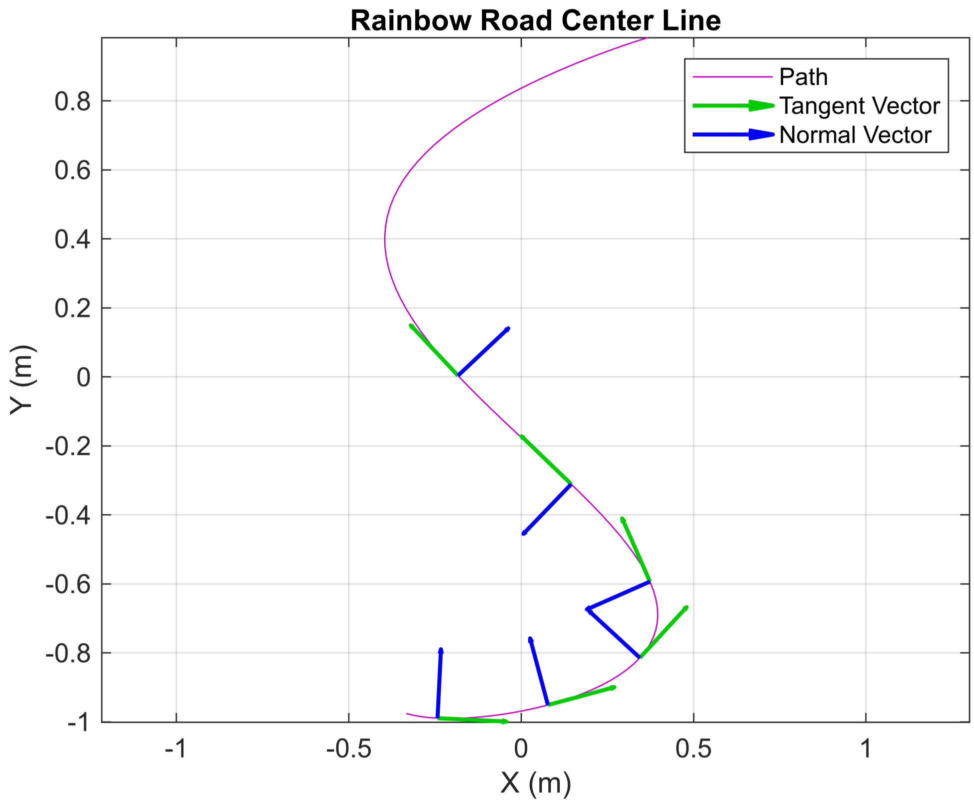
```
syms t;
u = t;

t_vals = linspace(0, 30, 100);

rainbow_road_center_line = [
            0.3960 * cos(2.65 * (u + 1.4)), ...
            -0.99 * sin(u + 1.4)
];

neato_max_speed = 0.3;
neato_min_speed = -0.3;
wheel_dist = 0.245;

u_range = [0, 3.2];
full_u_range = linspace(u_range(1), u_range(2), 100);
magnitude = @(vec) simplify(sqrt(sum(vec.^2)));

rainbow_road_center_line_vel = diff(rainbow_road_center_line);
rainbow_road_center_line_speed = magnitude(rainbow_road_center_line_vel);
rainbow_road_center_line_length = int(rainbow_road_center_line_speed, u, [0, u]);
rainbow_road_center_line_tangent_vec = rainbow_road_center_line_vel ./
rainbow_road_center_line_speed;
rainbow_road_center_line_tangent_vec_deriv =
diff(rainbow_road_center_line_tangent_vec);
rainbow_road_center_line_norm_vec = rainbow_road_center_line_tangent_vec_deriv ./
norm(rainbow_road_center_line_tangent_vec_deriv);

rainbow_road_center_line_x(u) = rainbow_road_center_line(1);
rainbow_road_center_line_y(u) = rainbow_road_center_line(2);
rainbow_road_center_line_pos_data_x =
double(rainbow_road_center_line_x(full_u_range));
rainbow_road_center_line_pos_data_y =
double(rainbow_road_center_line_y(full_u_range));
rainbow_road_center_line_tangent_vec_x(u) = rainbow_road_center_line_tangent_vec(1);
rainbow_road_center_line_tangent_vec_y(u) = rainbow_road_center_line_tangent_vec(2);
rainbow_road_center_line_norm_vec_x(u) = rainbow_road_center_line_norm_vec(1);
rainbow_road_center_line_norm_vec_y(u) = rainbow_road_center_line_norm_vec(2);

rainbow_road_angular_velocity(u) = rainbow_road_center_line_tangent_vec_x
* diff(rainbow_road_center_line_tangent_vec_y)
- rainbow_road_center_line_tangent_vec_y *
diff(rainbow_road_center_line_tangent_vec_x);

rainbow_road_vel_left = (rainbow_road_center_line_speed - (wheel_dist / 2) .*
rainbow_road_angular_velocity);
rainbow_road_vel_right = (rainbow_road_center_line_speed + (wheel_dist / 2) .*
rainbow_road_angular_velocity);
```

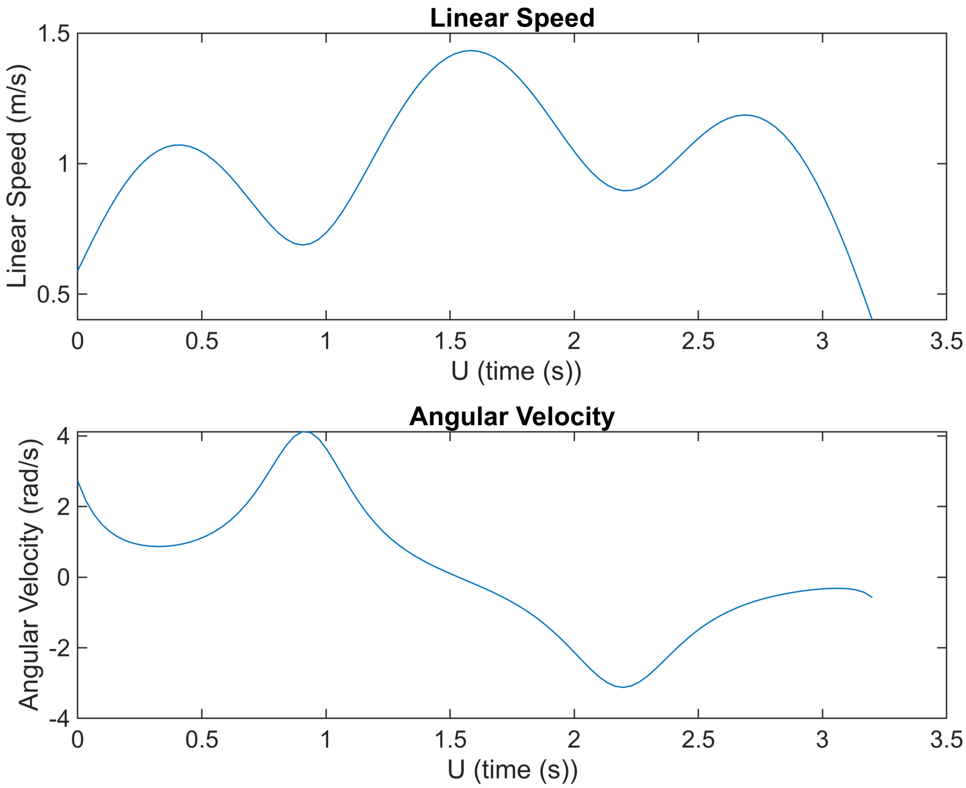
PLOTS (6.1, 6.2)

```
figure;
plot(rainbow_road_center_line_pos_data_x, rainbow_road_center_line_pos_data_y,
Color=[0.8, 0, 0.8]); hold on
    for data_val = 5:10:60
        quiver( ...
            rainbow_road_center_line_pos_data_x(data_val), ...
            rainbow_road_center_line_pos_data_y(data_val), ...
            double(rainbow_road_center_line_tangent_vec_x(full_u_range(data_val))),
...
            double(rainbow_road_center_line_tangent_vec_y(full_u_range(data_val))),
...
            0.20, ...
            LineWidth=1.5, ...
            Color=[0, 0.8, 0] ...
        )
        quiver( ...
            rainbow_road_center_line_pos_data_x(data_val), ...
            rainbow_road_center_line_pos_data_y(data_val), ...
            double(rainbow_road_center_line_norm_vec_x(full_u_range(data_val))), ...
            double(rainbow_road_center_line_norm_vec_y(full_u_range(data_val))), ...
            0.20, ...
            LineWidth=1.5, ...
            Color=[0, 0, 1] ...
        )
    end
    legend("Path", "Tangent Vector", "Normal Vector")
    title("Rainbow Road Center Line")
    xlabel("X (m)"); ylabel("Y (m)")
    grid on
    axis equal;
hold off
```



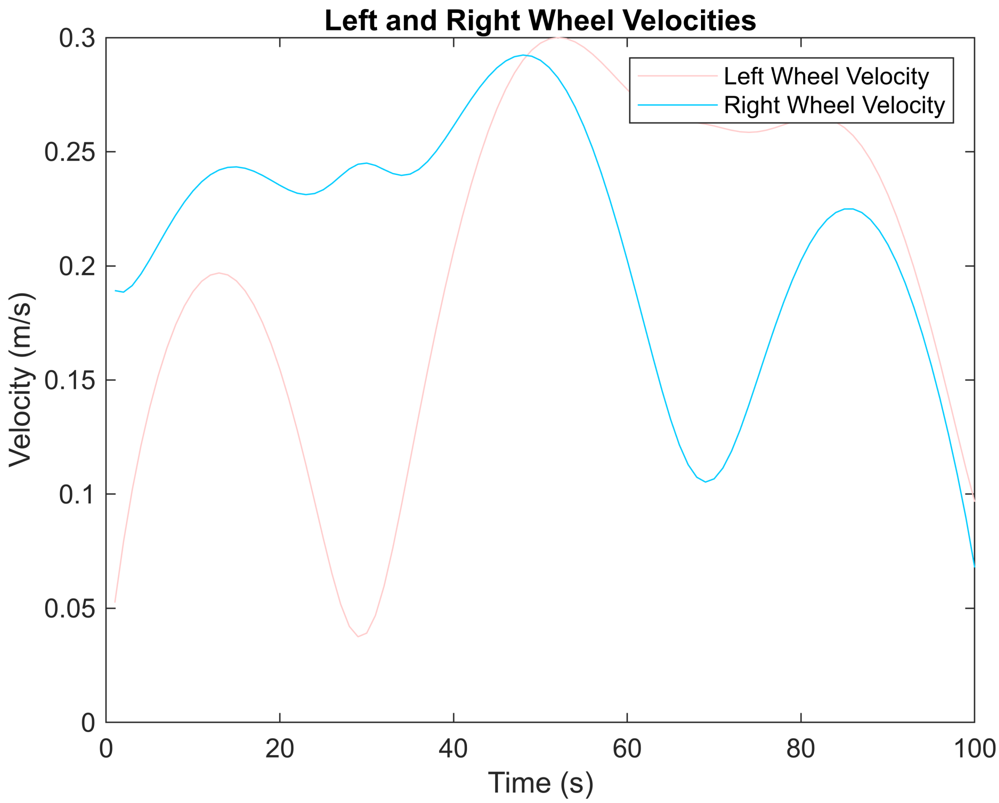
Plot of the path of the neato, with tangent and normal vectors all across the path.

```
figure;
subplot(2, 1, 1)
plot(full_u_range, double(subs(rainbow_road_center_line_speed, u, full_u_range)));
hold on
    title("Linear Speed")
    xlabel(" U (time (s))"); ylabel("Linear Speed (m/s)")
hold off

subplot(2, 1, 2)
plot(full_u_range, double(subs(rainbow_road_angular_velocity, u, full_u_range)));
hold on
    title("Angular Velocity")
    xlabel("U (time (s))"); ylabel("Angular Velocity (rad/s)")
hold off
```



A better view of the experimental angular velocity and linear speed of the Neato.

```
rainbow_road_vel_left_data = double(subs(rainbow_road_vel_left, u, full_u_range));
rainbow_road_vel_right_data = double(subs(rainbow_road_vel_right, u, full_u_range));
scaling_factor = max(rainbow_road_vel_left_data) / 0.3;

figure;
plot(rainbow_road_vel_left_data ./ scaling_factor, Color=[1, 0.8, 0.8]); hold on
    plot(rainbow_road_vel_right_data ./ scaling_factor, Color=[0, 0.8, 1]);
    title("Left and Right Wheel Velocities")
    legend("Left Wheel Velocity", "Right Wheel Velocity")
    xlabel("Time (s)"); ylabel("Velocity (m/s)")
hold off
```



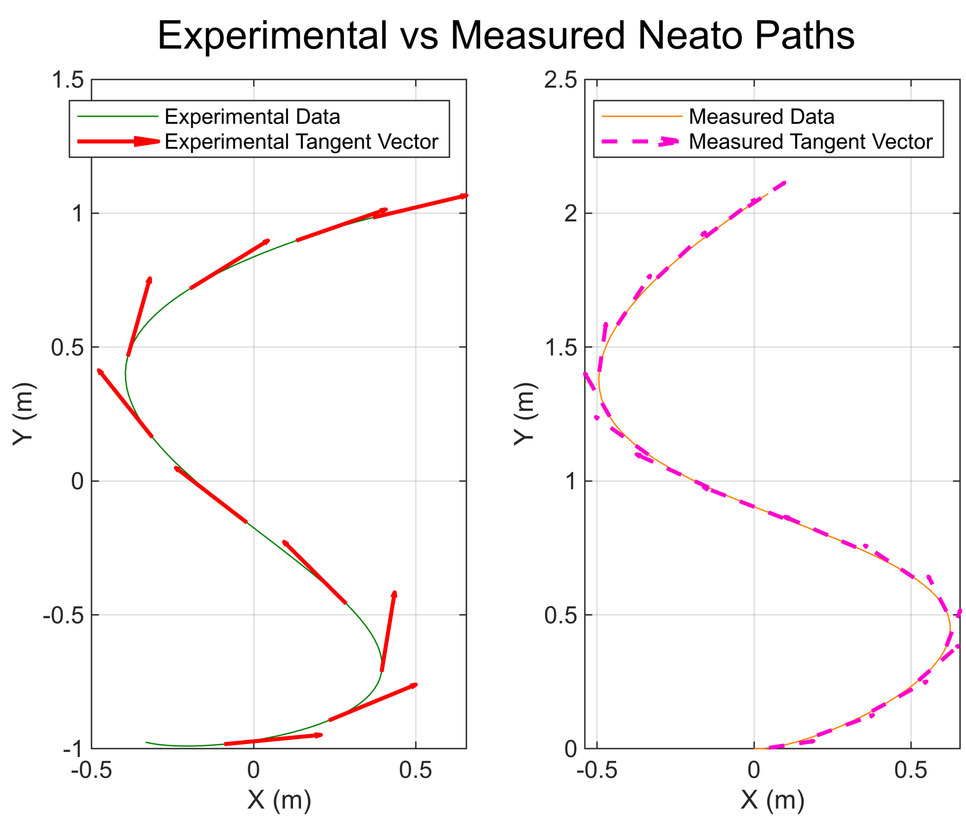
Left and Right Wheel Experimental Velocities, in a better view.

EXERCISE 6.4

```
load("neato_data.mat")

t_list = recorded_data(1:172, 1);
left_encoder_data = recorded_data(1:172, 2);
right_encoder_data = recorded_data(1:172, 3);
left_vel_data = recorded_data(1:172, 4);
right_vel_data = recorded_data(1:172, 5);

left_vel_approx = diff(left_encoder_data) ./ diff(t_list);
right_vel_approx = diff(right_encoder_data) ./ diff(t_list);

rotation_rate = (right_vel_data - left_vel_data) / wheel_dist;
orientation = [0; cumsum(rotation_rate(1:end-1) .* diff(t_list))];

neato_fwd_vel = (left_vel_data + right_vel_data) / 2;

vel_x = cos(orientation) .* neato_fwd_vel;
vel_y = sin(orientation) .* neato_fwd_vel;

x_pos =  [0; cumsum(vel_x(1:end-1) .* diff(t_list))];
y_pos = [0; cumsum(vel_y(1:end-1) .* diff(t_list))];
```

```
x_vel = diff(x_pos) ./ diff(t_list);
y_vel = diff(y_pos) ./ diff(t_list);

speed = norm([x_vel, y_vel]);

tangent_x = x_vel ./ speed;
tangent_y = y_vel ./ speed;

q_x = diff(tangent_x);
q_y = diff(tangent_x);

norm_x = q_x ./ norm(q_x);
norm_y = q_y ./ norm(q_y);

angular_velocity = tangent_x(1:end-1) .* q_y - tangent_y(1:end-1) .* q_x;

figure;
sgtitle("Experimental vs Measured Neato Paths")
subplot(1, 2, 1)
exp_plot = ...
plot(double(subs(rr_x, t, t_vals)), double(subs(rr_y, t, t_vals)), Color=[0, 0.5,
0]); hold on
    exp_vals = [];
    i = 1;
    for val = t_vals
        if (mod(i, 10) == 0)
            exp_vals(i) = quiver( ...
                double(subs(rr_x, t, val)), ...
                double(subs(rr_y, t, val)), ...
                double(subs(rr_tang_x, t, val)), ...
                double(subs(rr_tang_y, t, val)), ...
                0.30, ...
                Color=[1, 0, 0], ...
                LineWidth=1.5 ...
            );
        end
        i = i + 1;
    end

    legend("Experimental Data", "Experimental Tangent Vector", Location="North")
    xlabel("X (m)"); ylabel("Y (m)")
    grid on;
hold off

subplot(1, 2, 2)
measure_plot = ...
plot(x_pos, y_pos, Color=[1, 0.5, 0]); hold on
    measure_vals = [];
    i = 1;
```

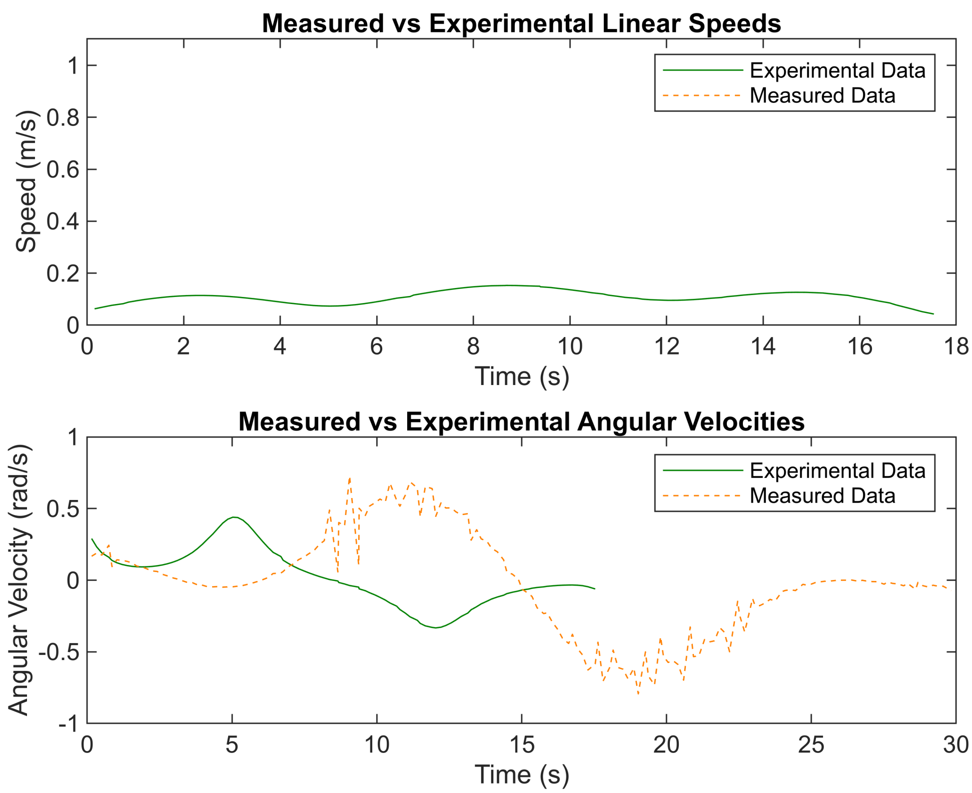
```
    for val = 5:10:length(tangent_x)
        measure_vals(i) = quiver( ...
            x_pos(val), ...
            y_pos(val), ...
            tangent_x(val), ...
            tangent_y(val), ...
            2, ...
            Color=[1, 0, 0.8], ...
            LineStyle="--", ...
            LineWidth=1.5 ...
        );
        i = i + 1;
    end

    legend([measure_plot, measure_vals(1)], "Measured Data", "Measured Tangent
Vector", Location="North")
    xlabel("X (m)"); ylabel("Y (m)")
    grid on;
hold off
```



Both of the Neato's paths, experimental and measured. Measured isn't perfect, but it follows the path relatively closely. The tangent vectors in the measured data line up a little more closely than the ones in the experimental data.

```
figure;
subplot(2, 1, 1)
```

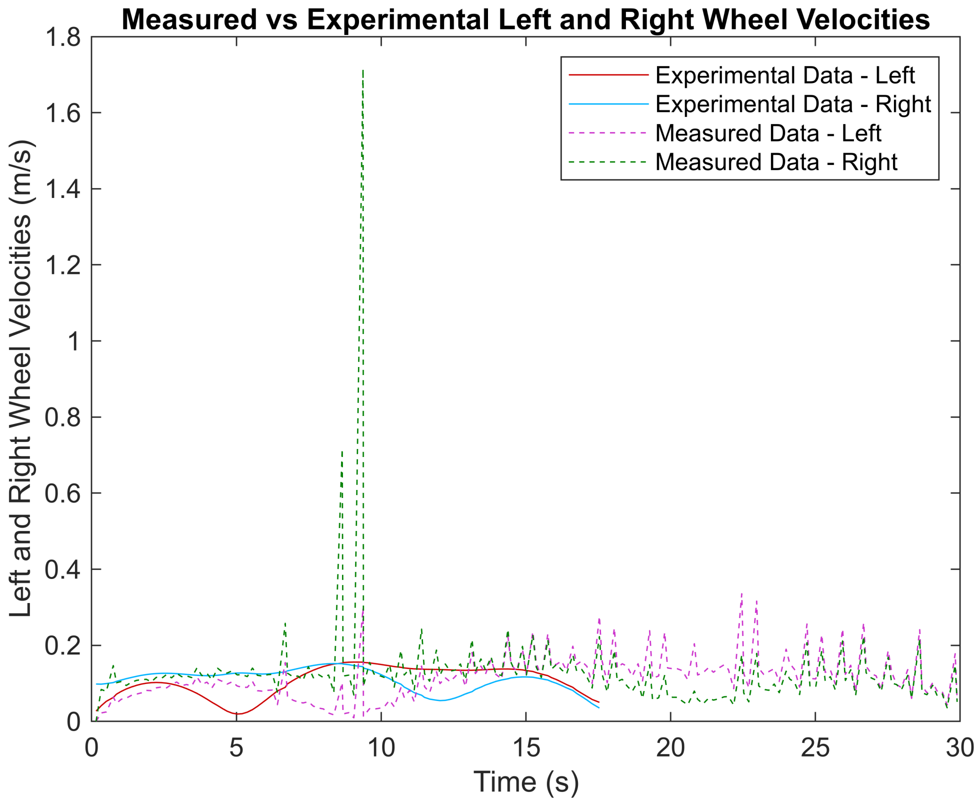
```
plot(t_list(1:100), double(subs(rr_speed, t, t_vals)), Color=[0, 0.5, 0]); hold on
    plot(speed, Color=[1, 0.5, 0], LineStyle="--");
    legend("Experimental Data", "Measured Data")
    xlabel("Time (s)"); ylabel("Speed (m/s)")
    title("Measured vs Experimental Linear Speeds")
hold off

subplot(2, 1, 2)
plot(t_list(1:100), double(subs(rr_ang_vel, t, t_vals)), Color=[0, 0.5, 0]); hold on
    plot(t_list(1:170), angular_velocity * 10^3, Color=[1, 0.5, 0], LineStyle="--");
    legend("Experimental Data", "Measured Data", Location="Northeast")
    ylabel("Angular Velocity (rad/s)"); xlabel("Time (s)")
    title("Measured vs Experimental Angular Velocities")
hold off
```



Subplots of the Neato's linear speeds, and angular velocities. The linear speeds are the exact same, but the angular velocities are extremely different, but following the same general curve.

```
figure;
plot(t_list(1:100), double(subs(rr_vel_left, t, t_vals)), Color=[0.8, 0, 0]); hold
on
    plot(t_list(1:100), double(subs(rr_vel_right, t, t_vals)), Color=[0, 0.7, 1]);
    plot(t_list(1:171), left_vel_approx, Color=[0.8, 0.2, 0.8], LineStyle="--");
    plot(t_list(1:171), right_vel_approx, Color=[0, 0.5, 0], LineStyle="--");
    legend("Experimental Data - Left", "Experimental Data - Right", "Measured Data
- Left", "Measured Data - Right")
    xlabel("Time (s)"); ylabel("Left and Right Wheel Velocities (m/s)")
```

```
    title("Measured vs Experimental Left and Right Wheel Velocities")
hold off
```



All of the Neato's experimental and measured left and right wheel velocities. The right wheel in the measured data peaks, but otherwise the measured and experimental data stay within the same range (0-0.3).

EXERCISE 6.5

Rainbow Road Video

Github

u Function

As u is a function of t, we determined it to be a scalar and decided to base it on the total time we wanted the neato to cross the rainbow road. Our function scales the time, which we chose as 30 seconds, between the u boundaries of 0 and 3.2.

$$u = \frac{t}{\frac{30}{3.2}}$$

Left and Right Wheel Velocity Commands

The given rainbow road equation was first split into its x and y components. For each point, we calculated a velocity and speed, and used them to find a tangent vector at each of those points. The x and y components of these tangent vectors were then used to find the angular velocity, using the equation $\dot{\theta} = T_x \cdot \frac{\mathrm{dT}_y}{\mathrm{dt}} - T_y \cdot \frac{\mathrm{dT}_x}{\mathrm{dt}}$. Finally, the velocities of each wheel were calculated using the speed, distance between the Neato's wheels, and angular velocity, with the left wheel using the equation $v_L = \text{ speed} - \frac{\text{distance}}{2}\theta$, and the right wheel using the equation $v_R = \text{ speed} + \frac{\text{distance}}{2}\theta$. Velocity commands were calculated using a spread of values between the time values (0 to 30).

Encoder Data

To reconstruct the path of our Neato, we first estimated the velocities of the wheels using the left and right encoder values. These were used in tandem to calculate the estimated forward velocity of the Neato using $\frac{v_r + v_l}{2}$. Finally, we split the velocity into its x and y components using the orientation of the Neato with $x = \cos(\theta) \cdot v$ and $y = \sin(\theta) \cdot v$, then multiplied the values by their time elapsed to get the x and y positions at each time.