Image Restoration

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Noise modeling

- An image gets corrupted by noise, which may arise in the process of acquiring the image or during its transmission, or even during reproduction of the image
- Removal of noise from an image is one of the most important tasks in image processing
- Depending on the nature of the noise, such as additive or multiplicative type of noise, there are several approaches towards removing noise from an image

Noise modeling

- Mathematically the imaging procedure including corruption by noise can be represented by

$$v(x, y) = g[u(x, y) + \eta(x, y)]$$

- Here u (x, y) represents the original image, and v (x, y) is the observed image, g() is generally nonlinear function and $\eta(x, y)$ represent the additive noise

Types of noise in an image and their Characteristics

- ***Additive noise:*** Sometime the noises generated from sensors are white Gaussian, which is essentially additive and signal independent, i.e.

$$g(x,y) = f(x,y) + \eta(x,y)$$

- Where g (x, y) is the result of the original image function f (x, y) corrupted by the additive Gaussian noise η(x, y)

Cont.

- ***Multiplicative noise:*** The graininess from photographic plates essentially multiplicative in nature. The speckle noise from the imaging system as in coherent, ultra sound imaging etc are also multiplicative

$$g(x,y) = f(x,y) * \eta(x,y)$$

- where $\eta(x, y)$ is the multiplicative noise

Cont.

- Impulse noise: Quite often the noisy sensors generate impulse noise
- Sometimes the noise generated from digital image transmission system is impulsive in nature, which can be model as

$$g(x,y) = (1-p)f(x,y) + p.i(x,y)$$

- Where i(x, y) is the impulsive noise and p is a binary parameter that assumes the values of either 0 or 1.

Cont.

- The impulsive noise may be easily detected from the noisy image because of the contrast anomalies.
- Once the noise impulses are detected, these are replaced by the signal samples

Image Restoration

- Improving the quality of images acquired by optical, electro-optical or electronic means is one of the basic tasks in digital image processing. Images may be degraded due to several reasons,
  - Imperfection of the imaging system
  - Imperfection in the transmission channel
  - Degradation due to atmospheric condition
  - Degradation due to relative motion between the object and the camera

Cont.

- The original scene is usually blurred due to convolution of the degrading mechanism as maintain before
- A random noise is usually added to the degraded data. The random noise may originate from the image formation process, trans mission medium, recording process etc.

Cont.

- A typical cause of degradation may be defocusing of the camera lens system, sudden jerking motion of the imaging system. The additive Gaussian noise results in the degraded image

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

- where f(x, y) is the pixel value of the original image at the point (x, y), H is the degradation model, and g (x, y) is the degraded image; η(x, y) is the additive noise.

Cont.

- A simple degradation model may be achieved by convolving a 3×3 window with the original image. Thus g (x, y) becomes

$$g(x,y)=\sum_{k,l\in w} H(k,l)\,f(x-k,y-l)+\eta(x,y)$$

- Here w represent the convolving window.

- There are conventional methods like inverse filtering, winner filtering, kalman filtering, etc., to restore the original image.
- In all these cases we assume that the blurring function H is known.
- If H is known, then obviously we can get back or restore original image by simply by convolving the degraded image with the inverse of the blurring function.
- To remove the noise, it is important that the noise statistics should be known prior

Inverse Filtering

- Inverse filtering approach toward image restoration assumes that the degradation of the image is caused by a degradation function h (i, j)
- The additive noise is assumed to be independent of the image signal
- Thus the degradation can be expressed in the Fourier domain as

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

- Where F (u, v) is the Fourier transform of the uncorrupted image, G (u, v) is the Fourier transform of the degradation image, H (u, v) is the Fourier transform of the degradation process h (i, j).

Cont.

- Therefore from the above equation we can write

$$F(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{\mathrm{N}(\mathrm{u},\mathrm{v})}{\mathrm{H}(\mathrm{u},\mathrm{v})}$$

- From the above equation it is evident that the original image cannot be restored in case the nature of the random noise function is unknown
- In case there is no additive noise N (u, v), the inverse filtering performs very well
- If noise is present, then its influence becomes significant at higher frequency (u, v), where magnitude of H (u, v) becomes very low.
- The effect of noise in such cases influences the result of restoration

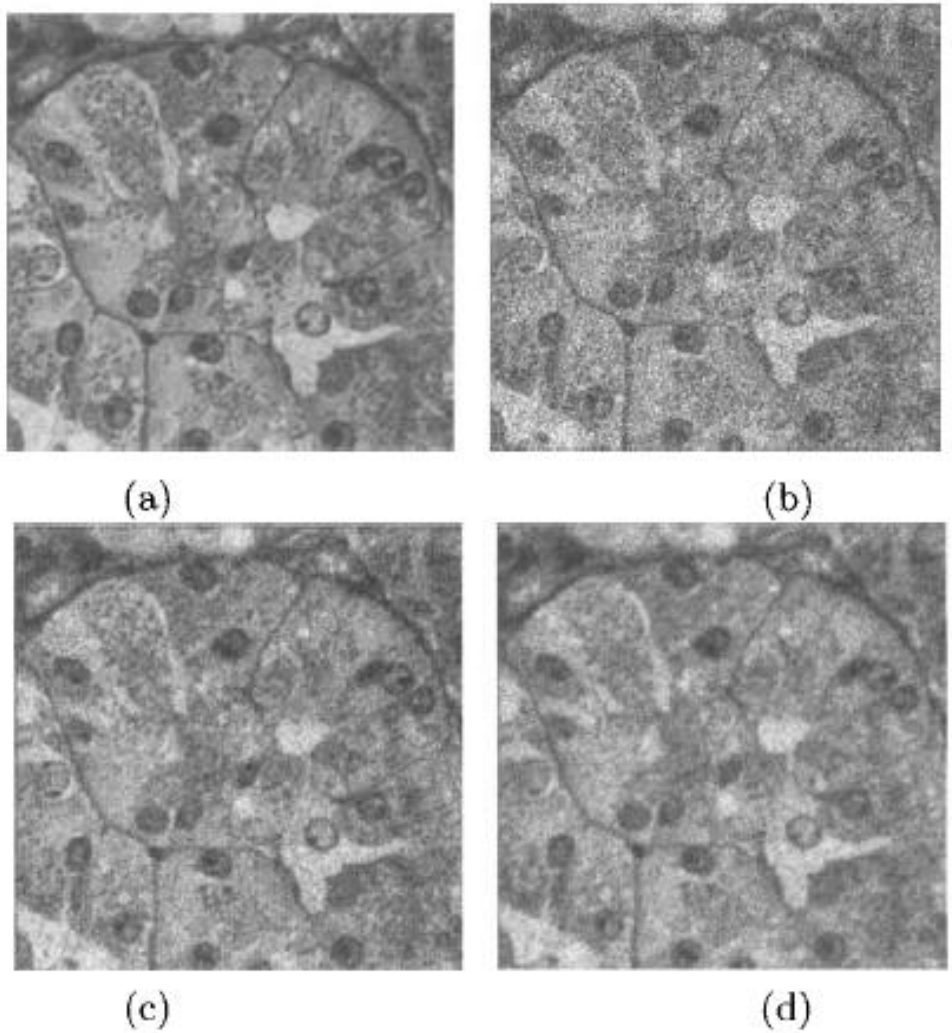


Fig. 6.6 The results of restoration: (a) original image, (b) degraded image, (c) inverse filtered result, (d) Wiener filtered result

Wiener Filter

- The Wiener is a minimum mean-square estimator which theoretically minimizes average error
- Wiener filter a method that restores an image in the presence of blur as well as noise
- Here, both the image f (x, y) and noise η(x, y) are the zero mean random processes and the objective is to obtain an estimate $\hat{f}(x,y)$ of the original image f (x, y) such that the mean square error is minimized. The resulting estimate is also MMSE estimate.

$$e^2 = E\left\{\left[f(x,y) - \hat{f}(x,y)\right]^2\right\}$$

Cont.

$$g(x,y) = h(x,y) * f(x,y) + n(x,y)$$

$$\widehat{f}(x,y) = w(x,y) * g(x,y)$$

$Where\,$ w(x, y) is the wiener filter.

$$E\left[\left\{f(x,y) - \widehat{f}(x,y)\right\}^2\right]$$

$$= E\left[f(x,y)^2\right] + E\left[\widehat{f}(x,y)^2\right] - 2.E\left[f(x,y)f(x,y)\right]$$

$$.........\,..........\,.$$

$Winner\ filter\ equation\ for\ an\ N \times N\ image:$

$$W(u,v) = \frac{H^*(u,v)}{\left|H(u,v)\right|^2 + \left[\dfrac{S_n(u,v)}{S_I(u,v)}\right]}$$

$Where:$

$H^*(u,v) = complex\ conjugate\ of\ H(u,v)$

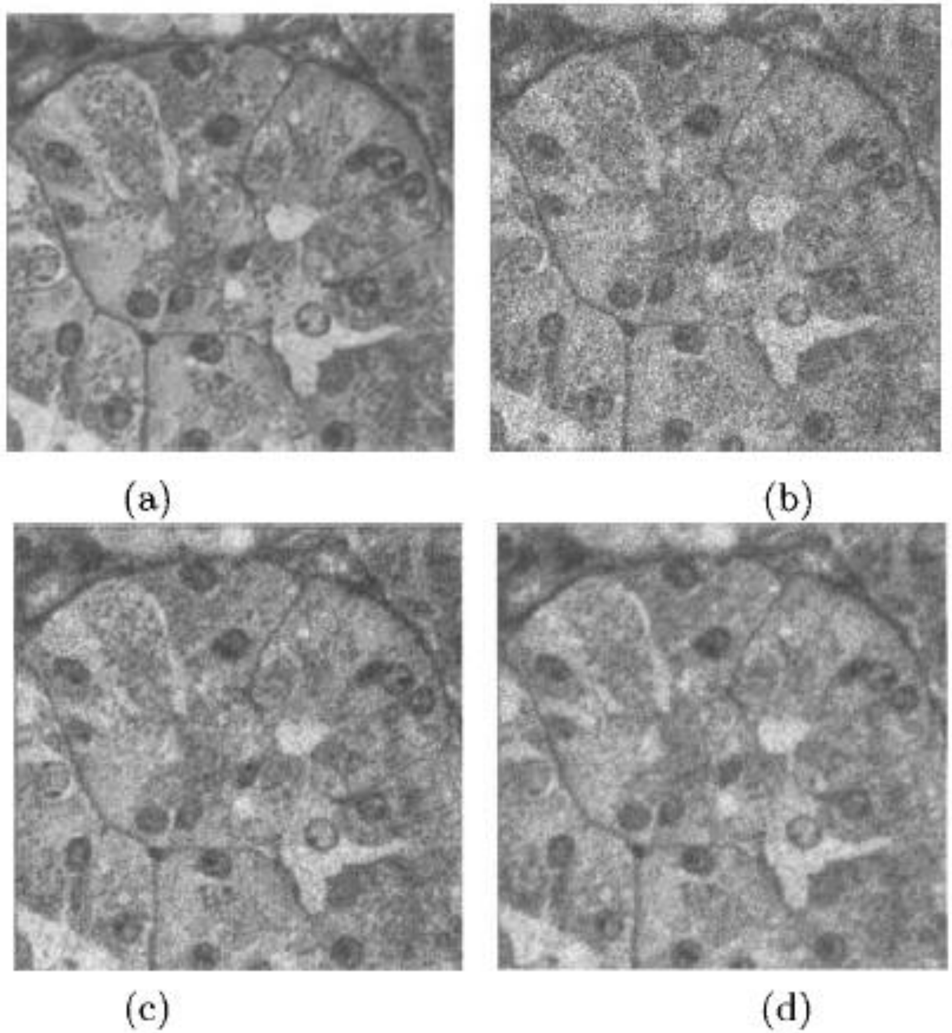
$S_n(u,v) = \left|N(u,v)\right|^2 = power\ spectrum\ of\ the\ noise$

$S_I(u,v) = \left|I(u,v)\right|^2 = power\ spectrum\ of\ the\ orginal\ image$

Cont.

- The Wiener filter response is reduced at high frequencies compared to an inverse filter due to the power spectrum ratio, $\left[\frac{\mathrm{S_n}(u,v)}{\mathrm{S_I}(u,v)}\right]$
- The practical wiener filter replaces the power spectrum ratio with an experimentally determine constant, because in real applications $S_I(u,v)$ cannot be determined

$$W(u,v)=\frac{H^*(u,v)}{|H(u,v)|^2+k}$$



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