Ü 10

Aufgabe 1

$$Tf(h) = h \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right), \quad h = \frac{b-a}{2^i}$$

a) ~~Also~~: $\int_0^{\pi} \cos(x^2)\,dx$

T_{00}: $f(a) = f(0) = \cos(0^2) = 1$

$f(b) = f(\pi) = \cos(\pi^2) = -0.902685$

T_{10}: $f\left(\frac{a+b}{2}\right) = f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi^2}{4}\right) = -0.781212$

T_{20}: $f\left(a+\frac{b-a}{4}\right) = f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi^2}{16}\right) = 0.815705$

$f\left(a+3\frac{b-a}{4}\right) = f\left(\frac{3\pi}{4}\right) = \cos\left(\frac{9\pi^2}{16}\right) = 0.744151$

T_{30}: $f\left(a+\frac{b-a}{8}\right) = f\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi^2}{64}\right) = 0.988133$

$f\left(a+\frac{b-a}{4}+\frac{b-a}{8}\right) = f\left(\frac{3\pi}{8}\right) = \cos\left(\frac{9\pi^2}{64}\right) = 0.181865$

$f\left(\frac{a+b}{2}+\frac{b-a}{8}\right) = f\left(\frac{5\pi}{8}\right) = \cos\left(\frac{25\pi^2}{64}\right) = -0.755931$

$f\left(a+\frac{3(b-a)}{4}+\frac{b-a}{8}\right) = f\left(\frac{7\pi}{8}\right) = \cos\left(\frac{49\pi^2}{64}\right) = 0.273194$

T_{40}: $f\left(a+\frac{b-a}{16}\right) = f\left(\frac{\pi}{16}\right) = \cos\left(\frac{\pi^2}{256}\right) = 0.999257$

$f(\ldots) = f\left(\frac{3\pi}{16}\right) = \cos\left(\frac{9\pi^2}{256}\right) = 0.940405$

$= f\left(\frac{5\pi}{16}\right) = \cos\left(\frac{25\pi^2}{256}\right) = 0.570879$

$= f\left(\frac{7\pi}{16}\right) = \cos\left(\frac{49\pi^2}{256}\right) = -0.31296$

$= f\left(\frac{9\pi}{16}\right) = \cos\left(\frac{81\pi^2}{256}\right) = -0.999824$

$= f\left(\frac{11\pi}{16}\right) = \cos\left(\frac{121\pi^2}{256}\right) = -0.047441$

$= f\left(\frac{13\pi}{16}\right) = \cos\left(\frac{169\pi^2}{256}\right) = 0.97314$

$= f\left(\frac{15\pi}{16}\right) = \cos\left(\frac{225\pi^2}{256}\right) = -0.73147$

$$\Rightarrow T_{00} = \pi \cdot \left(\frac{f(a)+f(b)}{2} \right) = 0.152862$$

$$T_{10} = \frac{\pi}{2} \cdot \left(\frac{f(a)+f(b)}{2} + f\left(\frac{a+b}{2}\right) \right) = -1.15063$$

$$T_{20} = \frac{\pi}{4} \cdot \left(\frac{f(a)+f(b)}{\cancel{4}\, 2} + f\left(\frac{a+b}{2}\right) + f\left(a + \frac{b-a}{4}\right) + f\left(a + 3\,\frac{b-a}{4}\right) \right)$$

$$= 0.649761$$

$$T_{30} = \frac{\pi}{8} \cdot \left(\frac{f(a)+f(b)}{8} + f\left(\frac{a+b}{2}\right) + f\left(a + \frac{b-a}{4}\right) + f\left(a + 3\,\frac{b-a}{4}\right) \right.$$

$$\left. + f\left(a + \frac{b-a}{8}\right) + f\left(a + \frac{b-a}{4} + \frac{b-a}{8}\right) + f\left(\frac{a+b}{2} + \ldots\right) + f(\ldots) \right)$$

$$= 0.60265$$

$$T_{40} = \frac{\pi}{16} \cdot \big(0.152862 + (-0.781212) + 0.815705$$

$$+ 0.744151 + 0.988153 + 0.181865 + (-0.755931)$$

$$+ 0.293194 + 0.999257 + 0.940405 + 0.570379$$

$$+ (-0.31296) + (-0.99582) + (-0.047441)$$

$$+ 0.97314 + (-0.73147) \big)$$

$$= 0.574525$$

$$\Rightarrow T_{01} = \frac{4T_{10} - T_{00}}{3} = \cancel{41.75807}\ -1.58521$$

$$T_{11} = \frac{4T_{20} - T_{10}}{3} = 1.24951$$

$$T_{21} = \frac{4T_{30} - T_{20}}{3} = 0.586946$$

$$T_{31} = \frac{4T_{40} - T_{30}}{3} = 0.56515$$

$$\Rightarrow T_{02} = \frac{16T_{11} - T_{01}}{15} = 1.43832$$

$$T_{12} = \frac{16T_{21} - T_{11}}{15} = 0.542748$$

$$T_{22} = \frac{16T_{31} - T_{21}}{15} = 0.563697$$

$$\Rightarrow T_{03} = \frac{64 T_{12} - T_{02}}{63} = 0.528523$$

$$T_{13} = \frac{64 T_{22} - T_{12}}{63} = \cancel{0.6}\ 0.56403$$

$$\Rightarrow T_{04} = \frac{256 T_{13} - T_{03}}{255} = \underline{0.564167}$$

b) $\int_{10}^{170} x \cdot \ln\left(\frac{1}{x}\right) dx$

T_{00} : $f(a) = f(10) = -23.0259$

$f(b) = f(170) = -873.086$

T_{10}: $f\left(\frac{a+b}{2}\right) = f(90) = -404.983$

T_{20}: $f\left(a + \frac{b-a}{4}\right) = f(50) = -195.601$

$f\left(a + 3\frac{b-a}{4}\right) = f(130) = -632.779$

T_{30} : $f(30) = -102.036$

$f(70) = -297.395$

$f(110) = -517.053$

$f(150) = -751.595$

T_{40} : $f(20) = -59.9146$

$f(40) = -147.555$

$f(60) = -245.661$

$f(80) = -350.562$

$f(100) = -460.517$

$f(120) = -574.493$

$f(140) = -691.83$

$f(160) = -812.028$

$$\Rightarrow T_{00} = 160 \cdot \left(\frac{f(10)+f(170)}{2} \right) = -71688.9$$

$$T_{10} = \frac{160}{2} \cdot \left(\frac{f(10)+f(170)}{2} + f(90) \right) = -68243.1$$

$$T_{20} = \frac{160}{4} \cdot \left(\frac{f(10)+f(170)}{2} + f(90) + f(50) + f(130) \right)$$

$$= -67256.8$$

$$T_{30} = \frac{160}{8} \cdot \left(\frac{f(10)+f(170)}{2} + f(90) + f(50) + f(130) \right.$$

$$\left. + f(30) + f(70) + f(110) + f(150) \right) = -66990.0$$

$$T_{40} = \frac{160}{16} \cdot \left(\frac{f(10)+f(170)}{2} + f(90) + f(50) + f(130) \right.$$

$$+ f(30) + f(70) + f(110) + f(150) + f(20)$$

$$+ f(40) + f(60) + f(80) + f(100) + f(120)$$

$$\left. + f(140) + f(160) \right) = -66920.6$$

$$T_{01} = \frac{4T_{10} - T_{00}}{3} = -67034.5$$

$$T_{11} = \frac{4T_{20} - T_{10}}{3} = -66928.0$$

$$T_{21} = \frac{4T_{30} - T_{20}}{3} = -66901.1$$

$$T_{31} = \frac{4T_{40} - T_{30}}{3} = -66897.5$$

$$\Rightarrow T_{02} = \frac{16T_{11} - T_{01}}{15} = -66916.9$$

$$T_{12} = \frac{16T_{21} - T_{11}}{15} = -66899.3$$

$$T_{22} = \frac{16T_{31} - T_{21}}{15} = -66897.3$$

$$\Rightarrow T_{03} = \frac{64T_{12} - T_{02}}{63} = -66899.0$$

$$T_{13} = \frac{64T_{22} - T_{12}}{63} = -66897.3$$

$$\Rightarrow T_{04} = \frac{256 \cdot T_{13} - T_{03}}{255} = \underline{\underline{-66897.3}}$$