your own phone enclosure (a box); e.g., using 3d printing.

Horizontal option (use a cart)

Use a low-friction cart (available in the classrooms) with two attached springs to collect data (one on either side). Calculate an effective spring constant for your system (from two springs in series). Use low-tech means (e.g., duct tape) to attach your phone to the cart.

2.3 Deliverables

Your final report must include the following:

1. Table of estimated values:
   - Moving mass, m, in kilograms.
   - Spring constant, k, in Newtons/meter.
   - Damping coefficient, c (sometimes called b), in Newton-seconds/meter (kilograms/second).
   - Exponential decay rate, σ, in seconds^{-1}.
   - Damped frequency, ω_d, in radians/second.
   - Natural frequency, ω_n, in radians/second, computed from the measured mass and stiffness.
   - Natural frequency, ω_n, in radians/second, computed from the acceleration data.
   - Damping ratio, ζ, (dimensionless).
2. Photo of your experimental setup.
3. Plot: Experimental data from spring characterization (with a fit line):
   - Force on the vertical axis
   - Displacement on the horizontal axis.
4. Plot: Experimental data from phone:
   - Acceleration on the vertical axis.
   - Time on the horizontal axis.
5. Plot: the complex exponential coefficients, $\lambda = -\sigma \pm i\omega_d$ on the complex plane:
   - $\pm\omega_d$ on the vertical axis.
   - $-\sigma$ on the horizontal axis.
6. Your raw experimental data in a .csv file. This should be a two column spreadsheet:
   - The first column should be time (in seconds) of each measurement.
   - The second column should contain the acceleration measurements (in meters/second^2). Phyphox gathers motion data for each of the x-y-z directions. Only include the acceleration data (either x, y, or z) that you actually used in your analysis.
   - This data will be submitted as a separate Canvas assignment.
7. Abstract: 1-3 paragraphs summarizing your experimental approach and findings
   - This abstract should include a brief discussion comparing the estimate of the natural frequency you computed from the acceleration data with estimate your computed from the mass and stiffness. Were the estimated values similar or significantly different?