BCT 2205 - Lecture 4

✓ **DC circuit theory - I**

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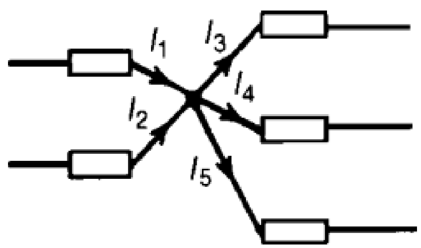
Introduction

- Laws which determine **currents** and **voltage** drops in DC networks are:
  i. Ohm's law,
  ii. Laws for resistors in series and in parallel
  iii. Kirchhoff's laws.
  iv. Superposition theorem
  v. Thévenin's theorem
  vi. Norton's theorem
  vii. Maximum power transfer theorem.

Kirchhoff's Current Law

- At any junction in an electric circuit, the **total current flowing into** that junction is equal to the **total current flowing out of** the junction, i.e., $\sum I = 0$.

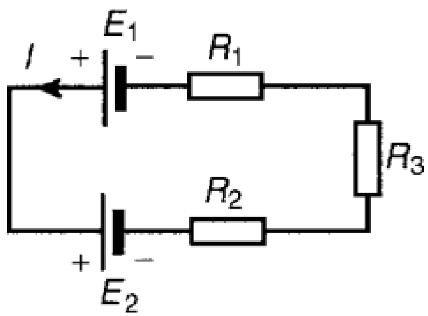


$$I_1 + I_2 = I_3 + I_4 + I_5$$

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

Kirchhoff's Voltage Law

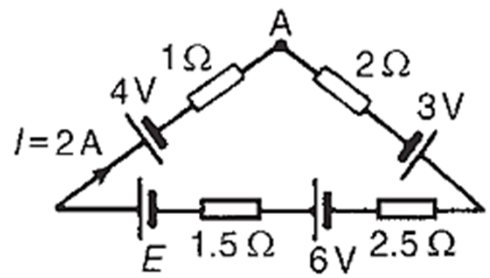
- In any closed loop in a network, the **algebraic sum** of voltage drops taken around the loop is equal to the **resultant** e.m.f. acting in that loop.



$$E_1 - E_2 = IR_1 + IR_2 + IR_3$$

- If current flows away from the positive terminal of a source, that source is considered positive.

- Determine the value of E



- Applying KVL and moving clockwise starting at point A:

$$3 + 6 + E - 4 = I(2 + 2.5 + 1.5 + 1)$$

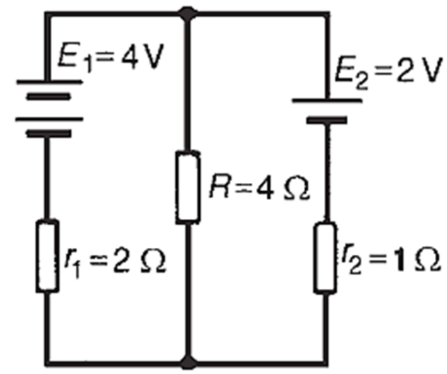
$$5 + E = I(7)$$

- Since $I = 2A$,

$$E = 14 - 5 = 9\,V$$

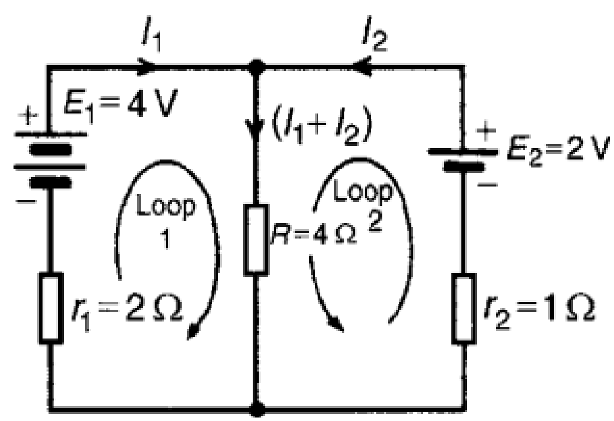
Example

- Use Kirchhoff's laws to determine the currents flowing in each branch of the network shown.



Solution

- Use KCL and label current directions on the original circuit diagram.
- **Current** flows **from the positive terminals** of the batteries.
- Branch currents are expressed in terms of I_1 and I_2 only, since the current through R is $(I_1 + I_2)$.

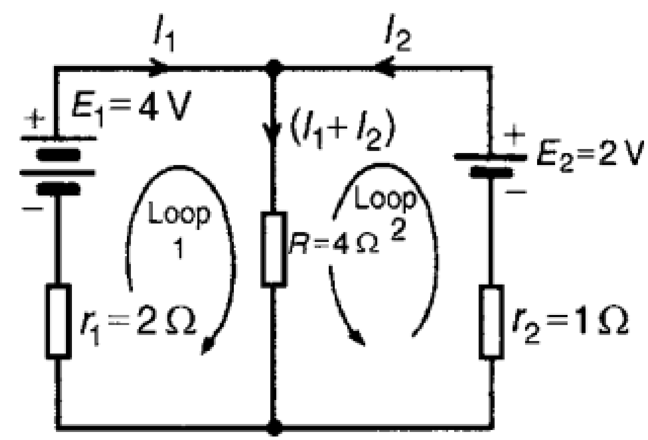


- Apply KVL voltage law to each loop. From loop 1, moving in a clockwise direction,

$$E_1 = I_1 r_1 + (I_1 + I_2)R$$

$$4 = 2I_1 + 4(I_1 + I_2)$$

$$6I_1 + 4I_2 = 4 \qquad (1)$$



- From loop 2, moving in an anticlockwise direction,

$$E_2 = I_2 r_2 + (I_1 + I_2)R$$

$$2 = 1I_2 + 4(I_1 + I_2)$$

$$4I_1 + 5I_2 = 2 \qquad (2)$$

- Solve Equations (1) and (2) for I_1 and I_2

$$2 \times (1) \text{ gives: } 12I_1 + 8I_2 = 8 \qquad (3)$$

$$3 \times (2) \text{ gives: } 12I_1 + 15I_2 = 6 \qquad (4)$$

$$(3) - (4) \text{ gives: } -7I_2 = 2 \ ; \ I_2 = -\frac{2}{7} = -0.286A$$

- From (1)

$$6I_1 + 4(-0.286) = 4$$

$$6I_1 = 4 + 1.144$$

$$I_1 = \frac{5.144}{6} = 0.857A$$

- Current flowing through resistance R is

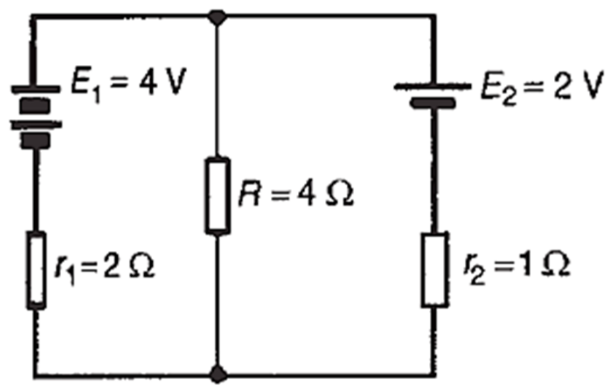
$$(I_1 + I_2) = 0.857 + (-0.286) = 0.571A$$

Superposition theorem

- In any network made up of linear resistances and containing more than one source of e.m.f., the resultant current flowing in any branch is the algebraic sum of the currents that would flow in that branch if each source was considered separately, all other sources being replaced at that time by their respective internal resistances.

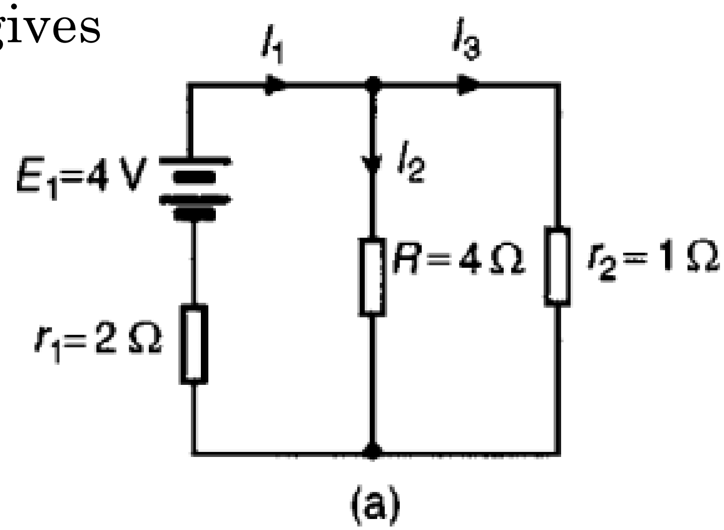
Example

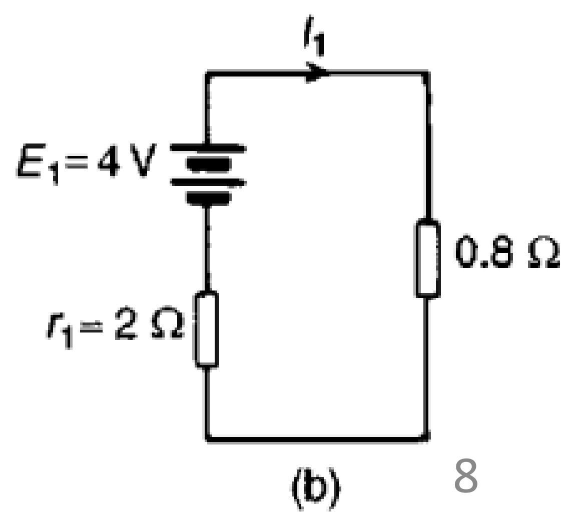
- Determine the current in each branch of the network, using the superposition theorem.



Example

- Redraw the original circuit with source E_2 removed, being replaced by r_2 only.
- Label the currents in each branch
- Current direction depends on the battery polarity, i.e., flows from the positive battery terminal.
- R in parallel with r_2 gives
- $\frac{4 \times 1}{4+1} = 0.8\Omega$





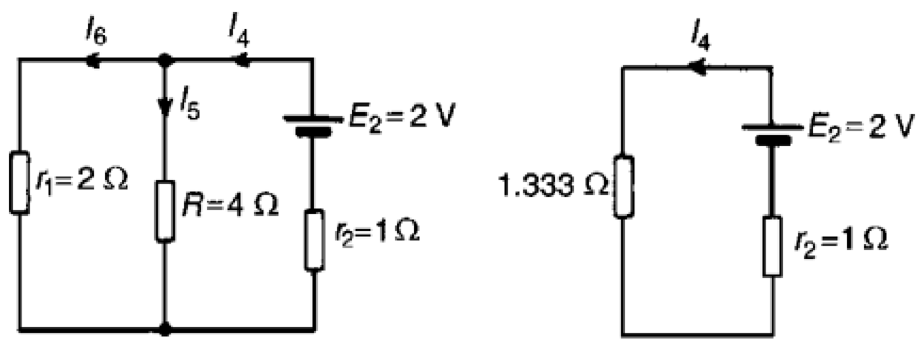
- From circuit (b),

$$I_1 = \frac{E_1}{r_1 + 0.8} = \frac{4}{2 + 0.8} = 1.429\ \text{A}$$

$$I_2 = \left(\frac{1}{4 + 1}\right) I_1 = \frac{1}{5}(1.429) = 0.286\ \text{A}$$

$$I_3 = \left(\frac{4}{4 + 1}\right) I_1 = \frac{4}{5}(1.429) = 1.143\ \text{A}$$

- Redraw the original circuit with E_1 replaced by r_1 only, as shown in Figure (a-b).



(a) (b)

- r_1 in parallel with R gives

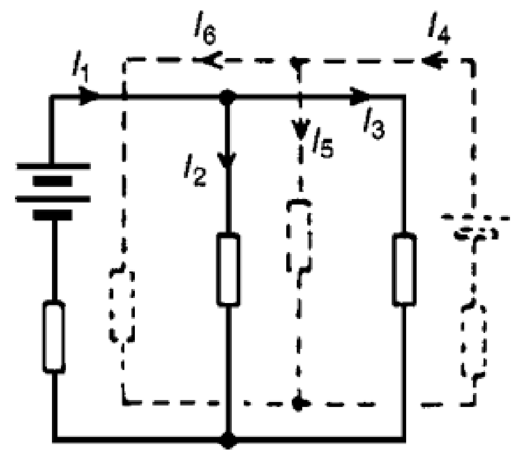
$$\frac{2 \times 4}{2 + 4} = \frac{8}{6} = 1.333\Omega$$

$$I_4 = \frac{E_2}{1.333 + r_2} = \frac{2}{1.333 + 1} = 0.857 \text{ A}$$

$$I_5 = \left(\frac{2}{2+4}\right) I_4 = \frac{2}{6}(0.857) = 0.286 \text{ A}$$

$$I_6 = \left(\frac{4}{2+4}\right) I_4 = \frac{4}{6}(0.857) = 0.571 \text{ A}$$

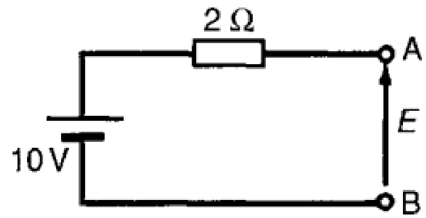
- Superimpose the two Figures:



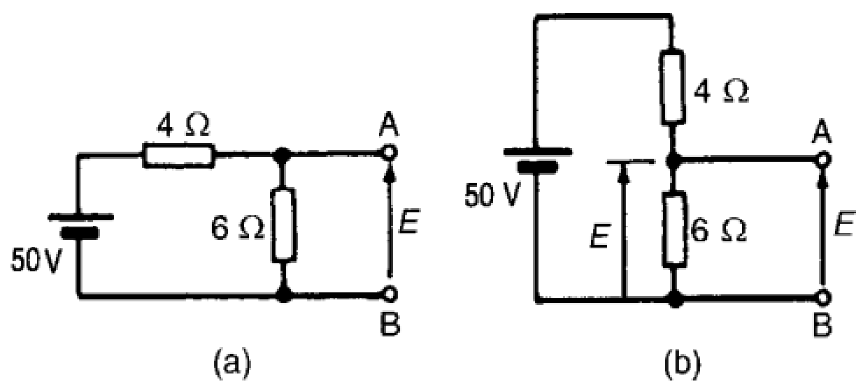
$$I_1 - I_6 = 1.429 - 0.571 = 0.858 \text{ A}$$
$$I_4 - I_3 = 0.857 - 1.143 = -0.286 \text{ A}$$
$$I_2 + I_5 = 0.286 + 0.286 = 0.572\text{A}$$

General DC circuit theory

- E across terminals AB is equal to 10V since no current flows through the 2 Ω resistor and hence no voltage drop occurs.



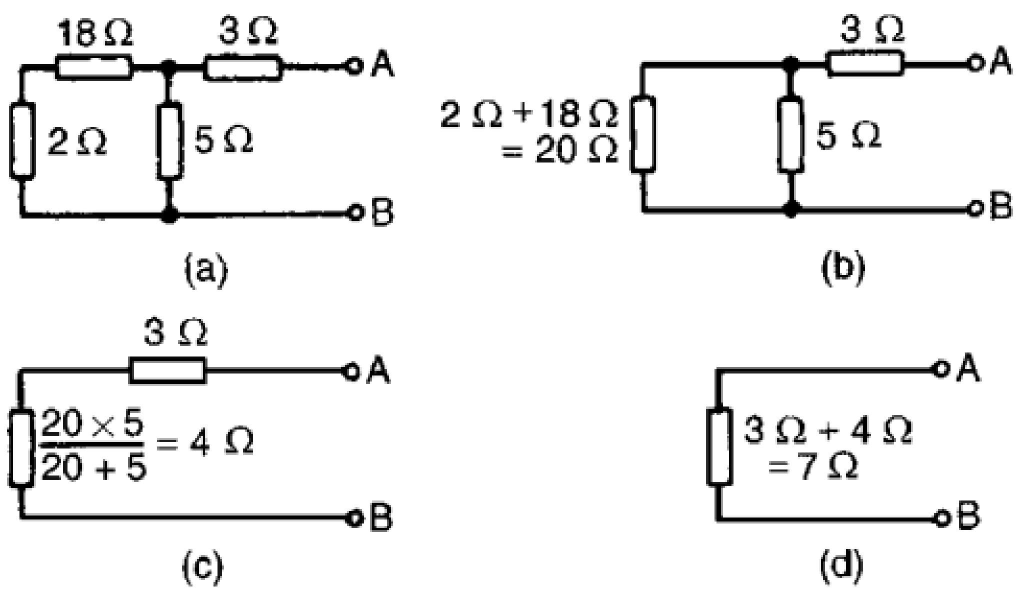
- E across terminals AB in (a) is the same as the voltage across the 6Ω resistor.



$$E = \left(\frac{6}{6+4}\right)50 = 30V$$

General DC circuit theory

- The resistance 'looking-in' at terminals AB in Figure (a) is obtained by reducing the circuit in stages as shown in Figures (b) to (d).



End of session

Questions….?