

FOUNDATION



Typeclass

Type and Operations

```
case class Person(name: String, age: Int)

def setAge(p: Person, newAge: Int): Person =
  p.copy(age = newAge)

def isAdult(p: Person): Boolean =
  p.age >= 18
```



Polymorphism(s)



Inheritance

```
sealed trait Shape

case class Circle(radius: Double) extends Shape
case class Rectangle(width: Double, height: Double) extends Shape

def double(shape: Shape): Shape =
  shape match {
    case Circle(x)      => Circle(2 * x)
    case Rectangle(w, h) => Rectangle(2 * w, 2 * h)
  }
```



Parametric Polymorphism

```
def headOption[A](xs: List[A]): Option[A] =  
  xs match {  
    case Nil      => None  
    case x :: _   => Some(x)  
  }
```

```
scala> headOption(List(1,2,3))  
res0: Option[Int] = Some(1)  
  
scala> headOption(List("Hello", "World"))  
res1: Option[String] = Some(Hello)
```



Row Polymorphism

```
case class Person(name: String, age: Int)

case class Employee(name: String, age: Int, companyName: String)

def isAdult(x: {val age: Int}): Boolean =
  x.age >= 18
```

```
scala> isAdult(Person("John", 14))
res2: Boolean = false

scala> isAdult(Employee("Eva", 22, "Foo Inc"))
res3: Boolean = true
```



Ad hoc Polymorphism

```
scala> 2 + 3  
res4: Int = 5
```

```
scala> 2.0 + 3.0  
res5: Double = 5.0
```

```
scala> BigDecimal(2) + BigDecimal(3)  
res6: scala.math.BigDecimal = 5
```

```
scala> List(1,2,3).map(_ + 1)  
res7: List[Int] = List(2, 3, 4)
```

```
scala> Vector(1,2,3).map(_ + 1)  
res8: scala.collection.immutable.Vector[Int] = Vector(2, 3, 4)
```

```
scala> Person("John", 34).toString  
res9: String = Person(John,34)
```

```
scala> false.toString  
res10: String = false
```



Plan

- Typeclass definition and comparison with other approaches
- Most common usage for **application developers**
- Use case: folding data



Use case: Equality



Top Level

```
class Object {  
  def equals(other: Any): Boolean =  
    this == other  
}
```

```
case class Foo(i: Int, s: String)
```



Top Level

```
class Object {  
  def equals(other: Any): Boolean =  
    this == other  
}
```

```
case class Foo(i: Int, s: String) extends Object {  
  override def equals(obj: Any): Boolean =  
    obj match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```



Top Level

```
class Object {  
  def equals(other: Any): Boolean =  
    this == other  
}
```

```
case class Foo(i: Int, s: String) extends Object {  
  override def equals(obj: Any): Boolean =  
    obj match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
scala> Foo(5, "Hello").equals(Foo(5, "Hello"))  
res11: Boolean = true
```

```
scala> 5.equals(6)  
res12: Boolean = false
```



Top Level

```
class Object {  
  def equals(other: Any): Boolean =  
    this == other  
}
```

```
case class Foo(i: Int, s: String) extends Object {  
  override def equals(obj: Any): Boolean =  
    obj match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
scala> Foo(5, "Hello").equals(3)  
res13: Boolean = false
```



Top Level

```
class Object {  
  def equals(other: Any): Boolean =  
    this == other  
}
```

```
case class Foo(i: Int, s: String) extends Object {  
  override def equals(obj: Any): Boolean =  
    obj match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
val inc1: Int => Int = _ + 1  
val inc2: Int => Int = _ + 1
```

```
scala> inc1.equals(inc2)  
res14: Boolean = false
```



Mixin Trait

```
trait Eq {  
  def eqv(other: Any): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq {  
  def eqv(other: Any): Boolean =  
    other match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```



Mixin Trait

```
trait Eq {  
  def eqv(other: Any): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq {  
  def eqv(other: Any): Boolean =  
    other match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
def contains[A <: Eq](xs: List[A], value: A): Boolean =  
  xs.foldRight(false)((x, acc) => x.eqv(value) || acc)
```



Mixin Trait

```
trait Eq {  
  def eqv(other: Any): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq {  
  def eqv(other: Any): Boolean =  
    other match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
scala> Foo(5, "Hello").eqv(7)  
res15: Boolean = false
```



Mixin Trait

```
trait Eq {  
  def eqv(other: Any): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq {  
  def eqv(other: Any): Boolean =  
    other match {  
      case x: Foo => i == x.i && s == x.s  
      case _      => false  
    }  
}
```

```
scala> Foo(5, "Hello").eqv(7)  
res15: Boolean = false
```

```
scala> 5.eqv(3)  
      5.eqv(3)  
      ^
```

On line 2: error: value eqv is not a member of Int



F-Bounded Trait

```
trait Eq[A <: Eq[A]]{  
  def eqv(other: A): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq[Foo] {  
  def eqv(other: Foo): Boolean =  
    i == other.i && s == other.s  
}
```



F-Bounded Trait

```
trait Eq[A <: Eq[A]]{  
  def eqv(other: A): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq[Foo] {  
  def eqv(other: Foo): Boolean =  
    i == other.i && s == other.s  
}
```

```
def contains[A <: Eq[A]](xs: List[A], value: A): Boolean =  
  xs.foldRight(false)((x, acc) => x.eqv(value) || acc)
```



F-Bounded Trait

```
trait Eq[A <: Eq[A]]{  
  def eqv(other: A): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq[Foo] {  
  def eqv(other: Foo): Boolean =  
    i == other.i && s == other.s  
}
```

```
scala> Foo(5, "Hello").eqv(7)  
      Foo(5, "Hello").eqv(7)  
                        ^  
On line 2: error: type mismatch;  
found   : Int(7)  
required: Foo
```



F-Bounded Trait

```
trait Eq[A <: Eq[A]]{  
  def eqv(other: A): Boolean  
}
```

```
case class Foo(i: Int, s: String) extends Eq[Foo] {  
  def eqv(other: Foo): Boolean =  
    i == other.i && s == other.s  
}
```

```
scala> Foo(5, "Hello").eqv(7)  
      Foo(5, "Hello").eqv(7)  
                        ^  
On line 2: error: type mismatch;  
found   : Int(7)  
required: Foo
```

```
scala> 5.eqv(7)  
      5.eqv(7)  
      ^  
On line 2: error: value eqv is not a member of Int
```



Overloading

```
def eqv(x: Int, y: Int): Boolean = x == y
def eqv(x: String, y: String): Boolean = x == y
def eqv(x: Foo, y: Foo): Boolean = eqv(x.i, x.i) && eqv(x.s, y.s)
```



Overloading

```
def eqv(x: Int, y: Int): Boolean = x == y
def eqv(x: String, y: String): Boolean = x == y
def eqv(x: Foo, y: Foo): Boolean = eqv(x.i, y.i) && eqv(x.s, y.s)
```

```
def contains[A](xs: List[A], value: A): Boolean = ???
```



Overloading

```
def eqv(x: Int, y: Int): Boolean = x == y
def eqv(x: String, y: String): Boolean = x == y
def eqv(x: Foo, y: Foo): Boolean = eqv(x.i, y.i) && eqv(x.s, y.s)
```

```
def contains[A](xs: List[A], value: A)(compare: (A, A) => Boolean): Boolean =
  xs.foldRight(false)((x, acc) => compare(x, value) || acc)
```



Overloading

```
def eqv(x: Int, y: Int): Boolean = x == y
def eqv(x: String, y: String): Boolean = x == y
def eqv(x: Foo, y: Foo): Boolean = eqv(x.i, y.i) && eqv(x.s, y.s)
```

```
def contains[A](xs: List[A], value: A)(compare: (A, A) => Boolean): Boolean =
  xs.foldRight(false)((x, acc) => compare(x, value) || acc)
```

```
scala> contains(List(1,2,3,4,5), 6)(eqv)
res19: Boolean = false
```

```
scala> contains(List("hello", "world"), "world")(eqv)
res20: Boolean = true
```

```
scala> contains(List(1,2,3,4,5), "hello")(eqv)
      contains(List(1,2,3,4,5), "hello")(eqv)
                                   ^
```

```
On line 2: error: type mismatch;
  found   : (x: Foo, y: Foo)Boolean <and> (x: String, y: String)Boolean <and> (x: Int, y: Int)Boolean
  required: (Any, Any) => Boolean
```



Overloading

```
def exact(x: Double, y: Double): Boolean =  
  x == y  
  
def approx(error: Double)(x: Double, y: Double): Boolean =  
  (x - y).abs < error
```



Overloading

```
def exact(x: Double, y: Double): Boolean =  
  x == y  
  
def approx(error: Double)(x: Double, y: Double): Boolean =  
  (x - y).abs < error
```

```
scala> val xs = List(1.0, 2.5, 3.3)  
xs: List[Double] = List(1.0, 2.5, 3.3)
```

```
scala> contains(xs, 3.2)(exact)  
res22: Boolean = false
```

```
scala> contains(xs, 3.2)(approx(0.2))  
res23: Boolean = true
```

```
scala> contains(xs, 3.2)(approx(0.001))  
res24: Boolean = false
```



Interface

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
val fooEq: Eq[Foo] = new Eq[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean = x.i == y.i && x.s == y.s  
}
```

```
val intEq: Eq[Int] = new Eq[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
}
```



Interface

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
val fooEq: Eq[Foo] = new Eq[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean = x.i == y.i && x.s == y.s  
}
```

```
val intEq: Eq[Int] = new Eq[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
}
```

```
scala> fooEq.eqv(Foo(1, "Hello"), Foo(5, "Hi"))  
res25: Boolean = false
```

```
scala> intEq.eqv(5, 5)  
res26: Boolean = true
```



Interface

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
val fooEq: Eq[Foo] = new Eq[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean = x.i == y.i && x.s == y.s  
}
```

```
val intEq: Eq[Int] = new Eq[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
}
```

```
scala> fooEq.eqv(Foo(1, "Hello"), 3)  
      fooEq.eqv(Foo(1, "Hello"), 3)  
                                ^
```

```
On line 2: error: type mismatch;  
found    : Int(3)  
required: Foo
```



Interface

```
def contains[A](xs: List[A], value: A)(eq: Eq[A]): Boolean =  
  xs.foldRight(false)((x, acc) => eq.eqv(x, value) || acc)  
  
val exact: Eq[Double] = new Eq[Double] {  
  def eqv(x: Double, y: Double): Boolean = x == y  
}  
  
def approx(error: Double): Eq[Double] = new Eq[Double] {  
  def eqv(x: Double, y: Double): Boolean = (x - y).abs < error  
}  
  
val xs = List(1.0, 2.5, 3.3)
```

```
scala> contains(xs, 3.2)(exact)  
res28: Boolean = false  
  
scala> contains(xs, 3.2)(approx(0.2))  
res29: Boolean = true  
  
scala> contains(xs, 3.2)(approx(0.001))  
res30: Boolean = false
```



Interface

```
trait Eq[A] {  
  def eqv (x: A, y: A): Boolean  
  def neqv(x: A, y: A): Boolean = !eqv(x, y)  
}
```



Interface

```
trait Eq[A] {  
  def eqv (x: A, y: A): Boolean  
  def neqv(x: A, y: A): Boolean = !eqv(x, y)  
}
```

```
trait Hash[A] extends Eq[A] {  
  def hash(a: A): Int  
}  
  
val fooHash: Hash[Foo] = new Hash[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean = x.i == y.i && x.s == y.s  
  def hash(a: Foo): Int = a.hashCode  
}  
  
val intHash: Hash[Int] = new Hash[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
  def hash(a: Int): Int = a  
}
```



Interface or record of functions

```
case class Hash[A](  
  eqv: (A, A) => Boolean,  
  hash: A => Int  
)  
  
val hashFoo: Hash[Foo] = Hash[Foo](  
  eqv  = (x, y) => x.i == y.i && x.s == y.s,  
  hash = _.hashCode  
)  
  
val intHash: Hash[Int] = Hash[Int](  
  eqv  = _ == _,  
  hash = identity  
)
```



Interface or record of functions

```
trait Eq[A] {  
  def eqv (x: A, y: A): Boolean  
  def neqv(x: A, y: A): Boolean = !eqv(x, y)  
}  
  
trait Hash[A] extends Eq[A] {  
  def hash(a: A): Int  
}  
  
val intHash: Hash[Int] = new Hash[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
  def hash(a: Int): Int = a  
}  
  
def hashToEq[A](hash: Hash[A]): Eq[A] = hash  
  
def reverseHash[A](current: Hash[A]): Hash[A] =  
  new Hash[A]{  
    def eqv(x: A, y: A): Boolean = current.eqv(x, y)  
    def hash(a: A): Int = - current.hash(a)  
  }
```

```
case class Eq[A](  
  eqv: (A, A) => Boolean  
)  
  
case class Hash[A](  
  eqv: (A, A) => Boolean,  
  hash: A => Int  
)  
  
val intHash: Hash[Int] = Hash[Int](  
  eqv = _ == _,  
  hash = identity  
)  
  
def hashToEq[A](hash: Hash[A]): Eq[A] = Eq(hash.eqv)  
  
def reverseHash[A](current: Hash[A]): Hash[A] =  
  current.copy(  
    hash = (x: A) => - current.hash(x)  
  )
```



Typeclass

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
implicit val intEq: Eq[Int] = new Eq[Int]{  
  def eqv(x: Int, y: Int): Boolean = x == y  
}
```

```
implicit val stringEq: Eq[String] = new Eq[String]{  
  def eqv(x: String, y: String): Boolean = x == y  
}
```

```
implicit val fooEq: Eq[Foo] = new Eq[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean =  
    implicitly[Eq[Int]].eqv(x.i, y.i) && implicitly[Eq[String]].eqv(x.s, y.s)  
}
```



Typeclass: Summoning

```
object Global {  
  val eqDictionary: Map[x: Type, Eq[x]] := Map(  
    Int    -> intEq,  
    String -> stringEq,  
    Foo    -> fooEq  
  )  
}
```



Typeclass: Summoning

```
object Global {  
  val eqDictionary: Map[x: Type, Eq[x]] := Map(  
    Int    -> intEq,  
    String -> stringEq,  
    Foo    -> fooEq  
  )  
}
```

```
scala> implicitly[Eq[Int]].eqv(5, 6)  
res31: Boolean = false
```

```
scala> implicitly[Eq[String]].eqv("Hello", "Hello")  
res32: Boolean = true
```

```
scala> implicitly[Eq[Double]].eqv(5.5, 5.6)  
      implicitly[Eq[Double]].eqv(5.5, 5.6)  
              ^
```

On line 2: error: could not find implicit value for parameter e: Eq[Double]



Typeclass: Constrain

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
def contains[A](xs: List[A], value: A)(implicit ev: Eq[A]): Boolean =  
  xs.foldRight(false)((x, acc) => ev.eqv(x, value) || acc)
```



Typeclass: Constrain

```
trait Eq[A]{  
  def eqv(x: A, y: A): Boolean  
}
```

```
def contains[A](xs: List[A], value: A)(implicit ev: Eq[A]): Boolean =  
  xs.foldRight(false)((x, acc) => ev.eqv(x, value) || acc)
```

```
scala> contains(List(1,2,3,4,5), 4)  
res34: Boolean = true
```

```
scala> contains(List("hello","world"), "world")  
res35: Boolean = true
```

```
scala> contains(List("hello","world"), "foo")  
res36: Boolean = false
```

```
scala> contains(List(1.0, 2.5, 3.3), 3.2)  
           contains(List(1.0, 2.5, 3.3), 3.2)  
                ^
```

On line 2: error: could not find implicit value for parameter ev: Eq[Double]



Syntactic sugar



Syntactic sugar: Summoning implicit

```
object Eq {  
  def apply[A](implicit ev: Eq[A]): Eq[A] = ev  
}
```

```
scala> implicitly[Eq[Int]]  
res38: Eq[Int] = $anon$1@1aeb0e23
```

```
scala> Eq[Int]  
res39: Eq[Int] = $anon$1@1aeb0e23
```

```
scala> Eq[Int].eqv(5, 5)  
res40: Boolean = true
```



Syntactic sugar: Context bound

```
def contains[A](xs: List[A], value: A)(implicit ev: Eq[A]): Boolean =  
  xs.foldRight(false)((x, acc) => ev.eqv(x, value) || acc)  
  
def contains[A: Eq](xs: List[A], value: A): Boolean =  
  xs.foldRight(false)((x, acc) => Eq[A].eqv(x, value) || acc)
```



Syntactic sugar: Extension methods

```
implicit class EqSyntax[A](self: A){  
  def eqv(other: A)(implicit ev: Eq[A]): Boolean =  
    ev.eqv(self, other)  
  
  def ===(other: A)(implicit ev: Eq[A]): Boolean =  
    eqv(other)  
}
```



Syntactic sugar: Extension methods

```
implicit class EqSyntax[A](self: A){  
  def eqv(other: A)(implicit ev: Eq[A]): Boolean =  
    ev.eqv(self, other)  
  
  def ===(other: A)(implicit ev: Eq[A]): Boolean =  
    eqv(other)  
}
```

```
scala> 5.eqv(5)  
res41: Boolean = true  
  
scala> 5 === 6  
res42: Boolean = false  
  
scala> "foo" === "hello"  
res43: Boolean = false
```

```
implicit val fooEq: Eq[Foo] = new Eq[Foo]{  
  def eqv(x: Foo, y: Foo): Boolean =  
    x.i === y.i && x.s === y.s  
}
```



	Mixin			Values		
Features	Top level	Trait	F-Bounded	Overload	Interface	Typeclass
Define comparable type	□	□	□	□	□	□
Same type equality	□	□	□	□	□	□
Support foreign type	□	□	□	□	□	□
Customizable / Unique	U	U	U	C	C	U
Bundle + Hierarchy	□	□	□	□	□	□
Code Generation	□	□	□	□	□	□



Folding



Folding

```
def monoFoldLeft[A](fa: List[A])(initial: A)(f: (A, A) => A): A =  
  fa match {  
    case Nil => initial  
    case x :: xs => monoFoldLeft(xs)(f(initial, x))(f)  
  }
```



Folding

```
def monoFoldLeft[A](fa: List[A])(initial: A)(f: (A, A) => A): A =  
  fa match {  
    case Nil      => initial  
    case x :: xs => monoFoldLeft(xs)(f(initial, x))(f)  
  }
```

```
def monoFoldLeft[A](fa: List[A])(initial: A)(f: (A, A) => A): A = {  
  var res: A = initial  
  val it = fa.iterator  
  while(it.hasNext) res = f(res, it.next())  
  res  
}
```



Folding

```
scala> monoFoldLeft(List(1, 2, 3, 4, 5))(0)(_ + _)
res44: Int = 15
```



Folding

```
scala> monoFoldLeft(List(1, 2, 3, 4, 5))(0)(_ + _)
res44: Int = 15
```

```
scala> monoFoldLeft(List("foo", "bar"))("")(_ + _)
res45: String = foobar
```

```
scala> monoFoldLeft(List(List(1,2,3), List(4,5)))(Nil)(_ ++ _)
res46: List[Int] = List(1, 2, 3, 4, 5)
```



Folding

```
scala> monoFoldLeft(List(1, 2, 3, 4, 5))(0)(_ + _)
res44: Int = 15
```

```
scala> monoFoldLeft(List("foo", "bar"))("")(_ + _)
res45: String = foobar
```

```
scala> monoFoldLeft(List(List(1,2,3), List(4,5)))(Nil)(_ ++ _)
res46: List[Int] = List(1, 2, 3, 4, 5)
```

```
val inc    : Int => Int = _ + 1
val double: Int => Int = _ * 2

val func = monoFoldLeft(List(inc, double))(identity)(_ andThen _)
```

```
scala> func(5)
res47: Int = 12
```



Monoid

```
trait Monoid[A] {  
  def combine(x: A, y: A): A  
  def empty: A  
}
```

```
def monoFoldLeft[A](fa: List[A])(implicit ev: Monoid[A]): A = {  
  var res: A = ev.empty  
  val it = fa.iterator  
  while(it.hasNext) res = ev.combine(res, it.next())  
  res  
}
```



Monoid instances

```
implicit val intMonoid: Monoid[Int] = new Monoid[Int] {  
  def combine(x: Int, y: Int): Int = x + y  
  def empty: Int = 0  
}  
  
implicit val stringMonoid: Monoid[String] = new Monoid[String] {  
  def combine(x: String, y: String): String = x + y  
  def empty: String = ""  
}
```

```
scala> monoFoldLeft(List(1, 2, 3, 4, 5))  
res48: Int = 15
```

```
scala> monoFoldLeft(List("foo", "bar"))  
res49: String = foobar
```



Monoid instances

```
implicit val intMonoid: Monoid[Int] = new Monoid[Int] {  
  def combine(x: Int, y: Int): Int = x + y  
  def empty: Int = 0  
}  
  
implicit val stringMonoid: Monoid[String] = new Monoid[String] {  
  def combine(x: String, y: String): String = x + y  
  def empty: String = ""  
}
```

```
scala> monoFoldLeft(List(1, 2, 3, 4, 5))  
res48: Int = 15
```

```
scala> monoFoldLeft(List("foo", "bar"))  
res49: String = foobar
```

```
scala> monoFoldLeft(List(true, true, false))  
      monoFoldLeft(List(true, true, false))  
                    ^
```

On line 2: error: could not find implicit value for parameter ev: Monoid[Boolean]



Exercises 1 (up to 1h)



Semigroup

```
trait Semigroup[A] {  
  def combine(x: A, y: A): A  
}  
  
trait Monoid[A] extends Semigroup[A] {  
  def empty: A  
}
```



Semigroup

```
trait Semigroup[A] {  
  def combine(x: A, y: A): A  
}  
  
trait Monoid[A] extends Semigroup[A] {  
  def empty: A  
}
```

```
import cats.data.NonEmptyList  
  
def fold[A: Monoid](fa: List[A]): A = ???  
  
def reduce[A: Semigroup](fa: NonEmptyList[A]): A = ???
```



Exercises 1 and 2



Uniqueness

```
implicit val intMonoid: Monoid[Int] = new Monoid[Int] {  
  def combine(x: Int, y: Int): Int = x + y  
  def empty: Int = 0  
}
```

```
scala> intMonoid  
res51: Monoid[Int] = $anon$1@3328025d
```

```
scala> implicitly[Monoid[Int]]  
res52: Monoid[Int] = $anon$1@3328025d
```



Uniqueness

```
implicit val intMonoid2: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = (x + y).abs  
  def empty: Int = 0  
}  
  
implicit val intMonoid3: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = x  
  def empty: Int = 99  
}
```



Uniqueness

```
implicit val intMonoid2: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = (x + y).abs  
  def empty: Int = 0  
}
```

```
implicit val intMonoid3: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = x  
  def empty: Int = 99  
}
```

```
scala> implicitly[Monoid[Int]]  
      implicitly[Monoid[Int]]  
                ^
```

On line 2: error: ambiguous **implicit** values:
 both value intMonoid of **type** => **Monoid**[Int]
 and value intMonoid2 of **type** => **Monoid**[Int]
 match expected **type** **Monoid**[Int]



Uniqueness

```
implicit val intMonoid2: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = (x + y).abs  
  def empty: Int = 0  
}
```

```
implicit val intMonoid3: Monoid[Int] = new Monoid[Int]{  
  def combine(x: Int, y: Int): Int = x  
  def empty: Int = 99  
}
```

```
scala> fold(List(1,2,3,4))  
      fold(List(1,2,3,4))  
            ^
```

On line 2: error: ambiguous implicit values:
 both value intMonoid of type => Monoid[Int]
 and value intMonoid2 of type => Monoid[Int]
 match expected type Monoid[Int]



How to ensure uniqueness?

1. Parametricity

```
def map[F[_], A, B](fa: F[A])(f: A => B): F[B]
```

2. Laws

```
"Monoid: combine is associative" in {  
  forAll((x: A, y: A, z: A) => ((x |+| y) |+| z) == (x |+| (y |+| z)))  
}
```



Monoid Laws

```
- Double.Monoid.associative *** FAILED ***  
  GeneratorDrivenPropertyCheckFailedException was thrown during property evaluation.  
  (Discipline.scala:14)  
  Falsified after 8 successful property evaluations.  
  Location: (Discipline.scala:14)  
  Occurred when passed generated values (
```

```
    arg0 = -4.498344780314773E208,  
    arg1 = -5.116678570398714E199,  
    arg2 = -1.0056301257545645E214  
  )
```

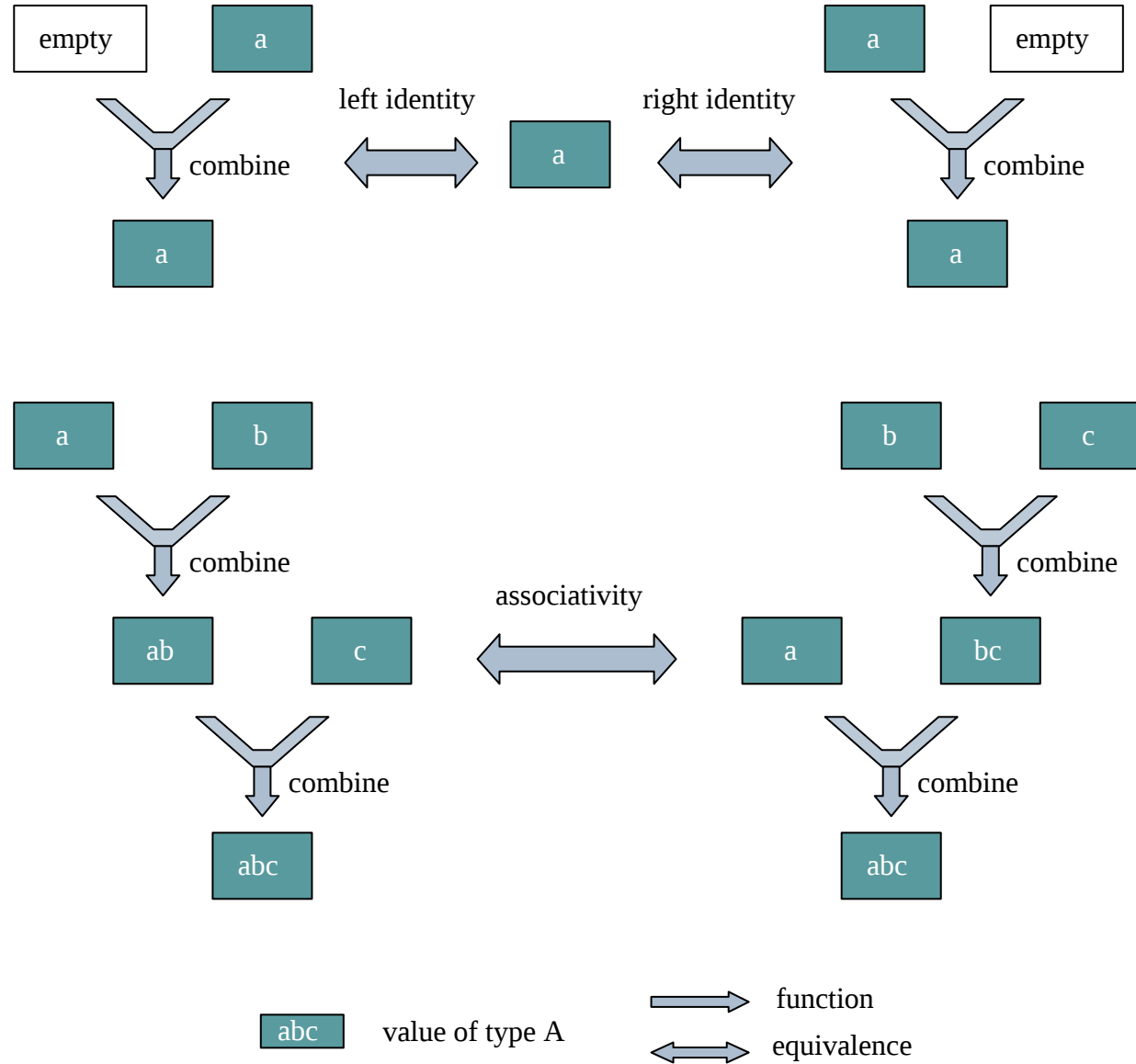
Failed after two weeks of CI build



Exercises 3



Monoid Laws



Use case: Distributed Map-Reduce

```
class RDD[A]{  
  def map[B](f: A => B): RDD[B]  
  def reduce(f: (A, A) => A): A  
}
```

Spark Accumulators (aka reduce)

Accumulators are variables that are only “added” to through an **associative** and **commutative** operation and can therefore be efficiently supported in parallel. They can be used to implement counters (as in MapReduce) or sums.



Exercises 4



Where to define instances?



Where to define instances?

1. Companion object of the target type

```
case class Metrics(ordersCreated: Long, ordersDeleted: Long)

object Metrics {
  implicit val Monoid: Monoid[Metrics] = new Monoid[Metrics]{
    def combine(x: Metrics, y: Metrics): Metrics =
      Metrics(
        ordersCreated = x.ordersCreated + y.ordersCreated,
        ordersDeleted = x.ordersDeleted + y.ordersDeleted
      )

    def empty: Metrics =
      Metrics(0, 0)
  }
}
```



Where to define instances?

2. Companion object of typeclass

```
object Monoid {  
  implicit val int: Monoid[Int] = new Monoid[Int] {  
    def combine(x: Int, y: Int): Int = x + y  
    def empty: Int = 0  
  }  
  
  implicit val string: Monoid[String] = new Monoid[String] {  
    def combine(x: String, y: String): String = x + y  
    def empty: String = ""  
  }  
}
```



Where to define instances?

3. Ad-hoc object (to avoid as much as possible!)

```
object MonoidInstance {  
  implicit val booleanMonoid: Monoid[Boolean] = new Monoid[Boolean]{  
    def combine(x: Boolean, y: Boolean): Boolean = x && y  
    def empty: Boolean = true  
  }  
}
```



Where to define instances?

3. Ad-hoc object (to avoid as much as possible!)

```
object MonoidInstance {  
  implicit val booleanMonoid: Monoid[Boolean] = new Monoid[Boolean]{  
    def combine(x: Boolean, y: Boolean): Boolean = x && y  
    def empty: Boolean = true  
  }  
}
```

```
scala> fold(List(true, false, false))  
      fold(List(true, false, false))  
            ^
```

On line 2: error: could not find implicit value for parameter ev: Monoid[Boolean]



Where to define instances?

3. Ad-hoc object (to avoid as much as possible!)

```
object MonoidInstance {  
  implicit val booleanMonoid: Monoid[Boolean] = new Monoid[Boolean]{  
    def combine(x: Boolean, y: Boolean): Boolean = x && y  
    def empty: Boolean = true  
  }  
}
```

```
scala> fold(List(true, false, false))  
      fold(List(true, false, false))  
            ^
```

On line 2: error: could not find implicit value for parameter ev: Monoid[Boolean]

```
import MonoidInstance._
```

```
scala> fold(List(true, false, false))  
res56: Boolean = false
```



Where to define instances?

3. Ad hoc object (to avoid as much as possible!)

```
object OtherMonoidInstance {  
  implicit val booleanMonoid: Monoid[Boolean] = new Monoid[Boolean]{  
    def combine(x: Boolean, y: Boolean): Boolean = x || y  
    def empty: Boolean = false  
  }  
}
```



Where to define instances?

3. Ad hoc object (to avoid as much as possible!)

```
object OtherMonoidInstance {  
  implicit val booleanMonoid: Monoid[Boolean] = new Monoid[Boolean]{  
    def combine(x: Boolean, y: Boolean): Boolean = x || y  
    def empty: Boolean = false  
  }  
}
```

```
import MonoidInstance._
```

```
scala> fold(List(true, false, false))  
res57: Boolean = false
```

```
import OtherMonoidInstance._
```

```
scala> fold(List(true, false, false))  
res58: Boolean = true
```



When should I use ad hoc object for typeclass instances

- In libraries when you have a typeclass hierarchy

```
trait Semigroup[A]

object Semigroup {
  implicit val int: Semigroup[Int] = ...
}

trait Monoid[A] extends Semigroup[A]

object Monoid {
  implicit val int: Monoid[Int] = ...
}

trait CommutativeMonoid[A] extends Monoid[A]

object CommutativeMonoid {
  implicit val int: CommutativeMonoid[Int] = ...
}
```

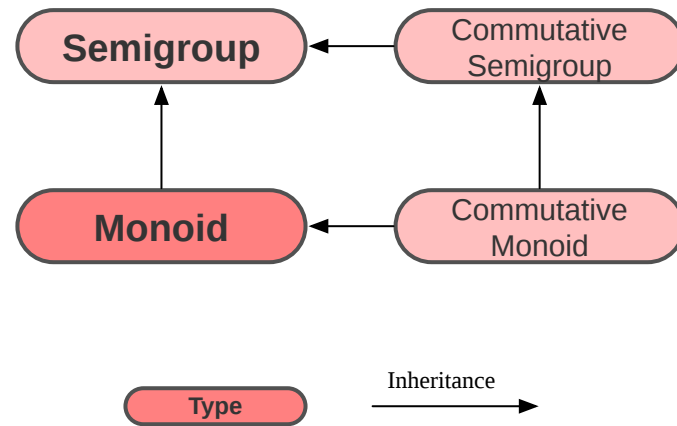
- If you control neither the typeclass or the type (Please make a PR!)



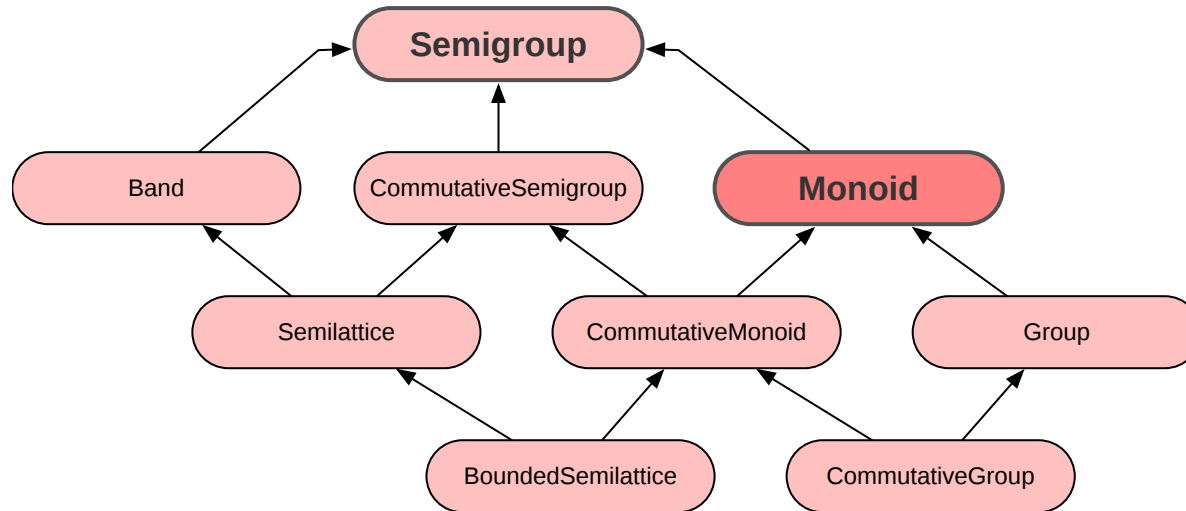
Exercises 5



Semigroups



Semigroups in Cats



Kinds



Kinds

```
:k Int  
Int :: Type
```



Kinds

```
:k Int  
Int :: Type
```

```
:k String  
String :: Type
```



Kinds

```
:k Int  
Int :: Type
```

```
:k String  
String :: Type
```

```
:k List  
List :: Type -> Type
```



Kinds

```
:k Int  
Int :: Type
```

```
:k String  
String :: Type
```

```
:k List  
List :: Type -> Type
```

```
:k List[Int]  
List[Int] :: Type
```



Kinds

```
:k Int  
Int :: Type
```

```
:k String  
String :: Type
```

```
:k List  
List :: Type -> Type
```

```
:k List[Int]  
List[Int] :: Type
```

```
:k Either  
Either :: Type -> Type -> Type
```



Kinds

```
:k Int  
Int :: Type
```

```
:k String  
String :: Type
```

```
:k List  
List :: Type -> Type
```

```
:k List[Int]  
List[Int] :: Type
```

```
:k Either  
Either :: Type -> Type -> Type
```

```
:k Either[String, ?]  
Either[String, ?] :: Type -> Type
```



Kinds

```
trait Semigroup[A]{  
  def combine(x: A, y: A): A  
}  
  
implicit val intSemigroup: Semigroup[Int] = new Semigroup[Int]{  
  def combine(x: Int, y: Int): Int = ???  
}
```



Kinds

```
trait Semigroup[A]{  
  def combine(x: A, y: A): A  
}  
  
implicit val intSemigroup: Semigroup[Int] = new Semigroup[Int]{  
  def combine(x: Int, y: Int): Int = ???  
}
```

```
scala> implicit val listSemigroup: Semigroup[List] = ???  
      implicit val listSemigroup: Semigroup[List] = ???  
                                   ^
```

On line 2: error: type List takes type parameters

```
implicit def listSemigroup[A]: Semigroup[List[A]] = new Semigroup[List[A]]{  
  def combine(x: List[A], y: List[A]): List[A] = ???  
}
```



Foldable

```
trait Foldable[F[_]] {  
  def foldLeft[A, B](fa: F[A], z: B)(f: (B, A) => B): B  
  def foldRight[A, B](fa: F[A], z: B)(f: (A, => B) => B): B  
}
```



Foldable

```
trait Foldable[F[_]] {  
  def foldLeft[A, B](fa: F[A], z: B)(f: (B, A) => B): B  
  def foldRight[A, B](fa: F[A], z: B)(f: (A, => B) => B): B  
}
```

```
implicit val listFoldable: Foldable[List] = new Foldable[List] {  
  def foldLeft[A, B](fa: List[A], z: B)(f: (B, A) => B): B = ???  
  def foldRight[A, B](fa: List[A], z: B)(f: (A, => B) => B): B = ???  
}
```



Foldable

```
trait Foldable[F[_]] {  
  def foldLeft[A, B](fa: F[A], z: B)(f: (B, A) => B): B  
  def foldRight[A, B](fa: F[A], z: B)(f: (A, => B) => B): B  
}
```

```
implicit val listFoldable: Foldable[List] = new Foldable[List] {  
  def foldLeft[A, B](fa: List[A], z: B)(f: (B, A) => B): B = ???  
  def foldRight[A, B](fa: List[A], z: B)(f: (A, => B) => B): B = ???  
}
```

```
scala> implicit val intFoldable: Foldable[Int] = ???  
      implicit val intFoldable: Foldable[Int] = ???  
                                ^
```

On line 2: error: Int takes no **type parameters**, **expected**: one



Exercises 6



Use Typeclass when

1. You have only one valid implementation for (almost) all types
2. You require uniqueness per type e.g. Eq, Hash, Ord

Use Interface / Record of functions when

1. You have many valid / interesting implementations per type
2. You need to package several functions together

Use Overloading when

1. You have many valid / interesting implementations per type
2. Single function



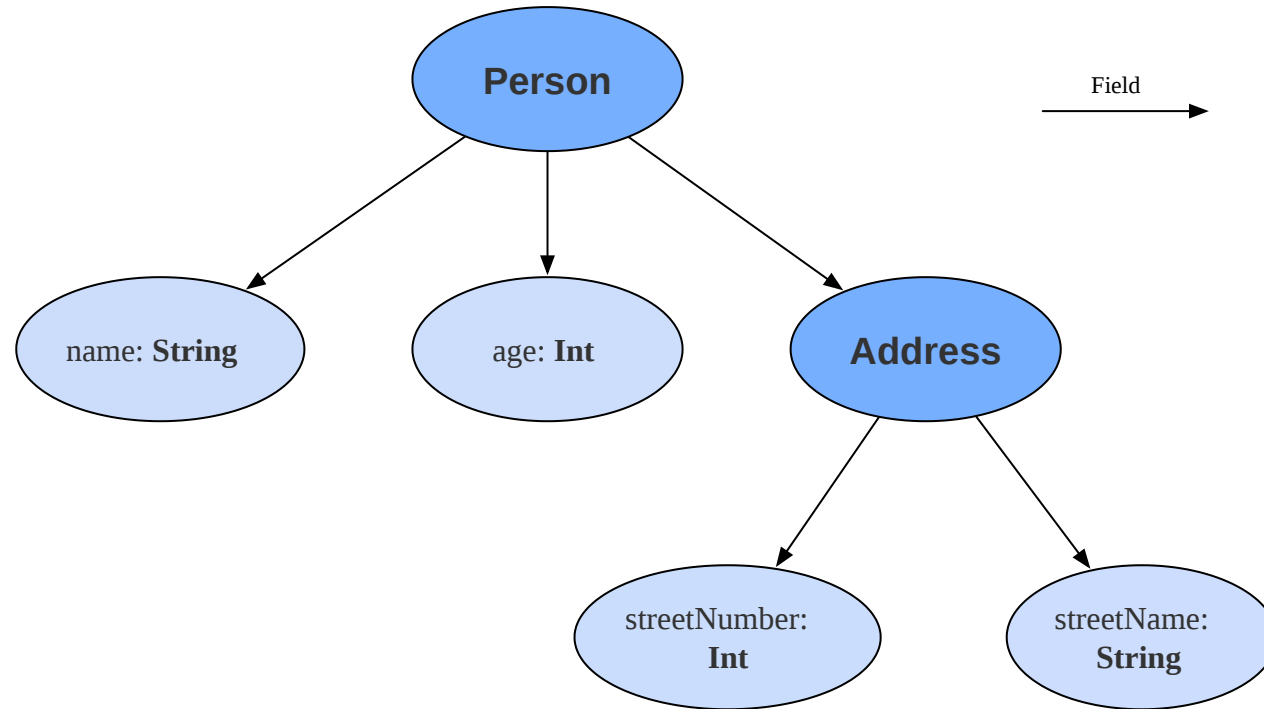
Two reasons why typeclasses are overused



1. Typeclass Derivation



Typeclass Derivation: Product



Typeclass Derivation: Product

```
case class Person(name: String, age: Int)
```

```
Person <==> (String, (Int, Unit))
```

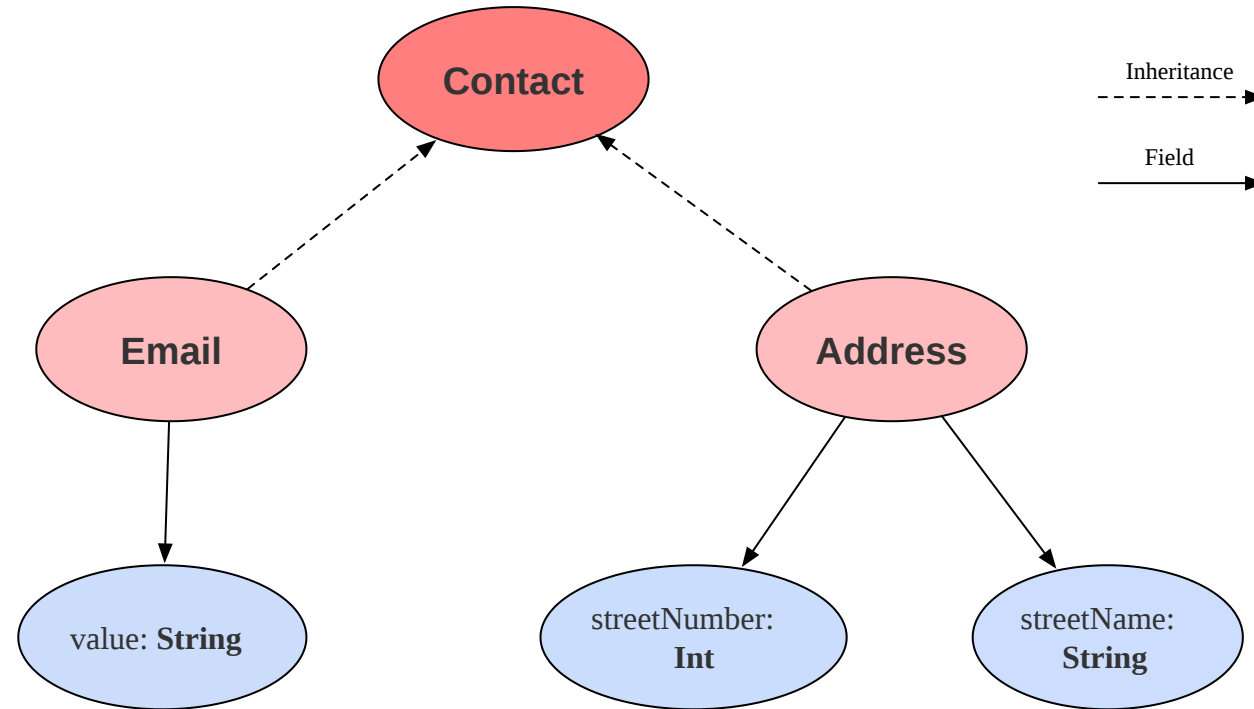
```
implicit val intEq : Eq[Int] = (x: Int, y: Int) => x == y
implicit val stringEq: Eq[String] = (x: String, y: String) => x == y
implicit val unitEq : Eq[Unit] = (x: Unit, y: Unit) => true
```

```
implicit def tuple2[A: Eq, B: Eq]: Eq[(A, B)] = new Eq[(A, B)] {
  def eqv(x: (A, B), y: (A, B)): Boolean =
    x._1 == y._1 && x._2 == y._2
}
```

```
Eq[Person] => Eq[(String, (Int, Unit))]
            => tuple2(Eq[String], Eq[(Int, Unit)])
            => tuple2(Eq[String], tuple2[Int, Unit])
            => tuple2(Eq[String], tuple2(Eq[Int], Eq[Unit]))
```



Typeclass Derivation: Sum



Typeclass Derivation: Sum

```
sealed trait Contact
case class Email(value: String) extends Contact
case class Address(streetNumber: Int, StreetName: String) extends Contact
```

```
Contact <==> Either[Email, Either[Address, Nothing]]
```

```
implicit val nothingEq: Eq[Nothing] = (x: Nothing, y: Nothing) => true

implicit def either[A: Eq, B: Eq]: Eq[Either[A, B]] = new Eq[Either[A, B]] {
  def eqv(e1: Either[A, B], e2: Either[A, B]): Boolean =
    (e1, e2) match {
      case (Left(x), Left(y)) => x === y
      case (Right(b1), Right(b2)) => b1 === b2
      case _ => false
    }
}
```

```
Eq[Contact] => Eq[Either[Email, Either[Address, Nothing]]]
=> either(Eq[Email], Eq[Either[Address, Nothing]])
=> ...
```



2. Better syntax



Better syntax

```
"foo" === "hello"  
stringEq.eqv("foo", "hello")
```

```
List("hello", "world", "!").foldMap(_.  
size)  
foldMap(List("hello", "world", "!"))(_.  
size)(intMonoid)
```

```
val json = """{ "name" : "John", "age" : 34 }"""  
  
json.as[Person]  
Person.restJsonDecoder.parse(json)
```



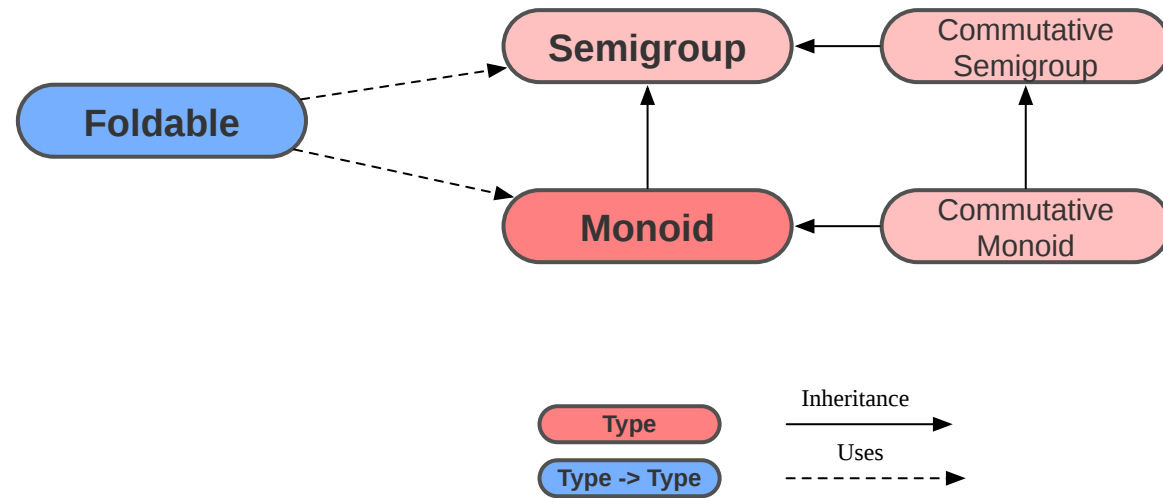
Future

```
List(1,2,3,4,5).fold (@via Sum)      // 15  
List(1,2,3,4,5).fold (@via Product) // 120
```

See [@Iceland_jack](#) work on #DerivingVia for Haskell



Review



Module 7: Functors

