Aufgabe 2 $\quad h = 15$

a) $D_1 f = \dfrac{f(x_0+h) - f(x_0)}{h} = \dfrac{e^{(1+h)^2} - e}{h}$

$h = \sqrt{2 \cdot 10^{-14} \dfrac{\overbrace{e}^{f(1)}}{\underbrace{4e}_{f''(1)}}} = \sqrt{\frac{1}{2} \cdot 10^{-14}} = \frac{1}{4} \cdot 10^{-7}$

$\Rightarrow D_1 f(1) = \dfrac{e^{(1+\sqrt{\frac{1}{2}\cdot 10^{-14}})^2} - e}{\sqrt{\frac{1}{2}\cdot 10^{-14}}} \Rightarrow D_1 f(1) = \underline{5.4366}$

$|D_1 f(x_0, h) - f'(x_0)| \leq \dfrac{|f''(x_0)|}{2} \cdot h$

$= \dfrac{4 \cdot e}{2} \cdot \sqrt{\frac{1}{2} \cdot 10^{-14}} = \underline{3.8442 \cdot 10^{-7}}$

b) $D_1 f = \dfrac{\ln((2+h)^2) - \ln(4)}{h}$

$h = \sqrt{2 \cdot 10^{-14} \dfrac{\overbrace{\ln 4}^{f(2)}}{\underbrace{\frac{2}{4} 4 \cdot 2^{-3}}_{|f''(2)|}}} = \sqrt{4 \cdot 10^{-14} \cdot \ln 4} = 2 \cdot 10^{-7} \cdot \sqrt{\ln 4}$

$|f''(2)| = \frac{2}{4} 4 \cdot x^{-3}$

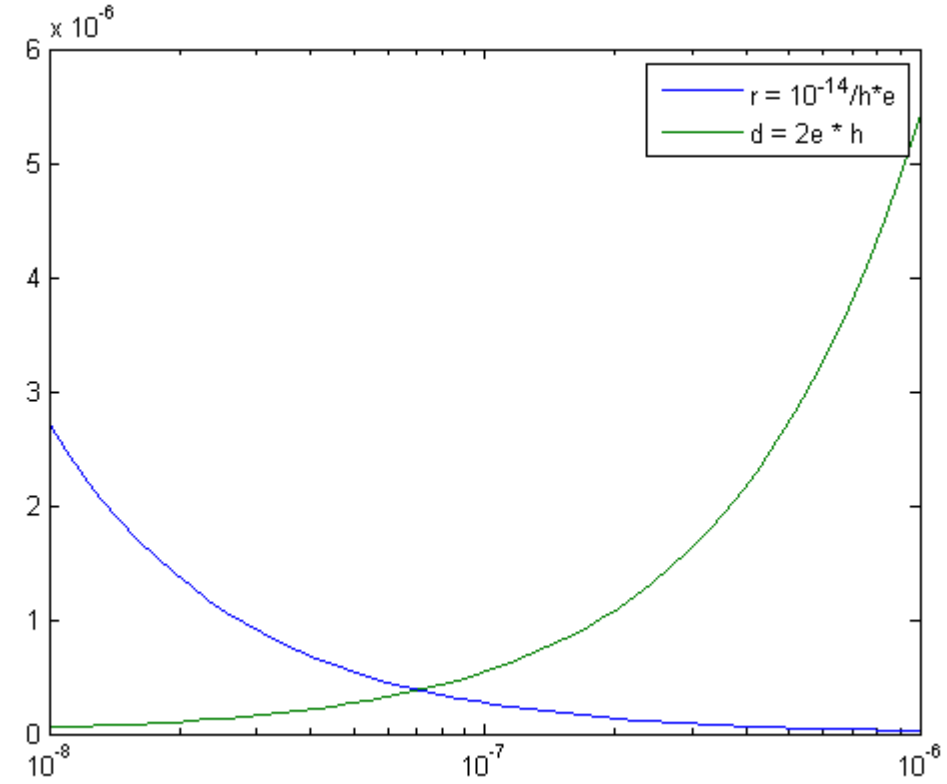
$\Rightarrow D_1 f(2) = \dfrac{\ln((2+\sqrt{4 \cdot 10^{-14} \ln 4})^2) - \ln 4}{\sqrt{4 \cdot 10^{-14} \cdot \ln 4}} = \underline{1.0000}$

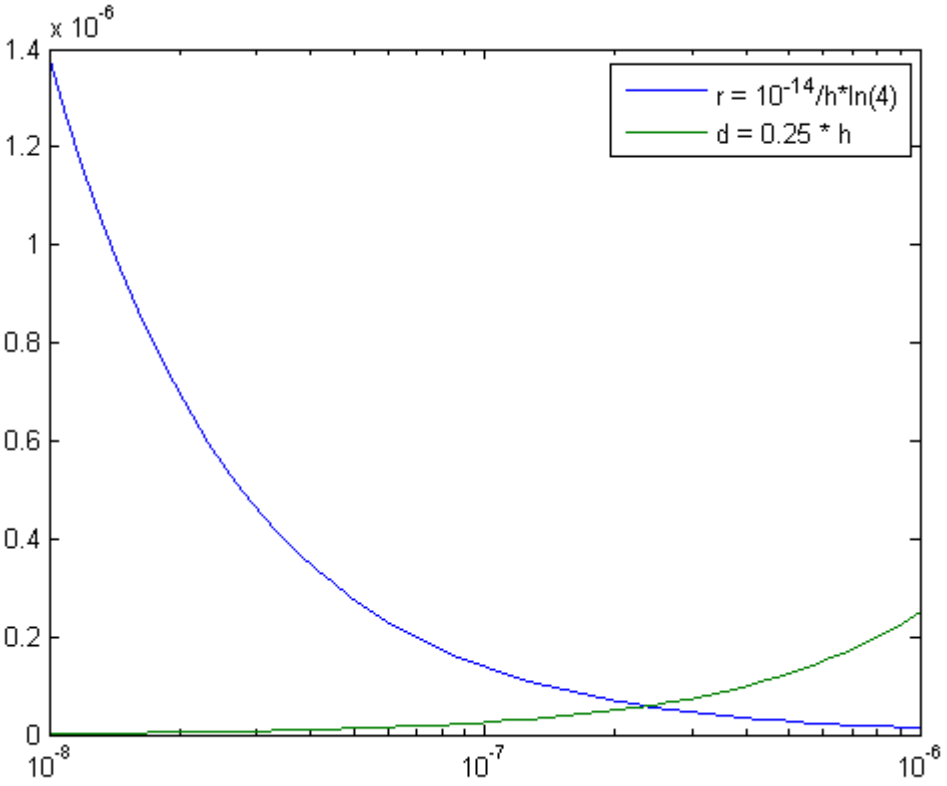
$|D_1 f(x_0, h) - f'(x_0)| \leq \dfrac{|f''(x_0)|}{2} \cdot h = \frac{1}{4} \cdot h = \underline{5.8871 \cdot 10^{-8}}$

Tools: $r = \dfrac{2 \cdot eps}{h} \cdot |f(x_0)|$, $d = \dfrac{|f''(x_0)|}{2} \cdot h$

a) $r = \dfrac{2 \cdot eps}{h} e = \dfrac{10^{-14}}{h} e$, $d = 2e \cdot h$

b) $r = \dfrac{2 \cdot eps}{h} \cdot \ln 4$, $d = \frac{1}{4} h$


x 10^-6
6
5
4
3
2
1
0
10^-8
10^-7
10^-6
r = 10^-14/h*e
d = 2e * h


x 10^-6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10^-8
10^-7
10^-6
r = 10^-14/h*ln(4)
d = 0.25 * h