BCT 2205 - Lecture 2

✓ Resistors and Capacitors

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Resistance

- Resistance is the **opposition to the flow of current** offered by various components in a circuit.

- The resistance of an electrical conductor is **directly proportional to length**, L, and **inversely proportional to cross-sectional area**, A, i.e.

$$R \propto \frac{L}{A}$$

- Introducing a constant of proportionality,

$$R = \boldsymbol{\rho} \frac{L}{A}$$

- The **constant of proportionality** is known as the resistivity of the material given by the symbol ρ, in ohmmeter, Ωm.

Examples

- The resistance of 1.5km wire of cross-sectional area 0.17mm^2 is 150Ω. Determine the resistivity of the wire. (*ensure you convert all quantities to their S.I. Units)

$$R = \rho \frac{L}{A}; \qquad \rho = \frac{RA}{L} = \frac{(150)(0.17\mu)}{1500} = 0.017\mu\Omega\text{m}$$

- Calculate the resistance of 10m length copper wires with a cross-section area of 1mm^2. Resistivity of copper is $1.76\ x\ 10^{-8}\Omega m$.

$$R = \rho \frac{L}{A} = \frac{1.76x10^{-8}x10}{1x10^{-6}} = 0.176\Omega$$

- Calculate the cross-sectional area of a piece of aluminum wire 100m long and having a resistance of 2 Ω, given resistivity of aluminum is $0.03x10^{-6}\Omega m$.

$$R = \rho \frac{L}{A}; \qquad A = \rho \frac{L}{R}; \qquad A = \frac{0.03\mu\ x\ 100}{2} = 1.5mm^2$$

Resistor Color Coding

- Resistor **color coding represents resistance** using color bands.
- The bands are **nearer to each other on one end** of the resistor than the other; this is the **end we start counting**.

<table>
<tr><th>Resistor color codes</th><th>Color numerical value</th><th>Tolerances</th></tr>
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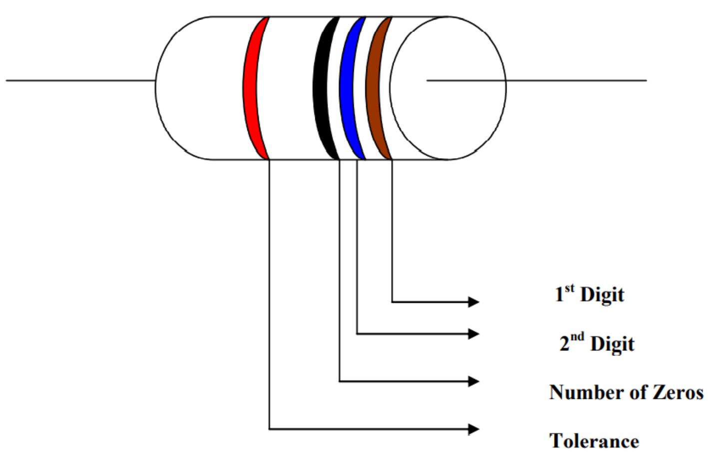
<td>
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<td><table>
<tr><td>Black - 0</td><td>Green - 5</td></tr>
<tr><td>Brown – 1</td><td>Blue – 6</td></tr>
<tr><td>Red – 2</td><td>Violet – 7</td></tr>
<tr><td>Orange -3</td><td>Grey – 8</td></tr>
<tr><td>Yellow – 4</td><td>White – 9</td></tr>
</table></td>
<td><table>
<tr><td>Brown</td><td>±1%</td></tr>
<tr><td>Red</td><td>±2%</td></tr>
<tr><td>Gold</td><td>±5%</td></tr>
<tr><td>Silver</td><td>±10%</td></tr>
<tr><td>None</td><td>±20%</td></tr>
</table></td>
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Converting from resistance to color bands.

i. $36 \times 10^4\ \Omega \pm 1\%$ Equals: Orange, Blue, Yellow, Brown

ii. $4500\Omega \pm 2\%$ Equals: Yellow, Green, Red, Red.

Converting from color bands to resistance.

Band	*Color*	*Numeric value*
1	Red	2
2	Black	0
3	Orange	10^3
4	Brown	±1%
	Equivalent resistance	$20 \times 10^3\ \Omega \pm 1\%$

Band	*Color*	*Numeric value*
1	Green	5
2	Brown	1
3	Yellow	10^4
4	-	±20%
	Equivalent resistance	$51 \times 10^4\ \Omega \pm 20\%$

Resistors in Series

- The source **current is the same** throughout the circuit, i.e.,

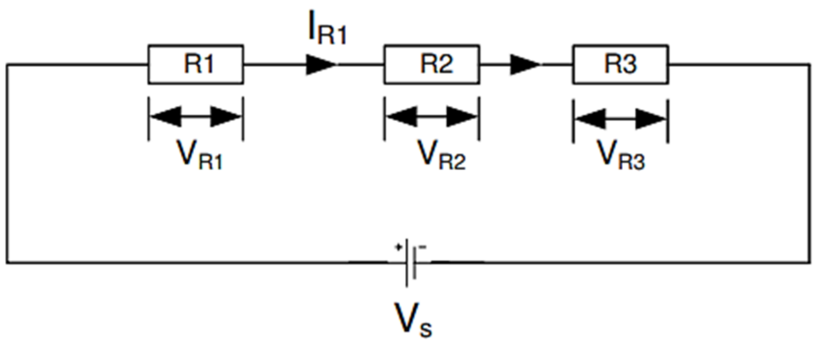
$$I = I_{R1} = I_{R2} = I_{R3}$$

- The source **voltage** is equal to the **sum** of all p.d.s across the resistors in the circuit i.e.,

$$V_S = V_{R1} + V_{R2} + V_{R3}$$
$$IR_t = I_{R1}R_1 + I_{R2}R_2 + I_{R3}R_3$$
$$IR_t = I(R_1 + R_2 + R_3)$$

- The total circuit resistance equals the sum of all resistances in the circuit i.e.,

$$R_t = R_1 + R_2 + R_3$$



Example

- Resistors R_1= 900Ω, R_2= 5K8Ω, R_3= 7M4Ω are connected in series across voltage supply of 200V.
  i. Calculate the total resistance.
  ii. Calculate the total current.
  iii. Determine the current passing through each of the resistors.
  iv. Voltage across each resistor.

$$R_t = 900 + 5800 + 7.4M\Omega$$

$$= 7406700\Omega$$

$$I = \frac{V_s}{R_t} = \frac{200}{7406700} = 2.7\ x\ 10^{-5}A$$

$$V = IR$$

$$V_1 = 2.7\ x\ 10^{-5}\ x\ 900 = 0.0243\ V$$

$$V_2 = 2.7\ x\ 10^{-5}\ x\ 5.8k = 0.1566\ V$$

$$V_3 = 2.7\ x\ 10^{-5}\ x\ 7.4M = 199.8\ V$$

Resistors in Parallel

- The voltage in a parallel circuit remains the same across any branch i.e.,

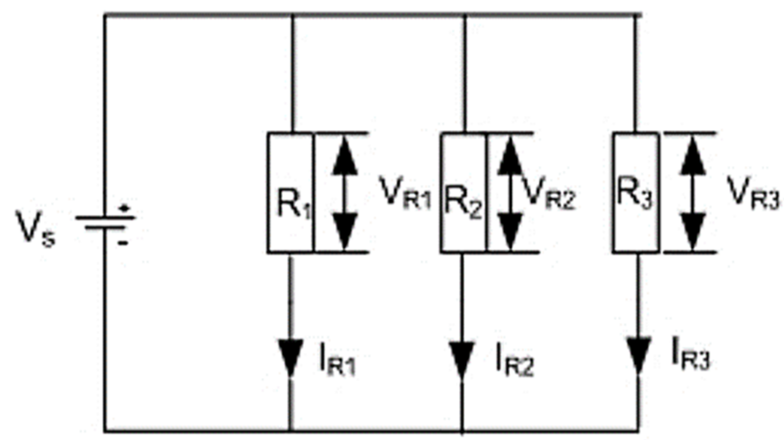
$$V_s = V_{R1} = V_{R2} = V_{R3}$$

- The total circuit current is the sum of all the branch currents i.e.,

$$I_t = I_{R1} + I_{R2} + I_{R3}$$

$$I_t = \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} + \frac{V_{R3}}{R_3}$$

$$I_t = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$



- The circuit resistance is obtained using

$$\frac{1}{R_t} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Example

- Resistors $R_1 = 900\Omega$, $R_2 = 5K8\Omega$, $R_3 = 7M4\Omega$ are connected in parallel across voltage supply of 200V.

i. Calculate the total resistance.

ii. Calculate the total current.

iii. Determine the current passing through each of the resistors.

iv. Current through each resistor.

$$\frac{1}{R_t} = \left(\frac{1}{900} + \frac{1}{5800} + \frac{1}{7.4M}\right)$$

$$R_t = 779.022\Omega$$

$$I_t = \frac{V_s}{R_t} = \frac{200}{779.022} = 0.257\ A$$

$$V_{Rt} = V_{R1} = V_{R2} = V_{R3} = 200V$$

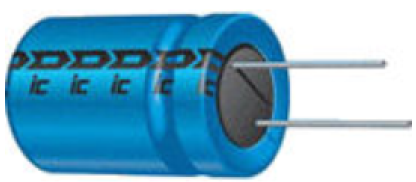
$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{200}{900} = 0.222\ A$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{200}{5800} = 0.034\ A$$

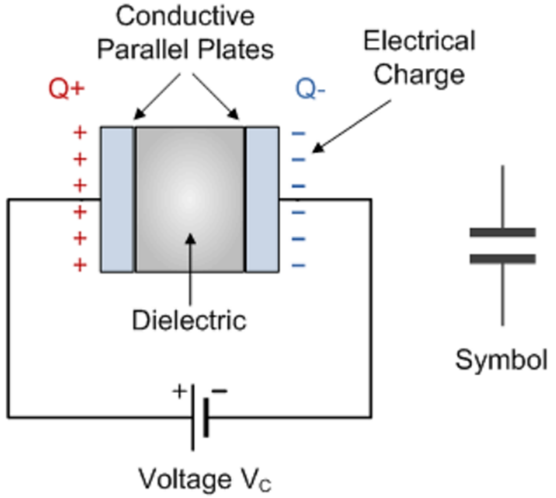
$$I_{R3} = \frac{V_{R3}}{R_3} = \frac{200}{7.4M} = 2.703\ x\ 10^{-5}\ A$$

Capacitors

- A capacitor consists of two plates separated by an insulating material known as a dielectric.
- Capacitors are simple passive devices used to store electric charge on their plates when connected to a voltage source.
- Capacitors can be used in different applications and circuits mostly to maintain the voltage at a certain level.



A Typical Capacitor



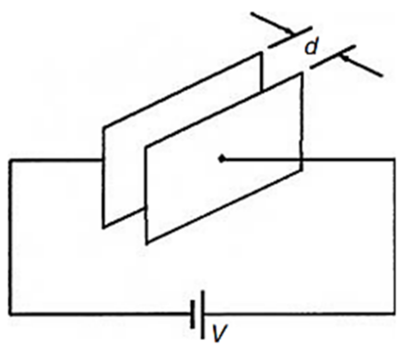
Capacitance

- The plates of capacitor are connected to a battery of V volts, where d is the distance between the plates.
- The property of a capacitor to store charge is called capacitance, C.

$$C = \frac{Q}{V}$$

- The unit of capacitance is the farad F.
- The charge Q stored in a capacitor is given by,

$$Q = I \times t \; Coulombs$$



Examples

- Determine the p.d. across a 4 µF capacitor when charged with 5 mC.

$$V = \frac{Q}{C} = \frac{5m}{4\mu} = 1.25kV$$

- Find the charge on a 50pF capacitor when the voltage applied to it is 2 kV.

$$Q = CV = 50\ x\ 10^{-12} x\ 2k\ =\ 100\ x\ 10^{-9} C\ =\ 0.1\mu C$$

- A direct current of 4A flows into a previously uncharged 20µF capacitor for 3ms. Determine the p.d. between the plates.

$$Q\ =\ It = 4\ x\ 3m = 12mC$$

$$Q = CV; \quad V = \frac{Q}{C} = \frac{12m}{20\mu} = 600V$$

Capacitors in parallel

- The applied voltage is common to all capacitors i.e.,

$$V_s = V_{c1} = V_{c2} = V_{c3}$$

- However, the current through each capacitor is different, thus, the charge developed across each capacitor is different.

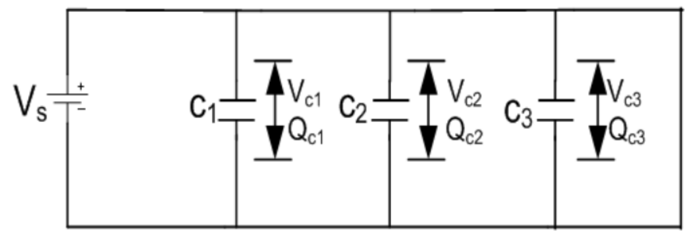
- The total charge is the sum of charges in each capacitor, i.e.,

$$Q_t = Q_{c1} + Q_{c2} + Q_{c3}$$

- Using $Q = CV$

$$V_s C_t = V_s(C_1 + C_2 + C_3)$$

$$C_t = C_1 + C_2 + C_3$$



Example

- Capacitances $1\mu F$, $3\mu F$, $5\mu F$ and $6\mu F$ are connected in parallel to a voltage supply of 100V. Determine the:

i. Equivalent circuit capacitance

ii. Total charge

iii. Charge on each capacitor

$$C = 1 + 3 + 5 + 6 = 15\mu F$$

$$Q = CV = 15\mu \ x\, 100 = 1.5mC$$

$$Q_1 = C_1V = 1\mu \, x\, 100 = 0.1mC$$

$$Q_2 = C_2V = 3\mu \, x\, 100 = 0.3mC$$

$$Q_3 = C_3V = 5\mu \, x\, 100 = 0.5mC$$

$$Q_4 = C_4V = 6\mu \, x\, 100 = 0.6mC$$

Capacitors in series

- In a series circuit, charge Q is constant since current flowing is the same through the capacitors, i.e.,

$$Q_t = Q_{c1} = Q_{c2} = Q_{c3}$$

- However, voltage across the capacitors is different. The applied voltage,

$$V_s = V_{c1} + V_{c2} + V_{c3}$$

- From $V = Q/C$

$$V_s = \frac{Q_t}{C_t} = \frac{Q_{c1}}{C_1} + \frac{Q_{c2}}{C_2} + \frac{Q_{c3}}{C_3}$$

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Example

i. Capacitances of $3\mu F$, $6\mu F$ and $12\mu F$ are connected in series across a 350V supply. Calculate:

a. The equivalent capacitance

b. Charge on each capacitor

c. Voltage across each capacitor

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{7}{12} \qquad ; C = \frac{12}{7}\mu F$$

$$Q = CV = \frac{12}{7}\mu \, x \, 350 = 0.6mC$$

- Voltage across each capacitor

$$V_1 = \frac{Q}{C_1} = \frac{0.6m}{3\mu} = 200V$$

$$V_2 = \frac{Q}{C_2} = \frac{0.6m}{6\mu} = 100V$$

$$V_3 = \frac{Q}{C_3} = \frac{0.6m}{12\mu} = 50V$$

End of session

Questions….?