DIGITAL ELECTRONICS

CHAPTER THREE: BOOLEAN ALGEBRA & LOGIC GATES (PART II):

*Laws of Boolean Algebra

*Standard Forms of Boolean Functions (SOP and POS)

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Previously on NUMBER SYSTEMS AND CODES

Number systems conversions examples:

Example 1: Convert the decimal number 12.625 to its binary equivalent.

	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
	16	8	4	2	1	.	0.5	0.25	0.125
12.625	0	1	1	0	0	.	1	0	1

ANS = 1100.101

Example 2: Convert 125.36_8 to binary.

Step 1: Convert the number to decimal first

$$(8^2 \times 1) + (8^1 \times 2) + (8^0 \times 5) + (8^{-1} \times 3) + (8^{-2} \times 6) = 85.46875_{10}$$

Number systems conversions examples:

Step 2: Convert the obtained decimal number to binary

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	2^{-4}
	128	64	32	16	8	4	2	1	.	0.5	0.25	0.125	0.0625
85.46875	0	1	0	1	0	1	0	1	.	0	1	1	1

ANS = 1010101.0111 {to 4 d.p}

Example 3: Convert 746.6875 to hexadecimal.

	16^2	16^1	16^0	.	16^{-1}	16^{-2}
	256	16	1		0.0625	0.00390625
746.6875	2	14	10	.	11	
Hex	2	E	A	.	B	

The Exclusive-OR gate explained…

- Defined as:

$$x = A \oplus B$$

A	B	x
0	0	0
0	1	1
1	0	1
1	1	0

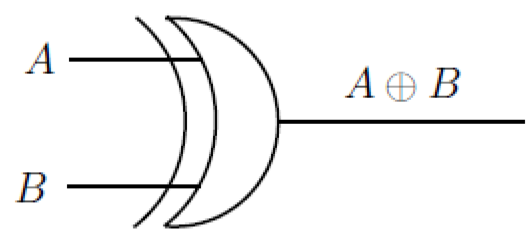


Figure 2.6: An Exclusive-OR gate

- The Exclusive-OR operation is sometimes abbreviated as XOR or EXOR.
- *Note that the XOR gate has only two inputs.*

The internal connection of the **Exclusive OR gate** is as below:

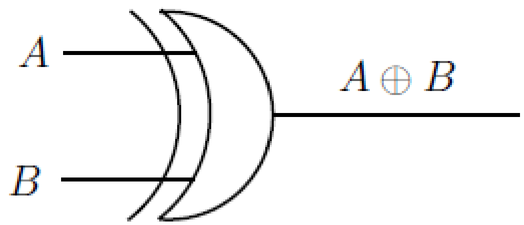
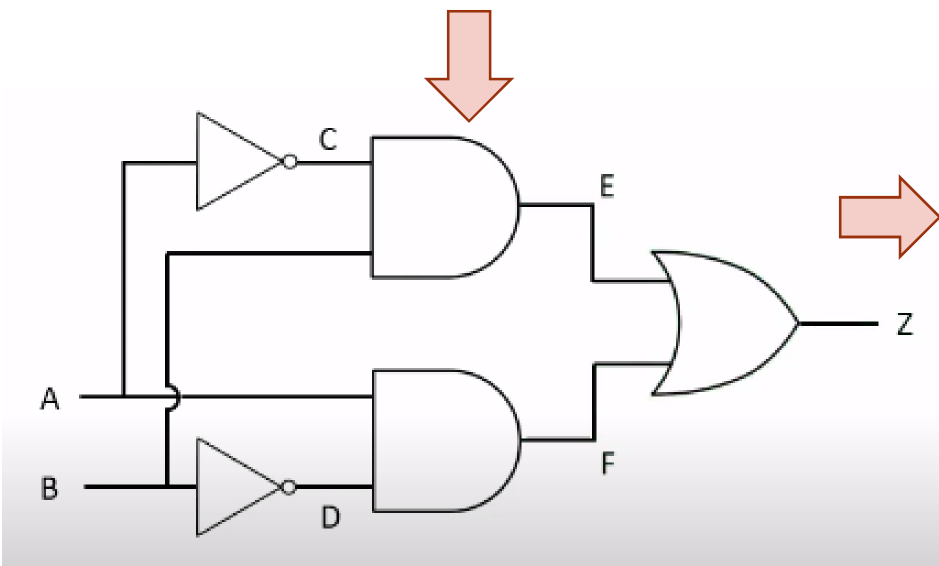


Figure 2.6: An Exclusive-OR gate



Truth Table

A	B	C	D	E (C.B)	F (A. D)	Z (E+F)
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0

The Exclusive-NOR gate explained…

- This is usually abbreviated as XNOR or EXNOR gate. It is the complement of the XOR operation.

$$x = A \odot B = \overline{A \oplus B}$$

A	B	x
0	0	1
0	1	0
1	0	0
1	1	1

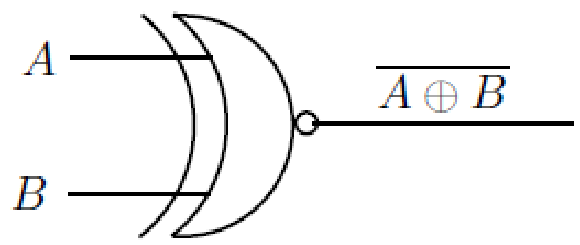


Figure 2.7: An Exclusive-NOR gate

The internal connection of the **Exclusive NOR gate** is as below:

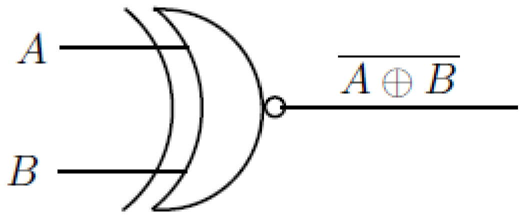
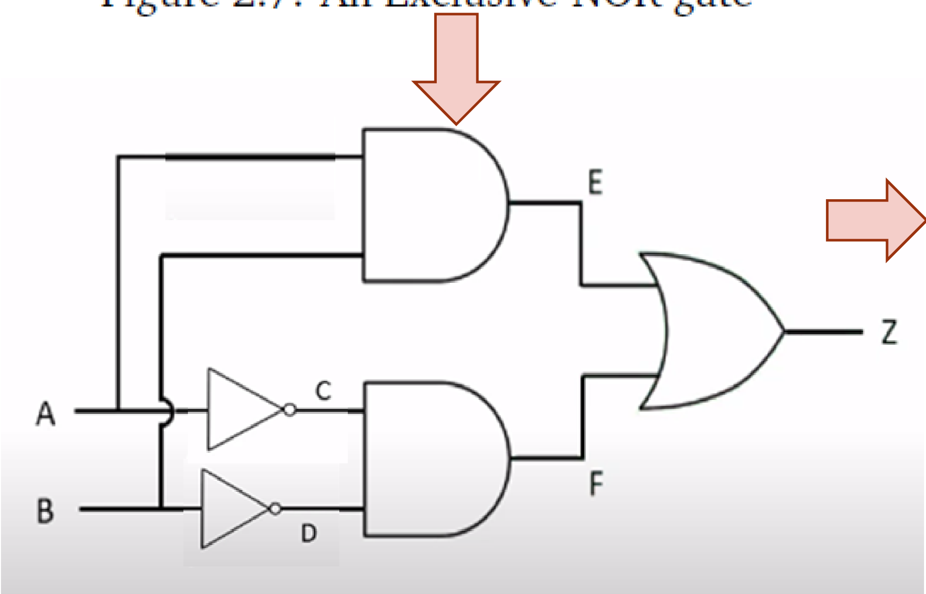


Figure 2.7: An Exclusive-NOR gate



Truth Table

A	B	C	D	E (A.B)	F (C. D)	Z (E+F)
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

2. BOOLEAN ALGEBRA AND LOGIC GATES

2.4 Laws of Boolean Algebra

Basic Theorems

$A + 0 = A$	$A \cdot 1 = A$
$A + 1 = 1$	$A \cdot 0 = 0$
$A + A = A$	$A \cdot A = A$
$A + \bar{A} = 1$	$A \cdot \bar{A} = 0$

- Looking at the above table, we can see that the corresponding laws on either side are related by:

i. Interchanging + and . Symbols

ii. Interchanging 0 and 1

- Theorems which are related to another by this double interchange are known as duals.

2.4 Laws of Boolean Algebra

Other theorems, each listed along with its dual, are tabulated below:

1:	$A + B = B + A$	$A \cdot B = B \cdot A$
2:	$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
3:	$A + B \cdot C = (A + B) \cdot (A + C)$	$A \cdot (B + C) = A \cdot B + A \cdot C$
4:	$A + A \cdot B = A$	$A \cdot (A + B) = A$
5:	$A + \overline{A} \cdot B = A + B$	$A \cdot (\overline{A} + B) = A \cdot B$
6:	$A \cdot B + A \cdot \overline{B} = A$	$(A + B) \cdot (A + \overline{B}) = A$
7:	$A \cdot B + \overline{A} \cdot C = (A + C) \cdot (\overline{A} + B)$	$(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot B$
8:	$A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$	$(A + B) \cdot (\overline{A} + C) \cdot (B + C) = (A + B) \cdot (\overline{A} + C)$
9:	$\overline{A + B + C + \cdots} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdots$	$\overline{A \cdot B \cdot C \cdots} = \overline{A} + \overline{B} + \overline{C} + \cdots$

Law 1 is the commutative law, 2 the associative law, 3 the distributive law, 4 is commonly referred to as the absorption theorem, 5 the simplification theorem, 6 the reduction theorem and 9 are the De Morgan's Theorems.

2.5 Proving Boolean Theorems

2.5.1 Proof by Truth-Table/Proof by Perfect Induction

Use a truth-table to prove that $AB + \overline{A}C + BC = AB + \overline{A}C$.

A	B	C	AB	$\overline{A}C$	BC	$AB+\overline{A}C+BC$	$AB+\overline{A}C$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	0	0	0	0
0	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	1	0	1	1	1

2.5.2 Proof by Algebraic means

- This requires a mastery of the laws of Boolean algebra so a lot of practice is needed to be able to use this technique effectively.
- The Priority of solving Boolean algebra follows the order:
  - NOT
  - AND
  - OR

2.5.2 Proof by Algebraic means

- This requires a mastery of the laws of Boolean algebra so a lot of practice is needed to be able to use this technique effectively.

Example:

- Use algebraic means to show that $A + A.B = A$

Solution:

Taking the LHS, the term A can be rewritten as A= A.1 hence the equation becomes:

LHS $= A.1 + A.B$ (i)

The term A is common and is factored out giving us:

$A\,(1 + B)$ (ii)

But $(1 + B) = 1$ hence equation (ii) becomes:

$A\,(1) = \mathbf{A}$

Example

- Use algebraic means to show that $A + \bar{A}B = A + B$

Solution:

Taking the LHS, rewrite the term $\mathrm{A} = \mathrm{A}.1$

LHS= $\mathrm{A}.1 + \bar{\mathrm{A}}\mathrm{B}$

Introduce a B to replace 1 i.e 1=(1+B)

LHS = $\mathrm{A}(1 + \mathrm{B}) + \bar{\mathrm{A}}\mathrm{B} = \mathrm{A} + \mathrm{AB} + \bar{\mathrm{A}}\mathrm{B}$

Factor out the B:

LHS= $\mathrm{A} + \mathrm{B}(\mathrm{A} + \bar{\mathrm{A}})$

But $\mathrm{A} + \bar{\mathrm{A}} = 1$.

Hence the equation simplifies to:

$$= \mathrm{A} + \mathrm{B}(1)$$

A + B = RHS hence proved

Example:

- Minimize $A.B + A\overline{B}$

<u>Solution:</u>

Note the A is common and factor it out:

$$A(B + \overline{B})$$

But $B + \overline{B} = 1$

Hence the equation becomes:

$A(1) = A$

Example:

- Minimize $AB + A\overline{B}C + A\,\overline{B}\,\overline{C}$

Solution:

A is common in all three variables. Factor it out:

$$A(B + \overline{B}C + \overline{B}\overline{C})$$

Note that $\overline{B}$ is common in two terms and factor it out:

$$A\left(B + \overline{B\{C + \overline{C}\}}\right)$$

But $C + \overline{C} = 1$

$A\left(B + \overline{B\{1\}}\right) = A(B + \overline{B})$

But $B + \overline{B} = 1$

$=A(1) = \mathbf{A}$

2.6 Standard Forms for Boolean Functions

- There are two standard forms for Boolean expressions: Standard sum of products form and Standard product of sums form.

2.6.1 Standard Sum of Products form

Given a function:

$$f(A, B, C) = (AB + C)(B + AC)$$

we can use the distributive rule (informally known as opening the brackets) to write:

$$f(A, B, C) = ABB + CB + ABAC + CAC$$

2. BOOLEAN ALGEBRA AND LOGIC GATES

- By use of Boolean rules, we can simplify the above expression to:

  $$f(A, B, C) = AB + BC + ABC + AC$$

- From the expression above, the terms AB, BC, ABC and AC are products, and they are all combined with an OR operation (logical addition or summation). The expression is said to be in Sum of Products form.
- This is a function of variables A, B and C, but not all the product terms contain all these variables e.g. the product term AB lacks the variable C, BC lacks the variable A, and AC lacks B.

- To express the function in **Standard Sum of Product form**, we must add the missing variables to all the product terms so that every variable appears in each product term (either in its true form or in its complement form). To do this, we use the Boolean algebra laws:

$$(A+\overline{A})=1 \qquad and \qquad A\cdot 1=A$$

*Means the function remains the same.

- The expression is now rewritten as:

$$f(A,B,C)=AB(C+\overline{C})+(A+\overline{A})BC+ABC+AC(B+\overline{B})$$

$$=ABC+AB\overline{C}+ABC+\overline{A}BC+ABC+ABC+A\overline{B}C$$

$$=ABC+AB\overline{C}+\overline{A}BC+A\overline{B}C$$

2. BOOLEAN ALGEBRA AND LOGIC GATES

- This form, in which a sum of products appears, each term involving all variables is called the *Standard Sum of Products Form* or *Canonical Sum of Products form*. Each individual term in the expression is known as a *MINTERM*, e.g. ABC is a minterm.
- Each minterm will have a logical value of 1 only when all the terms have a **LOGICAL VALUE OF 1**.

YOU NEED TO DRAW THIS MINTERM TABLE FOR SUCH QUESTIONS

A	B	C	$minterm$
0	0	0	$m_0 = \bar{A}\bar{B}\bar{C}$
0	0	1	$m_1 = \bar{A}\bar{B}C$
0	1	0	$m_2 = \bar{A}B\bar{C}$
0	1	1	$m_3 = \bar{A}BC$
1	0	0	$m_4 = A\bar{B}\bar{C}$
1	0	1	$m_5 = A\bar{B}C$
1	1	0	$m_6 = AB\bar{C}$
1	1	1	$m_7 = ABC$

2. BOOLEAN ALGEBRA AND LOGIC GATES

- Going back to the function we started off with and using the above table, we can write:

$$f(A,B,C) = m_7 + m_6 + m_3 + m_5$$

- Sometimes the above expression is also written as:

$$f(A,B,C) = \Sigma m(3,5,6,7)$$

2.6.2 Standard Product of Sums form

- Given a function:

$$f(A,B,C) = (AB + C)(B + AC)$$

- We can use the distributive rule to write:

$$f(A,B,C) = (A + C)(B + C)(B + A)(B + C)$$

$$= (A + B)(A + C)(B + C)$$

- The above expression is said to be in product of sums form.
- To convert this to the Standard product of sums form, we add the missing variables in each term, using the Boolean rules:

$$A \cdot \overline{A} = 0 \qquad and \qquad (A + 0) = A$$

*Means the function remains the same.

- We can therefore write:

$$f(A,B,C) = (A + B + C\bar{C})(A + B\bar{B} + C)(A\bar{A} + B + C)$$

- Again using the distributive rule:

$$f(A,B,C) = (A+B+C)(A+B+\bar{C})(A+B+C)(A+\bar{B}+C)(A+B+C)(\bar{A}+B+C)$$

$$f(A,B,C) = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + C)$$

- This is known as the *Standard Product of Sums form* or *Canonical Product of Sums form*. Each of the factors in the expression above is known as a **MAXTERM**, e.g. (A + B + C) is a maxterm.

- Each maxterm will have a **logical value 0** only when all the terms in it have a logical value 0, e.g. maxterm $(A + \overline{B} + C)$ will have a logical value 0 when A = 0, B = 1 and C = 0.

A	B	C	$maxterm$
0	0	0	$M_0 = A + B + C$
0	0	1	$M_1 = A + B + \bar{C}$
0	1	0	$M_2 = A + \bar{B} + C$
0	1	1	$M_3 = A + \bar{B} + \bar{C}$
1	0	0	$M_4 = \bar{A} + B + C$
1	0	1	$M_5 = \bar{A} + B + \bar{C}$
1	1	0	$M_6 = \bar{A} + \bar{B} + C$
1	1	1	$M_7 = \bar{A} + \bar{B} + \bar{C}$

We can therefore write:

$$f(A, B, C) = M_0 \cdot M_1 \cdot M_2 \cdot M_4$$

Sometimes this is also written as:

$$f(A, B, C) = \Pi M(0, 1, 2, 4)$$

YOU NEED TO DRAW THIS MAXTERM TABLE FOR SUCH QUESTIONS

End of session

Questions….?