Edge Detection

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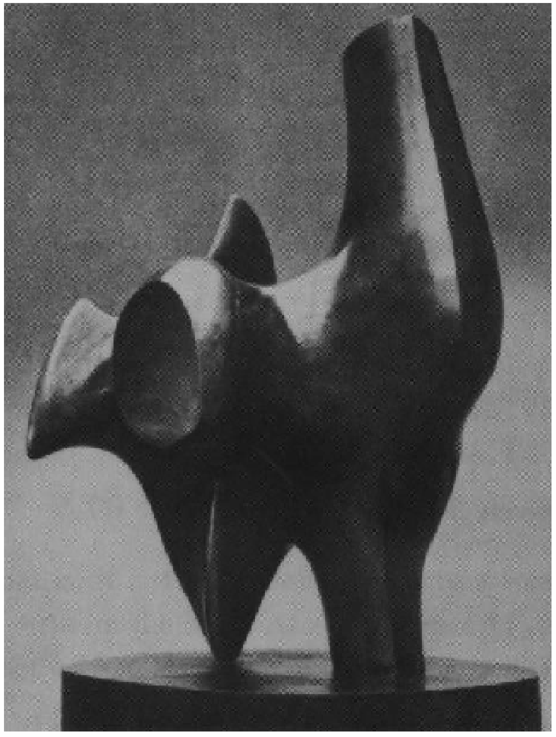
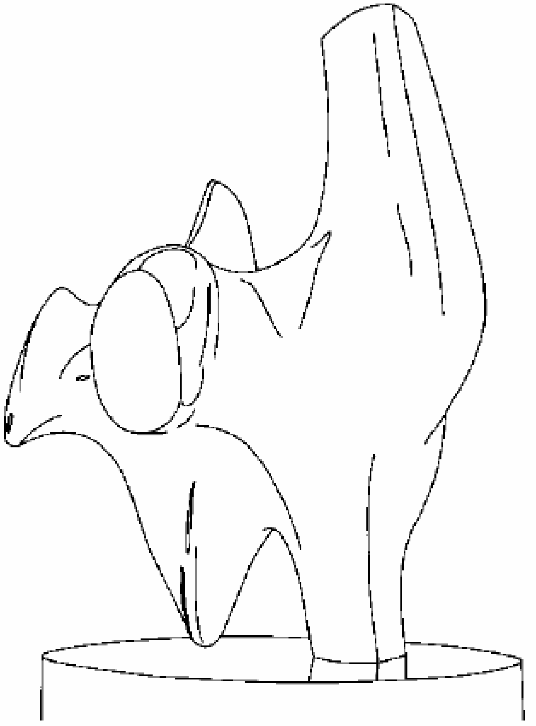
Edge detection

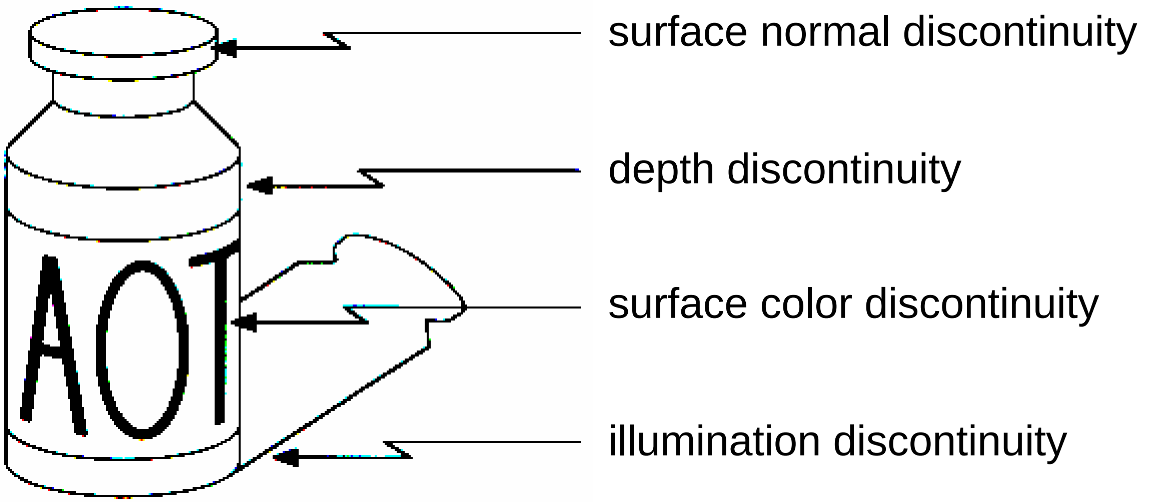




Convert a 2D image into a set of curves

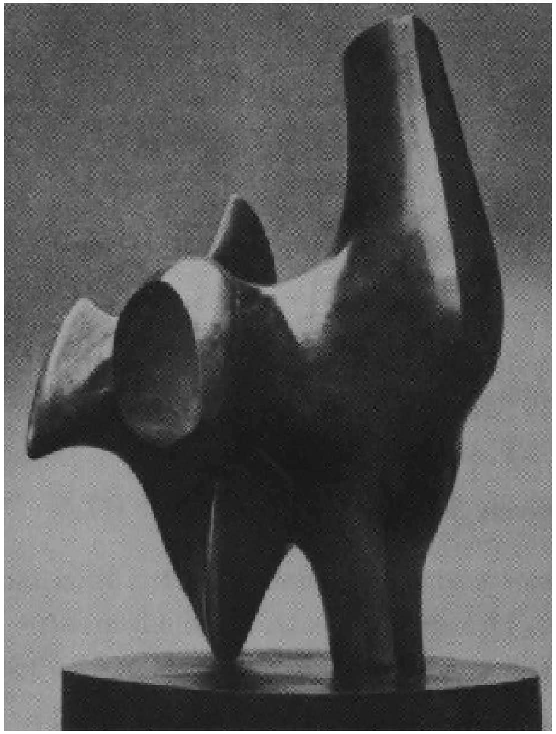
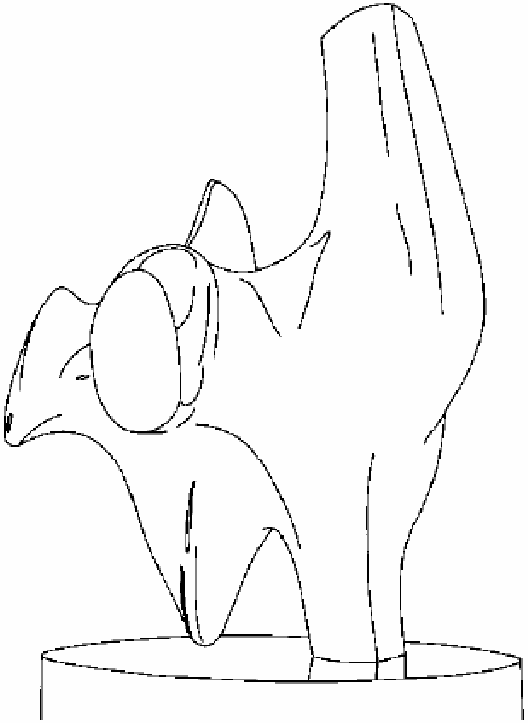
- Extracts salient features of the scene
- More compact than pixels

Origin of Edges



Edges are caused by a variety of factors

Edge detection

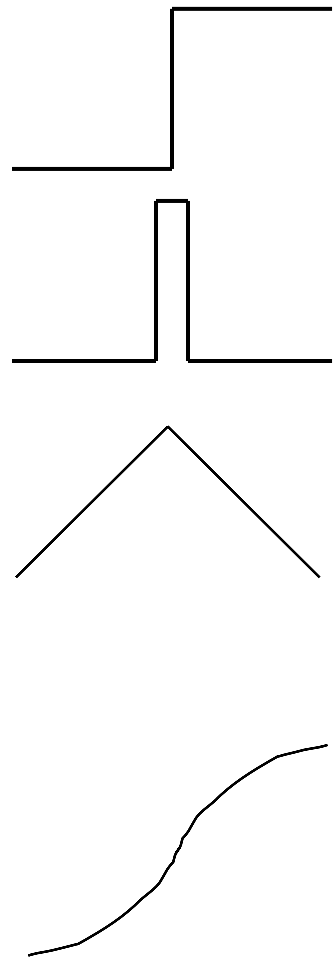
 

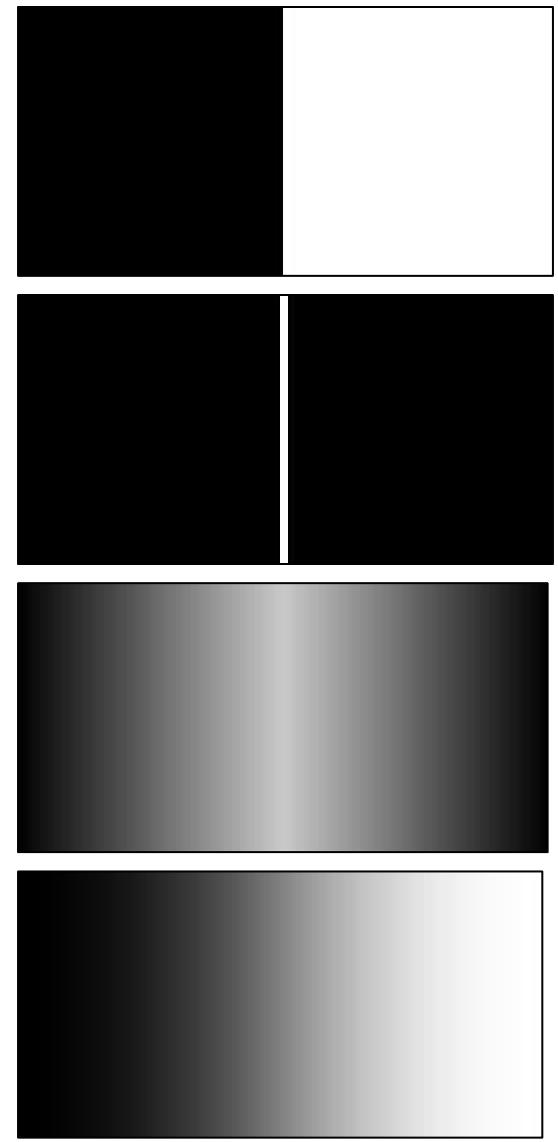
How can you tell that a pixel is on an edge?

Edges: types

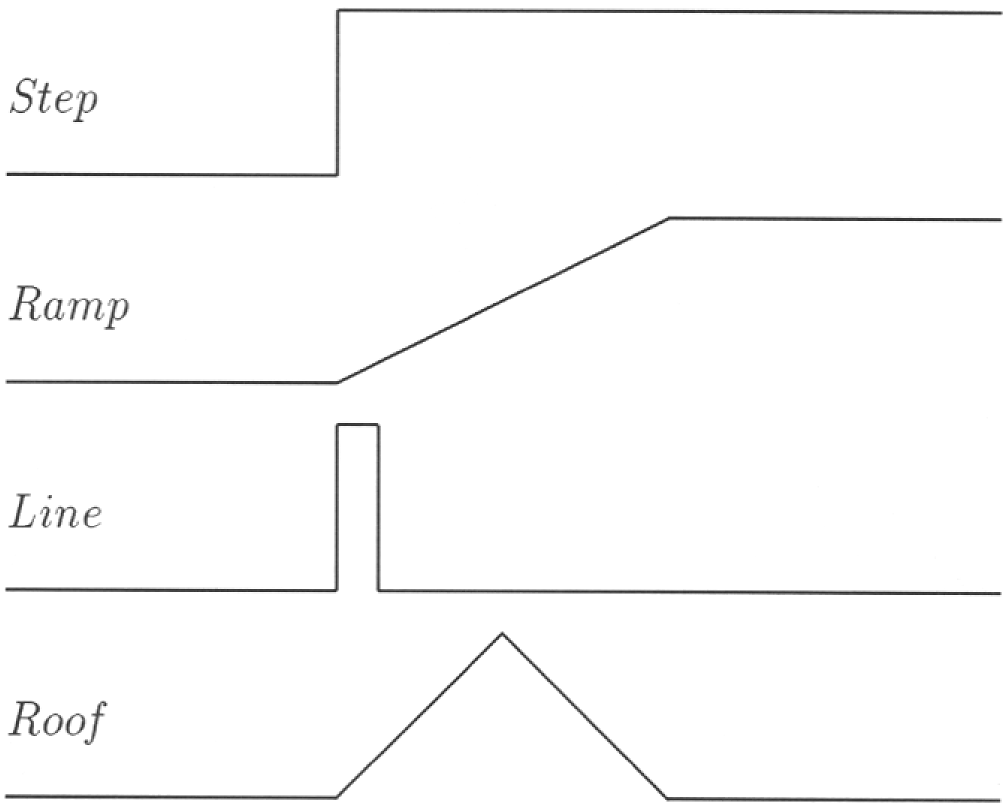
Discontinuities in intensity profile

- Step Edge
- Line
- Roof Edge
- "2nd Deriv. Edge"





Edge Profiles



Edge detection

1. Detection of short linear edge segments (edgels)

2. Aggregation of edgels into extended edges
   (maybe parametric description)

Edgel detection

- Difference operators
- Parametric-model matchers

Edge is Where Change Occurs

Change is measured by derivative in 1D

Biggest change, derivative has maximum magnitude

Or 2^{nd} derivative is zero.

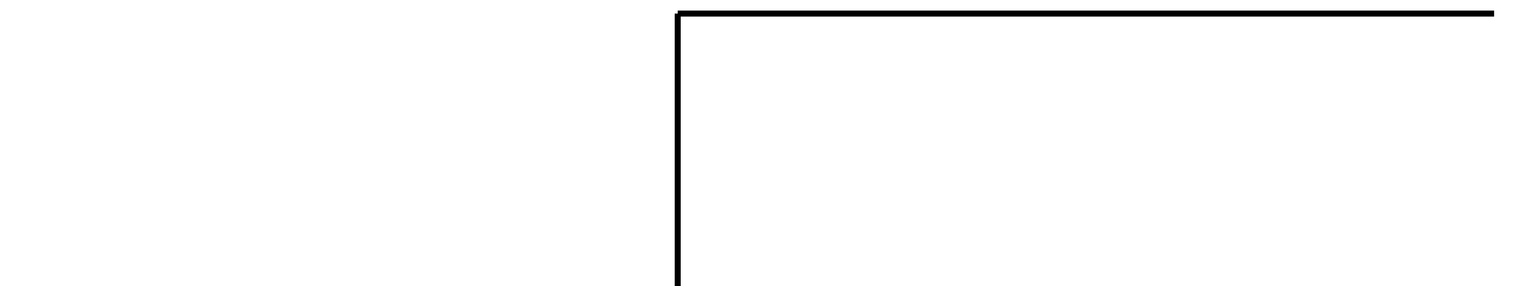
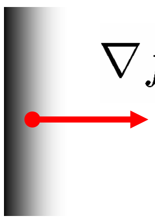


Image gradient

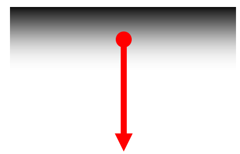
The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

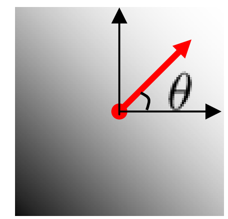
The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

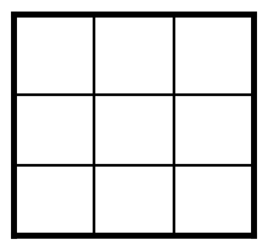
The discrete gradient

How can we differentiate a *digital* image f[x,y]?

- Option 1: reconstruct a continuous image, then take gradient
- Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx f[x+1, y] - f[x, y]$$

How would you implement this as a cross-correlation?



H

The Sobel operator

Better approximations of the derivatives exist

- The *Sobel* operators below are very commonly used

-1	0	1
-2	0	2
-1	0	1

s_x

1	2	1
0	0	0
-1	-2	-1

s_y

Gradient operators

(a)

$$\Delta_1 = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} \qquad \Delta_2 = \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$$

(b)

$$\Delta_1 = \begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix} \qquad \Delta_2 = \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{matrix}$$

(c)

$$\Delta_1 = \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix} \qquad \Delta_2 = \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

(d)

$$\Delta_1 = \begin{matrix} -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \end{matrix} \qquad \Delta_2 = \begin{matrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{matrix}$$

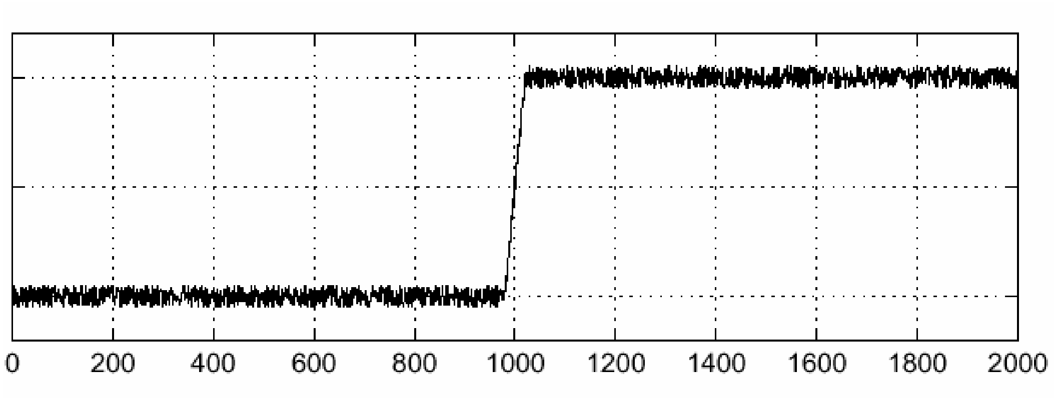
(a): Roberts' cross operator (b): 3x3 Prewitt operator
(c): Sobel operator (d) 4x4 Prewitt operator

Effects of noise

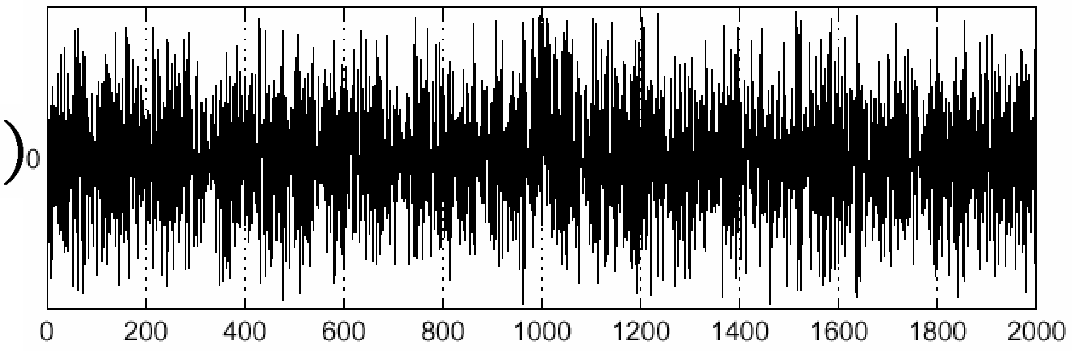
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

$f(x)$

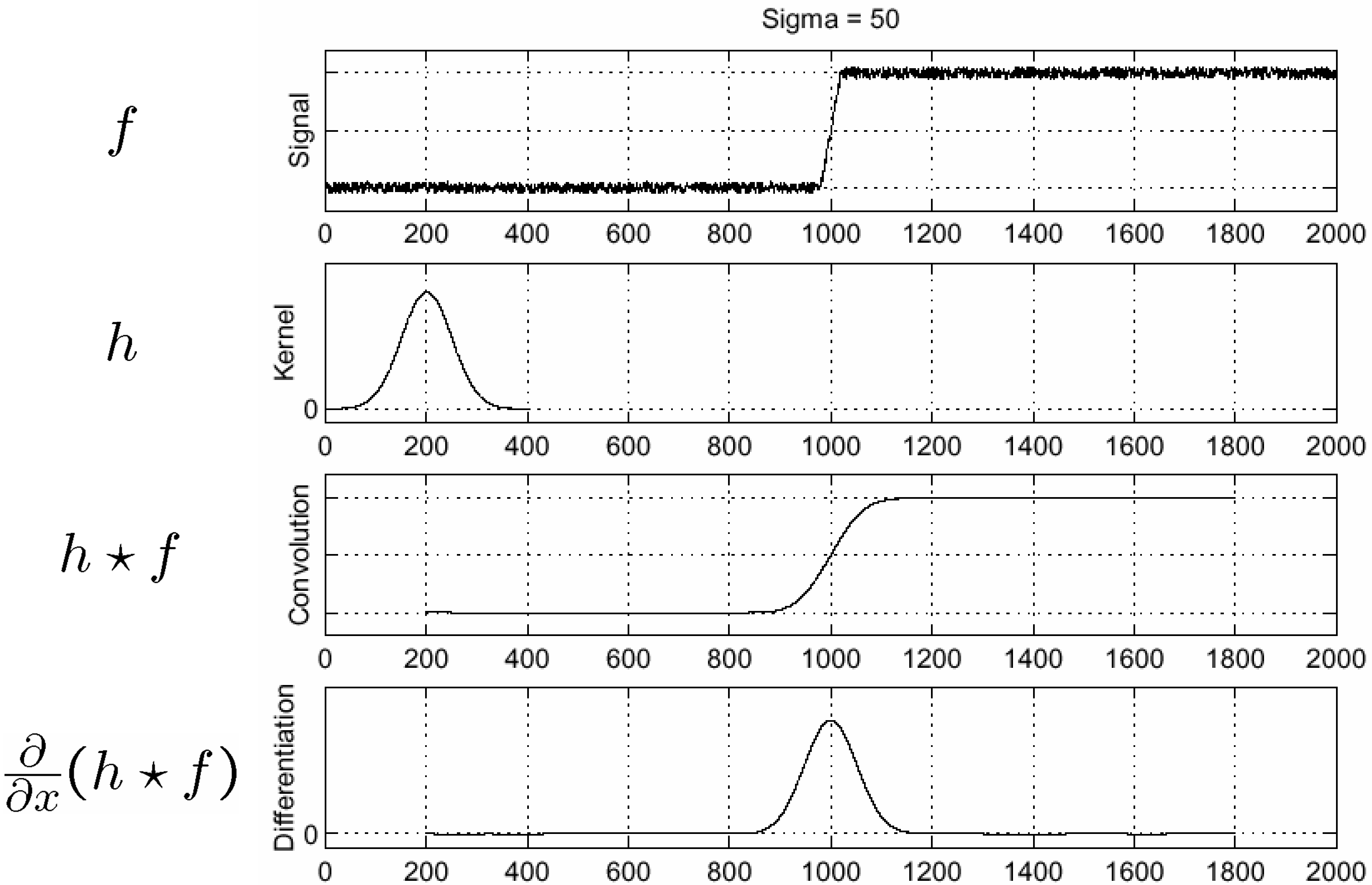


$\frac{d}{dx}f(x)$



Where is the edge?

Solution: smooth first



Where is the edge?   Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

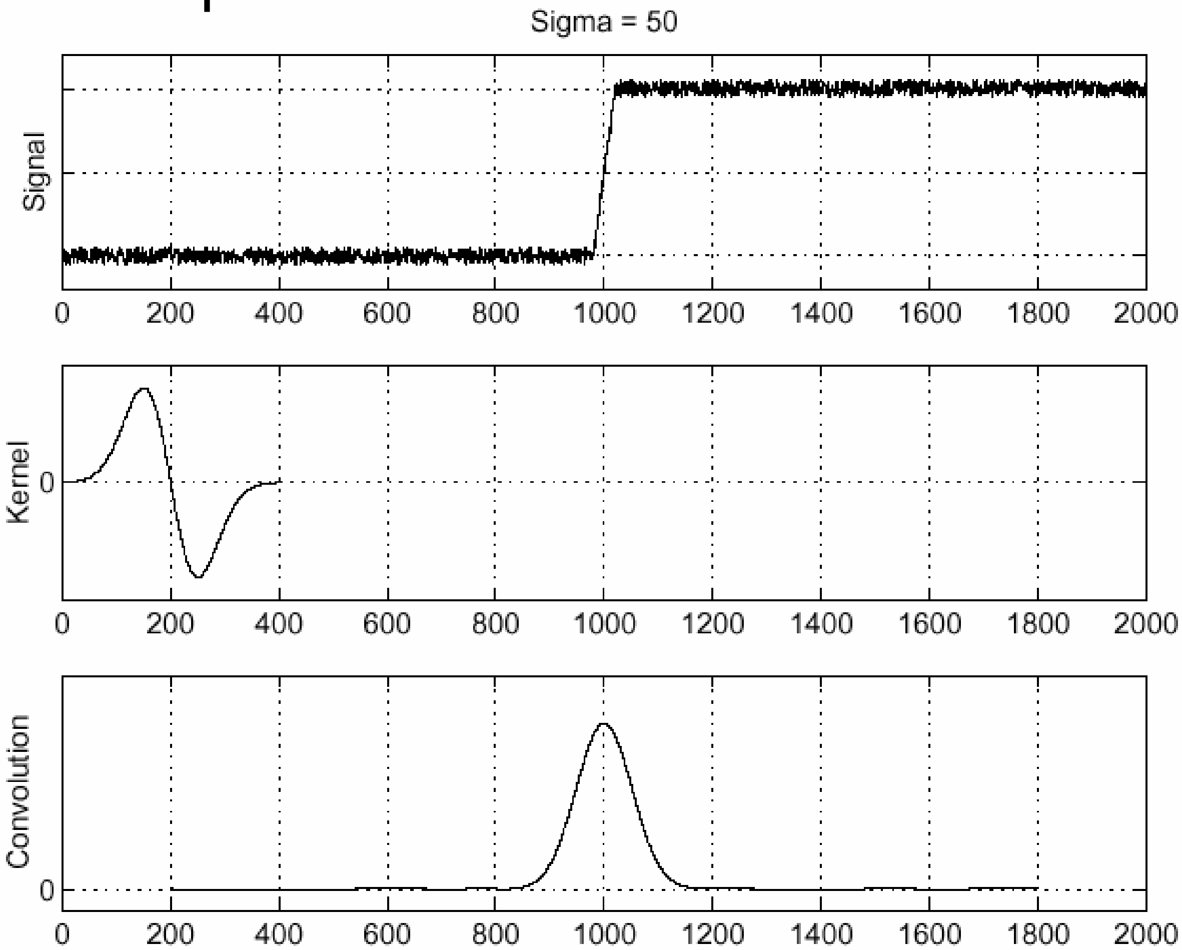
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:

f

$\frac{\partial}{\partial x}h$

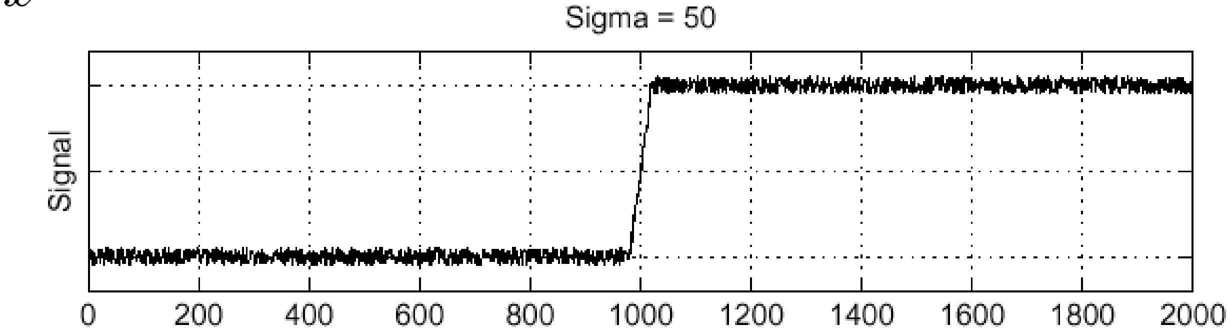
$(\frac{\partial}{\partial x}h) \star f$



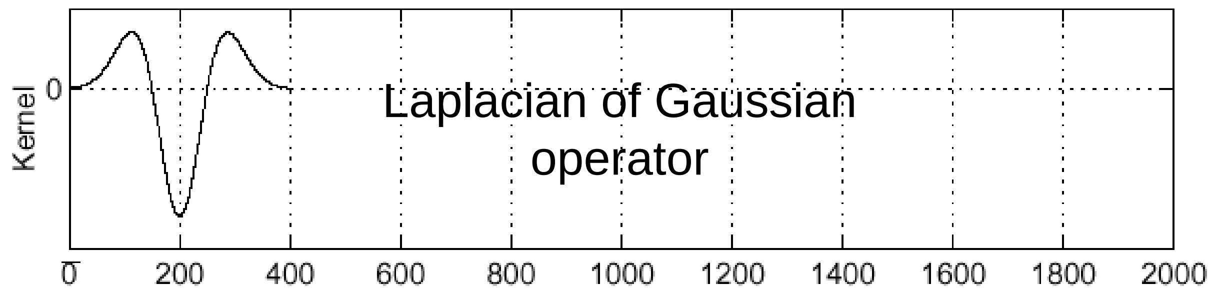
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

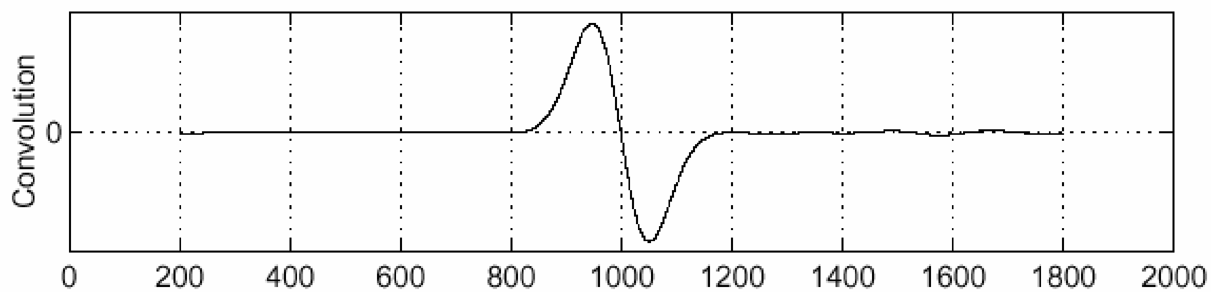
f



$\frac{\partial^2}{\partial x^2}h$

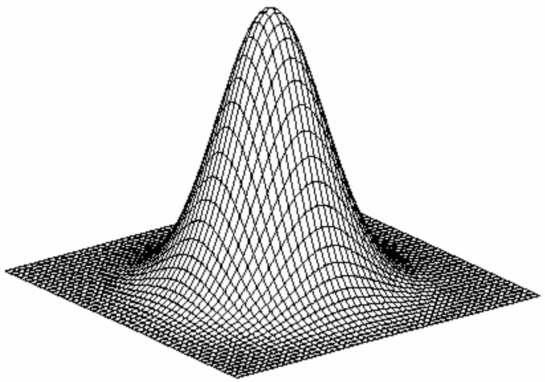


$(\frac{\partial^2}{\partial x^2}h) \star f$



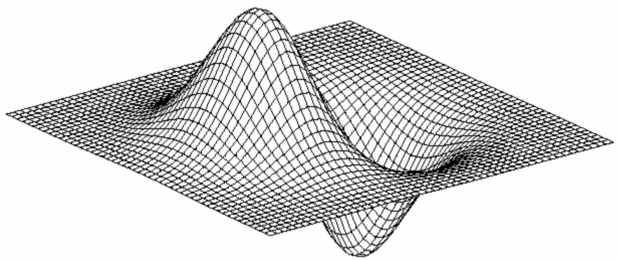
Where is the edge? Zero-crossings of bottom graph

2D edge detection filters



Gaussian

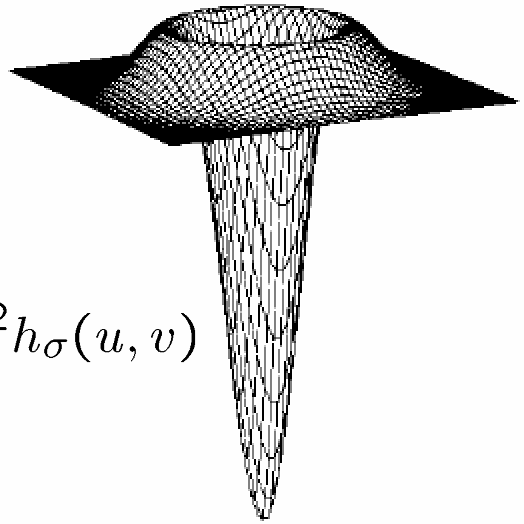
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Optimal Edge Detection: Canny

Assume:

- Linear filtering
- Additive Gaussian noise

Edge detector should have:

- Good Detection. Filter responds to edge, not noise.
- Good Localization: detected edge near true edge.
- Single Response: one per edge.

Optimal Edge Detection: Canny (continued)

Optimal Detector is approximately Derivative of Gaussian.

The Canny edge detector

original image (Lena)

The Canny edge detector

norm of the gradient

The Canny edge detector

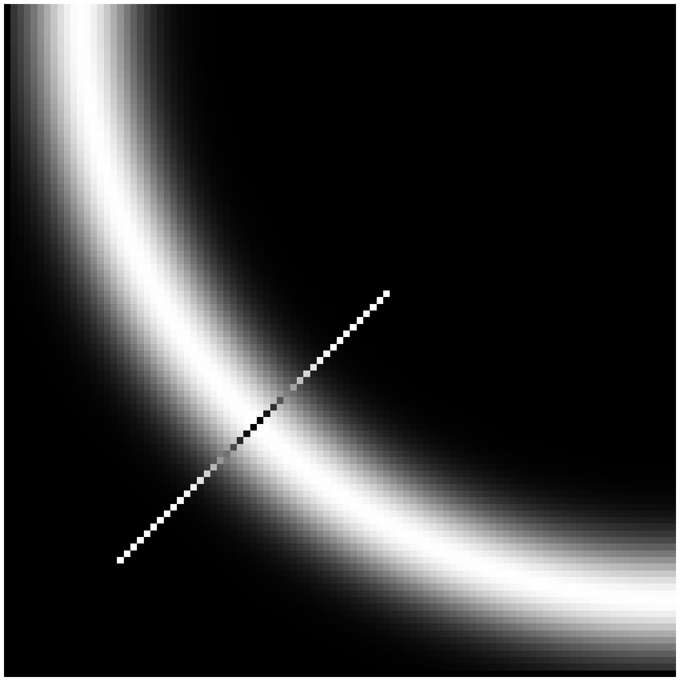
thresholding

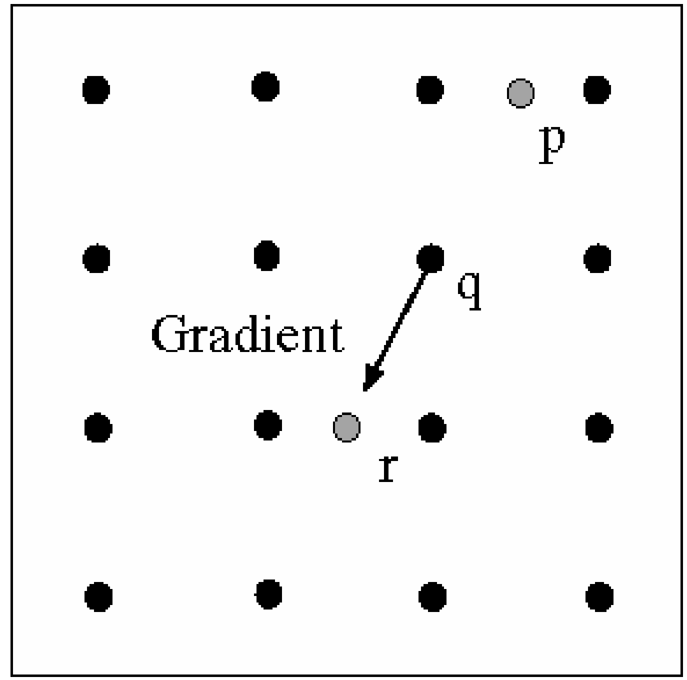
The Canny edge detector

thinning

(non-maximum suppression)

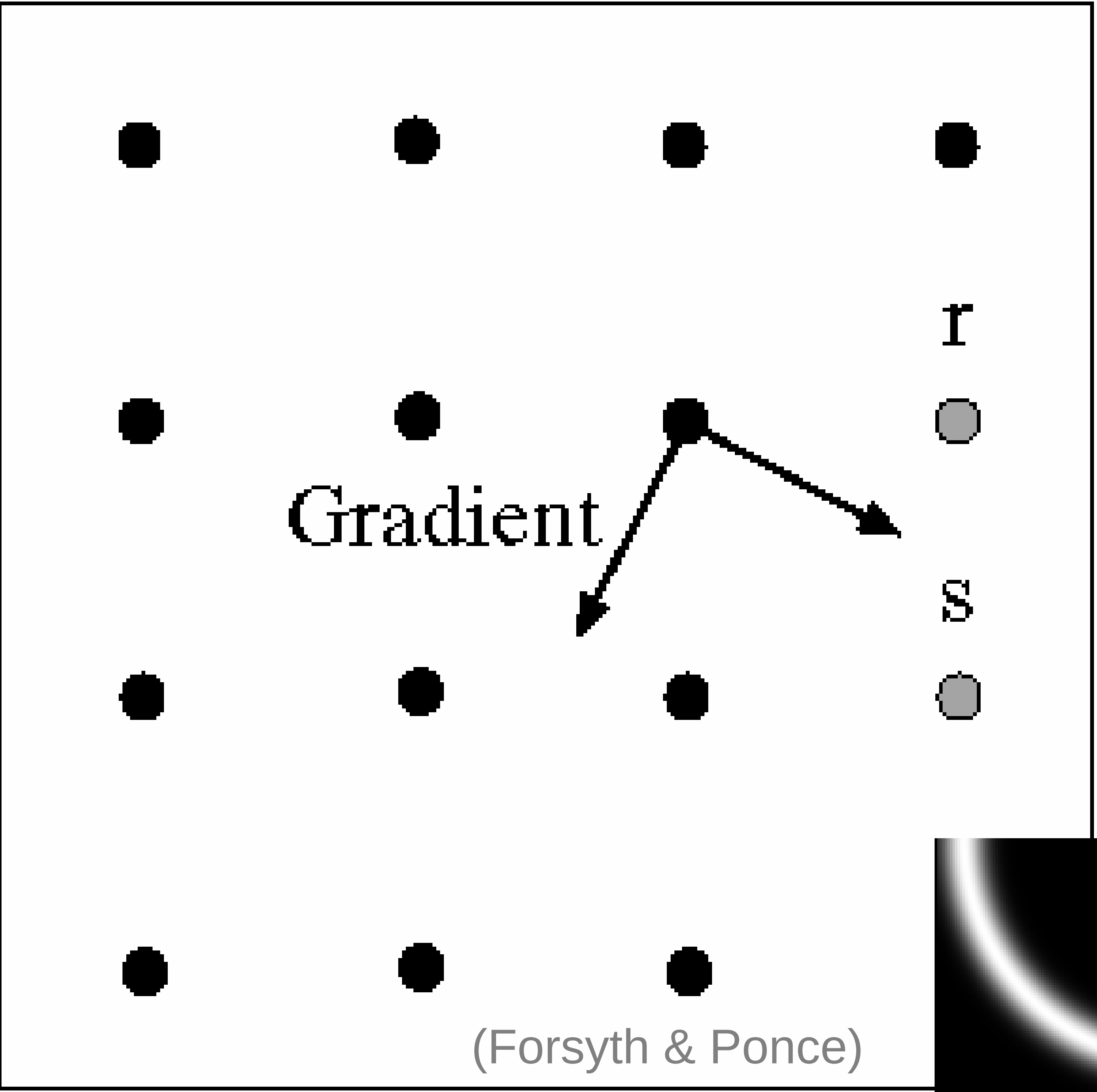
Non-maximum suppression





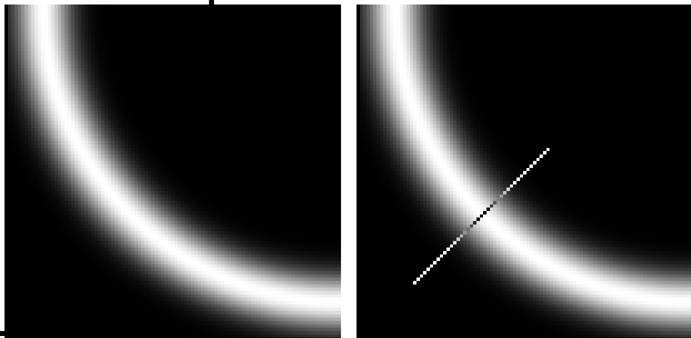
Check if pixel is local maximum along gradient direction

- requires checking interpolated pixels p and r



Predicting the next edge point

Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either r or s).



Effect of σ (Gaussian kernel size)

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features