Ü7 Florin Lüthi [illegible]

Aufgabe 1

a) $f(x) = e^x \cong \sum_{i=0}^{n} \left(\frac{f^{(i)}(x_0)}{i!} (x - x_0)^i + R_{n+1}(x) \right) \quad , x_0 = 0$

$\Rightarrow f'(x) = f''(x) = \ldots = f^{(n)}(x) = e^x \quad , f^{(i)}(0) = 1$

$$\Rightarrow f(x) \cong \underbrace{\frac{1}{1} \cdot x^0}_{i=0} + \underbrace{\frac{1}{1} \cdot x^1}_{i=1} + \underbrace{\frac{1}{2} x^2}_{i=2} + \underbrace{\frac{1}{6} x^3}_{i=3} + \underbrace{\frac{1}{24} x^4}_{i=4}$$

$$+ \underbrace{\frac{1}{120} x^5}_{i=5}$$

$$\Rightarrow \check{f}(x) = 1^0 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$$

b) $f(1) = e \quad , \check{f}(1) = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = \frac{163}{60}$

$\Rightarrow |f(1) - \check{f}(1)| = |e - \frac{163}{60}| = 0.0016$

c) $e = \sum_{i=0}^{\infty} \frac{1}{i!}$