DIGITAL ELECTRONICS

CHAPTER THREE: *Simplification of Boolean Expressions Examples

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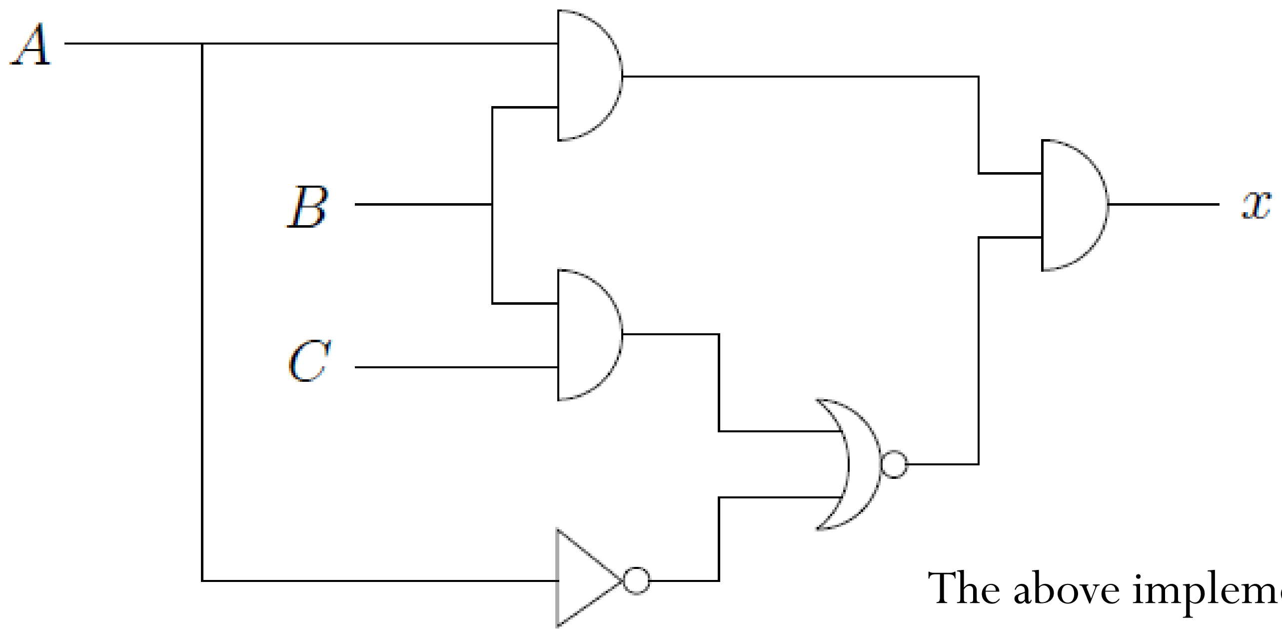
3.1 Simplification of Boolean Expressions

Introduction:

- Given the below Boolean function to implement:

$$x = AB\left(\overline{\overline{A} + BC}\right)$$

- It can be implemented as below:



The above implementation requires 5 logic gates.

- Suppose now we decided to simplify the expression before implementing it. From:

$$x = AB\left(\overline{\overline{A} + BC}\right)$$

i. Using De Morgan's theorems we get:

$$x = AB\left(\overline{\overline{A}} \cdot \overline{BC}\right)$$

$\overline{\bar{A} + BC} = \bar{\bar{A}}.\overline{BC}$

1:	$A + B = B + A$	$A \cdot B = B \cdot A$
2:	$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
3:	$A + B \cdot C = (A + B) \cdot (A + C)$	$A \cdot (B + C) = A \cdot B + A \cdot C$
4:	$A + A \cdot B = A$	$A \cdot (A + B) = A$
5:	$A + \overline{A} \cdot B = A + B$	$A \cdot (\overline{A} + B) = A \cdot B$
6:	$A \cdot B + A \cdot \overline{B} = A$	$(A + B) \cdot (A + \overline{B}) = A$
7:	$A \cdot B + \overline{A} \cdot C = (A + C) \cdot (\overline{A} + B)$	$(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot B$
8:	$A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$	$(A + B) \cdot (\overline{A} + C) \cdot (B + C) = (A + B) \cdot (\overline{A} + C)$
9:	$\overline{A + B + C + \cdots} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdots$	$\overline{A \cdot B \cdot C \cdots} = \overline{A} + \overline{B} + \overline{C} + \cdots$

ii. Simplifying further we get:

$$= AB\left(A(\overline{B}+\overline{C})\right)$$

$\overline{BC} = \bar{B} + \bar{C}$

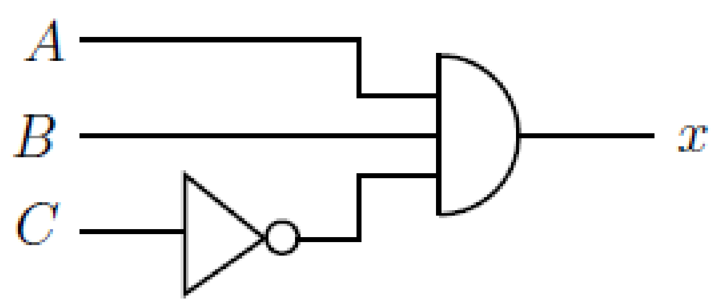
Opening the brackets $$= AB\left(A\overline{B} + A\overline{C}\right)$$

$$= ABA\overline{B} + ABA\overline{C}$$

Since $B\overline{B} = 0$ and AA=A

$$= AB\overline{C}$$

- The simplified circuit can now be implemented using two gates as below:



3.1 Simplification of Boolean Expression

- The previous example shows it is necessary to reduce Boolean expressions before implementing them as it makes the final circuit:

i. Cheaper - less gates used, needs a smaller circuit board.

ii. More reliable as there are fewer interconnections.

iii. Have a lower power consumption.

- There are two methods of simplifying logic expressions:

i. **Algebraic** Simplification - Uses theorems of Boolean Algebra.

ii. The **Karnaugh map** method - Graphical.

3.1.1 Algebraic Simplification

- To use this method, you need to know the theorems of Boolean algebra very well.
- You shall need a lot of practice to build your skills for Algebraic Simplification.
- Generally, there are two steps when using this method:

I. Put the expression in Sum of Products form (*not* Product of Sums form). This may require the use of De Morgan's theorem or the distributive rules.

II. Check for common factors and factor out whenever possible. Factoring usually results in the elimination of some of the terms.

Example 1 (Approach I)

Simplify algebraically:

$$ABC + AB\bar{C} + A\bar{B}C$$

- Since the expression is already in SOP form, so we shall just factor out the common terms:

$$AB\{C + \bar{C}\} + A\bar{B}C$$

- But, $AB + A\bar{B}C = \mathrm{A}\{B + \bar{B}C\}$
- Hence we have, $A\{B + \bar{B}C\}$
- This reduces to $A\{B + C\}$
- Opening the brackets we get: $AB + AC$

Using the theorems

$C + \bar{C} = 1$

5: $A + \overline{A} \cdot B = A + B$

6: $A \cdot B + A \cdot \overline{B} = A$

Example 1 (Approach II)

Simplify algebraically:

$$ABC + AB\bar{C} + A\bar{B}C$$

- Since the expression is already in SOP form, so we shall just factor out the common terms:

$$A\{BC + B\bar{C} + \bar{B}C\}$$

- But, $BC + B\bar{C} = B$
- Hence we have, $A\{B + \bar{B}C\}$
- This reduces to $A\{B + C\}$
- Opening the brackets we get: $AB + AC$

Using the theorems

5: $A + \overline{A} \cdot B = A + B$

6: $A \cdot B + A \cdot \overline{B} = A$

Example 2

Simplify ABC + ABC` + AB`C + A`BC

Since the expression is in the SOP we proceed to factor out common terms:

AB{C + C`} + AB`C + A`BC

AB{1} + AB`C + A`BC

AB+ AB`C + A`BC

B(A+A`C)+ AB`C

B{A+C}+AB`C

AB+BC+AB`C

A{B+B`C}+BC

A{B+C}+BC

AB+AC+BC

Using the theorems
$C + \bar{C} = 1$

Example 3

- **Show that**

ABC + ABC` + AB`C + A`BC = AB + AC + BC

It is easier to prove the RHS of the expression.

AB + AC + BC

We note that this is in SOP form. We now need to express it in the Standard SOP for it to match the expression on the right.

AB{C +C`} + AC{B +B`} + BC{A +A`}

ABC +ABC` + ABC +AB`C + ABC +A`BC

ABC +ABC`+AB`C+A`BC

The RHS=LHS hence proved.

Example 4

Simplify the expression:

ABC + AB`C + A`BC +A`B`C+A`B`C`

Solution:

AC{B + B`} + A`C{B +B`}+A`B`C`

AC{1} + A`C{1}+A`B`C`

AC + A`C+A`B`C`

C{ A+ A`}+A`B`C`

C{1}+A`B`C`

C+A`B`C`

C+A`B`

Using the theorems

5: $A + \overline{A} \cdot B = A + B$

6: $A \cdot B + A \cdot \overline{B} = A$

Example 5

Simplify $x = ABC + (\overline{A} + \overline{B})\left(\overline{\overline{A}} + \overline{\overline{C}}\right)$

X=ABC + (A`+B`)(A``+C``)

Since A``=**A and C``=C**

The expression becomes:

X=ABC + (A`+B`)(A+C)

Opening the brackets so that we have the SOP form

X=ABC + A`A+A`C+B`A+B`C

But A`A=0

Hence X=ABC +A`C+B`A+B`C

X=C(AB+A`)+B`A+B`C

X= C(A`+B)+B`A+B`C

X=CA`+CB+B`A+B`C

X=CA`+B`A+C{B+B`}=CA`+B`A+C

X=C{A`+1}+B`A=C+B`A

Using the theorems

5: $A + \overline{A} \cdot B = A + B$

6: $A \cdot B + A \cdot \overline{B} = A$

Example 6

- A student may register for course X only if he satisfies the following conditions:
- (1) Has completed at least 20 courses AND is a Computing student AND of good conduct, OR
- (2) Has completed at least 20 courses AND is a Computing student AND has departmental approval, OR
- (3) Has completed fewer than 20 courses AND is a Computing student AND not of good conduct, OR
- (4) Is of good conduct AND has departmental approval, OR
- (5) Is a Computing student AND does not have departmental approval.

Solution

Let the listed conditions be represented by the letters below:

- A: Has completed at least 20 courses
- B: Is a Computing student
- C: Is of good conduct
- D: Has departmental approval
- Y: Student may register for course X

Y=ABC+ABD+A`BC`+CD+BD`

We can now simplify this expression.

Solution

Y=ABC+ABD+A`BC`+CD+BD`

Y=ABC+A`BC`+CD+B{D`+DA}

Y=ABC+A`BC`+CD+B{D`+A}

Y=ABC+A`BC`+CD+BD`+BA

Y=AB{C+1}+A`BC`+CD+BD`

Y=AB+A`BC`+CD+BD`

Y=B{A+A`C`}+CD+BD`

Y=B{A+C`}+CD+BD`

Y=AB+BC`+CD+BD` *Rewrite **DC+D`B** to match theorem #8

Y=AB+BC`+DC+D`B+CB=AB+BC`+CD+BD`+CB

Y=AB+B{C`+C}+CD+BD`=AB+B+CD+BD`=B(A+1)+CD+BD`

Y=B+CD+BD`=B{D`+1}+CD = B+CD

Using the theorems

5: $A+\overline{A}\cdot B = A+B$

6: $A\cdot B + A\cdot\overline{B} = A$

8: $A\cdot B+\overline{A}\cdot C+B\cdot C = A\cdot B+\overline{A}\cdot C$

Hence a student may register for the course X if he is a Computing student OR he is of good conduct AND has departmental approval.

End of session

Questions….?