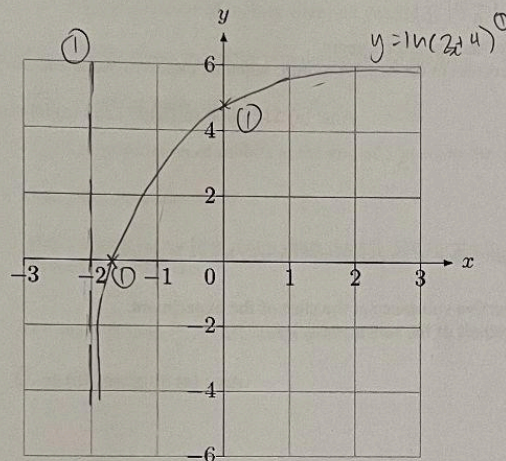


28. 2016/EOY/P2/Q6(Modified) [GDC is allowed]

Let $f(x) = \ln(2x+4) + 3$ for $-2 < x < 3$.

- (a) For the graph of f ,
 (i) find the x -intercept
 (ii) find the y -intercept
 (b) Hence, sketch the graph of f



Ans: (a)(i) $x = -1.98$ (ii) $y = 4.39$

29. 2013/CT/P2/Q6 [GDC is allowed]

A hot chemical was left to cool in the science laboratory. Its temperature, T is given by the formulae $T = 50 + 16e^{-kt}$ where k is a constant and t is the number of hours after the chemical has been placed there.

- (a) Given that the temperature of the chemical is 60° after 1 hour, find the value of k .

Ans: $k = 0.470$ (3sf)

- (b) Using your value of k , find the number of hours needed for the chemical to cool to at least 54° . Give your answer to the nearest hour.

Ans: $t = 2.95 \approx 3$ hours

30. 2014/CT/P2/Q3 [GDC is allowed]

A man buys a new car. After t months, its value, \$ V is given by $V = 125000e^{-pt}$, where p is a real constant.

- (a) Find the value of the car when the man first bought it.
- (b) The value of the car after 1 year is expected to be \$80 000.
- (i) Show that the value of p is $\frac{1}{6} \ln\left(\frac{5}{4}\right)$.
- (ii) Find the expected value of the car after 2 years.
- (c) Find the age of the car, correct to the nearest month, when its expected value becomes \$10 000.

Ans: (a) \$125 000 (b)(ii) \$51200 (c) 68 months

31. 2018/Test 1/Q7

The amount of radioactive substance in an experiment after t hours is given by $R = 50e^{-2t}$ grams.

- (a) Find the amount of radioactive substance at the start of the experiment.
- (b) Find the exact time where half of the substance is left.

Ans: (a) 50 grams (b) $\ln\sqrt{2}$

32. 2018/MYE/P2/Q5

Let $f(x) = \log_2(x+3) + 2$ where $-2.9 \leq x \leq 10$.

- (a) Write down the coordinates of the x and y intercepts of the graph of f . [2]
- (b) Find the range of f . [2]
- (c) Express $f(x)$ as $\log_2 k(x+3)$ where k is a constant to be found. [2]

Ans: (a) x -intercept is $(-2.75, 0)$, y -intercept is $(0, 3.58)$
(b) $\{y \mid -1.32 \leq y \leq 5.70\}$ (c) $f(x) = 4$

33. 2018/MYE/P2/Q6

In a forest, the population of wolves after x years is given by $W = 30(2^{\frac{x}{3}})$.

- (a) Find the value of x when the population of wolves reaches 50. [2]

In the same forest, the population of rabbits after x years is given by $R = 20e^{Rx}$. The population of rabbits after one year is 34.

- (b) (i) Write down the initial population of the rabbits. [2]
(ii) Find the value of B . [3]
(c) Find the population of rabbits at the end of 5 years to the nearest whole number. [2]
(d) Solve the equation $R = W$. [2]
(e) Hence, write down how many years does it take for the population of rabbits to first exceed wolves. [1]

Ans: (a) $x = 3\log_2 \frac{5}{3}$, (b)(i) 20, (b)(ii) $\ln 1.7$ or 0.531 (c) 284 (d) $x = 1.35$ (e) 1.35 years

34. 2020/EYE/P2/Q9

The temperature in $^{\circ}\text{C}$ of a pot of water is given by $T(m) = 80(2.64)^{-0.3m} + 10$, where m is the number of minutes after the pot is removed from the cooker.

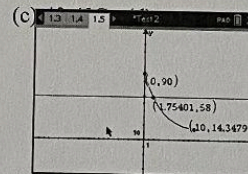
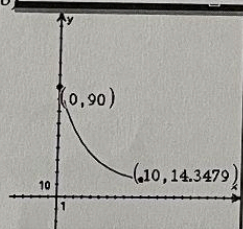
- Show that the temperature of water (when the pot has just been removed from the cooker) is 90°C .
- Sketch the graph of $T(m)$ for $0 \leq m \leq 10$.
- Find the temperature of the water after 3 minutes.
- Find the time taken for the temperature to reach 58°C .

Consider the function $S(m) = 8.5m + 10$ for $0 \leq m \leq 10$.

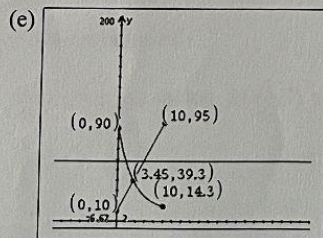
The function $S(m)$ represents the temperature of the soup in a pot placed on the cooker after the water is removed. The soup is then heated.

- On the same axes, sketch the graph of $S(m)$ for $0 \leq m \leq 10$.
- Solve $S(m) = T(m)$.
- In the context of the equation, explain what the answer in (f) represents.

Ans: (a) 90°C . (b)



$m = 1.75$ minutes



(f) $m = 3.45$ minutes

- It represents the time taken (in mins) for the soup and the water to reach the same temperature.

34. 2021/Test 1/Q4

The weight of a decaying material in a science experiment is given by $W = 1000 \times \left(\frac{4}{5}\right)^{\frac{x}{2}}$ grams where x is the number of hours from the start of the experiment.

- (a) Find the initial weight of the material. [2]
 (b) Find the number of hours taken for the weight of the material to be 640 grams. [5]

Ans: (a) 1000 (b) 4

35. 2021/Test 2/Q3

The mass, M , of carbon-14, in grams, remaining in a piece of fossilised wood is given by $M = 100e^{-kt}$, where k is a constant and t is measured in years.

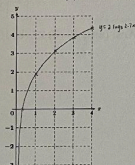
It takes 5730 years for the carbon-14 to be reduced to half its initial mass.

- (a) Find the initial mass. [2]
 (b) Find the value of k . [3]
 (c) Using the value of k found in (b), find the number of years for the mass of carbon-14 to be reduced to 30. [2]

Ans: (a) 100 (b) 0.000121 (c) 9950 Accept 9952 and 9953 years

36. 2021/Test 2/Q4

- (a) Given that $m = \log_3 5$ and $n = \log_3 7$, express $\log_3 175$ in terms of m and n . [2]
 (b) (i) On the grid given, sketch the graph of $y = 2 \log_3 2.7x$ for the domain of $0 < x \leq 4$.
 (ii) Solve the equation $3x - 2 = 2 \log_3 2.7x$. [6]



Ans: (a) $2m + n$ (b)(ii) $x = 0.161$ or 1.53 (3 s.f.)

36. 2021/EYE/P1/Q5

- (a) (i) Find the integer value of $\log 100 - \ln e$.
 (ii) Hence, find the integer value of $\log_3 (\log 100 - \ln e)$. [4]
 (b) Given that $2 \log_3 (x-1) - \log_3 (y+2) = 2$, express y in terms of x . [5]

Ans: (a)(i) 1 (ii) 0 (b) $y = \frac{1}{9}(x-1)^2 - 2$

36. 2021/EYE/P1/Q7

- (a) Simplify $5^{5x+2} \times 125^x$ to the form 5^{ax+b} , where a and b are constants to be determined. [2]
 (b) Hence, solve the equation $5^{5x+2} \times 125^x = \sqrt{5}$. [4]

Ans: (a) 5^{6x+2} (b) $x = -\frac{3}{16}$

37. 2021/EYE/P2/Q1

Crystal Blue lake originally had no fishes. A number of fishes were introduced to the lake and their population is modelled by

$$P(t) = 30 \times 1.2^{-t} + 10, \quad t \geq 0,$$

where t is the time in months since the fishes were introduced.

- (a) Find the number of fishes
 (i) which were introduced to the lake;
 (ii) that are in the lake after 5 months. [4]
 (b) Find the time, in months, for the population to decrease to 15 fishes. Give your answer to the nearest month. [2]

Ans: (a)(i) 40 (ii) 22 (b) 10

38. 2021/EYE/P2/Q2

It is given that $\log_a 2 = x$ and $\log_a 5 = y$.

- (a) Express $\log_a \left(\frac{10}{a^2} \right)$ in terms of x and/or y . [4]
 (b) Find the value of a^{x-y} . [2]

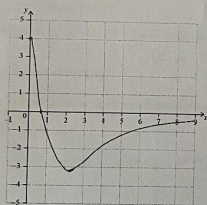
Ans: (a) $x + y - 3$ (b) $\frac{2}{5}$

39. 2021/EYE/P2/Q3

Let $f(x) = \frac{4-7x^2}{e^x}$, for $0 \leq x \leq 9$.

- (a) Find the zero of $f(x)$. [2]
 (b) The graph of f has a minimum at the point A. Find the coordinates of A. [2]
 (c) On the following grid, sketch the graph of f . [3]

Ans: (a) 0.759 (b) A(2.25, -3.31)



40. 2021/EYE/P2/Q4

Let $f(x) = \frac{1}{\sqrt{x+2}}$.

(a) Find

(i) the domain of f ;

(ii) the range of f .

[3]

Let $g(x) = 2 - \ln(x^2 - 1)$.

(b) Solve $f(x) = g(x)$.

[3]

Ans: (a)(i) $x > -2$ (ii) $f(x) > 0$ (b) $x = -1.59, x = 2.36$