Animal Gaits with CPG in semi-symmetrical systems

Computational Neuroscience

Final Presentation

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Outline

- Background on animal gaits
- Gaits symmetry and non-linearity
- Coupled oscillators, ODEs CPG network model
- Bipedal, quadruped, hexapodal
  - Example equations and simulation
- Extensions to other networks
  - Loss of symmetry, radially symmetric and semi-symmetric systems
- Conclusions

Animal gaits

"Several distinct periodic patterns of leg movements in animal locomotion"

"Animals typically employ their limbs in a number of periodic modes"

Typical symmetries:

- Bilateral
- Front-rear
- Walk symmetry
- Etc,..

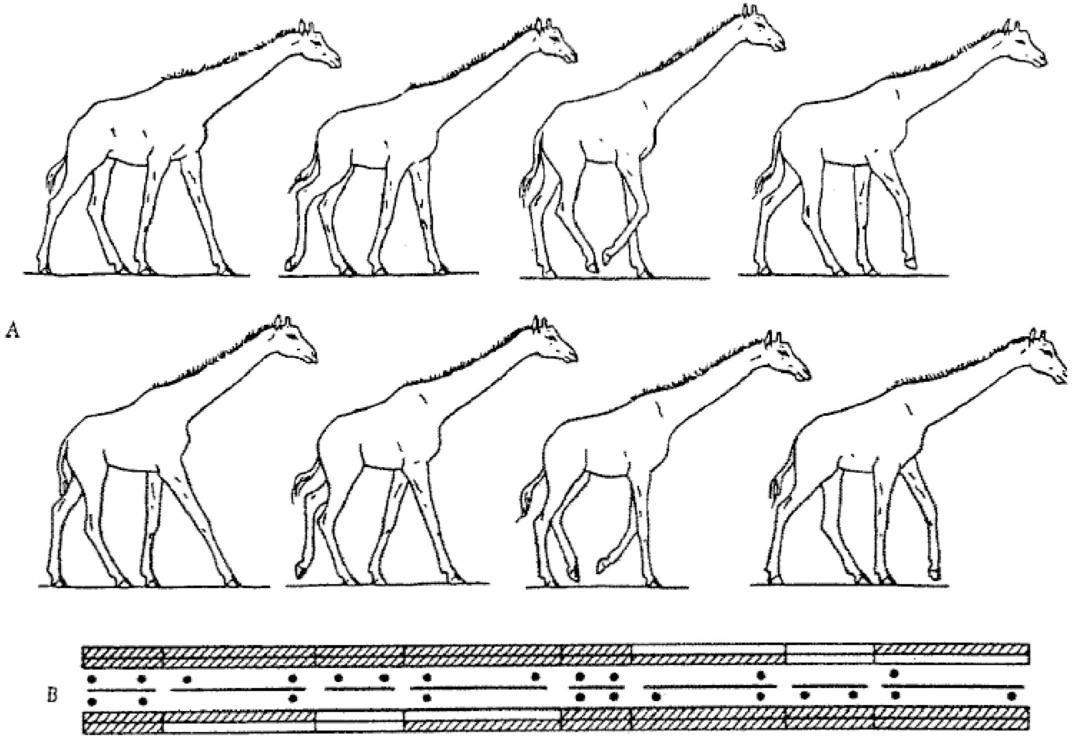
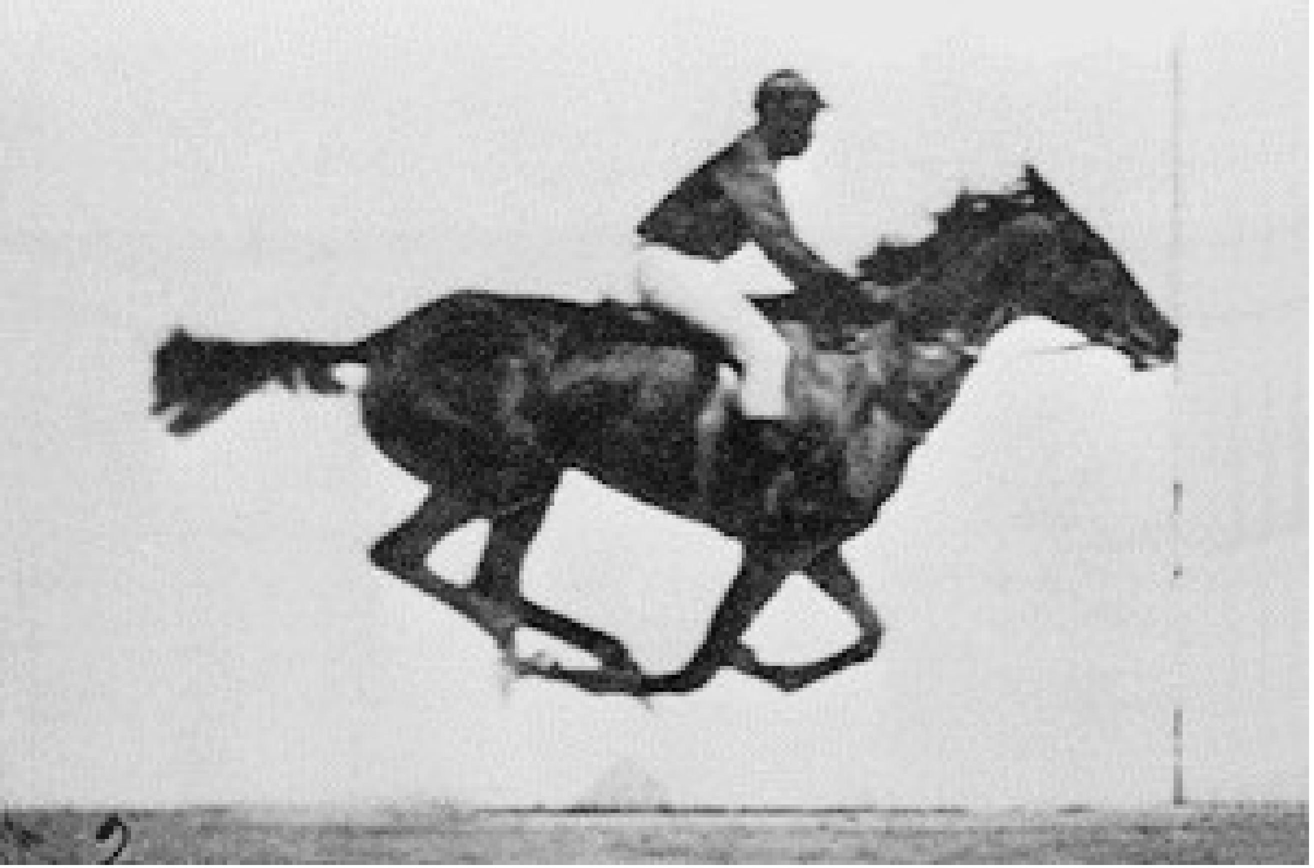


Fig. 1. Slow rack-like walk of giraffe is left-right reflected by a phase shift of half a period. The bars below are the *support graph* of the gait, and show when each foot is in contact with the ground. [From P. Gambaryan (1974), *How Mammals Run: Anatomical Adaptations*. Distributed by John Wiley & Sons, Inc., New York.]

Humans: walk, run, hop;
Horses: walk, run, canter, gallop;
Others: pace, trot, bound, jump, pronk,etc ...

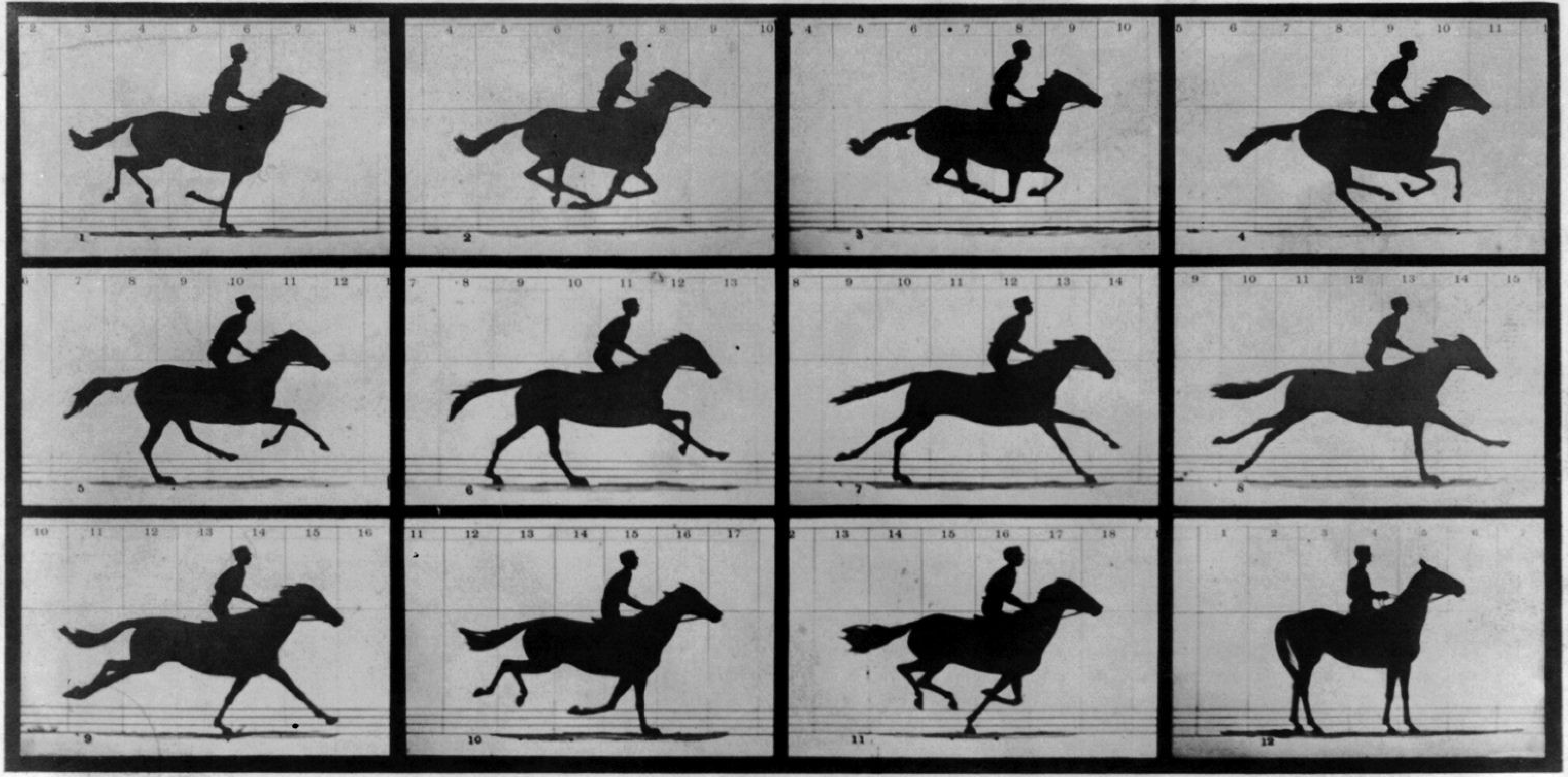
Animal Gaits



Gait analysis is an ancient Science

Aristotle describing the movement of a horse: “The back legs move diagonally in relation to the front legs;for after the right fore leg animals move the left hind leg, then the left fore leg, and after it the right hind leg.”

Modern Gait analysis also started with a horse:
According to legend in the 1870s Leland Standfor (ex-governor of California) had a bet with Frederick MacCrellish about the placement of the feet of a galloping horse.



Copyright, 1878, by MUYBRIDGE.

MORSE'S Gallery, 417 Montgomery St., San Francisco.

THE HORSE IN MOTION.

Illustrated by

MUYBRIDGE.

AUTOMATIC ELECTRO-PHOTOGRAPH.

"SALLIE GARDNER," owned by LELAND STANFORD; running at a 1.40 gait over the Palo Alto track, 19th June, 1878.

The negatives of these photographs were made at intervals of twenty-seven inches of distance, and about the twenty-fifth part of a second of time; they illustrate consecutive positions assumed in each twenty-seven inches of progress during a single stride of the mare. The vertical lines were twenty-seven inches apart; the horizontal lines represent elevations of four inches each. The exposure of each negative was less than the two-thousandth part of a second.

Central Patterns Generators (CPG) as a Network model

CPGs are networks of neurons in the central nervous system which produce cyclic patterns

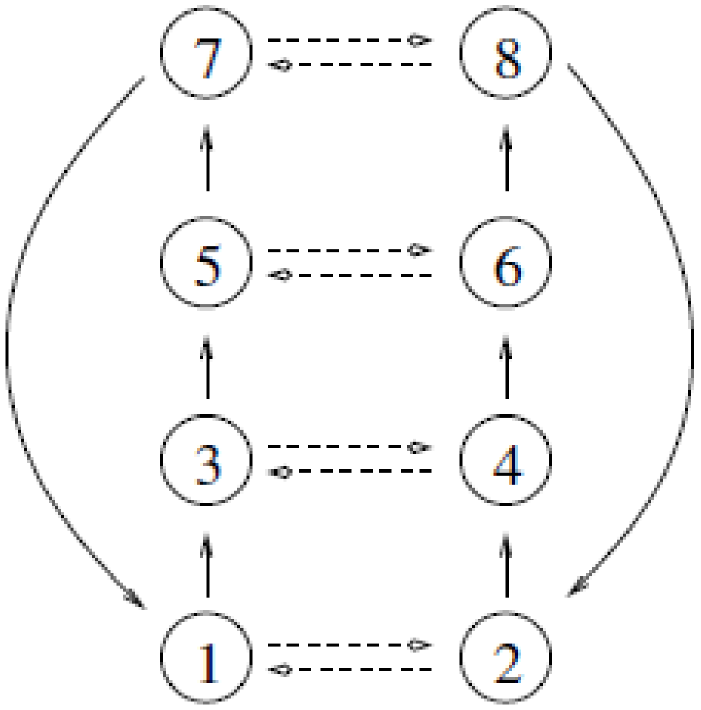


FIGURE 10. Eight-cell network for quadrupeds. Dashed lines indicate contralateral coupling; single lines indicate ipsilateral coupling.

Gaits Symmetry

		walk		jump		trot		pace		bound		pronk	
LF	RF	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
LH	RH	$\frac{1}{2}$	0	$\frac{3}{4}$	$\frac{3}{4}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
LF	RF	$\frac{1}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
LH	RH	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
Subgroup K		$\mathbf{Z}_2(\kappa\omega^2)$		$\mathbf{Z}_2(\kappa)$		$\mathbf{Z}_4(\kappa\omega)$		$\mathbf{Z}_4(\omega)$		$\mathbf{D}_2(\kappa, \omega^2)$		$\mathbf{Z}_2 \times \mathbf{Z}_4$	

TABLE 2. Phase shifts for primary gaits in the eight-cell network.

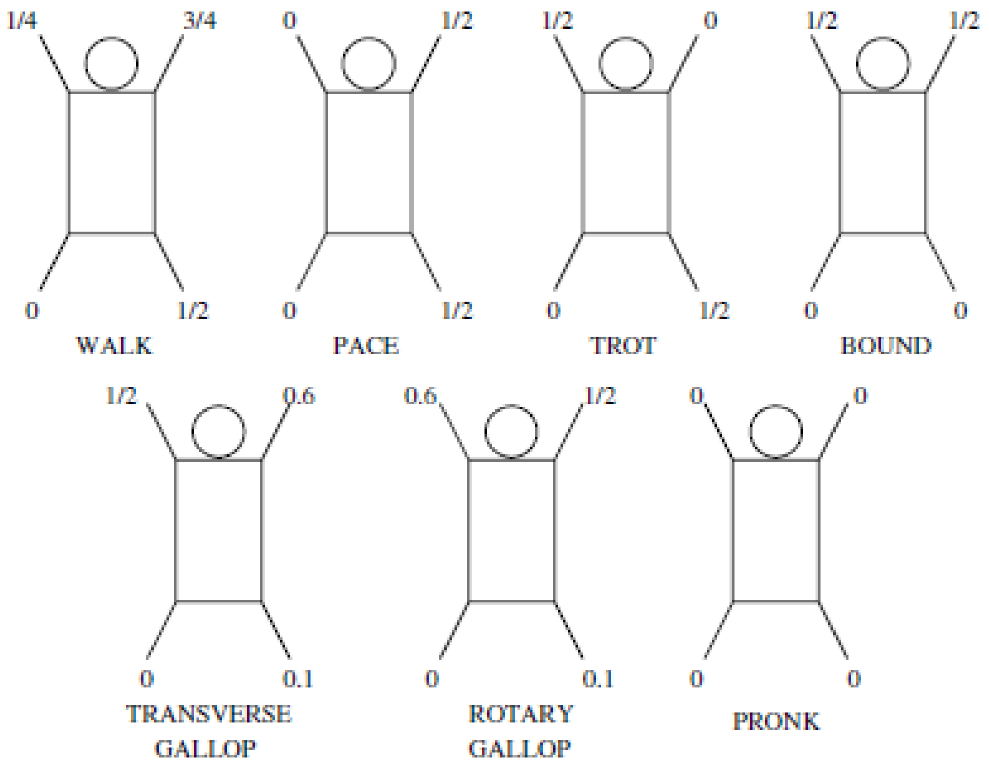


FIGURE 7. Seven quadrupedal gaits. Numbers indicate the percentage of the time through the gait when the associated leg first strikes the ground. Gaits begin when left hind leg strikes ground.

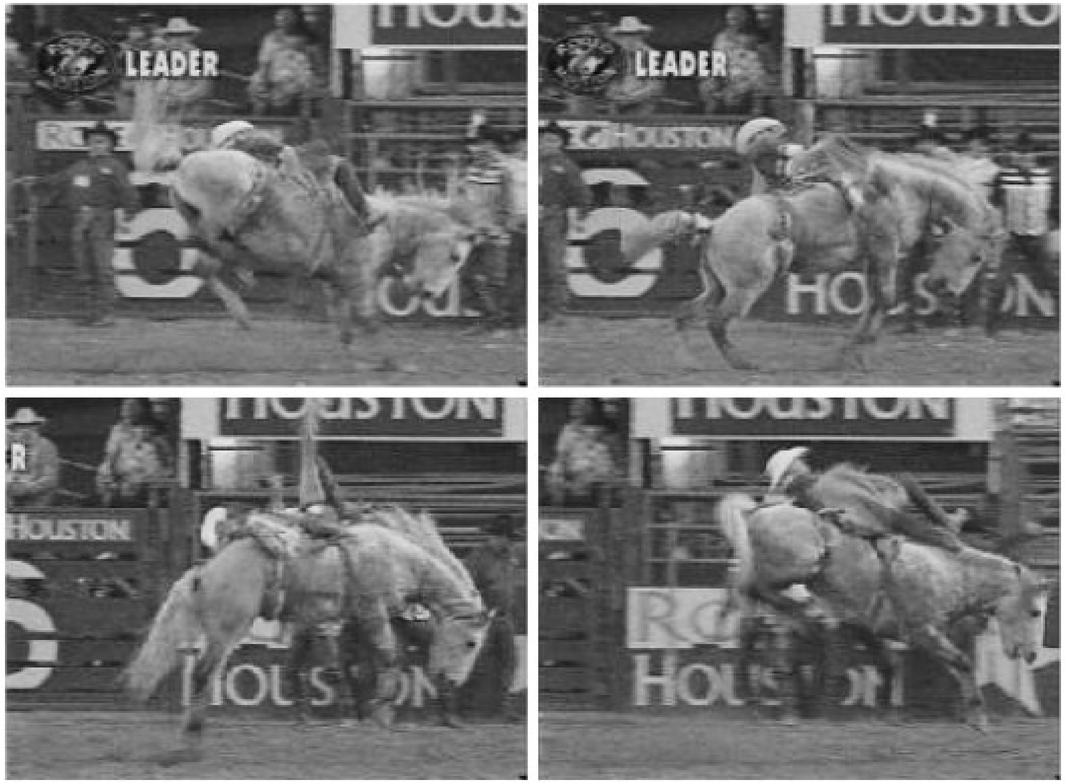


FIGURE 11. Quarter cycles of bareback bronc jump at Houston Livestock Show and Rodeo. (UL) fore legs hit ground, (UR) hind legs hit ground, (LL) and (LR) all legs in air.

Non-linearity

(b)

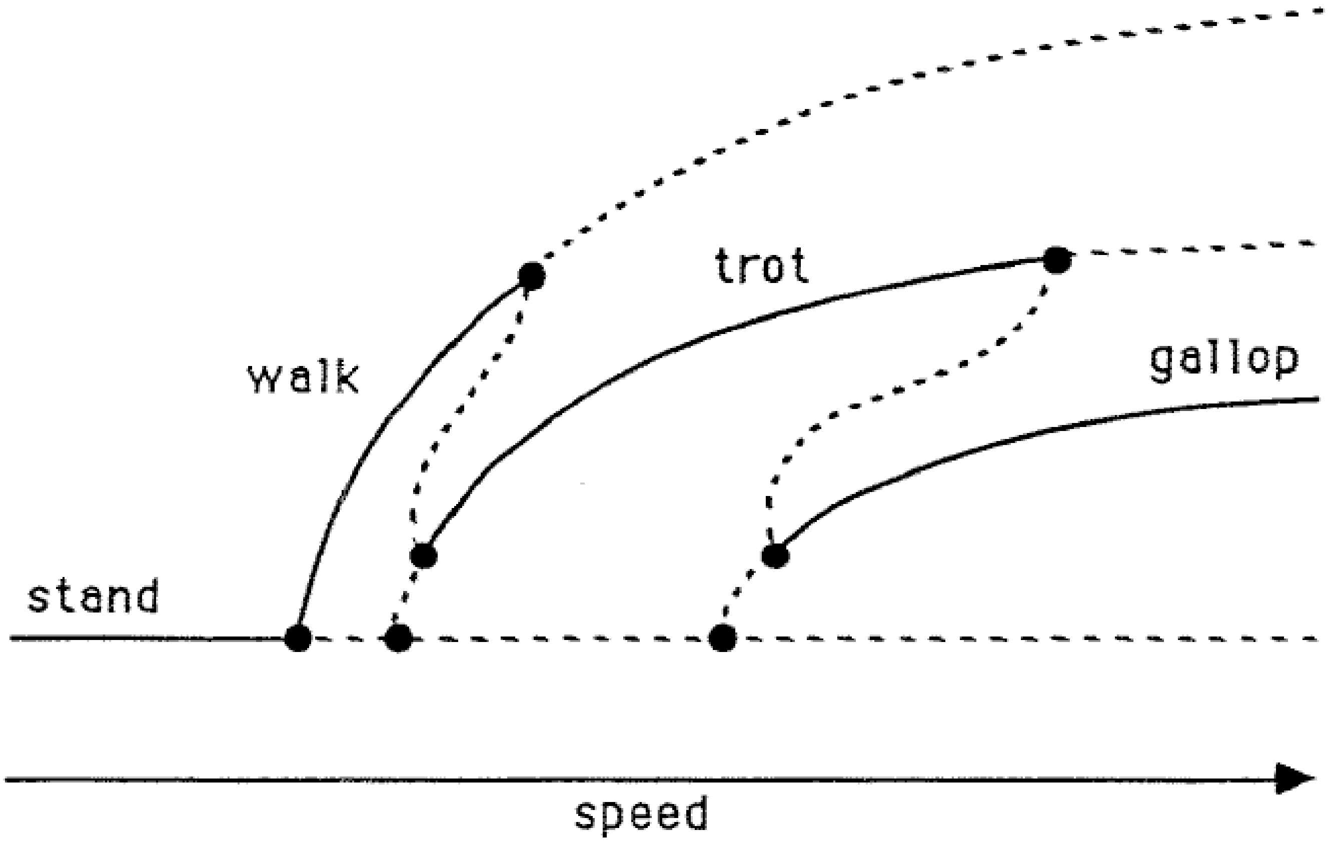


Fig. 9. (b) Interpretation as a bifurcation diagram.

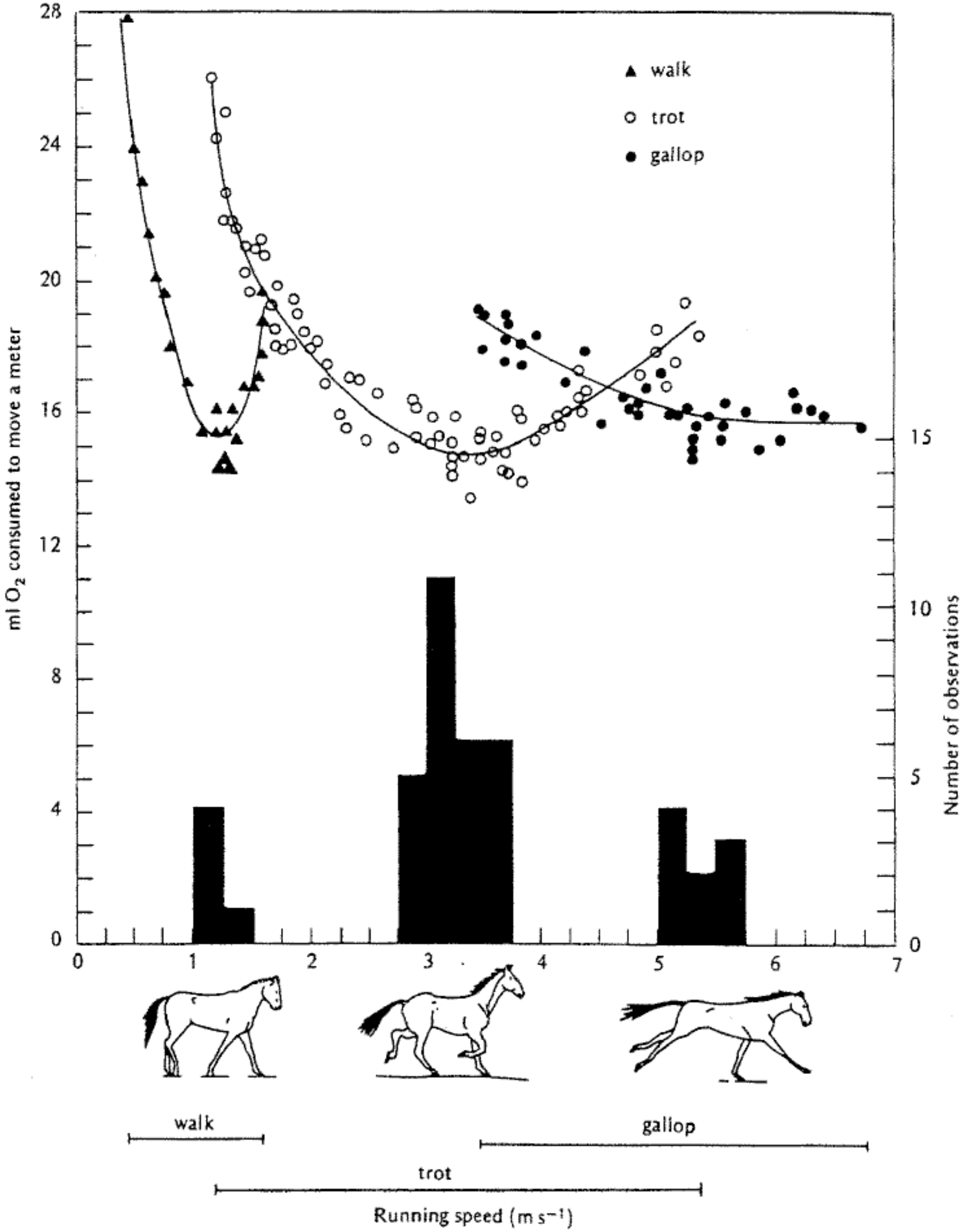


Fig. 9. (a) Graph of oxygen consumption versus speed for horses. [From Schmidt–Nielsen (1990), redrawn from Hoyt and Taylor (1981). Reprinted with permission from *Nature* (Gait and the energetics of locomotion in horses, *Nature* **292**, 239–240). Copyright 1981 Macmillan Magazines Limited, London.]

Coupled oscillators

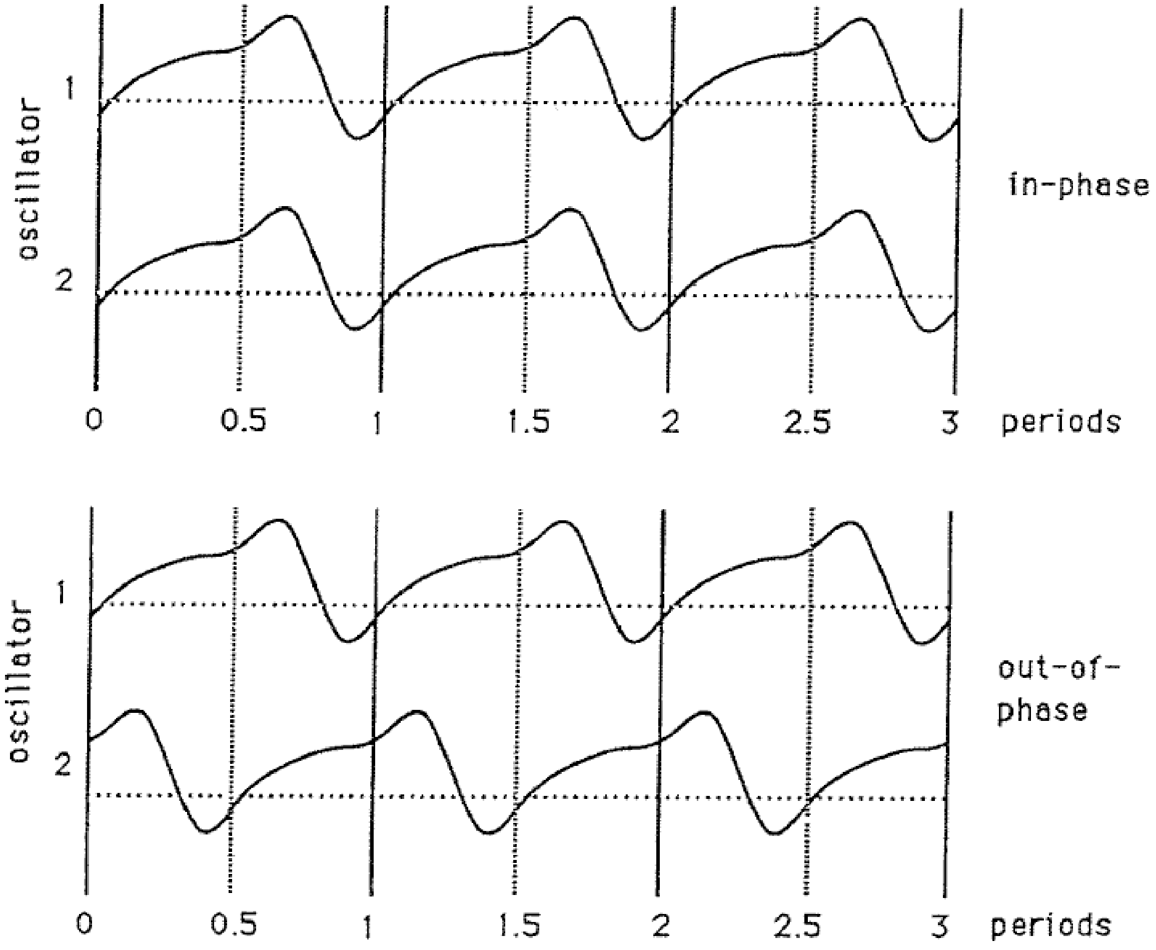


Fig. 11. Schematic of in-phase and out-of-phase motion for two identical coupled oscillators.

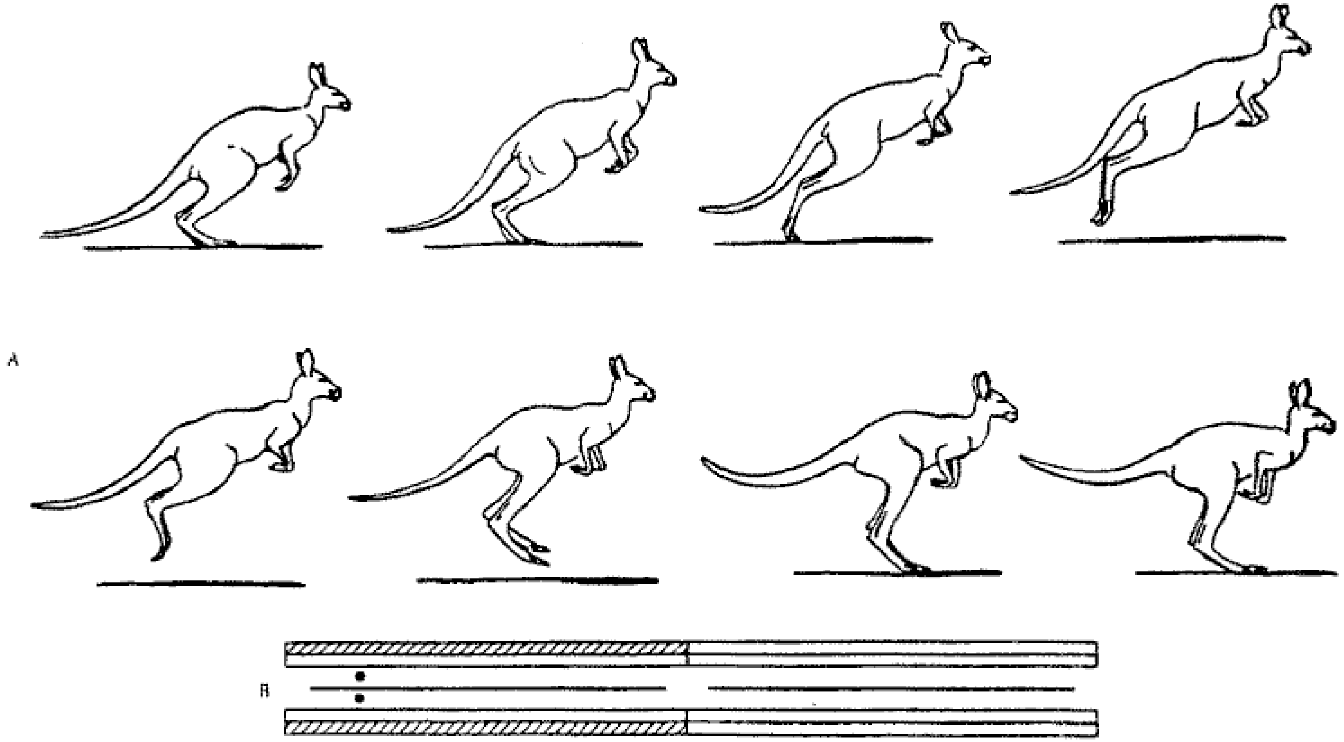


Fig. 12. Bound of kangaroo has in-phase left-right symmetry. [From P. Gambaryan (1974). *How Mammals Run: Anatomical Adaptations*. Distributed by John Wiley & Sons, Inc., New York.]

Quadruped example equations

$$\frac{dx_1}{dt} = y_1 - \alpha x_1\left(\frac{x_1^2}{3} - 1\right) + \beta + \epsilon x_1^2 + \gamma(x_1 - x_2) + \delta(x_1 - x_4),$$

$$\frac{dx_2}{dt} = y_2 - \alpha x_2\left(\frac{x_2^2}{3} - 1\right) + \beta + \epsilon x_2^2 + \gamma(x_2 - x_1) + \delta(x_2 - x_3),$$

$$\frac{dx_3}{dt} = y_3 - \alpha x_3\left(\frac{x_3^2}{3} - 1\right) + \beta + \epsilon x_3^2 + \gamma(x_3 - x_4) + \delta(x_3 - x_2),$$

$$\frac{dx_4}{dt} = y_4 - \alpha x_4\left(\frac{x_4^2}{3} - 1\right) + \beta + \epsilon x_4^2 + \gamma(x_4 - x_3) + \delta(x_4 - x_1),$$

$$\frac{dy_1}{dt} = -x_1, \qquad \frac{dy_2}{dt} = -x_2,$$

$$\frac{dy_3}{dt} = -x_3, \qquad \frac{dy_4}{dt} = -x_4.$$

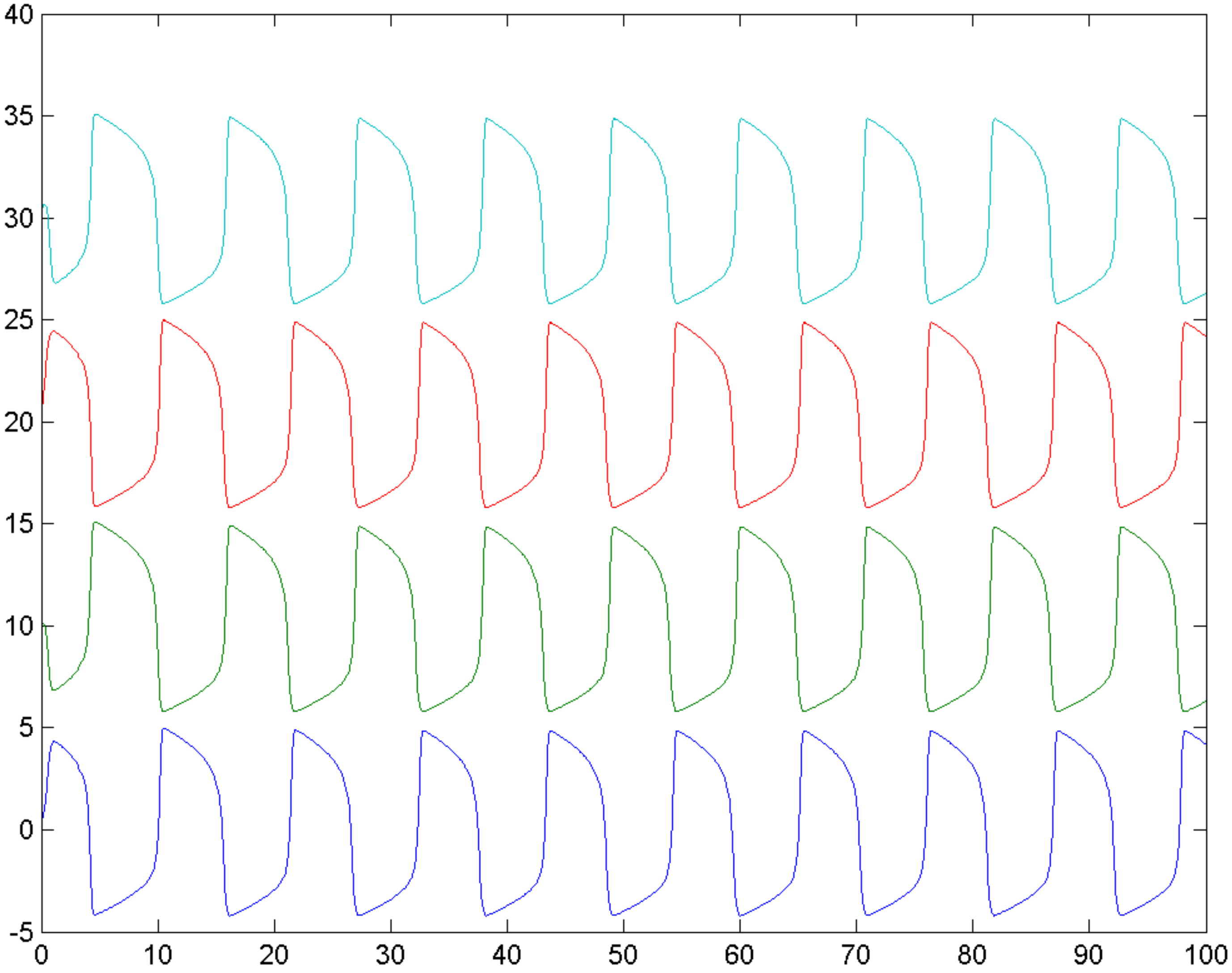
$$\frac{dx_1}{dt} = y_1 - \alpha x_1\left(\frac{x_1^2}{3} - 1\right) + \beta + \epsilon x_1^2 + \gamma(2x_1 - x_2 - x_4),$$

$$\frac{dx_2}{dt} = y_2 - \alpha x_2\left(\frac{x_2^2}{3} - 1\right) + \beta + \epsilon x_2^2 + \delta(2x_2 - x_1 - x_3),$$

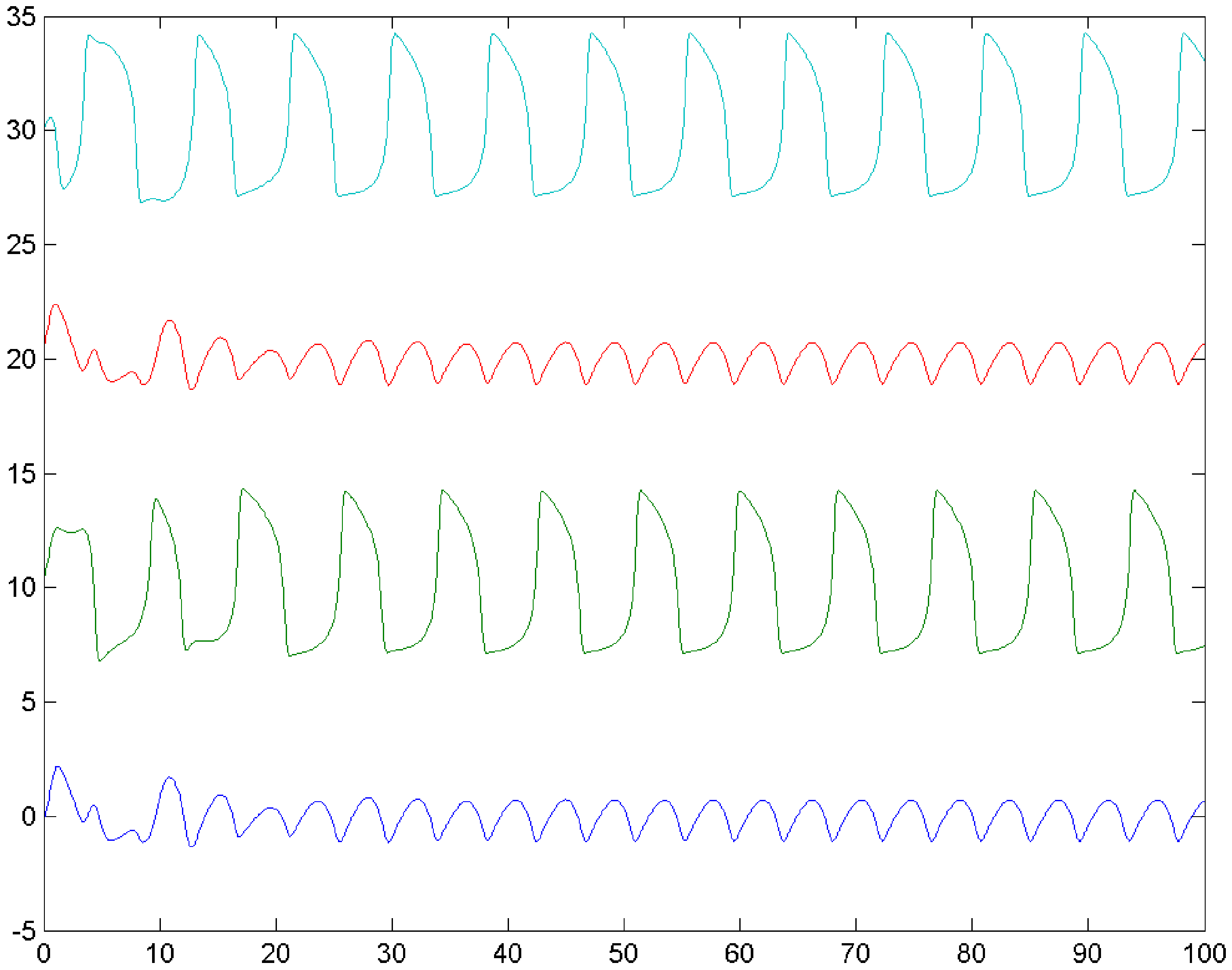
$$\frac{dx_3}{dt} = y_3 - \alpha x_3\left(\frac{x_3^2}{3} - 1\right) + \beta + \epsilon x_3^2 + \gamma(2x_3 - x_2 - x_4),$$

$$\frac{dx_4}{dt} = y_4 - \alpha x_4\left(\frac{x_4^2}{3} - 1\right) + \beta + \epsilon x_4^2 + \delta(2x_4 - x_3 - x_1).$$

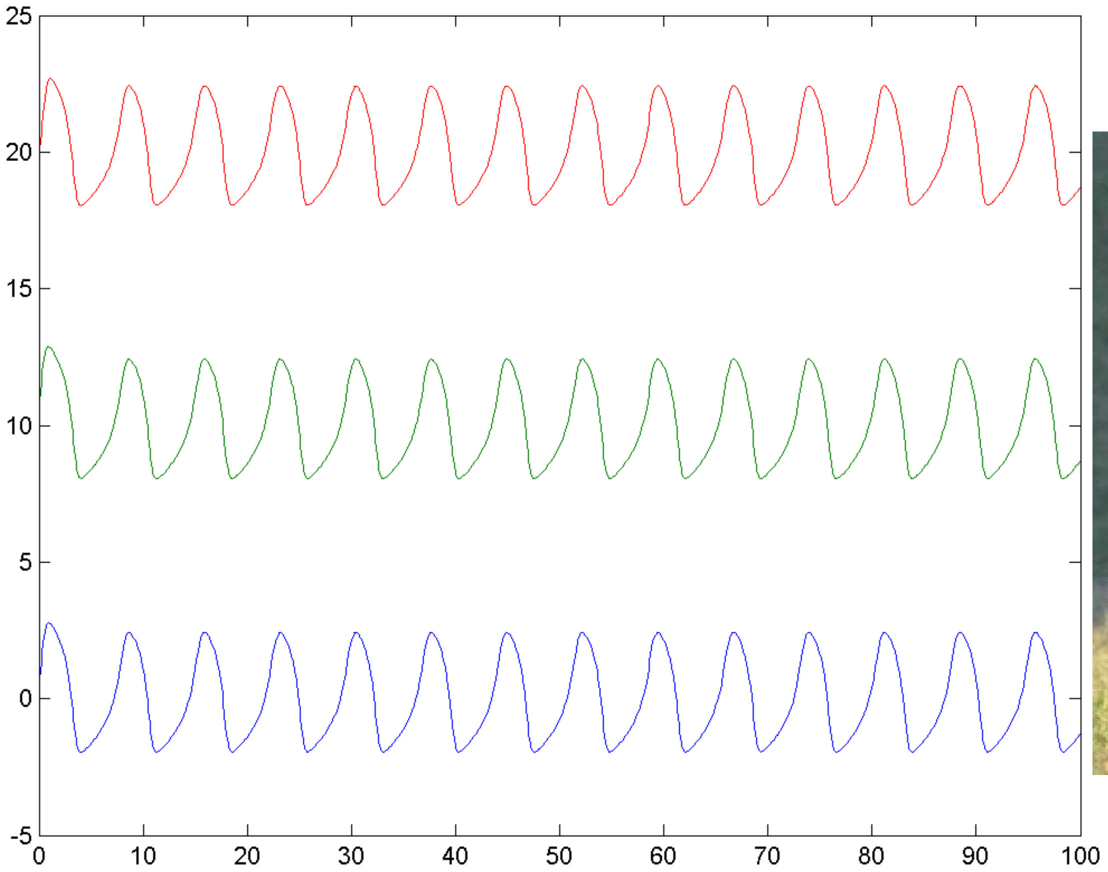
Example plot for system 1



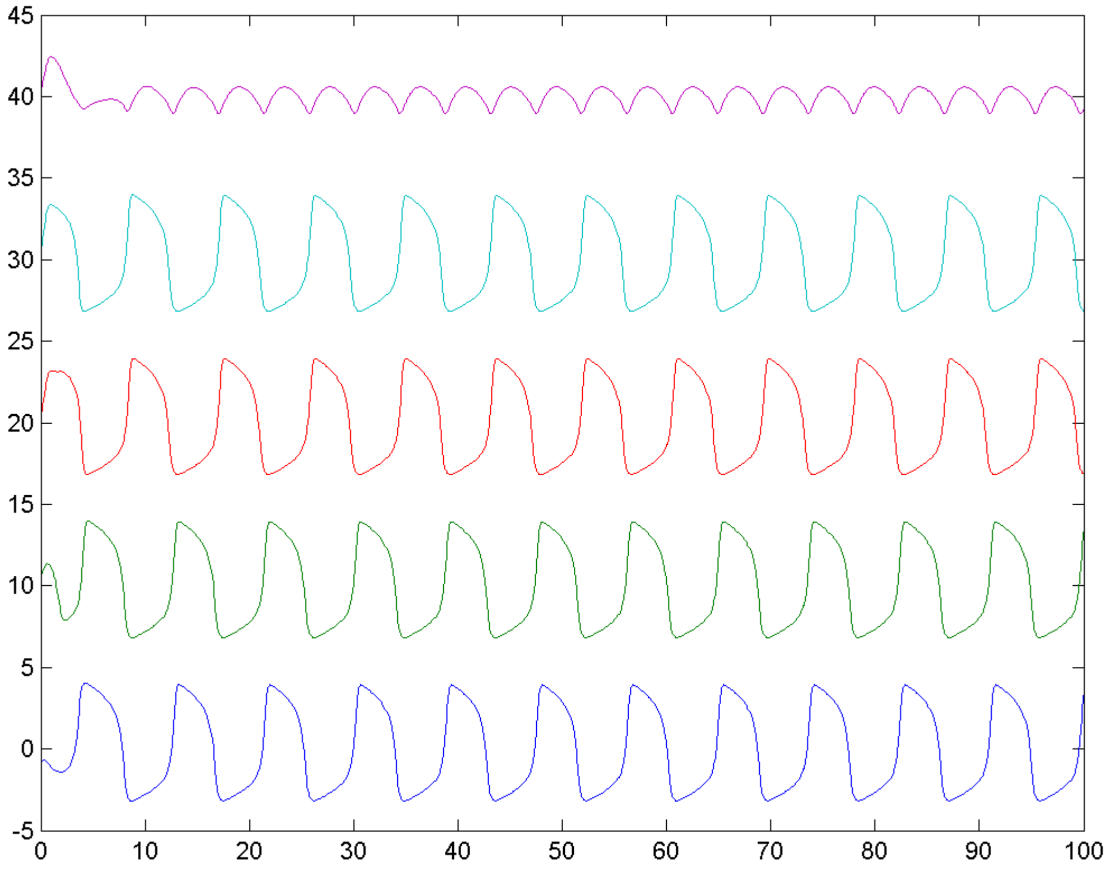
Example plot for system 2



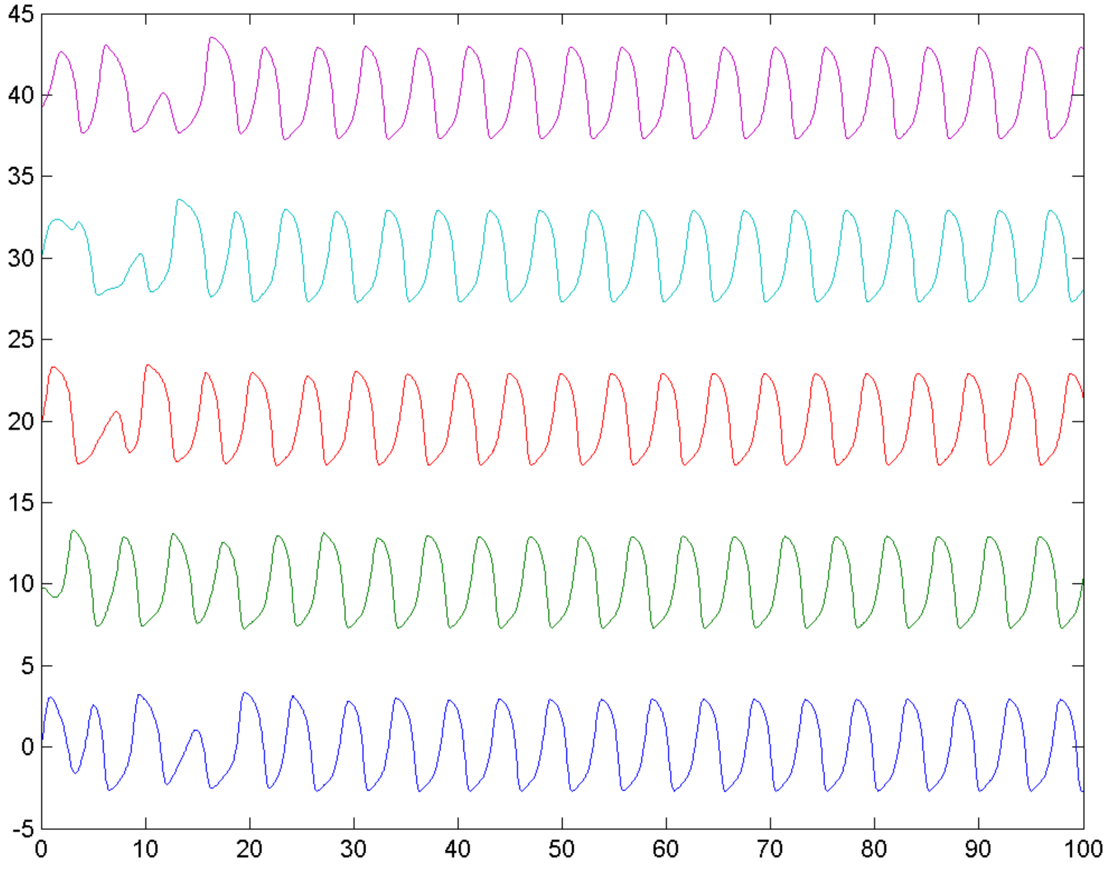
Kangaroo jump



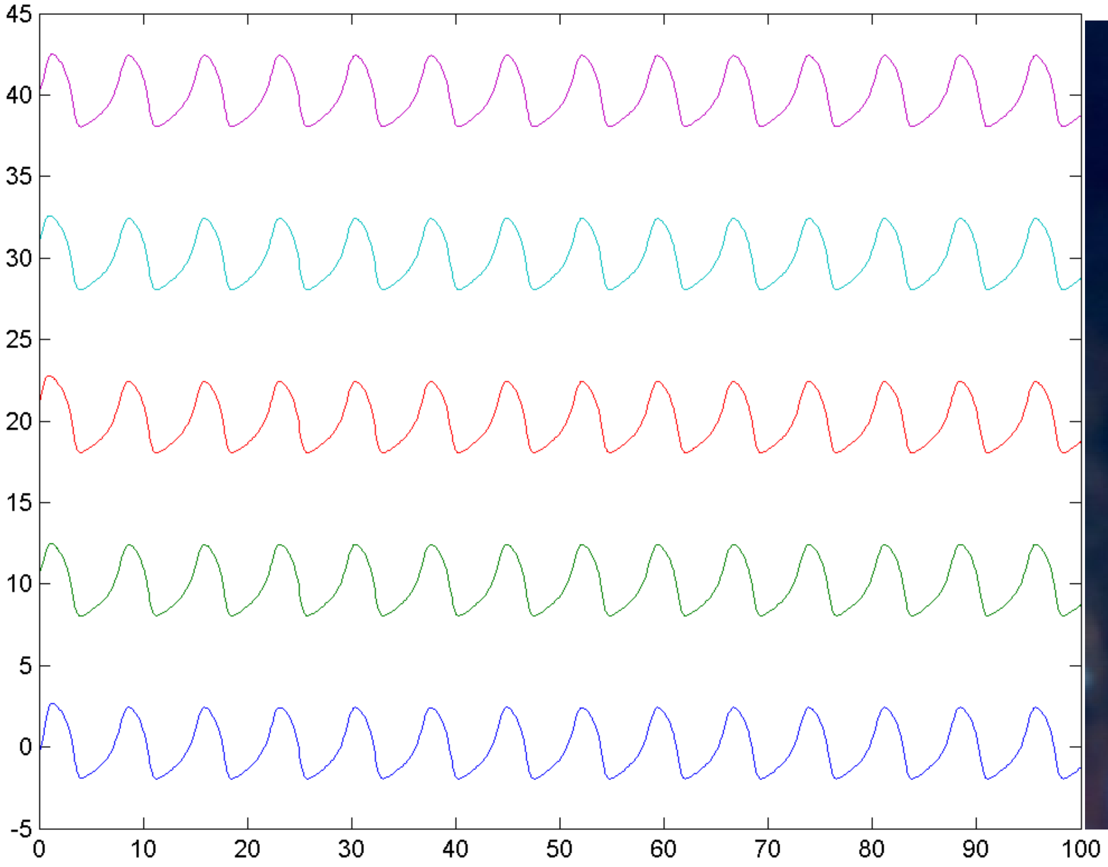
Mokey Arms-Tail-Legs-Tail leap



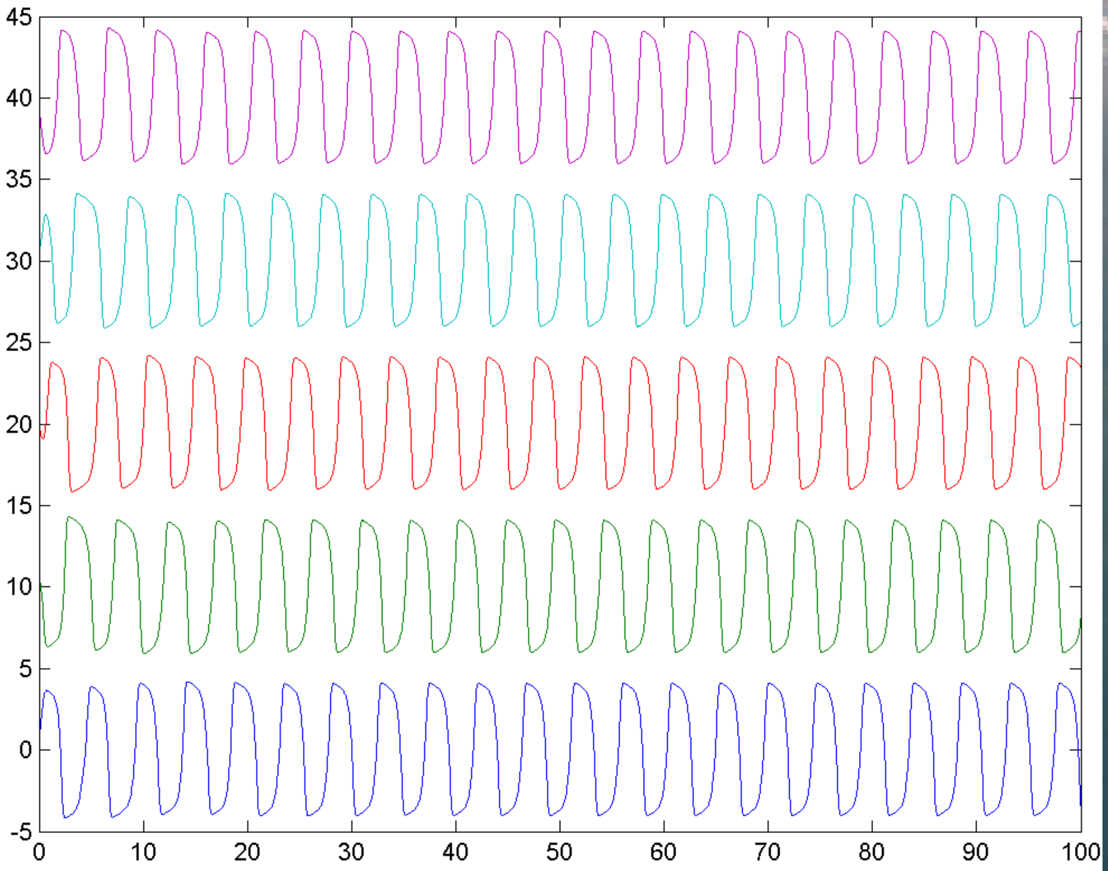
Starfishes crawl



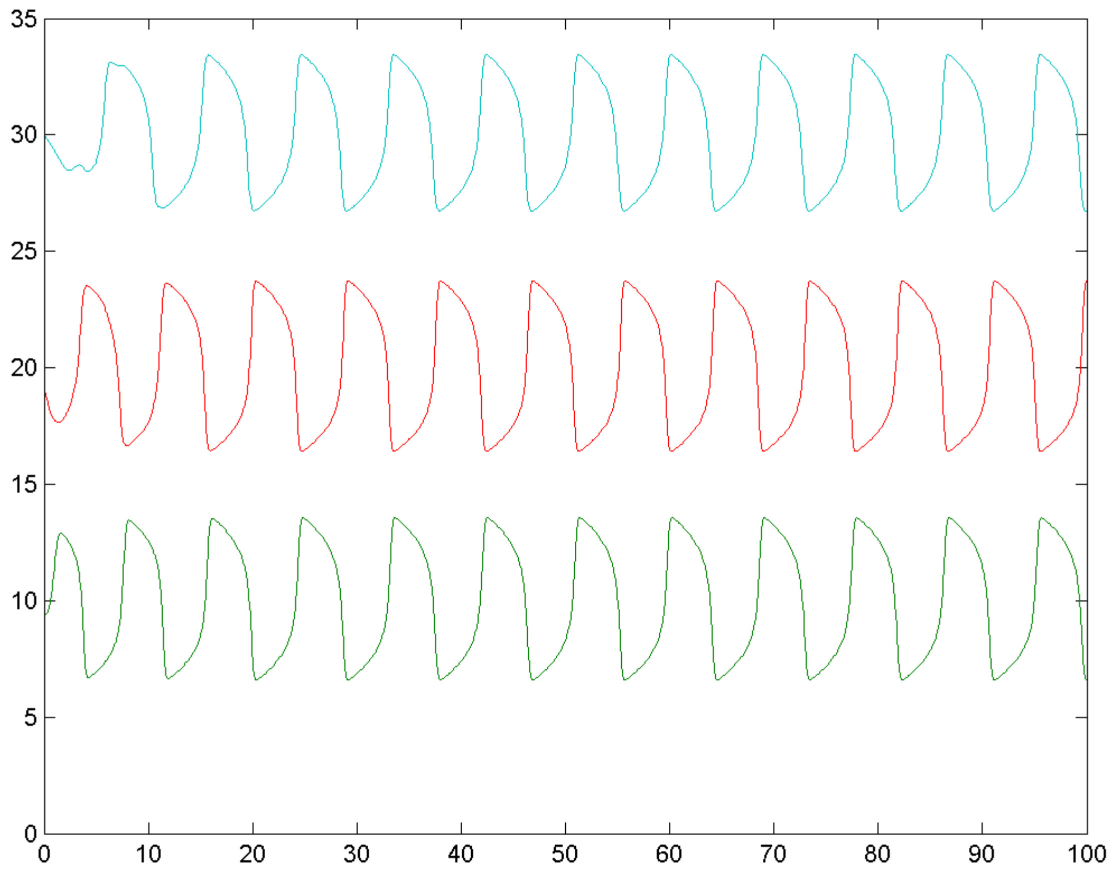
Starfishes pace-swim



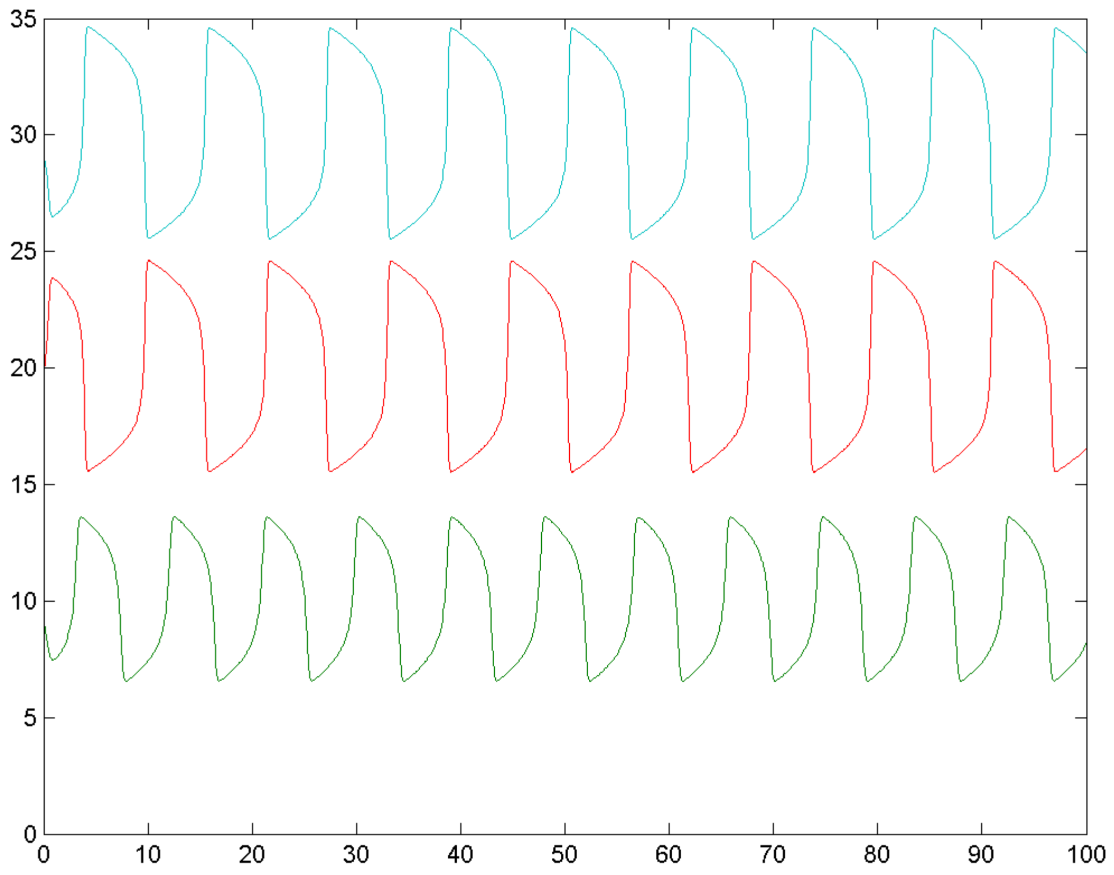
Starfishes swim-run



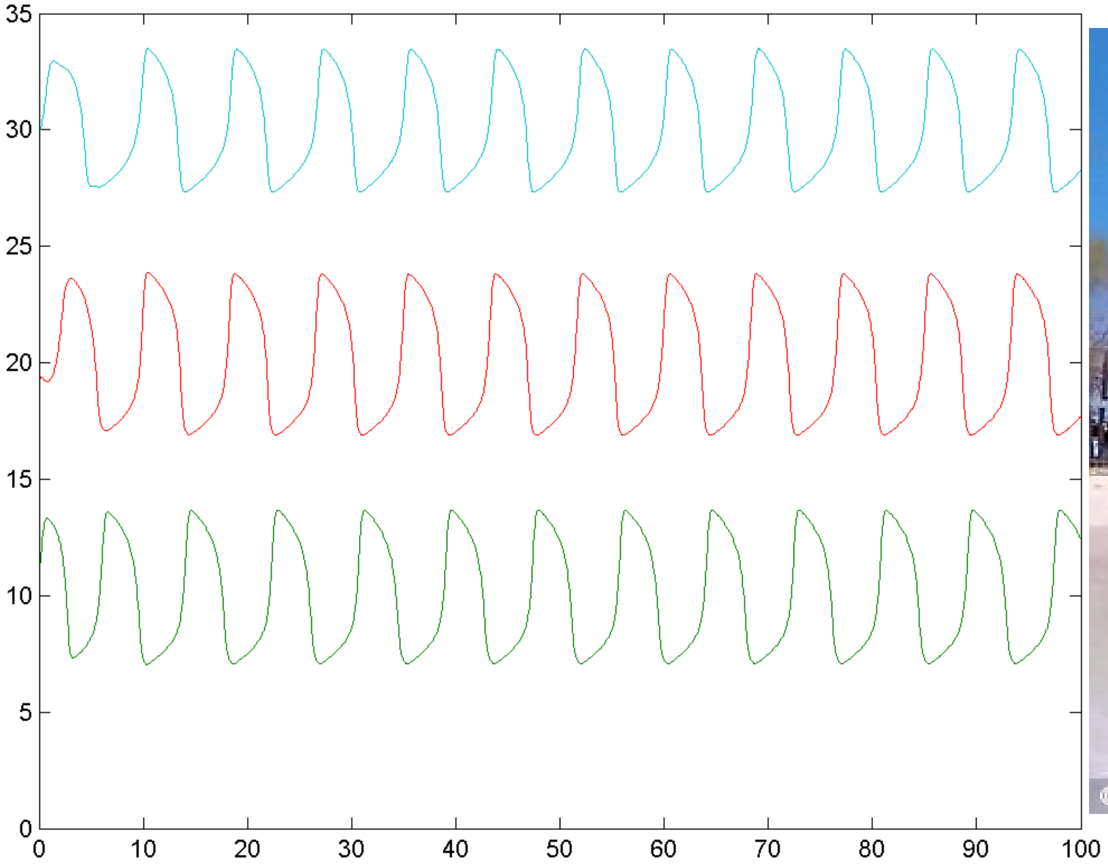
3-legged dog can walk and pace...



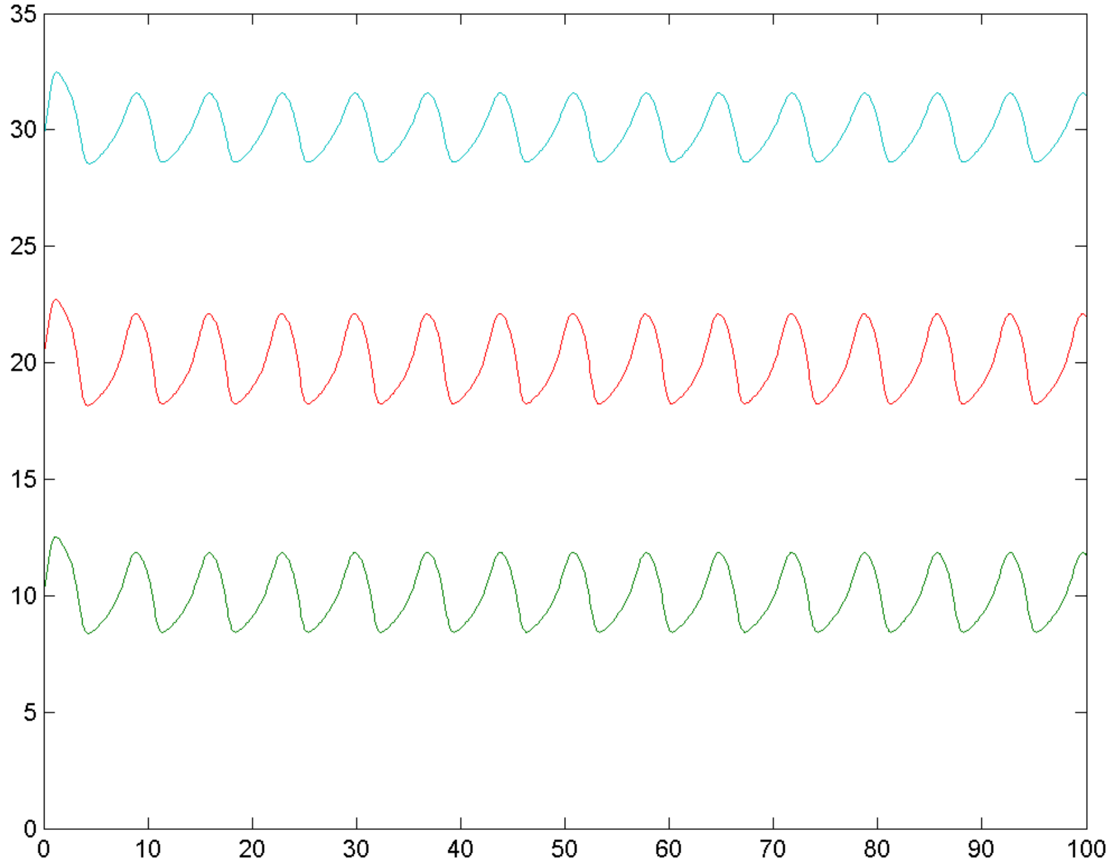
3-legged dog can trot and run…



3-legged dog can bound (front-back jump)...



3-legged dog can even pronk (all legs jump)



Conclusions

- Coupled oscillators reflects well the idea of CPG and the natural emergence of multiple gaits (in the same animal)
- Network symmetry determines the possible gaits
- Also when considering networks with weaker symmetries is still possible to obtain a significant number of gaits
- CPG networks can be robust enough to allow functional gaits also when a limb is lost (example 3 legged dog)

Future directions

- Consider different coupling networks
- Analise more specifically the parameter space for different gaits
- Find other interesting less symmetric network, or with different symmetries
- Determine biologically meaningful equations for coupled oscillators ODEs
- Couple with model for gait optimization

References

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6) NONLINEAR DYNAMICS OF NETWORKS: THE GROUPOID FORMALISM. MARTIN GOLUBITSKY AND IAN STEWART

7) Octopus Research Group: http://www.octopus.huji.ac.il/site/index.html

Thank you for your attention!