$\sigma_0(n) \leftarrow n$ এর কয়টি division আছে?

Odd divison count

⇓

কোন কোন সংখ্যার odd সংখ্যক divison আছে?

$25 = 1\times25$ | $49 = 1\times49$ | $10 = 1\times10$
$\boxed{5\times5}$ | 7×7 | $= 2\times5$
$a=b$ | $a=b$ |

যেসব $(a=b)$ তার odd সংখ্যক divison আছে।

অর্থাৎ, পূর্ণ বর্গ সংখ্যার divison odd সংখ্যক

composit number ⇒ NOT prime.

⇓

at least 3টি divison থাকতে হবে (1, n ছাড়া অন্যান্য divison থাকবে)

* $b \mid a \leftarrow$ b divides a.

* if $p \mid a*b$ ($p \leftarrow$ Primes) then $p \mid a$ and/or, $p \mid b$

Why?

$3 \mid 15\times10 = \frac{15\times10}{3} = \frac{3\times5\times10}{3} = 50$

$\frac{4\times3}{6} = \frac{2\times2\times3}{2\times3}$

SPF সবচেয়ে ছোট prime, যেটি দিয়ে ঐ সংখ্যাকে ভাগ যায়।

A সবচেয়ে ছোট division (n) hoy, তা অবশ্যই prime হবে।

$\sqrt{n}$ ← 10^{15} ← এর জন্য $\log_2(10^{15}) = 50$

$N\sqrt{n} \rightarrow n + \frac{n}{2} + \frac{n}{4} + \dots$

$\sqrt{n}$ → $\log_2 n$

$n^2 + n + 41$ / $n \leq 39$ all prime

□ no. of div. using PF:-

$$n = p^a \times q^b \times r^c \times s^d \quad , p, q, r, s = \text{all prime}$$

a, b, c, d = count each prime.

$\therefore$ no. of divison $= (a+1) \times (b+1) \times (c+1) \times (d+1)$

$\sqrt{n}$

□ exactly 1 div = 1

" 2 div = all prime

" 3 div = ~~compost~~ when? ⇒ শুধুমাত্র prime square

সংখ্যাগুলো 3 divison হবে।

$p^1 = (1+1) = 2$

$$\boxed{p^2 = (2+1) = 3}$$

$p^1 \times q^1 = (1+1) \times (1+1) = 4$ ×

exactly 4 div? when?

$p^1 \times q^1 = (1+1) \times (1+1) = 4$

$p^3 = (3+1) = 4$

2^{18}

30

10^7

worst case $\rightarrow$ no of division $\rightarrow$ $1 \times 10^7 = 10^7$

2%

$0\%0$

$0\%0$

$0\%0$

worst case $\rightarrow$ $\boxed{2\sqrt{n}}$

But actully $\rightarrow$ $\boxed{\sqrt[3]{n}}$ হয়ে থাকে।

n er divisor

$n = 10^9$ হলে divisor $\rightarrow 10^6$

Sum of divison :-

$\sigma_1(n)$ $\Leftarrow$ sum of divison

$\sigma_2(n)$ $\Leftarrow$ sum of divison square

$\sigma_3(n)$ $\Leftarrow$ sum of divison cuble

let, $n = 2^3 \times 5^2 \times 7^2$

$SOD = (1+2+2^2+2^3) \times (1+5+5^2) \times (1+7+7^2)$

$n = P_1^{e_1} \times P_2^{e_2} \times \ldots \times P_n^{e_n}$

$$SOD = (1+P_1+P_1^2+\cdots+P_1^{e_1}) \times (1+P_2+P_2^2+\cdots+P_2^{e_2})$$

$$SOD = \left(\frac{(P_1^{e_1+1}-1)}{(P_1-1)}\right) \times \left(\frac{P_2^{e_2+1}-1}{P_2-1}\right) \cdots$$

$\frac{P_1^{e_1+1}-1}{P_1-1} \times$

$\frac{P_2^{e_2+1}-1}{P_2-1}$

$\frac{P_1^{e_1+1}-1}{P_1-1}$

$k=2$

$$SOD(2) = \frac{(2^k)^{(e+1)} - 1}{2^k - 1} \times \frac{(3^k)^{e+1}}{3^k - 1} \ldots$$

$$n = p_1^{e_1} \times p_2^{e_2} \times p_3^{e_3} \times \ldots \times p_n^{e_n}$$

Hence SOD (divisors kth power sum)

$$= \frac{(p_1^k)^{(e_1+1)} - 1}{p_1^k - 1} \times \frac{(p_2^k)^{(e_2+1)} - 1}{p_2^k - 1} \times \ldots \times \frac{(p_n^k)^{(e_n+1)}}{p_n^k - 1}$$

$(p_1)^k$

a^k

---

divison [1 to n] count :-

$i = 1$ to 10

1 → 1
2 → 1 2
3 → 1 3
4 → 1 2 4
5 → 1 5
6 → 1 2 3 6
7 → 1 7
8 → 1 2 4 8
9 → 1 3 9
10 → 1 2 5 10

$n \to 10^6$

$n \to 10^6$

query 16 n prime factorize করো।

⇓

prime factor using sieve

$n = 6$, $k = 3$. $6 \to 2, 3$

$2 \to 1$

$3 \to 1$ $k = 3$

$(1+2) \times (1+3) = 1, 2, 3, 6$

$(p_1^k)^{(e_1+1)}$

p_1 (e_1)

$$\frac{2^6 - 1}{2^3 - 1} \times \frac{3^6 - 1}{3^3 - 1}$$

9×28

$$\boxed{\frac{a}{b} \bmod m}$$

$a \to p_1^{k \times (e_1+1)} - 1$

$b \to (p_1^k) - 1$

Prime gap :- if n is prime, then what is the next prime ?

⇓

difference between two prime.

$n \rightarrow$ এর পরের prime maximum $(\log(n))^2$ এর কাছাকাছি operation এর মধ্যে পাব !

complexity : $\Theta(\sqrt{n} \times \log(n)^2)$

GCD :-

$$n_1 = P_1^{a_1} \times P_2^{a_2} \times \dots \times P_n^{a_n}$$

$$n_2 = P_1^{x_1} \times P_2^{x_2} \times \dots \times P_n^{x_n}$$

$$\therefore \gcd(n_1, n_2) = P_1^{\min(a_1, x_1)} \times P_2^{\min(a_2, x_2)} \times \dots \times P_n^{\min(a_n, x_n)}$$

$$\text{lcm} = P_1^{\max(a_1, x_1)} \times P_2^{\max(a_2, x_2)} \times \dots \times P_n^{\max(a_n, x_n)}$$

$$\gcd \times \text{lcm} = P_1^{a_1+b_1} \times P_2^{a_2+b_2} \times \dots \times P_n^{a_n+b_n}$$

$$\boxed{\gcd \times \text{lcm} = n_1 \times n_2}$$

* দুটি সংখ্যার common divison যদি $\gcd(a,b) = x$ হলে x এর divison।

** let, $g = \gcd(a,b)$:

$a-b$;
$a+b$, g দ্বারা divisible কী?

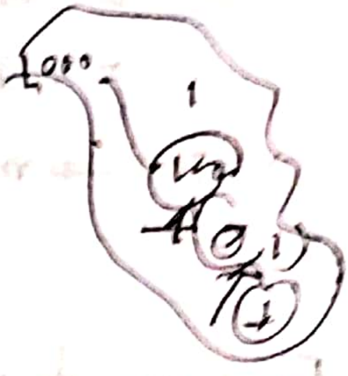
⇒ Yes, → $a+b$
$= xg + yg$
$= g(x+y)$

g → if $a>b$

$\gcd(a,b) = \gcd(a-b, b) = \gcd(a-2b, b)$
$= \gcd(a-3b, b)$

~~if a<b~~ , '0' আসার পর্যন্ত
নিচের কাজটি করবো।

NOT a good way



⇒ $\gcd(a,b) = \gcd(a \% b, b)$
if $a>b$

complexity :- $\log n$

coprimes ⇒ $\gcd(x,y) = 1$ হলে x,y = coprime।
→ common কোনো prime নেই।

NT hack ⇒ always think with primes numbers

$n!$ এর কয়টি divison আছে?

$n = 10^5$ 1 2 3 4 5 — —

⇒ prime factor বের করি।
⇒ then সমাধান ⇒ $(e_1+1) \times (e_2+1) \ldots$

□ no. trailing zero? → এটা করার জন্য prime factorize করবো।

মানে ~~10গুণ~~ 000

n^{100}

$n \Rightarrow \alpha \times 10 \times 10$

$= \alpha \times 2 \times 5 \times 2 \times 5$

$= \alpha \times 2^2 \times 5^2$

$\therefore$ zero $\to \min(2,5)$

$n = 2^a \times 5^b \times \cdots$

হবে $\boxed{\text{trailing zero} = \min(a,b)}$

□ every two consecutive numbers are coprime

□ EOF : end of file

```
while(scanf("%d", &x) != EOF){   // input শেষ
    // code
```

$O(n)$ G. LS

- Proper divisor = all natural division of n, that is strictly less than n

$t \to 10^5$

$n \to 10^5$

2, 2, 5, 5 — 20

100

1, 20
2, 10
4, 5

= 22

1, 5

10

- computes no. of different prime factors in a pos. int

int $n \to$

$n \to 10^6$

10^6

20

$2^2 \times 5$

output $\to$ 2, 5 = 2

$2 \to$ | 1 |

$3 \to$ | | 1 |

$4 \to$ | 1 | |

$6 \to$ | 1 | 1 |

$11 \to 2 \cdot 10^4$

$V_p(x) \leftarrow$ largest ~~power of~~ exponent of power of p that divides the number x. p is prime

$10890 = 2 \times 3^2 \times 5 \times 11^2$

$V_3(10890) = 2$.

$V_2(4) = 2$.

$V_p(n!) = \sum_{i=1}^{\infty} \left[\frac{n}{p^i}\right]$

$\Rightarrow 10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 2^? \times \ldots$

$V_2(10!) = 8$

floor

formula: $V_2(10!) = \left|\frac{10}{2^1}\right| + \left|\frac{10}{2^2}\right| + \left|\frac{10}{2^3}\right| + \underbrace{\left|\frac{10}{2^4}\right| + \left|\frac{10}{2^5}\right| + \ldots}_{0}$

$= (5 + 2 + 1) = 8$