

Aufgabe 1

$$I = \int_1^2 \ln(x^2) dx \Rightarrow f(x) = \ln(x^2)$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

$$\Rightarrow f''(x) = -\frac{2}{x^2} \Rightarrow |\max| = f''(1) = 2$$

$$\Rightarrow f'''(x) = \frac{4}{x^3}$$

$$\Rightarrow f^{(4)}(x) = -\frac{12}{x^4} \Rightarrow |\max| = f^{(4)}(1) = 12$$

Sum. Rechteckregel:

$$\left| \int_a^b f(x) dx - Rf(h) \right| \leq \frac{h^2}{24} (b-a) \cdot \max_{x \in [a,b]} |f''(x)|$$

$$10^{-5} \leq \frac{h^2}{24} \cdot 1 \cdot 2 = \frac{h^2}{12} \Rightarrow h^2 = 12 \cdot 10^{-5}$$

$$\Rightarrow h = \underline{\underline{0.010954}}$$

Trapez: $h = \frac{b-a}{n} \Rightarrow h = \frac{1}{n} \Rightarrow n = \frac{1}{h} \Rightarrow n = \underline{\underline{92}}$

$$10^{-5} \leq \frac{h^2}{12} \cdot 1 \cdot 2 = \frac{h^2}{6} \Rightarrow h^2 = 6 \cdot 10^{-5}$$

$$\Rightarrow h = \underline{\underline{0.007746}}$$

$$\Rightarrow n = \underline{\underline{130}}$$

Simpson:

$$10^{-5} \leq \frac{h^4}{2880} \cdot 1 \cdot 12 \Rightarrow h^4 = 0.221536$$

$$\Rightarrow h = \underline{\underline{\frac{1}{2}}}$$

Aufgabe 2

$$t = \int_{v_{t0}}^{v_t} \frac{m}{-v \cdot \sqrt{v}} dv \quad , m = 10 \text{ kg} , v_{t0} = 20 \frac{\text{m}}{\text{s}} , v_t = 5 \frac{\text{m}}{\text{s}}$$

$$= \int_{20}^5 \frac{10}{-v \cdot \sqrt{v}} dv \quad \Rightarrow f(t) = \frac{10}{-\sqrt{v}}$$

$$h = \frac{5-20}{5} = \frac{5-20}{5} = -3$$

a) Rechteck

$$Rf(h) = h \cdot \sum_{i=0}^{n-1} f(a + i \cdot h + \frac{h}{2}) = -3 \cdot \sum_{i=0}^{n-1} f(20 - 3i - \frac{3}{2})$$

| i | f |
|---|-----------|
| 0 | -0.125673 |
| 1 | -0.163871 |
| 2 | -0.226274 |
| 3 | -0.341519 |
| 4 | -0.603434 |

$$\Rightarrow -3 \cdot \sum = \underline{\underline{4.38231}}$$

$$\text{Fehler} = |4.447214 - 4.38...| = \underline{\underline{0.0878}}$$

c) Trapez

$$Tf(h) = h \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^{n-1} f(20 - 3i) \right)$$

| i | f |
|---|-----------|
| 0 | -0.111803 |
| 1 | -0.142668 |
| 2 | -0.190301 |
| 3 | -0.274101 |
| 4 | -0.441942 |

$$\Rightarrow -3 \cdot (-0.585115 + \sum) = \underline{\underline{4.97357}}$$

$$\text{Fehler} = \underline{\underline{0.521455}}$$

c) Simpson

$$Sf(h) = \frac{h}{3} \left(\frac{1}{2} f(a) + \overbrace{\sum_{k=1}^{n-1} f(a+k \cdot h)}^{1. \text{ Term}} + 2 \overbrace{\sum_{k=1}^n f\left(\frac{a+(k-1)h}{2} + \frac{f(a+k)h}{2}\right)}^{2. \text{ Term}} + \frac{1}{2} f(b) \right)$$

$$\frac{1}{2} f(-) = -0.051702$$

$$\frac{1}{2} f(1) = -0.447214$$

| k | 1. Term | 2. Term |
|-----|---------|---------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

\Rightarrow Side Aufgaben 3.