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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2024
Secondary Four Express and Five Normal

CANDIDATE NAME:

CLASS: [/]   INDEX NUMBER:

MATHEMATICS

4052/01

Paper 1

14 Aug 2024

2 hours 15 min

0800 – 1015 h

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
If working is needed for any question it must be **neatly and clearly** shown in the space below the question.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

This document consists of **27** printed pages.

Setter: Mr Azmi

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

3

1 Expand and simplify $a(5a-2b)(5a+2b)$.

Answer .... [2]

2 Given that $3^x + 3^{x+2} = 90$, find x.

Answer $x =$ .... [2]

4

3   Simplify $\dfrac{2x^2 + 4xy - 3x - 6y}{2x^2 + xy - 6y^2}$.

Answer .... [3]

4   Two integers, A and B, can be written as products of prime factors.

$$A = 3^{m+2} \times 7 \qquad B = q \times 3^m \times 7$$

The lowest common multiple (LCM) of A and B is $3^3 \times 5 \times 7$.
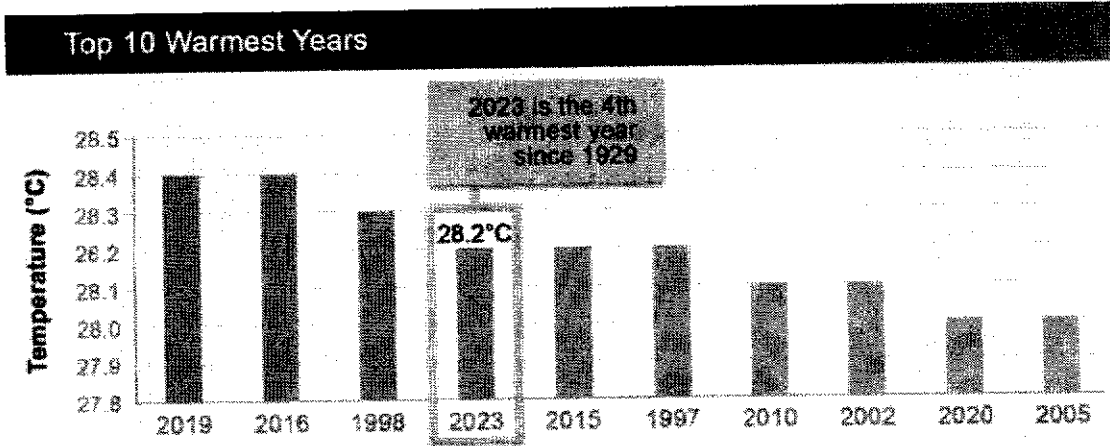
(a)   Write down the value of m and q.

Answer $m =$ .... [1]

$q =$ .... [1]

(b)   Find the highest common factor of A and B.

Answer   .... [1]

5   Below is a graph from the Annual Climate Singapore 2023, which shows the top 10 warmest years in Singapore since 1929.

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

....

....

....

....[2]

6 (a) Simplify $\left(\dfrac{y^9}{27x^{-6}}\right)^{-\frac{2}{3}}$, leaving your answer in positive index.

Answer .... [2]

(b) Given $\dfrac{25}{125^{2-x}} = 5^y$, express x in terms of y.

Answer .... [2]

8

7   **(a)**   Factorise $9x^2 + 24xy + 16y^2$ completely.

Answer .... [1]

(b)   Hence factorise $144a^8 - \left(9a^8 + 24a^4y + 16y^2\right)$ completely.

Answer .... [2]

8 The intensity of a sound detected by a receiver is inversely proportional to the square of the distance of the receiver from the source of the sound.
When the distance is r metres, the intensity of the sound detected is q decibels.
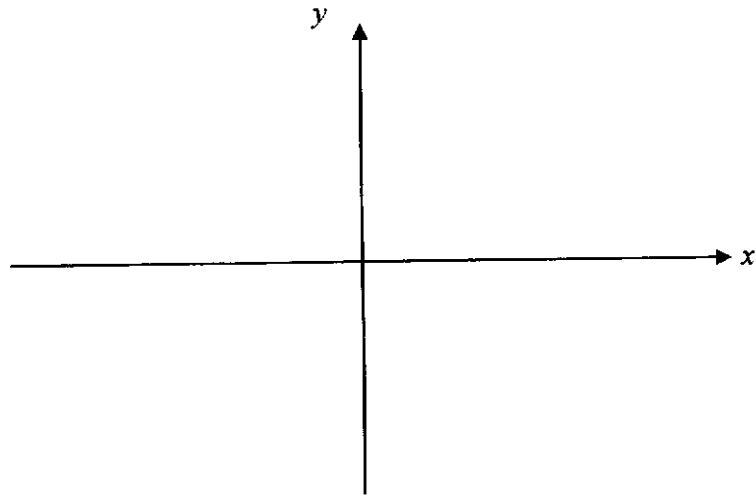
When the distance is reduced by 40%,
(a) find the intensity of the sound detected, in terms of q.

Answer .... decibels [2]

(b) calculate the percentage difference in the intensity of the sound detected.
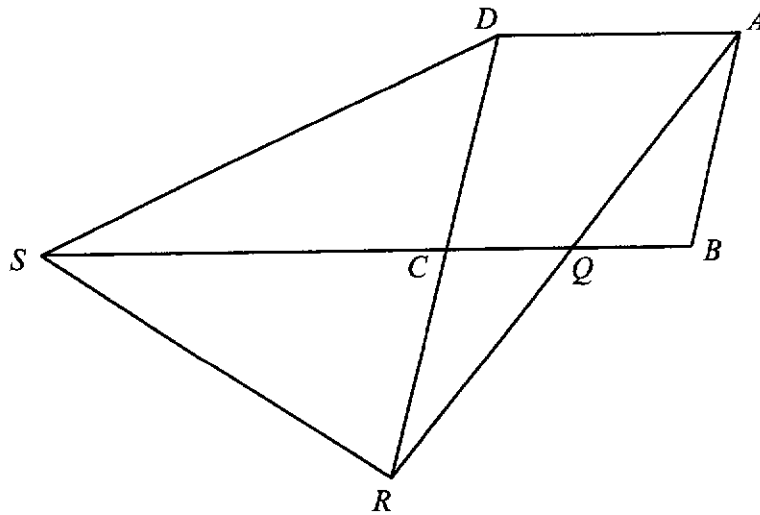
Answer .... % [1]

9   Sketch the graph of $y = 3^{-x}$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes.

[1]

---

10   *ABCD* is a rhombus and $RD = CS$.

SCB, DCR and *AQR* are straight lines.

Prove that triangles *SCD* and *RDA* are congruent.

....

....

....

....[3]

....

11 By expressing $x^2 - 10x - 2$ in the form $(x-h)^2 + k$, where h and k are constants, write down the coordinates of the minimum point of the graph of $y = x^2 - 10x - 2$.

Answer (.................. , ...) [2]

12 $\xi = \{(x,y): x \text{ and } y \text{ are integers, } -2 \le x < 1 \text{ and } -1 \le y \le 1\}$

$P = \{(x,y): x^2 + y^2 < 2\}$

$Q = \{(x,y): xy < 0\}$

(a) Find the number of elements in the universal set ξ.

Answer $\mathrm{n}(\xi) = $ .... [1]

(b) List the elements of the following in set notation.

(i) P

Answer $P = \{$ .... $\}$ [1]

(ii) Q

Answer $Q = \{$ .... $\}$ [1]

13   The diagram below shows a triangle *ABC*.
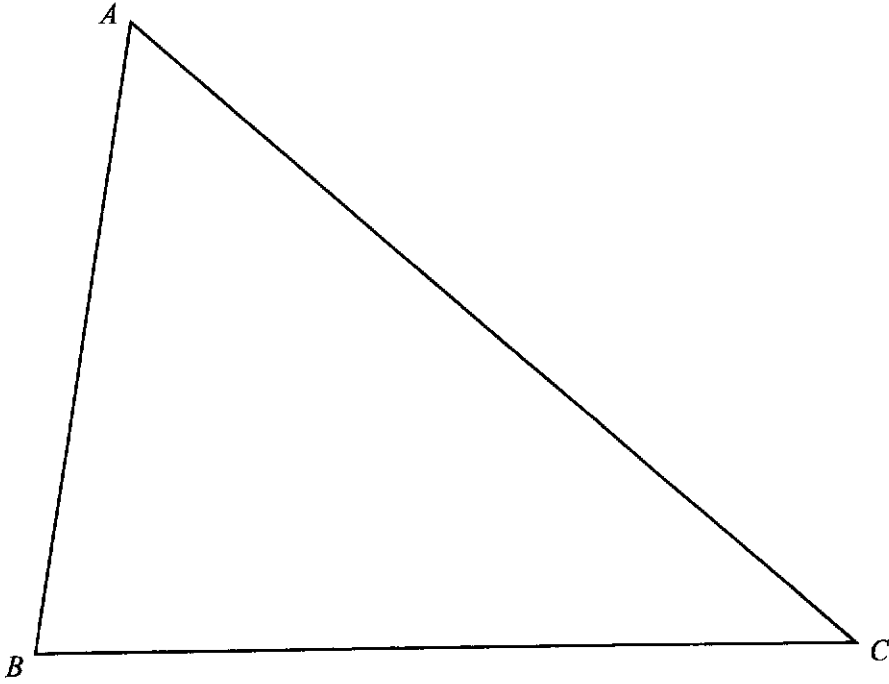
(a)   Construct the perpendicular bisector of *AC*.   [1]

(b)   Construct the bisector of angle *BCA*.   [1]
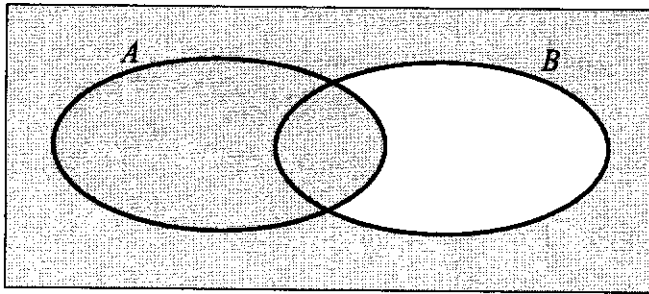
14 $W = \{0, \{0\}\}$.

(a) Find $n(W)$.

Answer $n(W) = $ .... [1]

(b) List all possible subsets of W.

Answer .... [1]

(c) Write down the set represented by the following shaded region.

Answer .... [1]

15 Solve the inequality $x - 3 \le \dfrac{5-x}{3} < \dfrac{x+1}{2}$.

Answer .... [3]

16 The diagram shows a semi-circle BAC, with diameter $BC = 34$ cm.
AD and BCD are straight lines.

34 cm
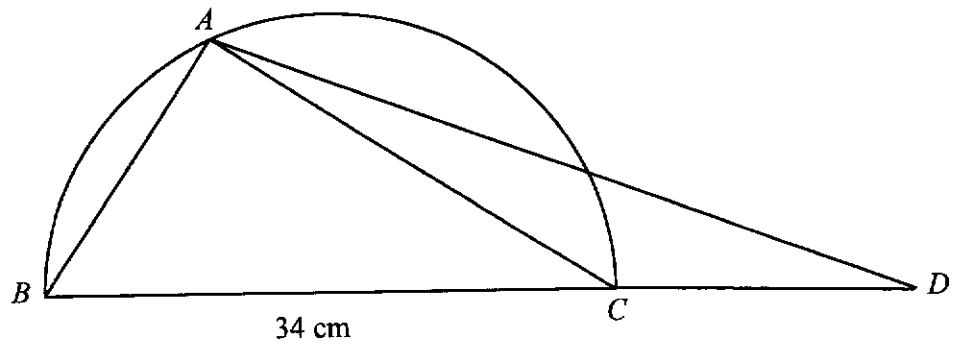
Given that $\cos \angle ABC = \dfrac{8}{17}$, without solving for angle ABC and angle ACD, find the exact value of $\cos \angle ACD$.

Answer .... [4]

17 The sequence T_1, T_2, T_3, T_4,... has the following first four terms.

$$T_1 = 2^2 - 1 = 3$$
$$T_2 = 3^2 - 2 = 7$$
$$T_3 = 4^2 - 3 = 13$$
$$T_4 = 5^2 - 4 = 21$$

(a) Find the 7th term of the sequence, T_7.

Answer T_7 = ... [1]

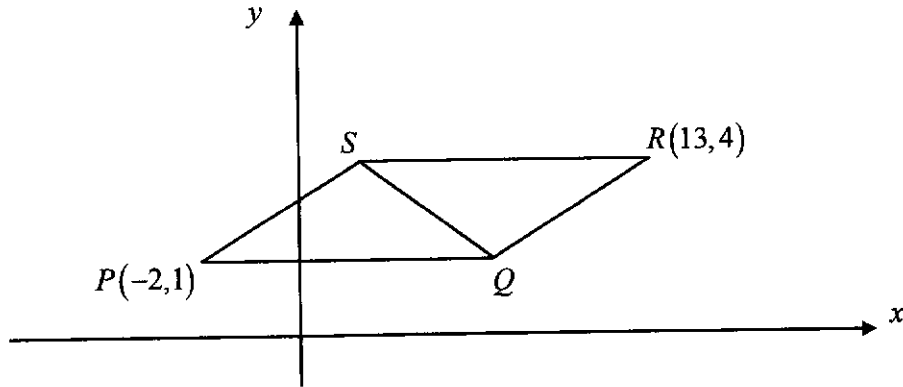(b) Write the expression for the nth term of the sequence.

Answer T_n = ... [1]

(c) Kelvin claims that the difference between 2 consecutive terms $T_{n+1} - T_n$, where n is a positive integer, will always be even. Do you agree? Explain your answer with clear workings.

....

....

....[3]

18 The diagram shows two congruent isosceles triangles, *PQS* and *RSQ*.
    PQ and *RS* are parallel to the *x*-axis.

(a) Write down the coordinates of the point *Q*.

Answer (.................. , ...) [1]

(b) Find the equation of the line *PR*.

Answer .... [2]

(c) *W* is a point on the *x*-axis such that *RW* is perpendicular to *RS*.
     Write down the coordinates of the point *W*.

Answer (.................. , ...) [1]

(d) Find the perimeter of *PQRS*.

Answer .... units [3]

---

19 In the year 2022, it was recorded that the average amount of waste generated in Singapore daily was 20.3 thousand tonnes. Assuming that there are 365 days in a year, find the total amount of waste generated in the year 2022, leaving your answer in standard form.

Answer .... tonnes [1]

---

20 A company sells chocolate muffins and banana muffins. These muffins are sold across three different outlets, which are Outlet A, Outlet B and Outlet C. The table shows the number of boxes of chocolate muffins and banana muffins produced at each of the three outlets in a particular day.

	Chocolate Muffin	Banana Muffin
Outlet A	36	40
Outlet B	48	39
Outlet C	45	x

The table can be represented by matrix $\mathbf{Q}$, where $\mathbf{Q} = \begin{pmatrix} 36 & 40 \\ 48 & 39 \\ 45 & x \end{pmatrix}$.

(a) (i) The production cost for each box of chocolate muffin and each box of banana muffin is \$3 and \$4.50 respectively. Represent this information in a 2×1 matrix, $\mathbf{P}$.

Answer $\mathbf{P}$ = $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(ii) Evaluate the matrix $\mathbf{R} = \mathbf{QP}$.

Answer $\mathbf{R}$ = $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(b) State what each of the elements in matrix **R** represents.

....

.... [1]

The company sells each box of muffins for 70% more than it costs to produce.

On that day, the company sold $\frac{5}{6}$ of each flavour of muffins in Outlet A, $\frac{4}{5}$ of each flavour of muffins in Outlet B, and all the muffins in Outlet C.

(c) By using the matrix **R**, find the total amount collected from the sale of muffins from all three outlets.

Answer $ .... [3]

(d) The total amount collected from the sales of muffins from all three outlets is $1454.52. Calculate the value of x.

Answer $x =$ .... [1]

21 In a class of 36 students, each student is either in a sports team or a performing arts club. Each student is also either a member or is a leader in their respective co-curricular activities.
One student is selected at random from the class. The probability that the student selected is in a sports team is $\dfrac{1}{3}$.

(a) Given that there are 4 sports team members in the class, find the number of students who are leaders in a sports team.

Answer .... [1]

Two students are selected at random from the class. The probability that both of them are members of a performing arts club is $\dfrac{1}{42}$.

(b) Find the number of students who are members in a performing arts club.

Answer .... [3]

In another class, the probability that a student is a leader in a co-curricular activity is k. The probability that a student is in a performing arts club is m.

(c) Derrick claimed that the probability of a student in this class being a leader or is in a performing arts club is $k + m$. Explain with reasons whether you agree with Derrick's claim.

....

....[1]

22 The scale of a map is $1 : n$.
A school garden measuring 8 cm^2 on a map has an actual area of 2048 m^2.
Find n.

*Answer* $n =$ .... [2]

23 4 technicians plan to repair 416 computers in 16 days.
However, after repairing for 10 days, one of the technicians fell sick and could not continue.
The remaining computers are to be repaired by the other three technicians.

Assuming that the technicians work at the same rate, calculate the total number of days needed for all 416 computers to be repaired.

Answer .... days [3]

24 The cumulative frequency diagram shows the time in hours that each of the 120 adults from Town A spend on exercise in one week.
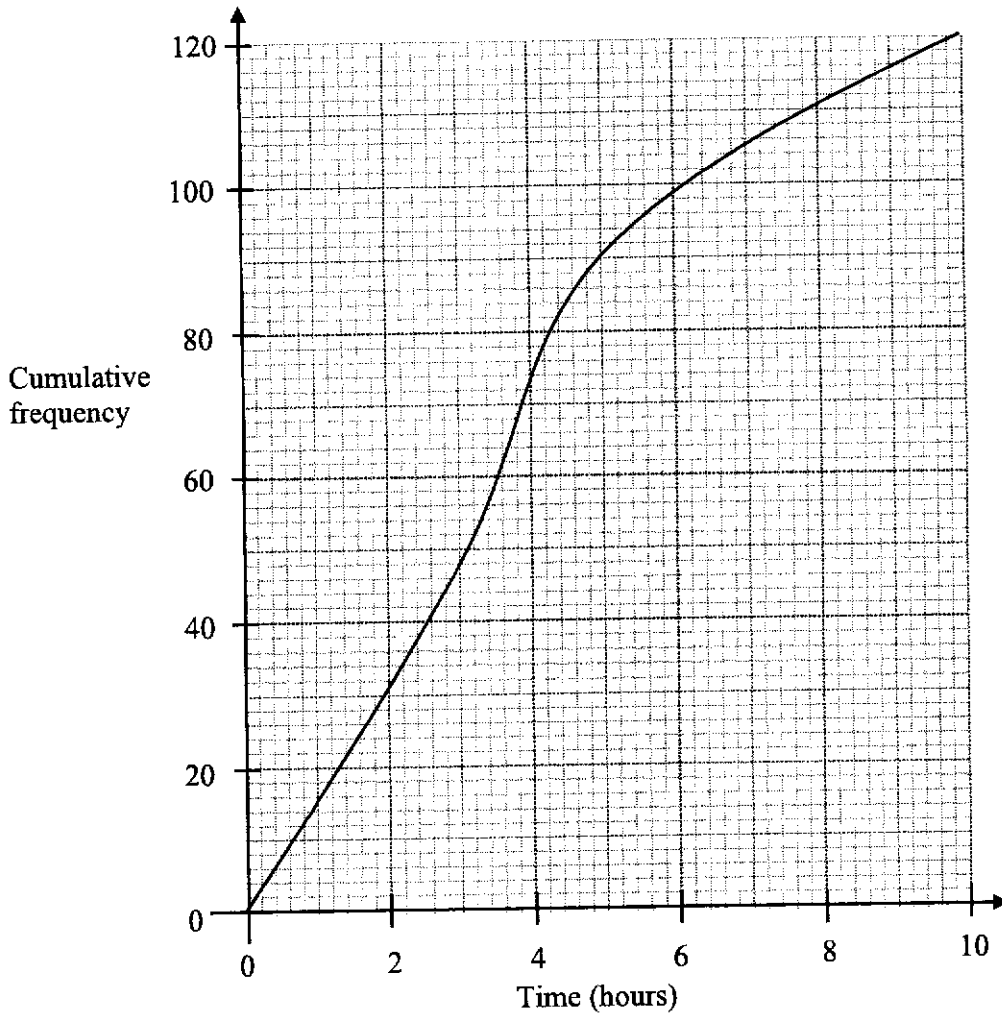

(a) Explain if you can find the probability of an adult spending exactly 5 hours on exercise weekly.

....

....[1]

(b) The health promotion board recommends at least k hours of weekly exercise for adults to stay fit and active. 60 % of the adults in Town A meet this recommended weekly exercise hours. Find the value of k.

Answer k = ... [1]

The box-and-whisker plot shows the time in hours that each of the 120 adults from Town B spend on exercise in one week.

Time (hours)
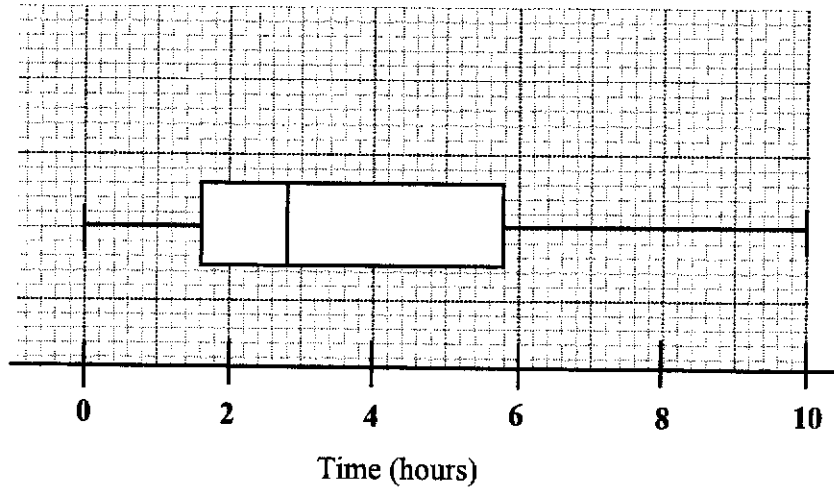
(c) Find the median and interquartile range of the weekly exercise hours of the adults in Town B.

Answer   Median = ... h [1]

Interquartile range = ... h [1]

(d) Make two comparisons between the weekly exercise hours of the adults from Town A and Town B. State your reasons.

....

....

....

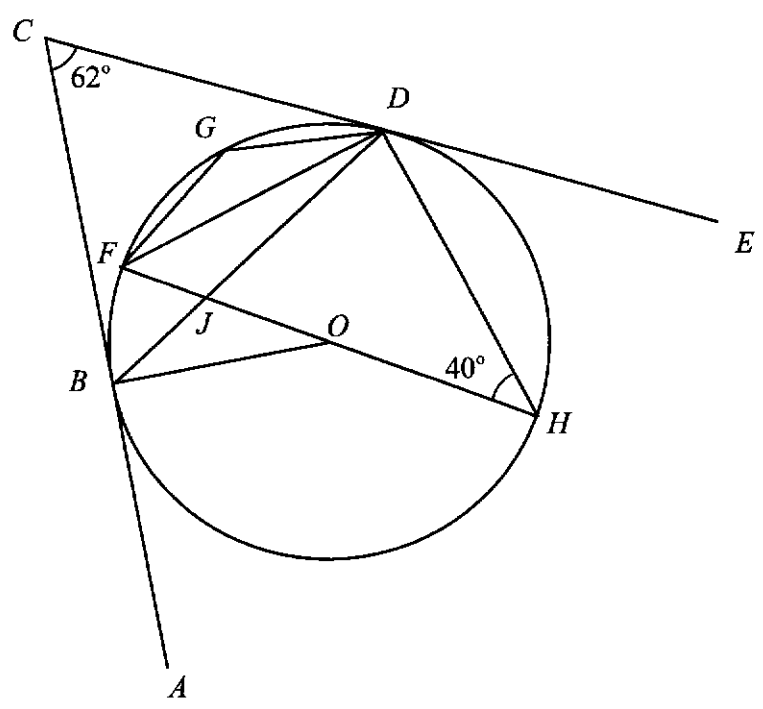...[2]

25 The lines *ABC* and *CDE* are tangents to the circle at *B* and *D* respectively.
F, *G* and *H* are points on the circle and the centre of the circle is *O*.
The straight lines *BD* and *HF* intersect at *J*.
It is given that $\angle ACE = 62°$ and $\angle DHF = 40°$.

Find, giving reasons for each answer,

(a) $\angle FGD$,

Answer ....° [1]

(b) $\angle CDO$,

Answer ....° [1]

(c) $\angle BOF$,

Answer ....° [2]

(c) $\angle FJD$.

Answer ....° [2]

26 A liquid container, 30 cm tall, can be modelled by a frustrum, A, placed below a cylinder B, as shown in **Figure 1**.

Figure 2 shows a sketch of the cone that the frustum A is a part of. The diameter QRS of the top of the frustrum, is 12 cm and the diameter VUT of the bottom is 8 cm. The height of the frustrum, RU is 9 cm. R and U are the midpoints of QS and VT respectively.
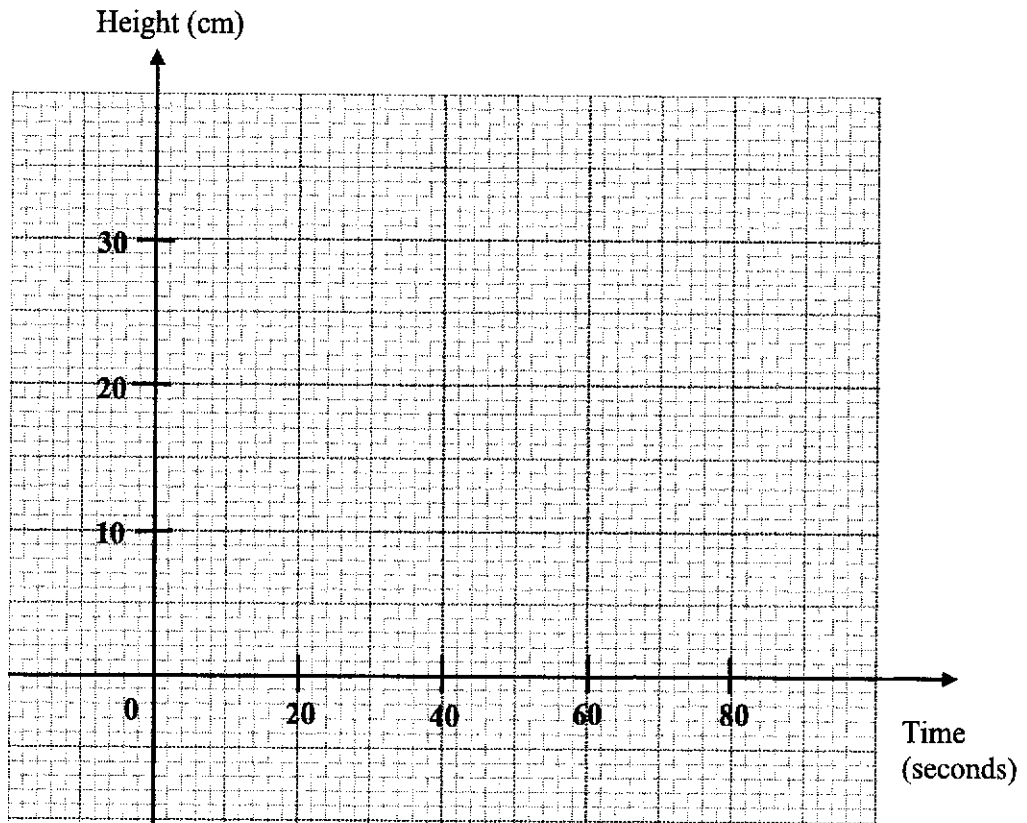
Figure 1	**Figure 2**
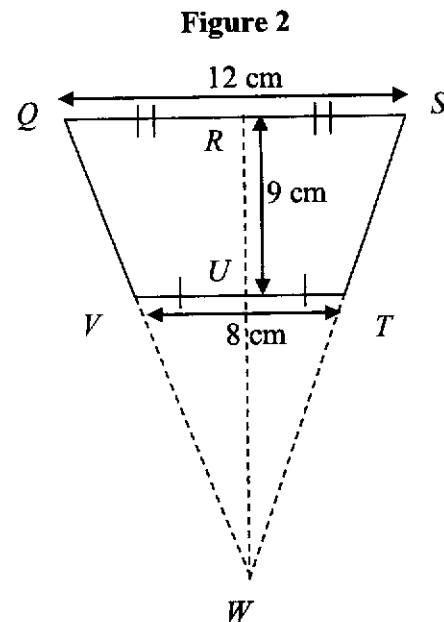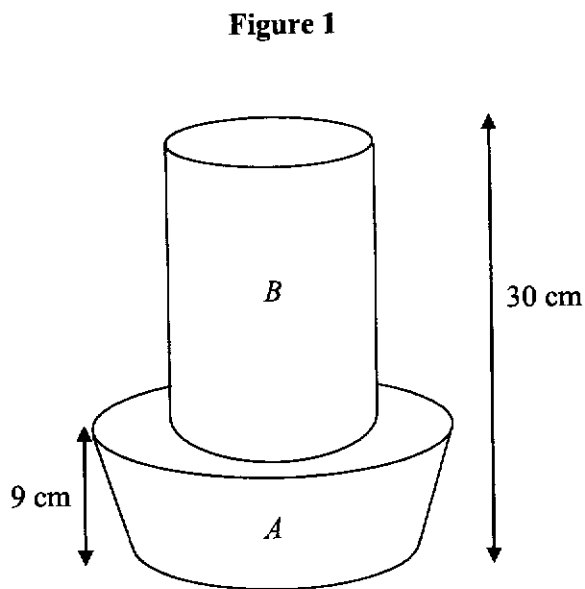
(a) Calculate the volume of the frustrum, A.

Answer .... cm^3 [3]
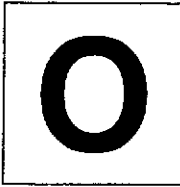
A liquid is poured into the container at a constant rate of q cm^3 per second.
It takes 50 seconds to fill up A completely and it takes another 30 seconds to fill up B completely.

(b) Sketch the height of the liquid in the container with respect to time.

Height (cm)

[2]

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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2024
Secondary Four Express and Five Normal

CANDIDATE NAME:

CLASS: /   INDEX NUMBER:

MATHEMATICS   4052/02

Paper 2   **15 August 2024**

2 hours 15 minutes

1100 – 1315h

Candidates answer on the Question Paper.

Additional Materials: Nil

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terms of π.

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Setter: Mr Colin Chen

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 A cone has a radius of 24 cm and a total surface area of $1536\,\pi$ cm^2.

(a) Show that the volume of the cone is $6144\,\pi$ cm^3. [3]

(b) The cone is recasted into a solid sphere. Find

(i) the radius of the sphere,

Answer .... cm [2]

(ii) the total surface area of the sphere.

Answer .... cm^2 [2]

4

2 (a) Matthew invests $20 000 into a fixed deposit at r % per month, compounded monthly. After 24 months he will be able to receive $21 337.05.
Find the value of r.

Answer $r =$ .... [2]

(b) Matthew buys a car on hire purchase. He made a 30% downpayment and takes up a 7 year loan which charges simple interest at 2.78% per annum. Given that he pays $1774.40 monthly instalment for the car, calculate the price of the car. Give your answer correct to the nearest dollar.

Answer $ .... [3]

(c) Matthew wants to invest in Japanese Yen (JPY) to make a profit. In April, he bought JPY with SGD 980. In June, he sold the JPY to get his money back in Singapore dollars.
The exchange rates were:
- April: SGD 1 = JPY 114.5
- June: SGD 1 = JPY 118.2

Calculate the percentage profit or loss that Matthew made.

Answer Matthew made a **profit/loss** of .... % [3]
(circle the correct option)

3 A chess club currently has 37 male members and 16 female members. The club hosted a Valentine's Day event to attract married couples to join the club as new members. (Both husband and wife must be totally new to the club.) After this event, the percentage of female members became 40%. Find the number of married couples who joined the club after the event.

Answer .... couples [3]

4   The marks of a group of 19 students in a test were recorded, as the marks are shown in the stem and leaf diagram. The mode is 42 marks.

Stem	Leaf						
3	*a*	0	1	2	6		
4	1	2	2	2	6	7	
5	0	2	4	4	*b*	5	8	9

Key: 4 | 1 means 41 marks

(i)   Find the value of *a* and *b* .

Answer a = ... [1]

b = ... [1]

(ii)   Find the median mark.

Answer .... [1]

(iii)   Calculate the mean and standard deviation of the marks.

Answer mean = ... [1]

Standard Deviation = ... [1]

(iv)   Marks for another group of 20 students was also recorded. The results are summarized in the table below.

Mean	48.2
Standard Deviation	7.22

Make two comparisons between the marks obtain by the group of 19 students and by the group of 20 students.

Answer ....

....

.... [2]

5   **(a)**   Adam has some 50-cent and 20-cent coins in his wallet. Given that he has a total of 73 coins adding up to a value of \$28.10, form 2 simultaneous equations to find the number of 50-cent and 20-cent coins he has.

Answer   He has .... 50-cent coins and .... 20-cent coins [5]

(b) Express $\dfrac{4x}{4x^2 - 25} + \dfrac{1}{5 - 2x}$ as a single fraction in its simplest form.

Answer .... [3]

6 The position vectors of points A and B are $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -12 \end{pmatrix}$ respectively.
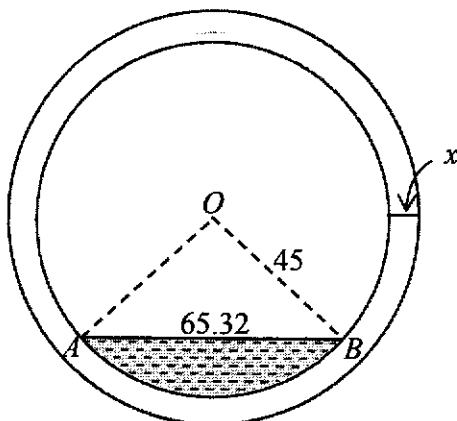
(a) Find the column vector $\overrightarrow{BA}$.

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(b) Find $\left| \overrightarrow{BA} \right|$.

*Answer* .... units [2]

7   The diagram shows the cross section of a plastic pipe. The arc AB is part of the circle with centre O and radius 45 mm. The pipe has an uniform thickness of x mm. The shaded area represents the cross-sectional area that is filled with water. $AB = 65.32$ mm.

(i)   Show that $\angle AOB = 1.6243$ radians.   [2]

(ii)   Given that the cross-sectional area of the pipe is 1398.29 mm^2, find x.

Answer $x =$ .... mm [2]

8
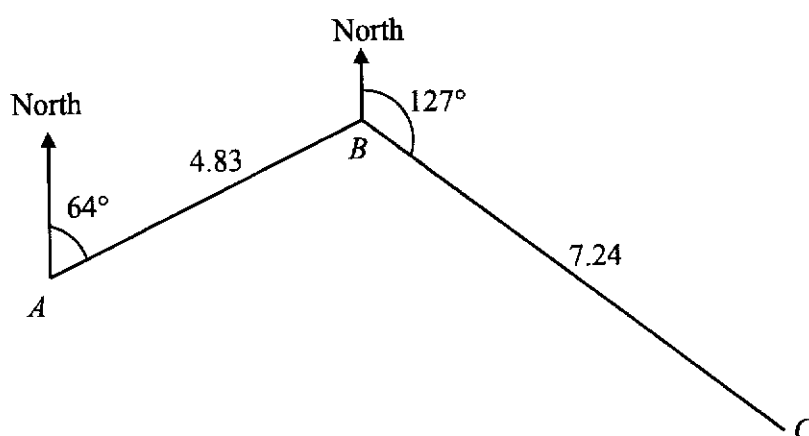
The diagram shows the positions of 3 points A, B, and C, at sea level.
$AB=4.83$ km and $BC=7.24$ km. The bearing of B from A is 064° and the bearing of C from B is 127°.

(i) Show that $AC=10.368$ km. [3]

(ii) Hence find the bearing of A from C.
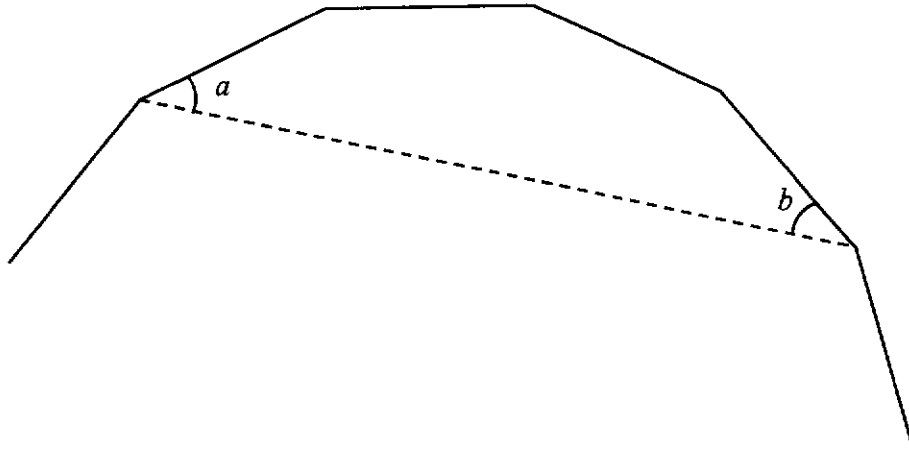
Answer .... ° [3]

(iii) A boat travels in a straight line from point *B* to reach the line *AC*. Find the shortest distance that the boat needs to travel.

Answer .... km [2]

(iv) An object was detected directly below point *C*. The angle of depression of the object from *B* is 12°. Find the distance of the object from *C*.

Answer .... km [2]

9   The diagram shows a part of a regular n-sided polygon.

Given $\angle a + \angle b = 72°$, find the value of n.

Answer $n =$ .... [3]

13

Blank Page

10 *CBEO* is a parallelogram. The position vectors of *C* and *E*, relative to *O*, are **a** and **b** respectively.
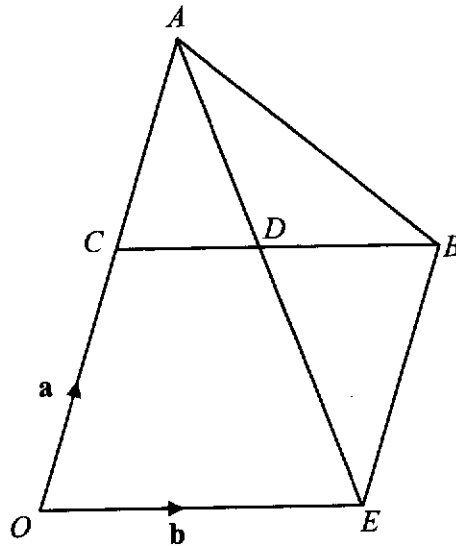
It is given that $3BC = 5BD$ and $AD = \dfrac{2}{3}DE$.

(a) Find, in terms of **a** and **b**,

(i) $\overrightarrow{CD}$,

Answer .... [1]

(ii) $\overrightarrow{AD}$.

Answer .... [3]

(b) Is $\overrightarrow{AC}$ parallel to $\overrightarrow{OC}$? Justify your answer with appropriate working.

Answer ....

.... [2]

(c) Find

(i) $\dfrac{\text{area of } \Delta ADB}{\text{area of } \Delta ACD}$,

Answer .... [1]

(ii) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta AOE}$,

Answer .... [2]

(iii) $\dfrac{\text{area of } \Delta ADB}{\text{area of } CDEO}$.

Answer .... [2]

16

11   The variables x and y are connected by the equation $y = 2x + \dfrac{6}{x} - 7$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	3	4	5	6	7
y	6	1	0	a	1	2.5	4.2	6	7.9

(a)   Calculate the value of a.

Answer $a =$ .... [1]

(b)   On the grid, draw the graph of $y = 2x + \dfrac{6}{x} - 7$ for $0.5 \le x \le 7$.

[2]

(c)   The point A has coordinates $(3, -1)$. A tangent to the curve can be drawn so that the tangent passes through A.

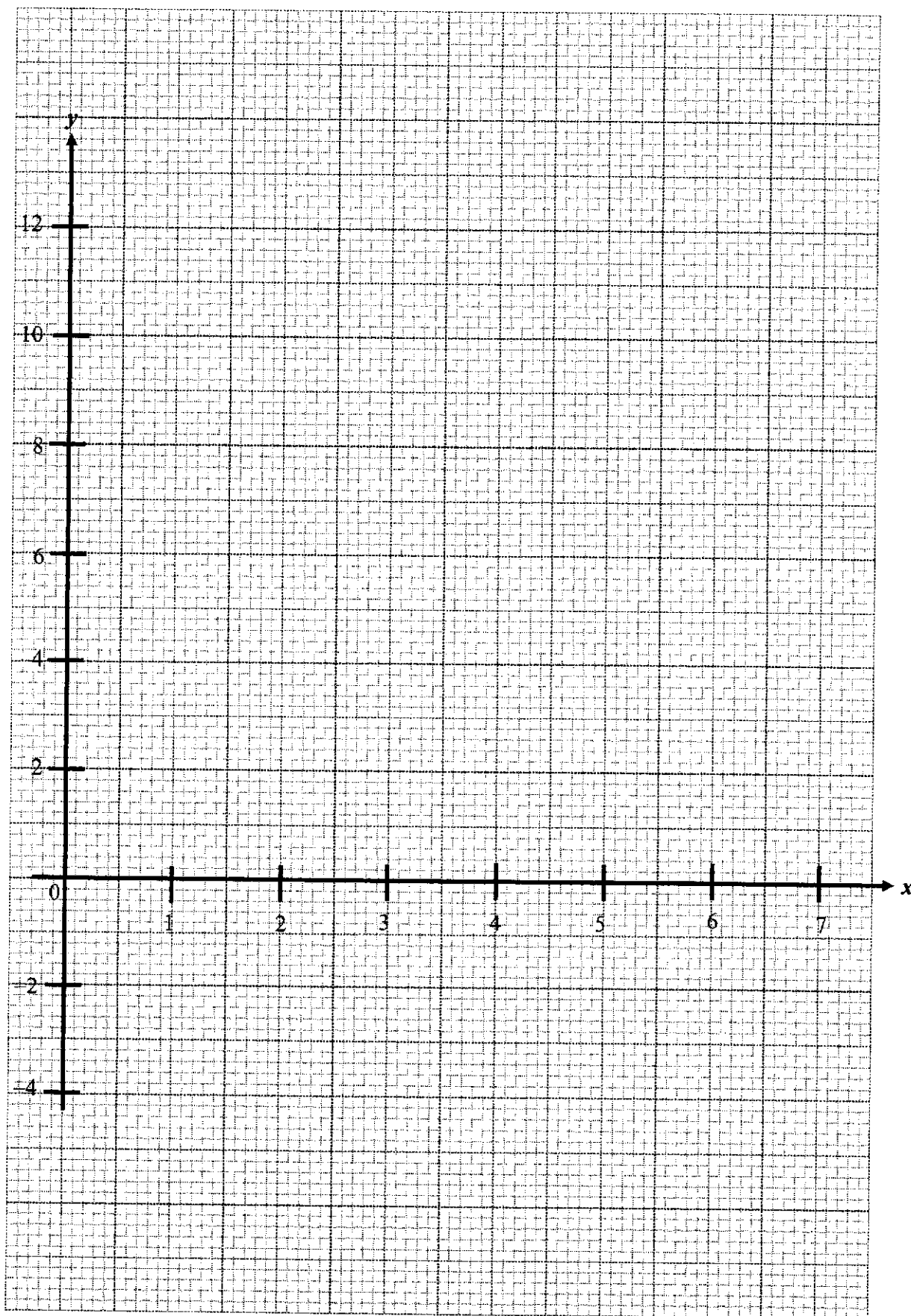(i)   On the same grid draw this tangent.   [1]

(ii)   Find the equation of this tangent.

Answer   .... [2]

(d)   By drawing a suitable straight line on your graph, explain why the equation $3x^2 - 8x + 6 = 0$ has no solution for $0.5 \le x \le 7$.

Answer ....

.... [3]

12   Mr Tan took part in a 42 km marathon run. He used x minutes to run the first 25 km at a constant speed.

(a)   Write down an expression, in terms of x, for his speed in km/h for the first 25 km.

Answer .... km/h [1]

(c)   For the last 17 km of the run, Mr Tan ran at a speed that is 2 km/h slower than his initial speed, and he took $(x-22.5)$ minutes.
Write down an equation to represent this information and show that it simplifies to $2x^2 - 525x + 33750 = 0$.   [3]

(c) Solve $2x^2 - 525x + 33750 = 0$.

Answer $x =$ .... or .... [3]

(d) Given that Mr Tan took more than 4 hours to complete his run, find the time he took for the whole run. Give your answer in hours and minutes.

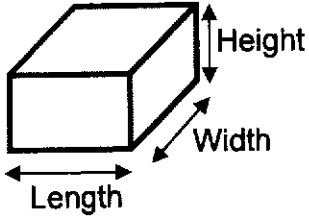*Answer* ....h.................... min [2]

13   A book writer intends to sell her books to either Australia or Brazil, and she is planning to use Singapore post office Speedpost service to mail her books. The following table shows the mailing prices and other requirements.

<div style="border:1px solid">

Speedpost (International) Service Rates

Destination	Weight Up To	Package (Speedpost)		
		Speedpost Express	Speedpost Priority	Speedpost Economy
Zone A Malaysia	Document	$25.00	$20.00	-
	2kg	$61.00	$39.00	$26.00
	5kg	$91.00	$55.00	$26.00
	10kg	$122.00	$78.00	$43.00
	20kg	$158.00	$115.00	$66.00
Zone B Asia	Document	$50.00	$45.00	-
	2kg	$108.00	$76.00	$37.00
	5kg	$180.00	$123.00	$37.00
	10kg	$269.00	$182.00	$64.00
	20kg	$398.00	$278.00	$106.00
Zone C Australia, New Zealand, Europe, USA & Canada	Document	$75.00	$70.00	-
	2kg	$159.00	$107.00	$80.00
	5kg	$259.00	$161.00	$80.00
	10kg	$400.00	$232.00	$135.00
	20kg	$569.00	$354.00	$202.00
Zone D Rest of the World	Document	$100.00	$95.00	-
	2kg	$393.00	$155.00	$86.00
	5kg	$630.00	$260.00	$86.00
	10kg	$929.00	$386.00	$160.00
	20kg	$1,388.00	$603.00	$282.00

- All rates are in Singapore dollars.
- GST is not applicable for international mail rates.
- The largest dimension should not exceed 400 mm, with length, width and height combined not exceeding 900 mm.

</div>

(a) The writer wants to send an 18 kg parcel using Speedpost Economy.

Calculate the percentage increase in the cost of sending to Brazil as compared to the cost of sending to Australia.

Answer .... % [2]

(b) All the books have the same dimension of 6 mm by 128 mm by 128 mm and weighs 75g each. The cost to manufacture each book is $1.80.

The author hopes to earn a profit of $3.00 for each book sold to Australia.

By considering the manufacturing cost and the mailing cost to Australia, find the lowest possible price at which the writer should sell each book. Give your answer in dollars and cents correct to the nearest ten cents. Show your working clearly.

(Answer space continues on the next page.)

Answer $ .... [6]

END OF PAPER

Sec 4 Express Mathematics 2024 Prelims Marking Scheme

1		$a(5a-2b)(5a+2b)$		
		$= a\left((5a)^2 - (2b)^2\right)$		
		$= a(25a^2 - 4b^2)$	M1	Correct expansion
		$= 25a^3 - 4ab^2$	A1	

2		$3^x + 3^{x+2} = 90$		
		$3^x + 3^x \times 3^2 = 90$	M1	
		$3^x(1+9) = 90$		
		$3^x = 9$		
		$3^x = 3^2$		
		$x = 2$	A1	

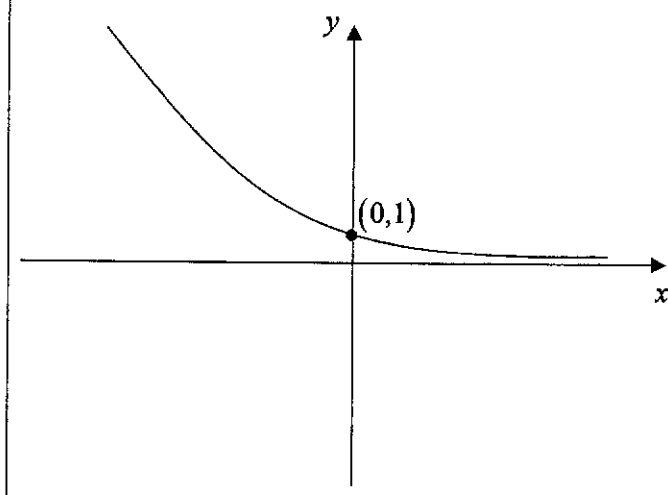| 3		$\dfrac{2x^2 + 4xy - 3x - 6y}{2x^2 + xy - 6y^2}$		
		$= \dfrac{2x(x+2y) - 3(x+2y)}{2x^2 + xy - 6y^2}$	M1	Factorise by grouping for numerator
		$= \dfrac{(x+2y)(2x-3)}{2x^2 + xy - 6y^2}$		
		$= \dfrac{(x+2y)(2x-3)}{(2x-3y)(x+2y)}$	M1	Factorise denominator
		$= \dfrac{2x-3}{2x-3y}$	A1	

4	(a)	LCM of A and B is $q \times 3^{m+2} \times 7$ and at the same time the LCM of A and B is $3^3 \times 5 \times 7$.		
		$m = 1$	B1	
		$q = 5$	B1	
	(b)	$A = 3^3 \times 7$ and $B = 3 \times 5 \times 7$		
		HCF of A and B $= 3 \times 7$ $= 21$	B1	

5		The size/height of the bar graph.		Stating that the graph does not start from zero is accepted too.
			B1	
		It can be misleading because for example, the size/height of the graph in 2019 is twice that of the graph in 2010 but this does not represent their actual temperatures.	B1	A relevant example has to be given.

6	(a)	$\left(\dfrac{y^9}{27x^{-6}}\right)^{-\frac{2}{3}}$		
		$=\dfrac{y^{9\times\left(-\frac{2}{3}\right)}}{3^{3\times\left(-\frac{2}{3}\right)}x^{-6\times\left(-\frac{2}{3}\right)}}$ $=\dfrac{y^{-6}}{3^{-2}x^4}$	M1	Multiply $\left(-\dfrac{2}{3}\right)$ to each index
		$=\dfrac{3^2}{x^4y^6}$	A1	$\dfrac{9}{x^4y^6}$ is also accepted
	(b)	$\dfrac{25}{125^{2-x}}=5^y$		
		$\dfrac{5^2}{\left(5^3\right)^{2-x}}=5^y$ $\dfrac{5^2}{5^{6-3x}}=5^y$ $5^{2-6+3x}=5^y$	M1	Any other equivalent form of $5^m=5^n$ will be accepted too
		$5^{3x-4}=5^y$ Comparing index, $3x-4=y$ $x=\dfrac{y+4}{3}$	A1	

7	(a)	$9x^2 + 24xy + 16y^2$		
		$=(3x+4y)^2$	**B1**	
	(b)	Let $x = a^4$, $144a^8 - \left(9a^8 + 24a^4 y + 16y^2\right)$ $= 144x^2 - \left(9x^2 + 24xy + 16y^2\right)$ $= \left(12x\right)^2 - \left(3x + 4y\right)^2$	**M1**	Attempt to factorise an expression in the form $a^2 - b^2$, after making use of part **(a)** answer
		$=(12x + 3x + 4y)(12x - 3x - 4y)$ $=(15x + 4y)(9x - 4y)$ $=\left(15a^4 + 4y\right)\left(9a^4 - 4y\right)$	**A1**	

| 8	(a)	$q = \dfrac{k}{r^2}$, where k is the proportionality constant.		
		q and r are the initial intensity and distance respectively.		
		When the distance is reduced by 40%, the new distance is $\dfrac{3r}{5}$. New intensity $= \dfrac{k}{\left(\dfrac{3r}{5}\right)^2}$	M1	
		$= \dfrac{k}{\dfrac{9r^2}{25}}$ $= \dfrac{25k}{9r^2}$ $= \dfrac{25}{9}q$	A1	
	(b)	Percentage difference $= \dfrac{\text{New Intensity} - \text{Initial Intensity}}{\text{Initial Intensity}} \times 100\%$ $= \dfrac{\dfrac{25}{9}q - q}{q} \times 100\%$ $= \dfrac{16}{9} \times 100\%$ $= 178\%$ (3 s.f.)	B1	$177\dfrac{7}{9}\%$ will be accepted too.

9			**B1**	Both shape and y-intercept needs to be drawn and indicated correctly

The graph shows a decreasing exponential curve passing through the point $(0,1)$ with axes labelled x and y.

10		$SC = RD$ (given)		No marks for this step
		$\angle SCD = \angle RDA$ (alternate angles)	**M1**	
		$CD = DA$ (sides of a rhombus are equal)	**M1**	
		By SAS congruency test, triangle SCD is congruent to triangle RDA.	**A1**	

11		$x^2 - 10x - 2$ $= (x-5)^2 - 25 - 2$ $= (x-5)^2 - 27$	**M1**	
		Minimum point is $(5, -27)$	**A1**	

12	(a)	$n(\xi)$ $= 3 \times 3$ $= 9$	B1	
	(b)	P $= \{(-1,0),(0,-1),(0,0),(0,1)\}$	B1	
	(c)	Q $= \{(-2,1),(-1,1)\}$	B1	

13   The diagram below shows a triangle *ABC*.
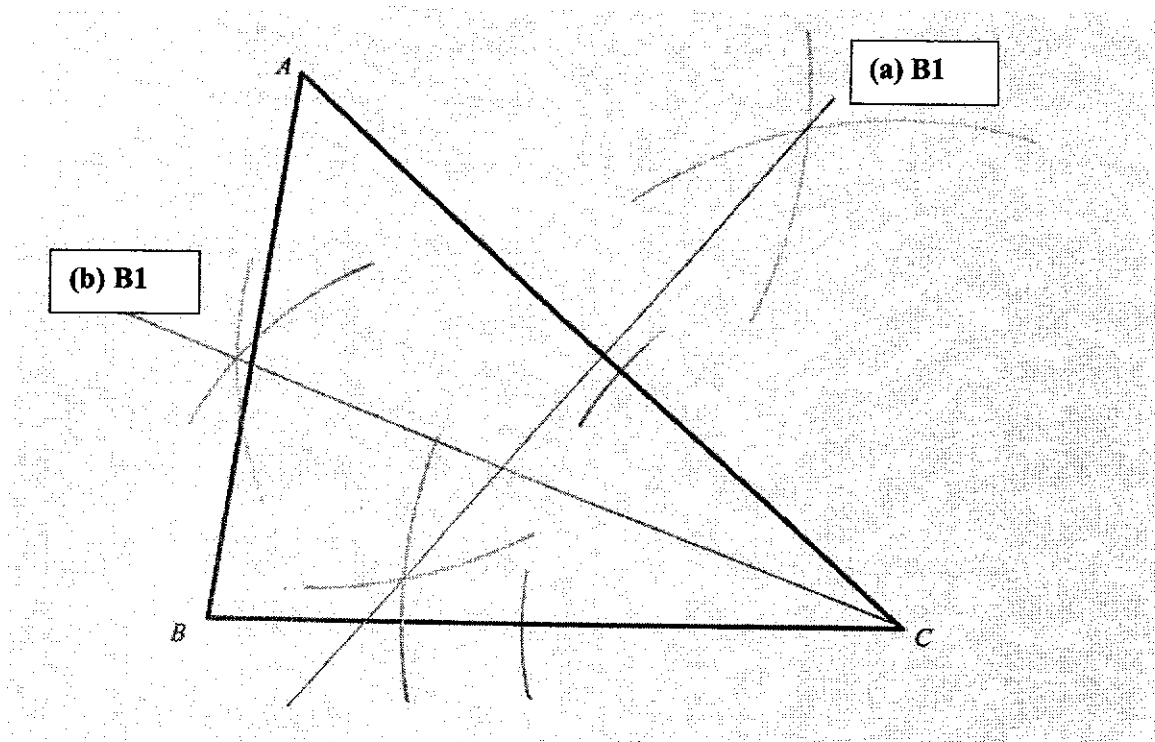
Note that marks will not be awarded if relevant arcs are not drawn for parts **(a)** and **(b)** respectively.

14	(a)	$n(W) = 2$	B1	
	(b)	$\varnothing, \{0\}, \{\{0\}\}, \{0, \{0\}\}$	B1	
	(c)	$A \cup B'$ / $(A' \cap B)'$ / $(A \cup B)' \cup A$ / $(A \cap B) \cup B'$	B1	

15		$x - 3 \leq \dfrac{5-x}{3} < \dfrac{x+1}{2}$		
		$x - 3 \leq \dfrac{5-x}{3}$ $3x - 9 \leq 5 - x$ $4x \leq 14$ $x \leq 3\dfrac{1}{2}$	M1	
		$\dfrac{5-x}{3} < \dfrac{x+1}{2}$ $10 - 2x < 3x + 3$ $7 < 5x$ $1\dfrac{2}{5} < x$	M1	
		$1\dfrac{2}{5} < x \leq 3\dfrac{1}{2}$	A1	

16		$\angle BAC = 90°$ (right angle in a semicircle)	M1	
		$\cos \angle ABC = \dfrac{AB}{BC}$ $\dfrac{8}{17} = \dfrac{AB}{34}$	M1	Only awarded with the relevant values substituted in this step.
		$AB = 16$ cm $AC = \sqrt{34^2 - 16^2}$ $= 30$ $\cos \angle ACD = -\cos \angle ACB$	M1	
		$= -\dfrac{AC}{BC}$ $= -\dfrac{30}{34}$ $= -\dfrac{15}{17}$	A1	

17	(a)	$T_7 = 8^2 - 7$ $= 57$	B1	
	(b)	$T_n = (n+1)^2 - n$	B1	
	(c)	$T_{n+1} - T_n$ $= (n+2)^2 - (n+1) - \left[(n+1)^2 - n\right]$	M1	
		$= n^2 + 4n + 4 - n - 1 - \left[n^2 + 2n + 1 - n\right]$ $= 2n + 2$ $= 2(n+1)$	M1	
		I agree. Since the difference $T_{n+1} - T_n$ is a multiple of 2, hence the difference will always be an even integer.	A1	

18	(a)	$15 \div 3 = 5$ $Q = (-2 + 5 \times 2, 1)$ $ = (8,1)$	B1	
	(b)	Gradient of PR $= \dfrac{4-1}{13-(-2)}$ $= \dfrac{1}{5}$	M1	
		Let the equation of the line PR be $y = mx + c$ $4 = \dfrac{1}{5} \times 13 + c$ $c = \dfrac{7}{5}$ Equation of line PR: $y = \dfrac{1}{5}x + \dfrac{7}{5}$	A1	
	(c)	$W = (13, 0)$	B1	
	(d)	Length of line segment QR $= \sqrt{(13-8)^2 + (4-1)^2}$ $= \sqrt{34}$	M1	
		Perimeter of $PQRS$ $= 2 \times \sqrt{34} + 2 \times 10$	M1	
		$= 31.7$ units (3 s.f.)	A1	

19		$20.3 \times 10^3 \times 365$ $= 7409500$ $= 7.41 \times 10^6$ (3 s.f.)	B1	

20	(a)(i)	$\mathbf{P} = \begin{pmatrix} 3 \\ 4.50 \end{pmatrix}$	B1	
	(a)(ii)	$\mathbf{R} = \begin{pmatrix} 36 & 40 \\ 48 & 39 \\ 45 & x \end{pmatrix} \begin{pmatrix} 3 \\ 4.50 \end{pmatrix}$ $= \begin{pmatrix} 288 \\ 319.5 \\ 135 + 4.5x \end{pmatrix}$	B1	
	(b)	The elements in matrix $\mathbf{R}$ represents the total production cost for the muffins sold at Outlets A, B and C respectively.	B1	
	(c)	$1.7\mathbf{R}$ $= 1.7 \begin{pmatrix} 288 \\ 319.5 \\ 135 + 4.5x \end{pmatrix}$	M1	
		$= \begin{pmatrix} 489.6 \\ 543.15 \\ 229.5 + 7.65x \end{pmatrix}$		
		$\begin{pmatrix} \dfrac{5}{6} & \dfrac{4}{5} & 1 \end{pmatrix} \begin{pmatrix} 489.6 \\ 543.15 \\ 229.5 + 7.65x \end{pmatrix}$	M1	
		$= (1072.02 + 7.65x)$ Total amount collected from sale $= \$(1072.02 + 7.65x)$	A1	
	(d)	$1072.02 + 7.65x = 1454.52$ $x = 50$	B1	

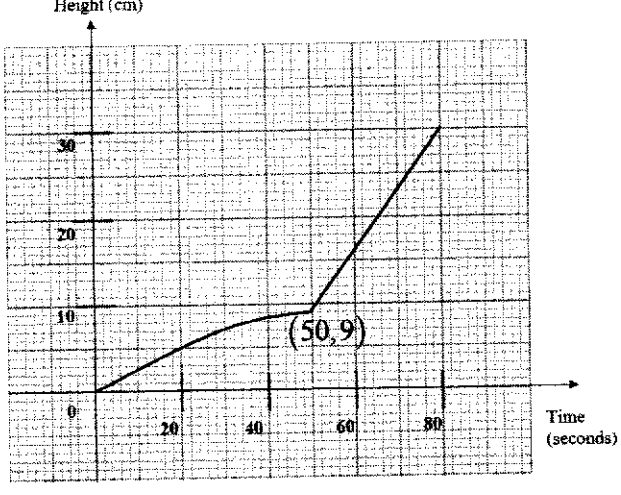| 21	(a)	Number of students who are in a sports team $=\dfrac{1}{3} \times 36$ $=12$ Number of leaders in a sports team $=12-4$ $=8$	B1	
	(b)	Let the number of students who are members in a performing arts club be n. P(both students selected are members in a performing arts club) $=\dfrac{n}{36} \times \dfrac{n-1}{35}$ Hence, $\dfrac{n(n-1)}{1260}=\dfrac{1}{42}$	M1	
		$n(n-1)=30$ $n^2-n-30=0$ $(n-6)(n+5)=0$	M1	Completing the square or quadratic formula is accepted too.
		$n=6$ or $n=-5$ (rej $\because n$ cannot be negative)	A1	
	(c)	I disagree with Derrick's claim because; a student <u>can be both a leader</u> and <u>in a performing arts club.</u> Or <u>being a leader and being in a performing arts club</u> are <u>not mutually exclusive.</u>	B1	

22		$8\ cm^2 : 2048\ m^2$ $=1\ cm^2 : 256\ m^2$ $=1\ cm^2 : 2560000\ cm^2$	M1	
		$=1\ cm : 1600\ cm$ $n=1600$	A1	

| 23 | | 4 technicians can repair 416 computers in 16 days

4 technicians can repair 26 computers in 1 day

4 technicians can repair 260 computers in 10 days | M1 | |
| | | 4 technicians can repair 156 computers in 6 days

1 technician can repair 156 computers in 24 days

3 technicians can repair 156 computers in 8 days | M1 | |
| | | Total number of days taken
$=10+8$
$=18$ | A1 | |
| | | | | |

24	(a)	No, you cannot because the cumulative frequency diagram only allows you to find probability of less than (less than or equals to) 5 weekly exercise hours or probability of at least (more than) 5 weekly exercise hours.	B1	
	(b)	$(100-60)\% \times 120 = 48$ $k = 3$	B1	

(c)

	Lower Quartile	Median	Upper Quartile	Interquartile Range
Town B	1.6	**2.8**	5.8	**4.2**

B1
B1

1 mark correct median and 1 mark for correct interquartile range.

	Lower Quartile	Median	Upper Quartile	Interquartile Range
Town A	1.9	**3.5**	5	**3.1**

(d)	On average, the people in Town A have a higher weekly exercise hours than Town B since its median is higher.	B1	
	There is a larger spread in weekly exercise hours in Town B since its interquartile range is higher as compared to that in Town A.	B1	

25	(a)	$\angle FGD = 180° - \angle FHD$ $\quad = 180° - 40°$ (angles in opposite segments) $\quad = 140°$	B1	
	(b)	$\angle CDO = 90°$ (tangent $\perp$ radius)	B1	
	(c)	$\angle ODH = 40°$ (base $\angle$ of isosceles triangle)	M1	
		$\angle FOD = 40° + 40°$ (external angle of a triangle) $\quad = 80°$		
		$\angle BOF = 360° - 90° - 90° - 62° - 80°$ ($\angle$ sum of quadrilateral $BODC$) $\quad = 38°$	A1	
	(d)	$\angle BDF = \dfrac{38°}{2}$ (angle at centre $= 2 \times$ angle at circumference) $\quad = 19°$	M1	
		$\angle OFD = \dfrac{180° - 80°}{2}$ (base $\angle$ of isosceles triangle) $\quad = 50°$ $\angle FJD = 180° - 50° - 19°$ $\quad = 111°$	A1	

26	(a)	Let the height of the cone in Figure 2 be h.		
		Since triangle QWS is similar to triangle VWT, $$\frac{h}{h-9}=\frac{12}{8}$$	M1	
		$12h-108=8h$ $h=27$		
		Volume of A $$=\frac{1}{3}\pi\left(6^2\right)h-\frac{1}{3}\pi\left(4^2\right)(h-9)$$	M1	
		$$=\frac{1}{3}\pi(36)(27)-\frac{1}{3}\pi(16)(18)$$ $$=\frac{1}{3}\pi[684]$$ $=228\pi$ $=716\text{ cm}^3$ (3 s.f.)	A1	
	(b)		B1 B1	1 mark is for correctly plotted point $(50,9)$. 1 mark is for the correct shapes for both graphs time from 0 to 50 and time 50 to 80 seconds.

4E EMath Prelim P2 2024 Mark Scheme

1a)	Since area $= 1536\pi$, $1536\pi = \pi(24)^2 + \pi(24)l$ $l = 40\,\text{cm}$	M1	
	height $= \sqrt{40^2 - 24^2} = 32\,\text{cm}$ Volume of cone $= \dfrac{1}{3} \times (24)^2 \times \pi \times 32$	M1	
	$= 6144\pi\,\text{cm}^3$ (Shown)	A1	
1bi)	$6144\pi = \dfrac{4}{3} \times \pi \times r^3$	M1	
	$r = \sqrt[3]{\dfrac{6144(3)}{4}}$ $= 16.641$ $= 16.6\,\text{cm}$ (to 3 s.f.)	A1	
1bii)	Surface area $= 4\pi(16.641)^2$	M1	
	$= 3479.91555$ $= 3480\,\text{cm}^2$ (to 3 s.f.)	A1	
2a)	$20000\left(1 + \dfrac{r}{100}\right)^{24} = 21337.05$	M1	
	$\left(1 + \dfrac{r}{100}\right)^{24} = \dfrac{21337.05}{20000}$ $1 + \dfrac{r}{100} = \sqrt[24]{\dfrac{21337.05}{20000}}$ $= 1.002700002$ $r = 0.270$ (to 3 s.f.)	A1	
2b)	loan amount $= \$1774.40 \times 84$ $= \$149049.60$ $100\% + 2.78\% \times 7 = 119.46\%$ Loan amount without interest $= \$149049.60 \times \dfrac{100\%}{119.46\%}$ $= \$124769.4626$	M1	
	Price of Car $= \$124769.4626 \times \dfrac{100\%}{70\%}$ $= \$178242$ (nearest dollar)	M1 A1	
2c)	April : SGD980 = JPY112210 June : JPY112210=SGD949.32318 Percentage loss $= \dfrac{949.32318 - 980}{980} \times 100\%$ $= -3.13\%$ (to 3 s.f.)	M1 M1 A1	Must circle loss.

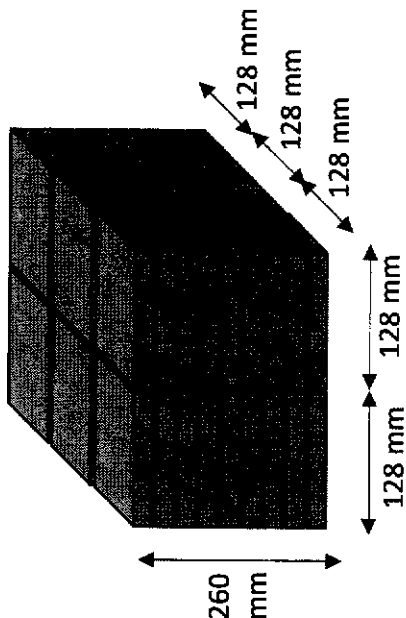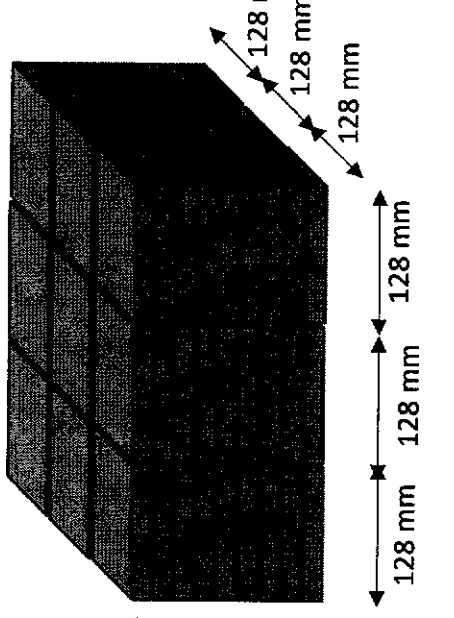| 3)	Let the number of couples needed be x. $\frac{16+x}{37+16+x+x}=0.4$ $16+x=0.4(53+2x)$ $16+x=21.2+0.8x$ $x=26$	M1 M1 A1	
4i)	$a=0$ $b=5$	B1 B1	
4ii)	46	B1	
4iii)	mean $=45.1$ Standard Deviation $=9.58$	B1 B1	
4iv)	The group of 20 students have higher marks since the mean is higher, and they have more consistent marks as their standard deviation is lower.	B1 B1	
5a)	Let y be the number of 50 cent coins and x be the number of 20 cent coins. $y+x=73$ --------(1) $(y)0.5+(x)0.2=28.10$ ------(2) From (2) $(y)=56.2-0.4(x)$ -----------(3) Sub (3) into (1) $56.2-0.4x+x=73$ $0.6x=16.8$ $x=28$ $y=73-28=45$ They have 45 50-cent coins and 28 20-cent coins.	M1 M1 M1 A1 A1	Forming 1st equation Forming 2nd equation For performing substitution/elimination A1 for 45 and A1 for 28
5b)	$\frac{4x}{(2x)^2-(5)^2}-\frac{1}{(2x-5)}$ $=\frac{4x}{(2x-5)(2x+5)}-\frac{1}{(2x-5)}$ $=\frac{4x-(2x+5)}{(2x-5)(2x+5)}$ $=\frac{4x-2x-5}{(2x-5)(2x+5)}$ $=\frac{2x-5}{(2x-5)(2x+5)}$ $=\frac{1}{2x+5}$	M1 M1 A1	• Factorise first denominator • Combine into single fraction

6a)	$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$ $= \begin{pmatrix} -1 \\ 12 \end{pmatrix} + \begin{pmatrix} -9 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} -10 \\ 15 \end{pmatrix}$	B1	
6b)	$\left|\overrightarrow{BA}\right| = \sqrt{(-10)^2 + (15)^2}$ $= 18.0$ units (to 3 s.f.)	M1 A1	
7i)	Let m be the mid point of AB. $MB = \frac{1}{2}(65.32)$ (property of chord) $= 32.66$ mm $\angle MOB = \sin^{-1}\left(\frac{32.66}{45}\right)$ $= 0.81216$ rad $\angle AOB = 2\angle MOB$ $= 1.624328$ $= 1.6243$ rad (shown)	M1 A1	M1 for using sine
7ii)	$\pi(45+x)^2 - \pi(45)^2 = 1389.29$ $(45+x)^2 - (45)^2 = 445.08953$ $x = 4.699$ $= 4.70$ mm (to 3 s.f.)	M1 A1	
8i)	$\angle ABN_B = 180° - 64° = 116°$ (int $\angle$s, $N_A // N_B$) $\angle ABC = 360° - 127° - 116°$ ($\angle$s at a pt.) $= 117°$ $AC = \sqrt{4.83^2 + 7.24^2 - 2(4.83)(7.24)\cos 117°}$ $= 10.368$ km (shown)	M1 M1 A1	Must give reason Use cosine rule
8ii)	$\dfrac{\sin 117°}{10.368} = \dfrac{\sin \angle BCA}{4.83}$ $\angle BCA = 24.52443°$ (5 d.p.) Bearing of A from C Reflex $\angle N_cCB = 360° - 24.52443° - (180° - 127°)$ $= 282.47557°$ Bearing of A from $C = 282.5°$ (1 d.p.)	M1 M1 A1	Use sine rule
8iii)	Shortest distance $= 7.24 \times \sin 24.52443°$ $= 3.00518$ $= 3.01$ km (to 3 s.f.)	M1 A1	
8iv)	Distance of object to $C = 7.24 \times \tan 12°$ $= 1.54$ km (to 3 s.f.)	M1 A1	

4E Prelim Math 2023

9	Let interior angle of polygon be x.		
	$3x + \angle a + \angle b = (5-2) \times 180°$	M1	
	$3x + 72° = 540°$		
	$\quad x = 156°$	M1	
	$n = \dfrac{360°}{180° - 156°}$		
	$\quad = 15$	A1	
		B1	
10ai)	$\dfrac{2}{5}\underset{\sim}{b}$		
10aii)	$\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CO} + \overrightarrow{OE}$		
	$= -\dfrac{2}{5}\underset{\sim}{b} - \underset{\sim}{a} + \underset{\sim}{b}$	M1	
	$= -\underset{\sim}{a} + \dfrac{3}{5}\underset{\sim}{b}$		
	$\overrightarrow{AD} = \dfrac{2}{3}\left(-\underset{\sim}{a} + \dfrac{3}{5}\underset{\sim}{b}\right)$	M1	
	$= \dfrac{2}{5}\underset{\sim}{b} - \dfrac{2}{3}\underset{\sim}{a}$	A1	
10b)	$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$		
	$= -\dfrac{2}{3}\underset{\sim}{a}$		
	$= -\dfrac{2}{3}\overrightarrow{OC}$	M1	
	therefor $\overrightarrow{AC}$ is parallel to $\overrightarrow{OC}$.	A1	
10ci)	Let h be the common height of $\triangle ADB$ and $\triangle ACD$ from A to CB.		
	$\dfrac{\text{area of } \triangle ADB}{\text{area of } \triangle ACD} = \dfrac{\frac{1}{2} \times DB \times h}{\frac{1}{2} \times CD \times h}$		
	$= \dfrac{3}{2}$	B1	
10cii)	Since $\dfrac{AC}{AO} = \dfrac{AD}{AE} = \dfrac{CD}{OE}$		
	$\triangle ADC$ is similar to $\triangle AOE$.		
	$\dfrac{\text{area of } \triangle ADB}{\text{area of } \triangle ACD} = \left(\dfrac{2}{5}\right)^2$	M1	
	$= \dfrac{4}{25}$	A1	
10ciii)	Area area of $\triangle ADB$: area of $\triangle ACD$: area of $\triangle AOE$		
	$= 6 : 4 : 25$		
	$\dfrac{\text{area of } \triangle ADB}{\text{area of CDEO}} = \dfrac{6}{25-4}$	M1	M1 for finding the ratio of area of CDEO relative to area of $\triangle ADB$
	$= \dfrac{2}{7}$	A1	

11a)	0	B1	
11b)	Refer to graph • plot all points correctly • smooth curve	B1 B1	
11ci)	Refer to graph	B1	
11cii)	Gradient $= \dfrac{0.8-(-1)}{0-3}$ $\qquad\qquad = -0.6 \qquad \pm 0.1$ $y = -0.6x + 0.8 \qquad$ y-intercept ± 0.1	M1 A1	
11d)	$3x^2 - 8x + 6 = 0$ $3x - 8 + \dfrac{6}{x} = 0$ $2x + \dfrac{6}{x} - 7 = -x + 1$ ⎫ $y = -x + 1$ ⎬ Draw graph $y = -x + 1$. ⎭ $3x^2 - 8x + 6 = 0$ has no solution because $y = 2x + \dfrac{6}{x} - 7$ and $y = -x + 1$ do not intersect for $0.5 \le x \le 7$	M1 M1 A1	M1 for either of these 2 working steps. M1 for drawing graph.
12a)	$\dfrac{1500}{x}$	B1	
12b)	$\dfrac{\dfrac{1500}{x} - \dfrac{17}{x-22.5}}{60} = 2$ $\dfrac{1500}{x} - \dfrac{1020}{x-22.5} = 2$ $1500(x-22.5) - 1020x = 2(x^2 - 22.5x)$ $2x^2 - 525x + 33750 = 0 \qquad$ (shown)	M1 M1 A1	
12c)	$x = \dfrac{-(-525) \pm \sqrt{(-525)^2 - 4(2)(33750)}}{2(2)}$ $\quad = \dfrac{525 \pm \sqrt{5625}}{4}$ $= 150 \qquad$ or $\quad = 112.5$	M1 A1 A1	A1 for each solution
12d)	Total time taken $= x + (x - 22.5) = 2x - 22.5$ minutes When $x = 112.5$ Total time $= 2(112.5) - 22.5$ $\qquad\qquad = 202.5$ mins (reject since total time >4 hours) When $x = 150$ Total time $= 2(150) - 22.5$ $\qquad\qquad = 277.5$ mins $\qquad\qquad = 4h\ 37.5min$	M1 A1	Must reject with reason.

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13a)	Percentage difference $= \dfrac{282-202}{202} \times 100\%$ $= 39.60396$ $= 39.6\%$ (to 3 s.f.)	M1 A1
13b)	The books can be stacked into columns. The configuration of the columns will fit into boxes.	
	Configuration 1 (1 column) Max height $= 900 - 128 - 128 = 644$ Exceed max 400mm Max books $= 400 \div 6 \approx 66$ books Total weight $= 66 \times 75 = 4950\,g$	M1
	Configuration 2 (2 columns) Max height $= 900 - 128 - 128 - 128 = 516$ Exceed max 400mm Max books per column $= 400 \div 6 = 66\frac{2}{3} \approx 66$ books Max books $= 66 \times 2 = 132$ books Total weight $= 132 \times 75 = 9900\,g$	M1 M1
	Configuration 3 (4 columns) Max height $= 900 - 128 - 128 - 128 - 128 = 388$ Max books per column $= 388 \div 6 = 64\frac{2}{3} \approx 64$ books Max books $= 64 \times 4 = 256$ books Total weight $= 256 \times 75 = 19200\,g$	M1

These 3 M1 can be allocated to any of the 5 configurations, M1 for max height, M1 for max books, M1 for max weight.

Configuration 1 — 400 mm, 128 mm, 128 mm

Configuration 2 — 400 mm, 128 mm, 128 mm, 128 mm, 128 mm

Configuration 3 — 388 mm, 128 mm, 128 mm, 128 mm, 128 mm

2

Configuration 4

260 mm

128 mm
128 mm
128 mm
128 mm
128 mm

Configuration 5

132 mm

128 mm
128 mm
128 mm

128 mm
128 mm
128 mm

M1

A1

Configuration 4 (6 columns)
Max height $= 900 - 128 - 128 - 128 - 128 - 128 = 260$

Max books per column $= 260 \div 6 = 43\frac{1}{3} = 43$ books

Max books $= 43 \times 6 = 258$ books
Total weight $= 258 \times 75 = 19350$ g

Configuration 5 (9 columns)
Max height $= 900 - 128 \times 6 = 132$
Max books per column $= 132 \div 6 = 22$ books
Max books $= 22 \times 9 = 198$ books
Total weight $= 198 \times 75 = 14850$ g

Max books by weight $= 20000 \div 75 \approx 266$ books

Pick configuration 4 as most books.
Total cost to manufacture and post to Australia
$= 202 + 258 \times 1.80$
$= \$666.40$

Price per book $= \dfrac{666.40 + 3 \times 258}{258}$
$= \$5.58294$
$= \$5.60$ (to nearest ten cents)