BCT 2205 - Lecture 5

✓ DC circuit theory - II

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Thévenin's theorem

- *The current in any branch of a network is that which would result if an e.m.f. equal to the p.d. across a break made in the branch, were introduced into the branch, all other e.m.f.'s being removed and represented by the internal resistances of the sources.*

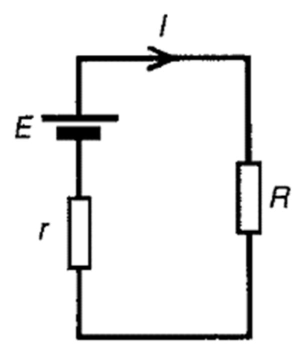
  i. remove the resistance R from that branch,

  ii. determine the open-circuit voltage, E, across the break,

  iii. remove each source of e.m.f. and replace them by their internal resistances and then determine the resistance, r, 'looking-in' at the break,

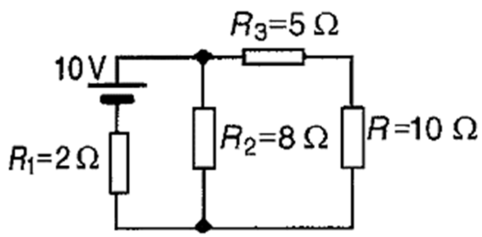
  iv. determine the value of the current from the equivalent circuit shown, i.e.,

$$I = \frac{E}{R+r}$$

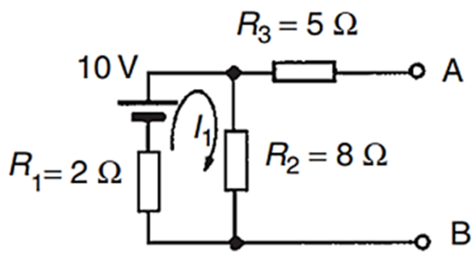


Example

- Use Thévenin's theorem to find current flowing in the 10 Ω resistor for the circuit

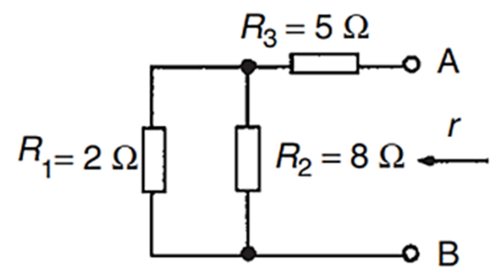


- The 10 Ω resistance is removed from the circuit



$$I_1 = \frac{10}{R_1 + R_2} = \frac{10}{2+8} = 1\ \text{A}$$

- P.d. across $R_2 = I_1 R_2 = 1 \times 8 = 8V$.
- Hence p.d. across AB, i.e., the open-circuit voltage across the break, $E = 8V$
- Removing the source of e.m.f. gives the circuit



- Resistance,

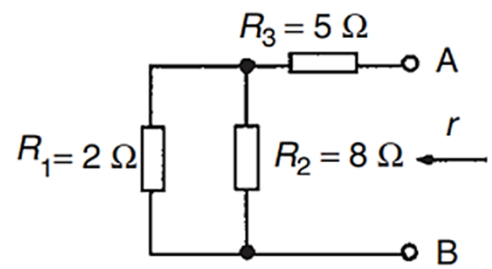
$$r = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

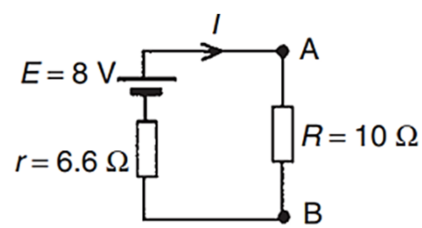
$$= 5 + \frac{2x8}{2+8} = 5 + 1.6 = 6.6\Omega$$

- The equivalent Thévenin's circuit is shown

$$I = \frac{E}{R+r} = \frac{8}{10+6.6}$$

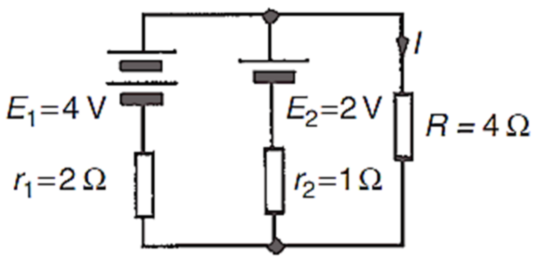
$$= \frac{8}{16.6} = 0.482\text{ A}$$



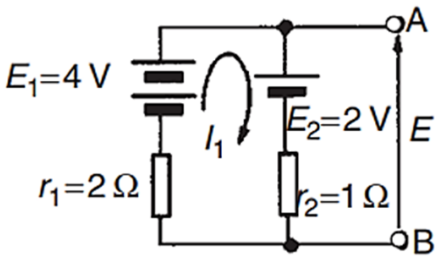


Example

- Use Thévenin's theorem to determine the current flowing in the 4 Ω resistor. Find also the power dissipated in the 4 Ω resistor.



- The 4 Ω resistor is removed from the circuit



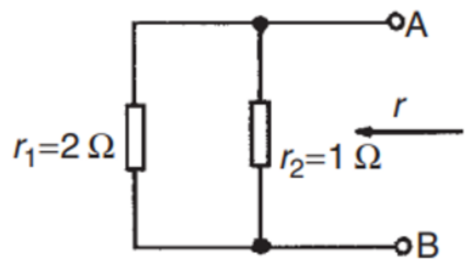
- Current

$$I_1 = \frac{E_1 - E_2}{r_1 + r_2} = \frac{4-2}{2+1} = \frac{2}{3}A$$

- P.d across AB,

$$E = E_2 + I_1 r_2 = 2 + \frac{2}{3}(1) = 2\left(\frac{2}{3}\right)V$$

- Removing the sources of e.m.f. gives



- Resistance

$$r = \frac{2x1}{2+1} = \frac{2}{3}\Omega$$

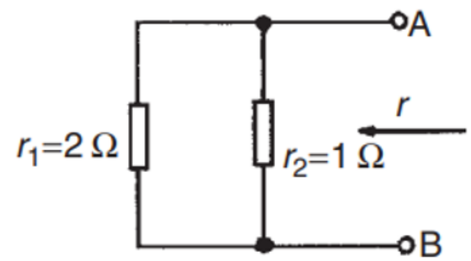
- Current in the 4Ω resistor

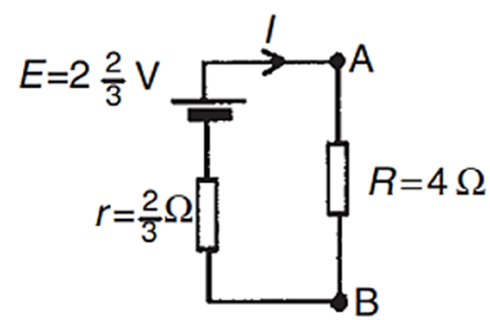
$$I = \frac{E}{r+R} = \frac{2\left(\frac{2}{3}\right)}{\frac{2}{3}+4}$$

$$= \frac{8/3}{14/3} = \frac{8}{14} = 0.571\ A$$

- Power dissipated in the 4Ω resistor,

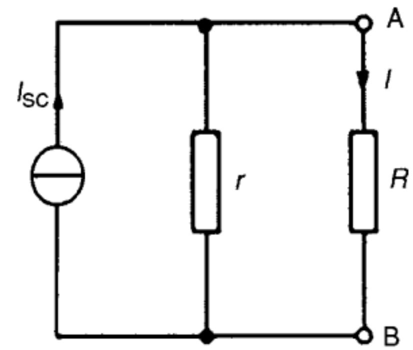
$$P = I^2R = (0.571^2)4 = 1.304\ W$$





Constant-current source

- Thévenin constant-voltage source consisted of a constant e.m.f. in series with an internal resistance.
- A source of electrical energy can also be represented by a constant-current source in parallel with a resistance.
  i. An ideal constant-voltage generator is one with zero internal resistance so that it supplies the same voltage to all loads.
  ii. An ideal constant-current generator is one with infinite internal resistance so that it supplies the same current to all loads.

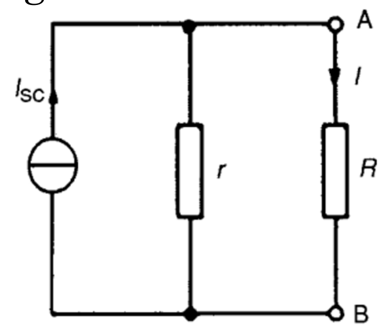


Norton’s theorem

- *The current that flows in any branch of a network is the same as that which would flow in the branch if it were connected across a source of electrical energy, the short-circuit current of which is equal to the current that would flow in a short-circuit across the branch, and the internal resistance of which is equal to the resistance which appears across the open-circuited branch terminals.*

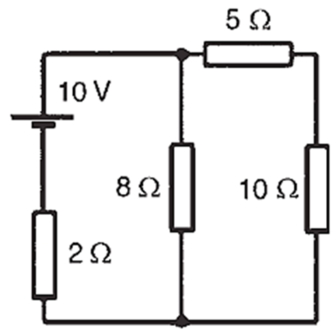
  i. short-circuit branch AB,
  ii. determine the short-circuit current I_{SC} flowing in the branch,
  iii. remove all sources of e.m.f. and replace them by their internal resistance, then determine the resistance r, ‘looking-in’ at a break made between A and B
  iv. determine the current flowing in resistance R from the Norton equivalent network.

$$I = \left(\frac{r}{r+R}\right) I_{sc}$$

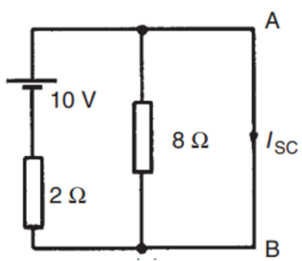


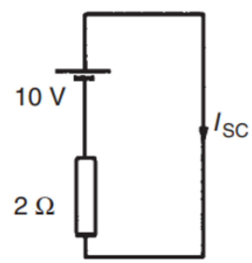
Example

- Use Norton's theorem to determine the current flowing in the 10 Ω resistance for the circuit



- The branch containing the 10 Ω resistance is short-circuited

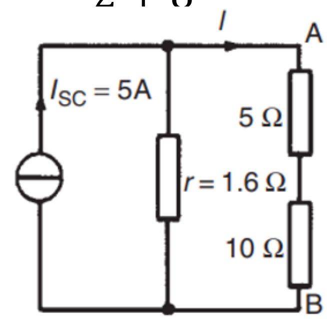




$$I_{SC} = \frac{10}{2} = 5\text{ A}$$

- If the 10 V source of e.m.f. is removed, the resistance 'looking-in' is given by

$$r = \frac{2\text{x}8}{2+8} = 1.6\Omega$$

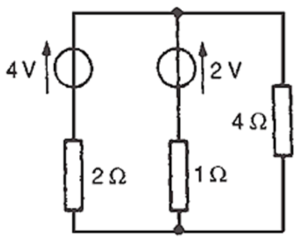


- Current in the 10 Ω resistance,

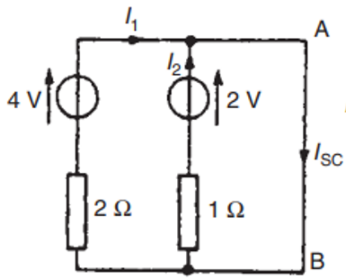
$$I = \left(\frac{1.6}{1.6+5+10}\right)(5) = 0.482\text{ A}$$

Example

- Use Norton's theorem to determine current flowing in the 4 Ω resistance



- The 4 Ω branch is short-circuited

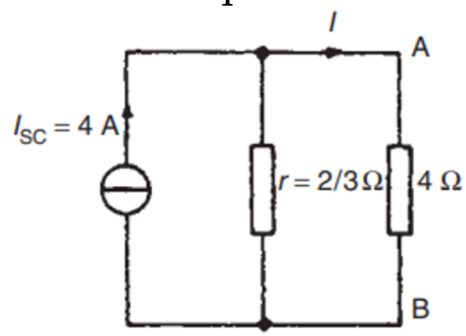


$$I_{SC} = I_1 + I_2 = \frac{4}{2} + \frac{2}{1} = 4\ \text{A}$$

- If the sources of e.m.f. are removed the resistance is:

$$r = \frac{2x1}{2+1} = \frac{2}{3}\Omega$$

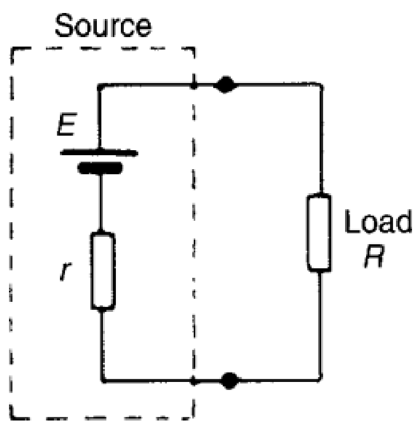
- From the Norton equivalent network



$$I = \left(\frac{\frac{2}{3}}{\frac{2}{3}+4}\right)(4) = 0.571\ \text{A}$$

Maximum power transfer theorem

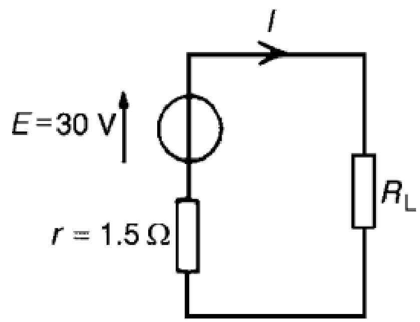
- *The power transferred from a supply source to a load is at its maximum when the resistance of the load is equal to the internal resistance of the source.*
- Hence, when R = r the power transferred from the source to the load is a maximum.



- Applications of the maximum power transfer theorem are found in
  i. Stereo amplifier design, seeking to maximise power delivered to speakers,
  ii. Electric vehicle design, seeking to maximise power delivered to drive a motor

Example

- A DC source has an open-circuit voltage of 30V and an internal resistance of 1.5 Ω. State the value of load resistance that gives maximum power dissipation and determine the value of this power.



- For maximum power dissipation, $R_L = r = 1.5\ \Omega$.

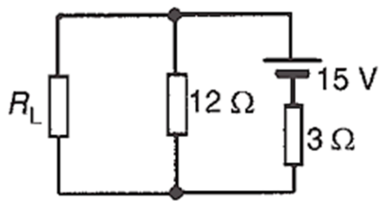
$$I = \frac{E}{(r + R_L)} = \frac{30}{(1.5 + 1.5)} = 10\ A$$

- Maximum power dissipated,

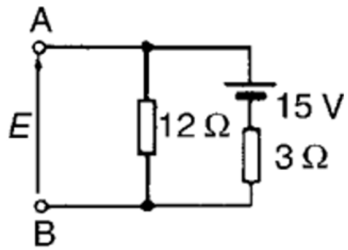
$$P = I^2 R_L = (10^2)(1.5) = 150W$$

Example

- Find the value of the load resistor R_L that gives maximum power dissipation and determine the value of this power.

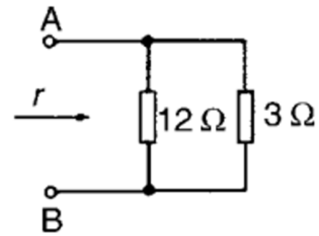


- Resistance R_L is removed from the circuit



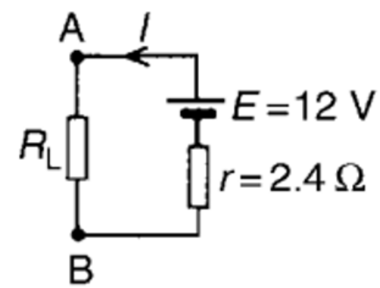
$$E = \left(\frac{12}{12+3}\right)(15) = 12V$$

- Removing the source of e.m.f.



$$r = \frac{12x3}{12+3} = 2.4\ \Omega$$

- Thévenin's circuit



$$I = \frac{12}{2.4+2.4} = 2.5A$$

$$P = I^2 R_L = 2.5^2(2.4) = 15\ W$$

End of session

Questions….?