Trigonometric, Differentiation ,Integration and Binomial Theorem Formulas

Important Trigonometric Formula

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$
6. $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$
7. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
8. $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
9. $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$
10. $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$
11. $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
12. $\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$
13. $\sin 2A = 2\sin A \cos A$
14. $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$
15. $\sin 3A = 3\sin A - 4\cos^3 A$
16. $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
17. $\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
18. $\cos 2A = \cos^2 A - \sin^2 A$
19. $2\cos^2 A = 1 + \cos 2A$
20. $2\sin^2 A = 1 - \cos 2A$
21. $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
22. $\cos 3A = 4\cos^3 A - 3\cos A$
23. $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$
24. $2\cos^2\frac{A}{2} = 1 + \cos A$
25. $2\sin^2\frac{A}{2} = 1 - \cos A$
26. $\cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
27. $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$
28. $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

29. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
30. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$
31. $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
32. $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
33. $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$
34. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$
35. $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
36. $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
37. $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$
38. $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$
39. $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$
40. $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\{xy - \sqrt{(1-x^2)(1-y^2)}\}$
41. $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\{xy + \sqrt{(1-x^2)(1-y^2)}\}$
42. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$
43. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$
44. $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx}$
45. $2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2}$
46. $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$
47. $2\tan^{-1} x = \sin^{-1}\frac{1-x^2}{1+x^2}$
48. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
49. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
50. $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$
51. $\cos A = \frac{b^2+c^2-a^2}{2bc}$
52. $\cos B = \frac{a^2+c^2-b^2}{2ac}$
53. $\cos C = \frac{a^2+b^2-c^2}{2ab}$
54. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
55. $a = b\cos C + c\cos B$
56. $b = c\cos A + a\cos C$

57. $c = a\cos B + b\ \cos A$

58. $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

59. $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

60. $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

Differentiation and integration formulas:

Limit formulas:

1. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
2. $\lim_{x\to 0}\frac{x}{\sin x} = 1$
3. $\lim_{x\to 0}\frac{\sin^{-1} x}{x} = 1$
4. $\lim_{x\to 0}\frac{\tan\ x}{x} = 1$
5. $\lim_{x\to 0}\frac{x}{\tan x} = 1$
6. $\lim_{x\to 0}\frac{\tan^{-1} x}{x} = 1$
7. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$
8. $\lim_{x\to 0}\frac{x^n - a^n}{x-a} = na^{n-1}$
9. $\frac{dy}{dx} = f'(x) = \frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Differentiation formulas:

1. $\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$
2. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(x) = 1$
5. $\frac{d}{dx}(c) = 0, where\ c\ is\ a\ constan.$
6. $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$
7. $\frac{d}{dx}(\sin x) = \cos x$
8. $\frac{d}{dx}(\cos x) = -\sin x$
9. $\frac{d}{dx}(\tan x) = sec^2 x$

10. $\frac{d}{dx}(\cot x) = -cosec^2\ x$
11. $\frac{d}{dx}(\sec x) = \sec x\ tan\ x$
12. $\frac{d}{dx}(cosec\ x) = -cosec\ x \cot x$
13. $\frac{d}{dx}(\ e^x) = e^x$
14. $\frac{d}{dx}(\ e^{mx}) = me^{mx}$
15. $\frac{d}{dx}(\ln x\ ) = \frac{1}{x}$
16. $\frac{d}{dx}(\log_m x\ ) = \frac{1}{x}\log_m e$
17. $\frac{d}{dx}(a^x\ ) = a^x \ln a$
18. $\frac{d}{dx}(a^x\ ) = a^x \log_e a$
19. $\frac{d}{dx}(\ \sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}(\cos^{-1} x\ ) = -\frac{1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}(\ \sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
22. $\frac{d}{dx}(\ cosec^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
23. $\frac{d}{dx}(\tan x\ ) = \frac{1}{1+x^2}$
24. $\frac{d}{dx}(\cot x\ ) = -\frac{1}{1+x^2}$

Common Integrals:

Polynomials:

1. $\int dx = x + c$
2. $\int kdx = kx + c$
3. $\int x^n dx = \frac{x^n}{n+1} + c, n \neq -1$
4. $\int \frac{1}{x} dx = \ln x + c$

Trigonometric functions

5. $\int \sin x\ dx = -\cos x + c$
6. $\int \cos x\ dx = \sin x + c$
7. $\int sec^2 x dx = \tan x + c$
8. $\int \sec x tan\ x\ dx = \sec x + c$
9. $\int cosec\ x \cot x\ dx = -cosec\ x + c$
10. $\int \tan x\ dx = -\ln(\cos x) + c = \ln(\sec x) + c$

11. $\int \cot x \, dx = \ln(\sin x) + c$
12. $\int \sec x \, dx = \ln(\sec x + \tan x) + c = \ln\left|tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$
13. $\int \operatorname{cosec} x \, dx = -\ln(\operatorname{cosec} x + \cot x) + c = ln\left|\tan\frac{x}{2}\right| + c$
14. $\int \sin mx \sin nx \; dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + c$, If $m^2 \neq n^2$
15. $\int \cos mx \cos nx \, x \; dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + c$, If $m^2 \neq n^2$

Exponential and Logarithm functions:

16. $\int e^x dx = e^x + c$
17. $\int e^{mx} dx = \frac{e^{mx}}{m} + c$
18. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(\mathrm{a} \sin bx - b \cos bx) + c$
19. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(\mathrm{a} \cos bx + b \sin bx) + c$
20. $\int a^x dx = \frac{a^x}{\ln a} + c$
21. $\int \ln x \, dx = x \ln x - x + c$

Inverse Trigonometric Functions:

22. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a} + c$
23. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
24. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$
25. $\int -\frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a} dx = +c$
26. $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + c$
27. $\int \cos^{-1} x \, dx = x \cos^{-1} x - \sqrt{1-x^2} + c$
28. $\int \tan^{-1} x \, dx = x \tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$

Miscellaneous:

29. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + c$
30. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + c$
31. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \ln\left(x + \sqrt{a^2+x^2}\right) + c$
32. $\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x + \sqrt{x^2-a^2}\right) + c$
33. $\int \sqrt{x^2-a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2-a^2}\right) + c$

34. $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + c$

35. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

36. $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + c$

37. $\int \frac{f'(x)}{f(x)}dx = \ln\{f(x)\} + c$

Standard substitutions -1 :

Function	Substitute
$a^2-x^2\ or\sqrt{a^2-x^2}$	$x = a\sin\theta\ or\ x = a\cos\theta$
$a^2+x^2\ or\sqrt{a^2+x^2}$	$x = a\tan\theta\ or\ x = a\cot\theta$
$x^2-a^2 or\sqrt{x^2-a^2}$	$x = a\sec\theta\ or\ x = a\ cosec\ \theta$

Standard substitutions -2 :

Form	Substitute
$\int \frac{dx}{(ax+b)\sqrt{ux+v}}$	$z^2 = ux+v$
$\int \frac{dx}{(ux+v)\sqrt{ax^2+bx+c}}$	$z = ux+v$

Permutation and Combination:

Factorial:

1. $0! = 1$
2. $(-n)! = \infty$
3. $n! = n(n-1)(n-2)\ldots\ldots 3.2.1 = 1.2.3\ldots\ldots n$

Permutation:

1. Permutation is an arrangement and we can write ${}^nP_r$
2. ${}^nP_r = \frac{n!}{(n-r)!}$ =n(n-1)(n-2)(n-3).....(n-r+1)
3. ${}^nC_r = \frac{n!}{r!(n-r)!}$
4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

5. If ${}^nC_x = {}^nC_y$ then n=x+y
6. ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots + {}^nC_n = 2^n$
7. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots + {}^nC_1 + {}^nC_3 + {}^nC_5 \ldots\ldots = 2^{n-1}$

Binomila theorem:

8. $(a+x)^n$ এর বিস্তৃতিতে $= a^n + {}^nC_1 a^{n-1}x^1 + {}^nC_2 a^{n-2}x^2 + \cdots$
9. $(a+x)^n = a^n + {}^nc_1 a^{n-1}x^1 + {}^nc_2 a^{n-2}x^2 + {}^nc_3 a^{n-3}x^3 + \cdots\cdots\cdots + {}^nc_r a^{n-r}x^r + \ldots\ldots\ldots + x^n$
10. $(r+1)$তম পদ, $T_{r+1} = {}^nc_r a^{n-r}x^r$
11. $(a+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\cdots\cdots + \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}x^r + \ldots\ldots\ldots$
12. (r+1)তম পদ, $T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}x^r$
13. $If\ |x| < 1$ then
    a) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \cdots + x^r + \cdots$
    b) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots + (-1)^r x^r + \cdots$
    c) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots + (r+1)x^r + \cdots$
    d) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots + (-1)^r(r+1)x^r + \cdots$
    e) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{1}{2}(r+1)(r+2)x^r + \cdots$
14. $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \cdots\cdots\cdots + (-1)^r \frac{1}{2}(r+1)(r+2)x^r + \cdots$

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