BCT 2205 - Lecture 6

✓DC Transients

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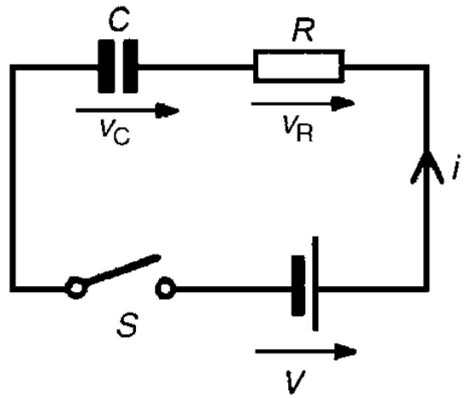
Introduction

- When a DC voltage is applied to a capacitor C and resistor R connected in series, there is a short period of time immediately after the voltage is connected, during which the current and voltage across C and R are changing.
- Similarly, when a DC voltage is connected to a circuit having inductance L connected in series with resistance R, there is a short period of time immediately after the voltage is connected, during which the current and the voltage across L and R are changing.
- These changing values are called **transients**.

C-R circuit



- When switch S is closed, by Kirchhoff's voltage law,

$$V = v_C + v_R \qquad (1)$$

- Written as,

$$V = \frac{q}{C} + iR \qquad (2)$$

- At the instant of closing S, assuming there is no initial charge on the capacitor, q_0 is zero, hence v_{C0} is zero.

$$V = 0 + v_{R0}$$
$$V = v_{R0}$$

- The initial current flowing,

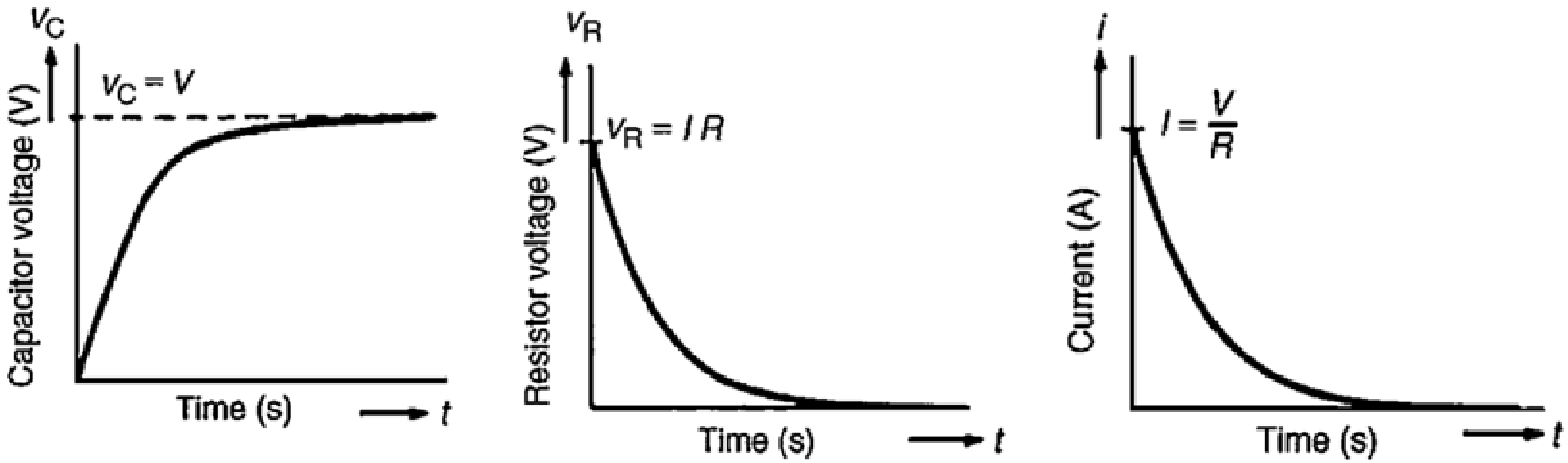
$$i_0 = I = \frac{V}{R}$$

- A short time later at time t_1 after closing S, the capacitor is partly charged to q_1 because current has been flowing.

$$V = \frac{q_1}{C} + i_1 R$$

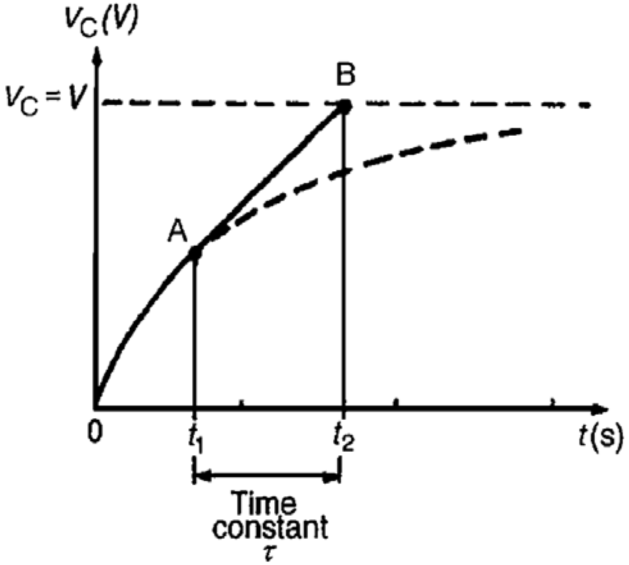
- A short time later, say t_2 after closing the switch, the charge has increased to q_2 and v_C has increased to $(\frac{q_2}{C})$ volts.
- Since $V = v_C + v_R$ and V is a constant, as v_C increase, v_R decreases.
- A few seconds after closing S, the capacitor is fully charged and current no longer flows,

$$i = 0, \qquad v_R = iR = 0, \qquad v_C = V$$



(a) Capacitor voltage transient (b) Resistor voltage transient (c) Current transient

Time constant for a $C - R$ circuit



- Let voltage be varied such that **current** flowing in the circuit is **constant**.
- The curve will follow tangent AB.
- Let the capacitor voltage reach its final value at t_2 seconds.
- $(t_2 - t_1)$ is called the **time constant** of the circuit, denoted by τ.

$$\tau = \boldsymbol{CR} \textbf{ seconds}$$

1. Growth of capacitor voltage

$$v_C = V\left(1 - e^{-\frac{t}{CR}}\right) = V(1 - e^{-t/\tau})$$

2. Decay of resistor voltage

$$v_R = Ve^{-\frac{t}{CR}} = Ve^{-t/\tau}$$

3. Decay of resistor current

$$i = Ie^{-\frac{t}{CR}} = Ie^{-t/\tau}$$

Example

- A circuit consists of a resistor connected in series with a 0.5μF capacitor and has a time constant of 12ms. Determine:

i. the value of the resistor,

ii. the capacitor voltage, 7 ms after connecting the circuit to a 10 V supply.

- The time constant

$$\tau = CR \quad ; \quad R = \frac{\tau}{C} = \frac{12m}{0.5\mu} = 24k\Omega$$

- The equation for growth of capacitor voltage

$$v_c = V\left(1 - e^{-\frac{t}{\tau}}\right) \quad ; \quad \tau = 12ms \quad ; \quad v_c = 10\left(1 - e^{-\frac{7m}{12m}}\right)$$

$$= 10(1 - e^{-0.583}) = 10(1 - 0.558) = 4.42V$$

- A 20 µF capacitor is connected in series with a 50 kΩ resistor and the circuit is connected to a 20 V, DC supply. Determine:

1. The initial value of the current flowing

$$I = \frac{V}{R} = \frac{20}{50K} = 0.4mA$$

2. Time constant of the circuit

$$\tau = CR = (20\mu)(50k) = 1s$$

3. Value of current one second after connection

$$i = Ie^{-t/\tau} = 0.4e^{-1/1} = 0.4\ x\ 0.368 = 0.147mA$$

4. Capacitor voltage two seconds after connection

$$v_c = V\left(1 - e^{-\frac{t}{\tau}}\right) = 20\left(1 - e^{-\frac{2}{1}}\right)$$

$$= 20(1 - 0.135) = 18.3V$$

5. The time after connection when resistor voltage is 15 V

$$v_R = Ve^{-t/\tau}\ ;\ 15 = 20e^{-t/1}$$

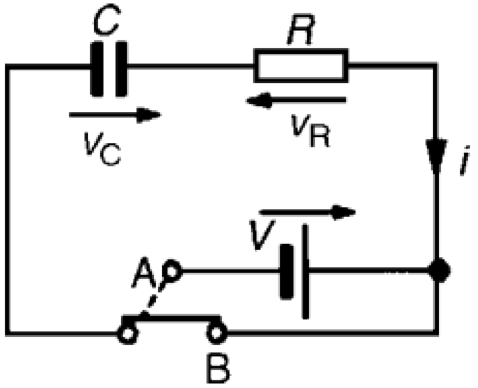
$$\frac{3}{4} = e^{-t} \quad ;\ \frac{4}{3} = e^{t}$$

- Taking natural logarithms of each side

$$\ln\frac{4}{3} = t\ln e \quad ;\ \ t = \ln\frac{4}{3} \ \ ; \quad t = 0.288\ s$$

Discharging a capacitor

- When a capacitor is charged and the switch is then moved to position B, the electrons stored in the capacitor keep the current flowing for a short time.

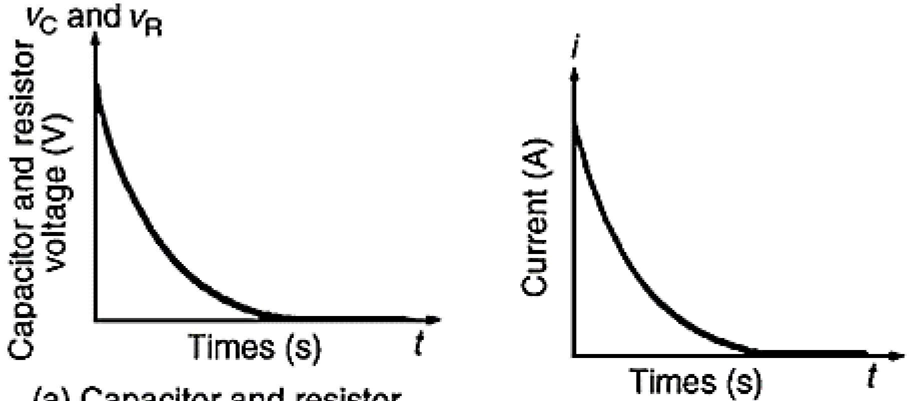


- Initially, $v_C = v_R = V$, and $i = I = V/R$.
- During the transient decay, $v_C = v_R$.
- Finally, the transients decay exponentially to zero, $v_C = v_R = 0$.
- Decay of voltage,

$$v_C = Ve^{(-t/CR)} = Ve^{(-t/\tau)}$$

- Decay of current,

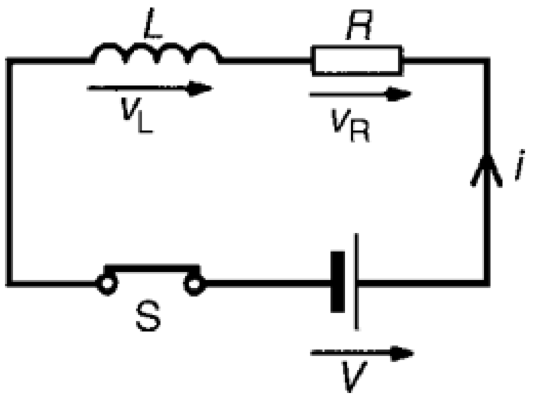
$$i = Ie^{(-t/CR)} = Ie^{(-t/\tau)}$$



(a) Capacitor and resistor voltage transient

(b) Current transient

Current growth in L – R circuit



$$V = v_L + v_R \qquad (1)$$

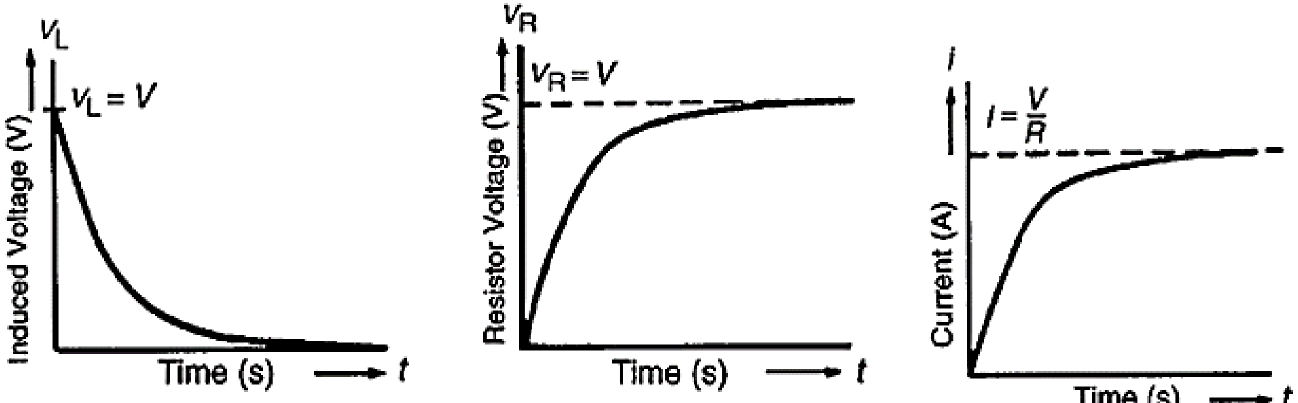
$$V = L\frac{di}{dt} + iR \qquad (2)$$

- At the instant of closing the switch,
  $V = v_L + 0 \quad ; \quad V = v_L$
- At t_1 after closing S,

$$V = L\frac{di_1}{dt_1} + i_1 R$$

- At t_2 seconds after closing the switch, the current flowing is i_2.
- Since v_R increases, v_L decreases.
- At steady state, current flow is entirely limited by R.

$$I = \frac{V}{R}, \qquad v_R = IR\,; \qquad v_L = 0$$



(a) Induced voltage transient (b) Resistor voltage transient (c) Current transient

Time constant for an L – R circuit

- The time constant of a series connected L – R circuit is given by:

$$time\ constant\ \tau = \frac{L}{R}\ seconds$$

1. Decay of induced voltage

$$v_L = Ve^{(-Rt/L)} = Ve^{(-t/\tau)}$$

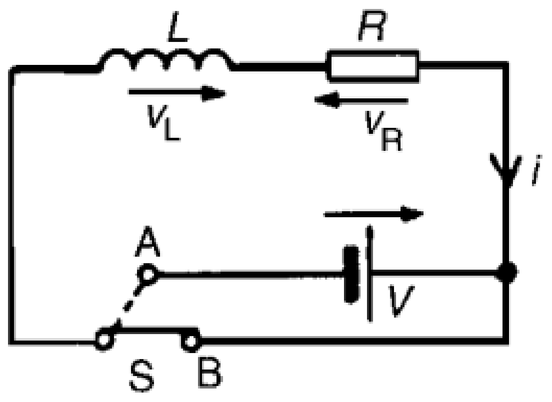
2. Growth of resistor voltage

$$v_R = V(1 - e^{\left(-\frac{Rt}{L}\right)}) = V(1 - e^{\left(-\frac{t}{\tau}\right)})$$

3. Growth of current flow

$$i = I(1 - e^{\left(-\frac{Rt}{L}\right)}) = I(1 - e^{\left(-\frac{t}{\tau}\right)})$$

Current decay in an L – R circuit



- When S is moved to position B, the current value decreases, causing a decrease in the strength of the magnetic field.
- Flux linkages occur, generating a voltage

$$v_L = L(di/dt)$$

- This voltage keeps current i flowing in the circuit, its value limited by R.

$$v_L = v_R$$

- The current decays exponentially to zero.
- v_L also decays exponentially to zero.
- Decay of voltages,

$$v_L = v_R = Ve^{(-Rt/L)} = Ve^{(-t/\tau)}$$

- Decay of current,

$$i = Ie^{(-Rt/L)} = Ie^{(-t/\tau)}$$

Example

- Find the current in a series RL circuit having R =2Ω and L = 10H when a DC voltage of 100V is applied.

$$\tau = 10/2 = 5\,s$$

1. The current in a series LR circuit after the sudden application of a DC voltage is,

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

2. Find the value of the current 5s after the application of the DC voltage.

$$i(t) = \frac{100}{2}\left(1 - e^{-\frac{5}{5}}\right) = 50\,(1 - e^{-1}) = 50\left(1 - \frac{1}{e}\right) = 31.48A$$

- An inductor has an inductance of 200 mH and is connected in series with a 1 kΩ resistor to a 24V, DC supply. Determine,

1. time constant of the circuit

$$\tau = \frac{L}{R} = \frac{0.2}{1000} = 0.2\,ms$$

2. steady-state value of the current flowing

$$I = \frac{V}{R} = \frac{24}{1000} = 24\,mA$$

3. current flowing in the circuit at a time equal to one time constant,

$$i = I\left(1 - e^{-\frac{t}{\tau}}\right) \;;\;\; t = 1\tau$$

$$i = 24\left(1 - e^{-\frac{1\tau}{\tau}}\right)$$

$$= 24(1 - e^{-1}) = 15.17\,mA$$

4. voltage drop across the inductor at a time equal to two-time constants

$$t = 2\tau, \qquad v_L = 24e^{-2\tau/\tau}$$

$$= 24e^{-2} = 3.248\,V$$

5. voltage drop across the resistor after a time equal to three-time constants.

$$v_R = V\left(1 - e^{-\frac{t}{\tau}}\right) \;;\;\; when\; t = 3\tau,$$

$$v_R = 24\left(1 - e^{-\frac{3\tau}{\tau}}\right) = 22.81\,V$$

End of session

Questions….?