A network model for multicellularity

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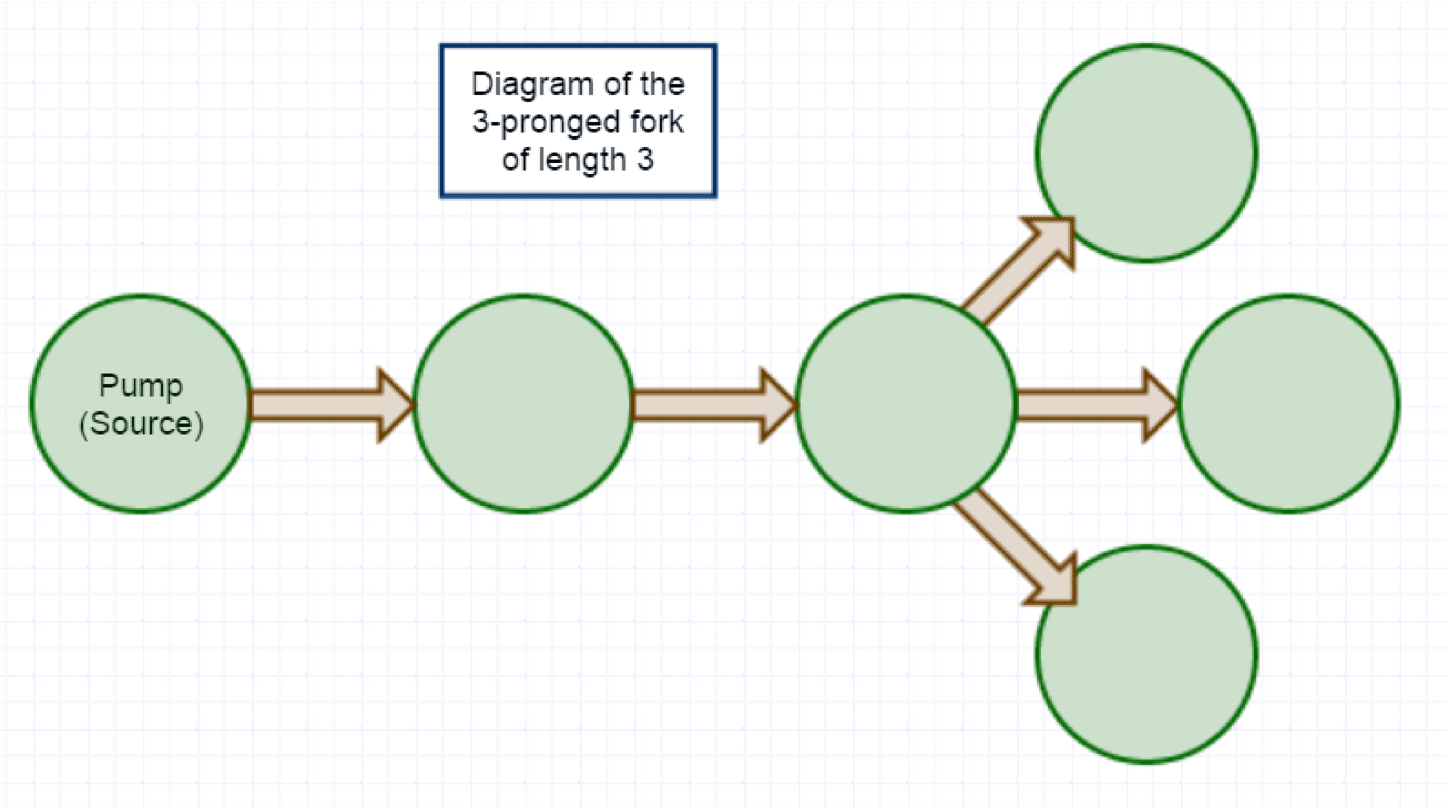

Outline

- Summary of previous direction
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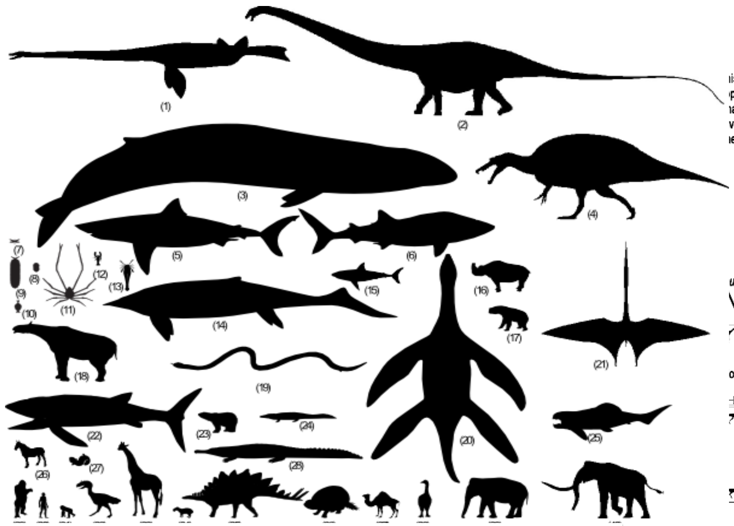
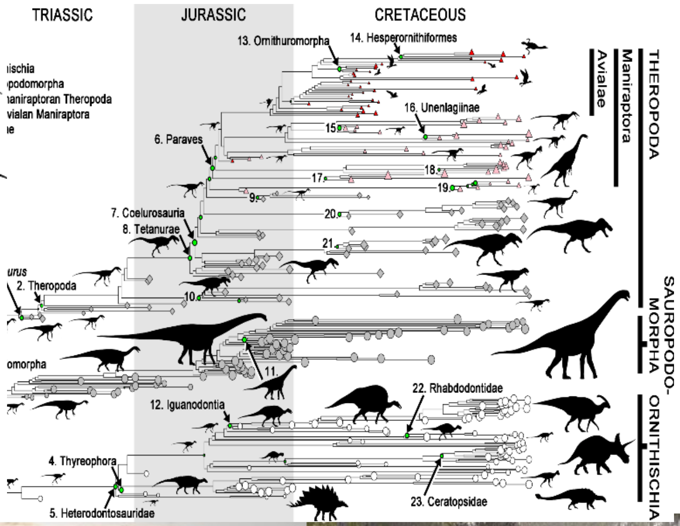
- Two canonical networks
- Critical energy
  - Example
- Extensions to other networks
- Robustness
  - Example
- Recursive formulation of critical energy (in progress)
- Future directions

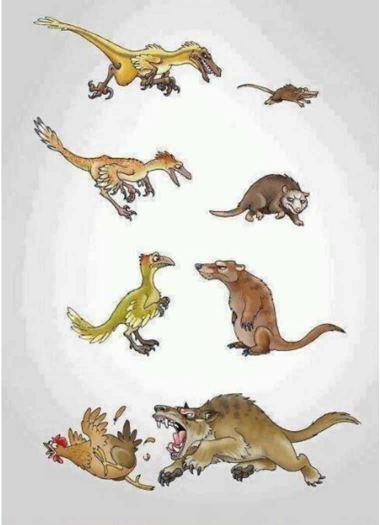
Summary of previous direction

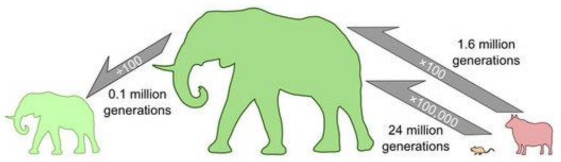
<u>Previous Goal</u>: Model evolution of animal size.

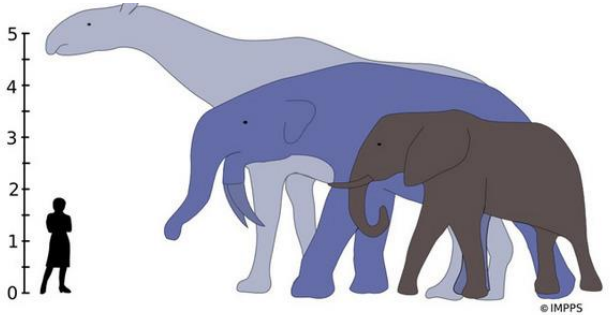
<u>New Goal</u>: Model of optimal sizes and structures for energy transport in biological networks.







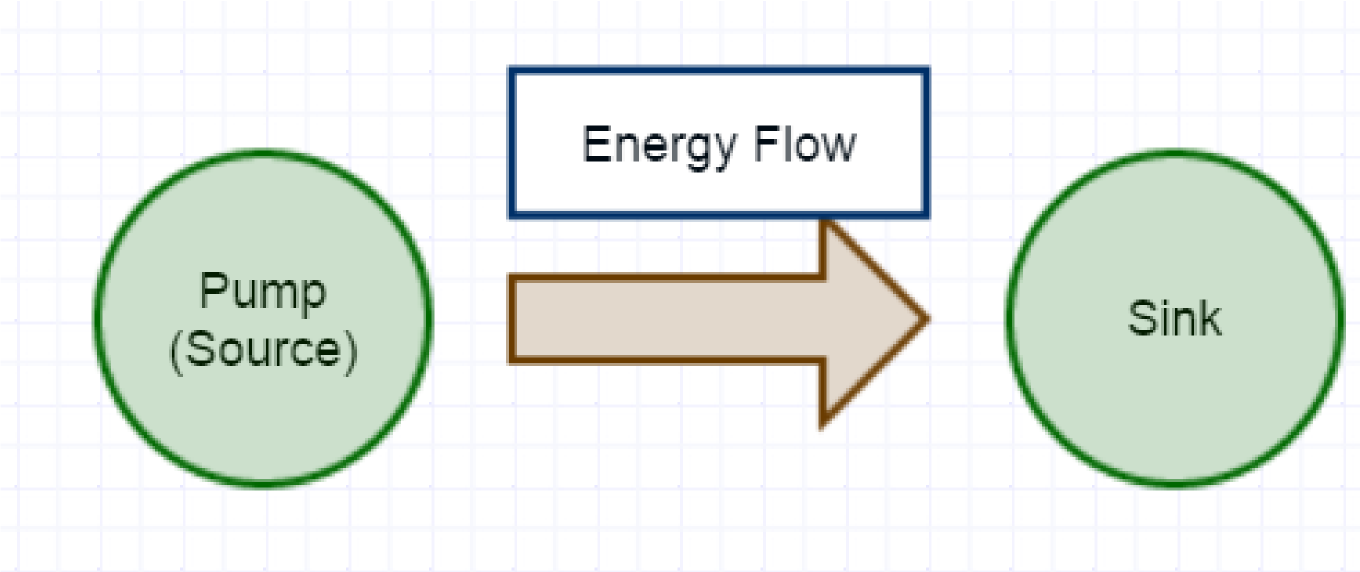




Basic network structure

- Begin with a connected, directed network of **N** nodes with energy source (pump).
- Energy, **E(N)**, is gathered by the pump
- Each node (including the pump) consumes a fixed amount of energy, **C**, that it receives.
- The remaining energy at that node, **E(N)-C**, is divided equally to its outgoing edges.
- A portion, 1-**p**, of the energy being transported is lost to the environment (with $0<p\leq1$).

Repeat until all nodes have consumed C energy or the energy remaining in the system is less than amount needed to sustain any unfed nodes. Unfed nodes at the end of a feeding cycle will be considered dead. A dead cell no longer consumes energy, but maintains connections.

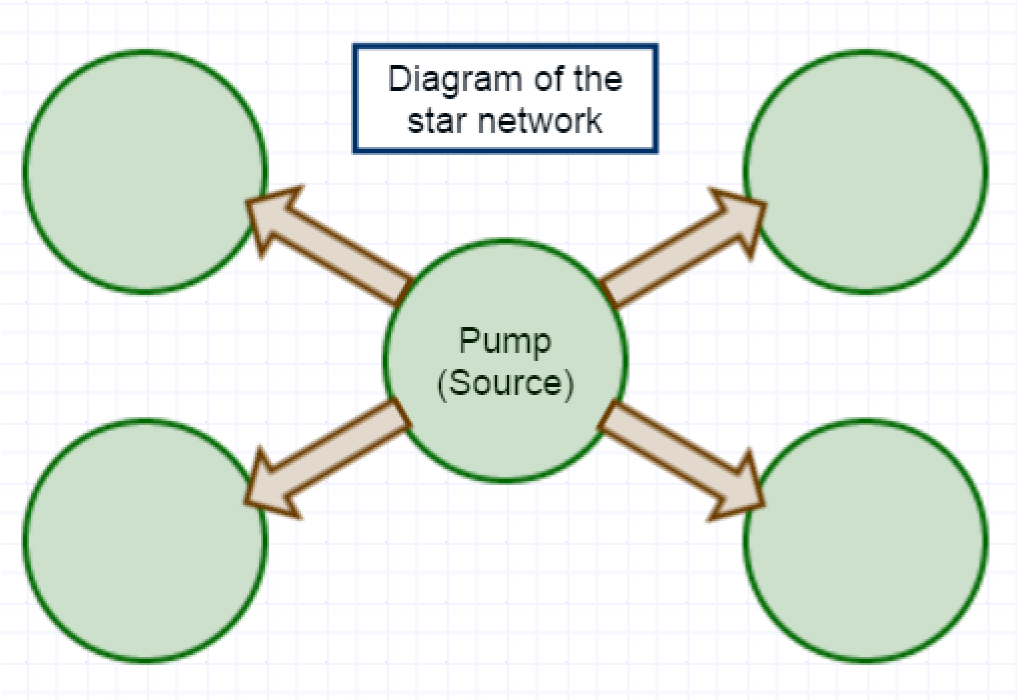


Assumptions of the model

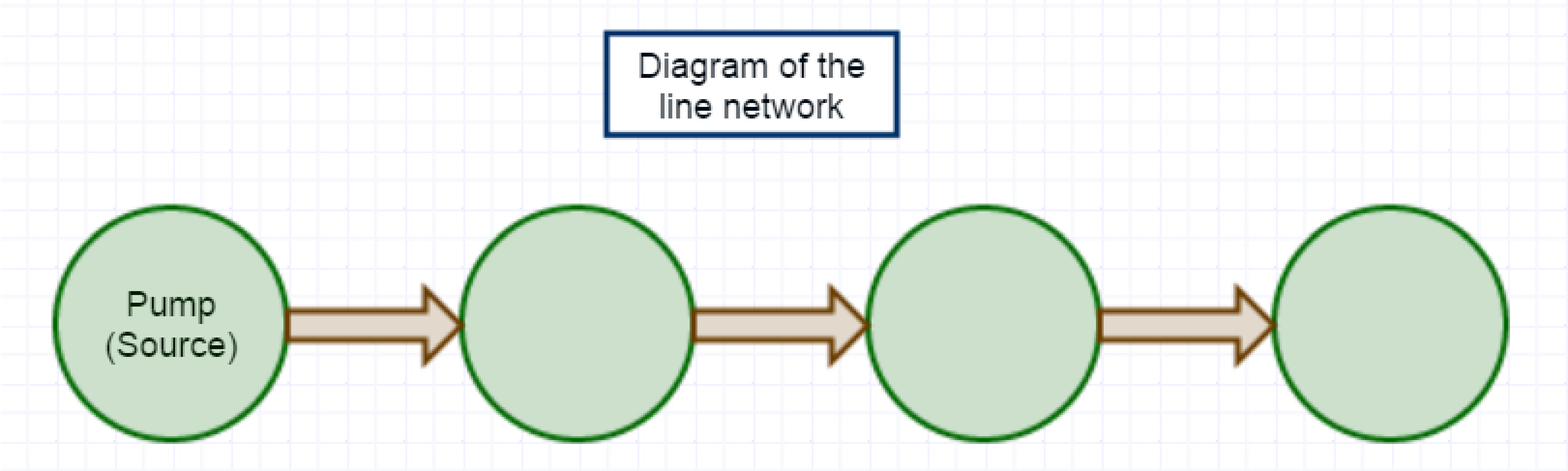
- Energy gathered at a given time step is monotonically increasing in the number of surviving nodes and is bounded by **$\bar{E}$**.
- The parameters of the energy function, **E(N),** and the bound **$\bar{E}$** can fluctuate with a certain random distribution (ex: Gaussian)
- After N feeding cycles, the network can reproduce and mutate at random, then the process is repeated (in progress).

Two canonical topologies

- The star connects the pump to all other nodes in the network and there are no other connections



- The line connects each node sequentially with the next one in a list.



Critical energy

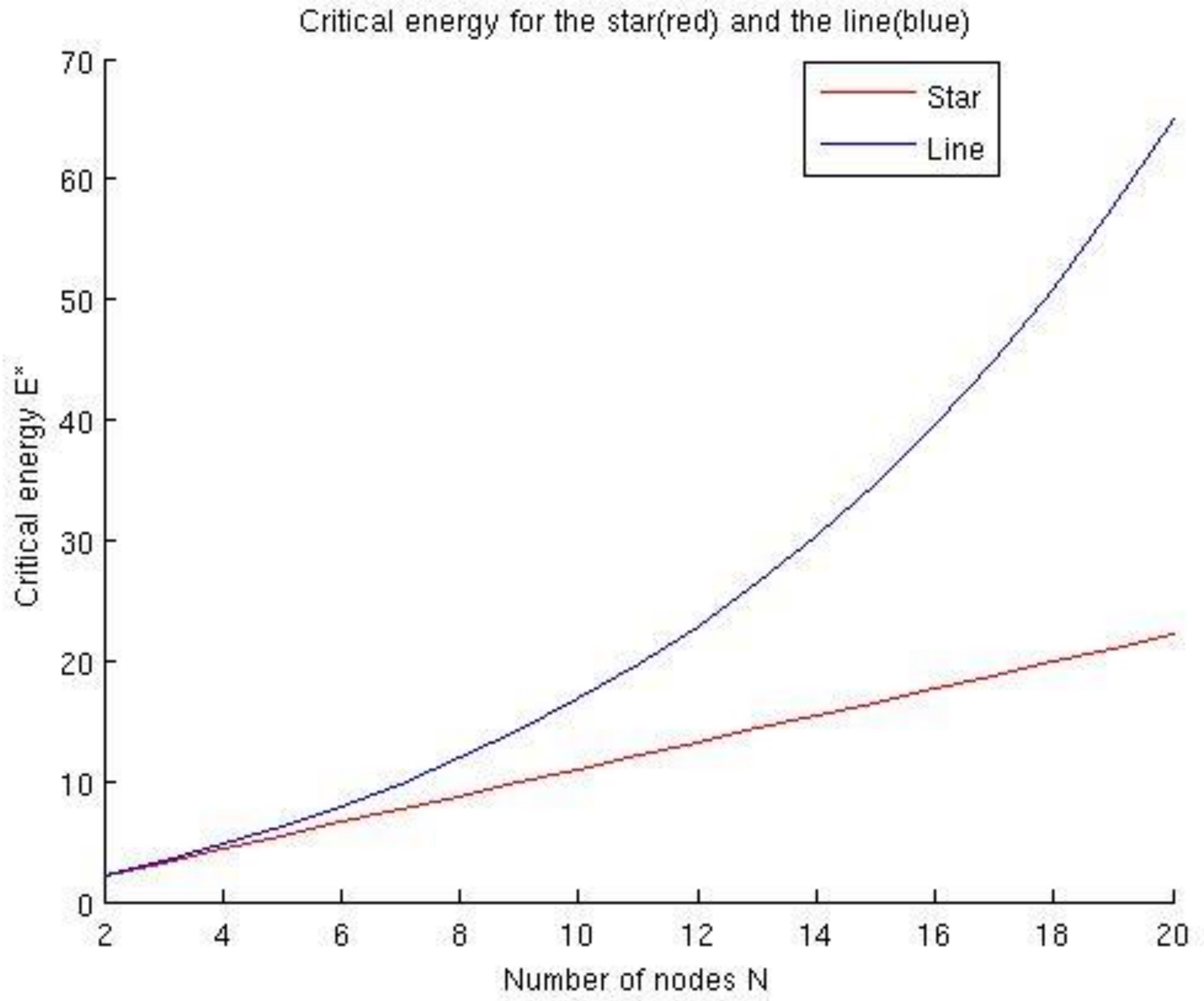
- The critical energy, **E*(N,T)**, is defined as the minimum energy needed to keep all nodes in the network alive with T being the structure of the network.
- In general, we calculate the critical energy by taking the maximum value of the energies needed for survival along every path leaving the pump.

Example: Calculating critical energy

- Let N=3
- Then there are 2 topologies the line, say T_L, and the star, T_S
- On the line there is only 1 path leaving the pump so we calculate $[(\mathrm{E}(3) - \mathrm{C}) * \mathrm{p} - \mathrm{C}] * \mathrm{p} - \mathrm{C} > 0$, then E*(3, T_L)=min(E(3)) such that the inequality remains true.
- On the star there are 2 equal paths so it suffices to calculate along one of them so that: $\left[\frac{E(3)-C}{2} * p\right] - C > 0$, then E*(3, T_S)=min(E(3)).

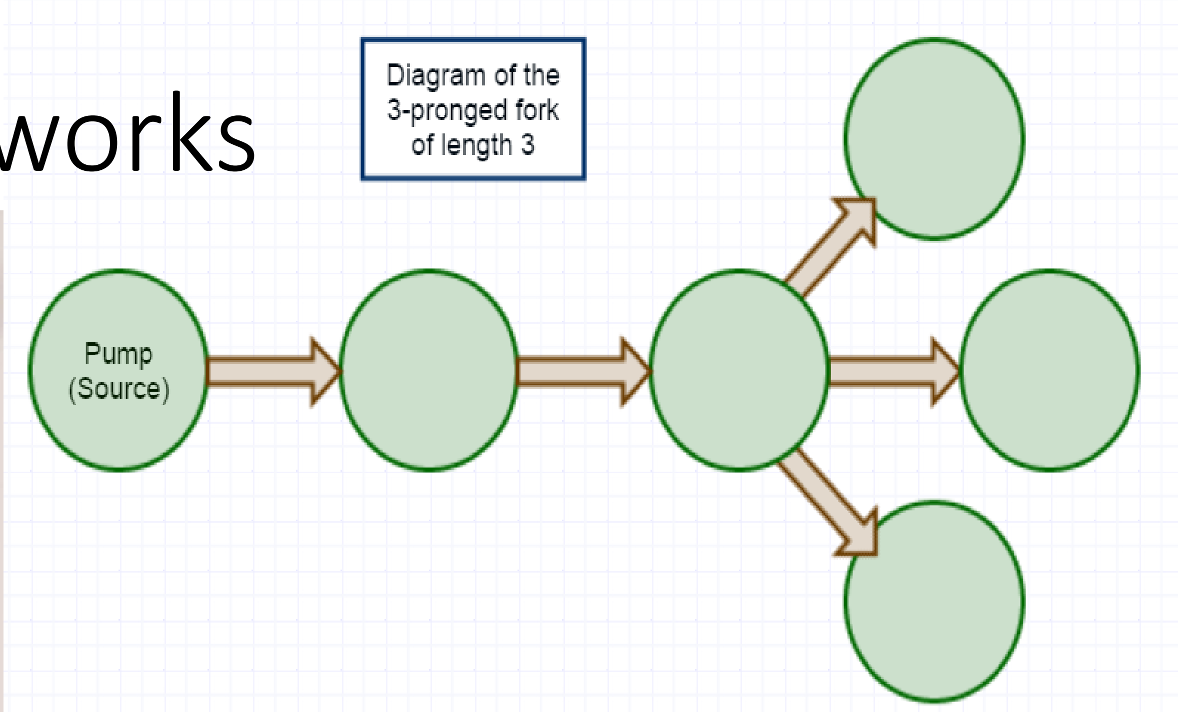
Critical energies by varying N

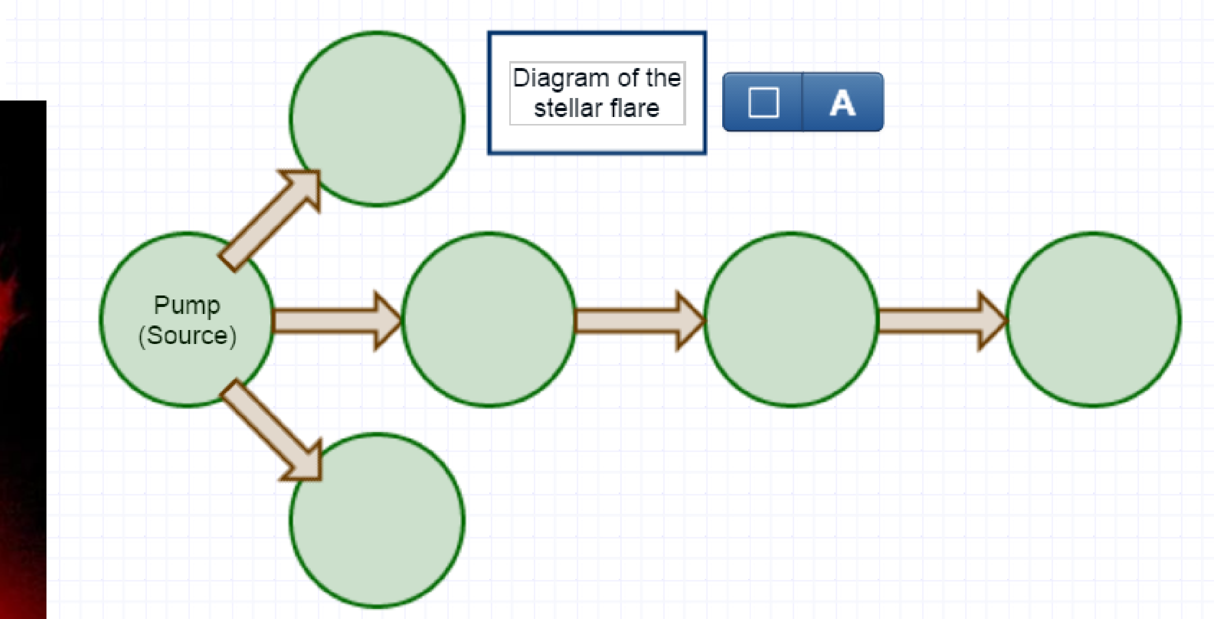
In this graph we use C=1 and p=0.9



Extending to other networks

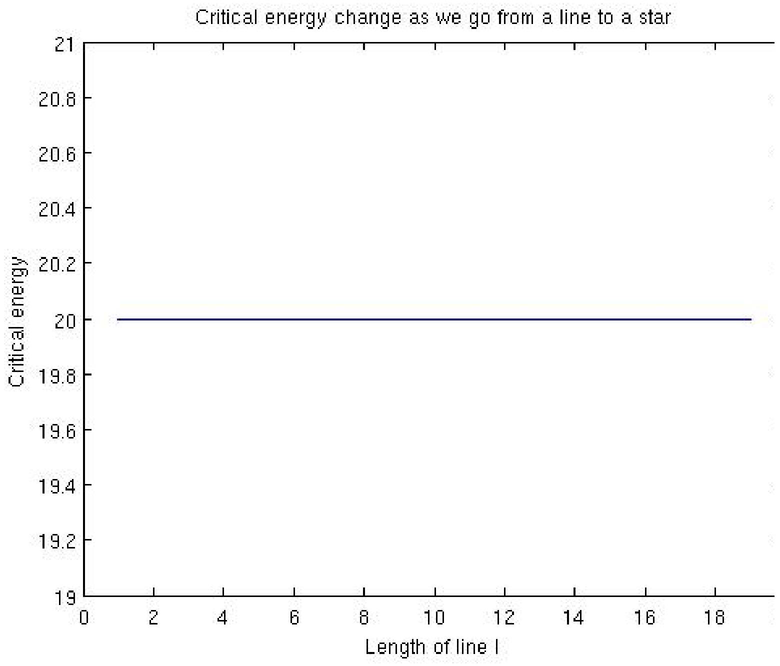
We define two quantities, S and L with S+L=N, where S and L go from 1 to N-1. S is the number of points in the star-like part. L is the length of the longest path without counting the center of the star.



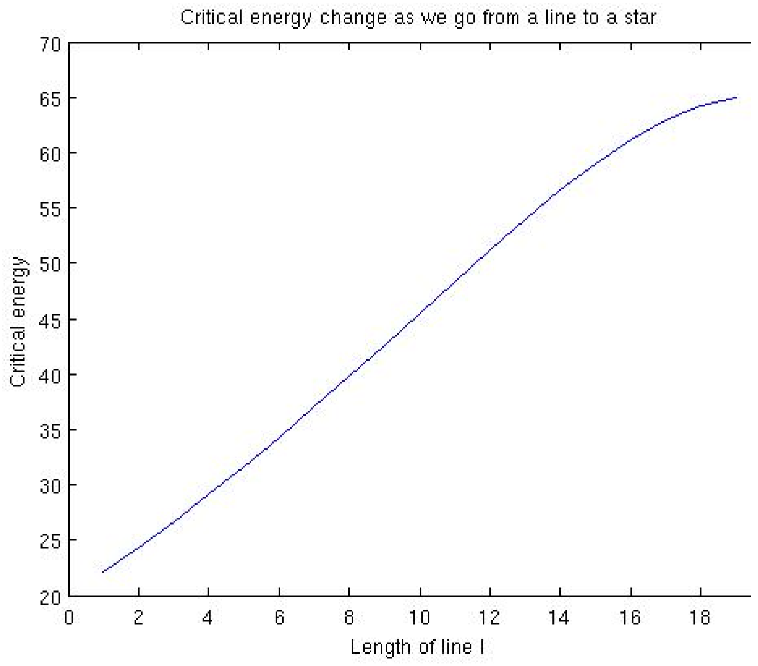


Plot of critical energy for N=20 of the fork

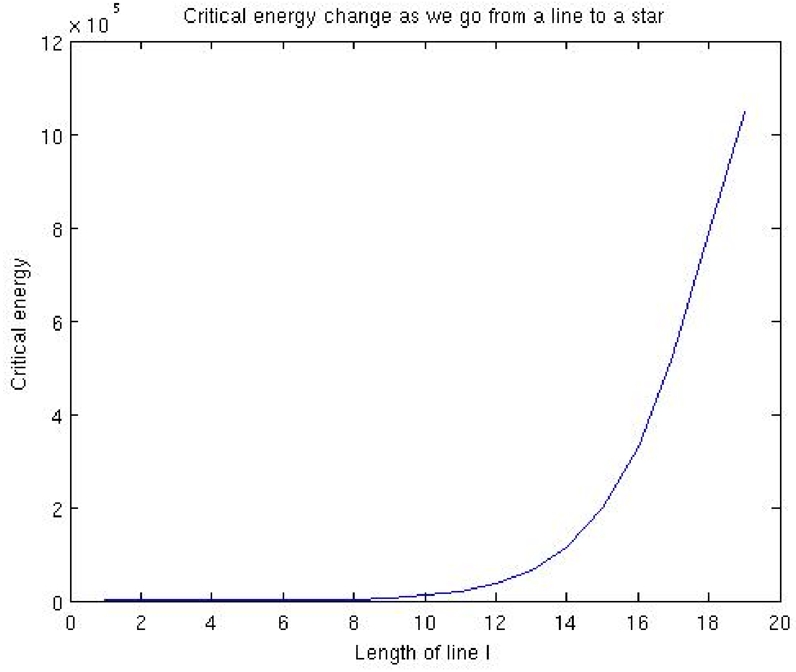
p=1



p=0.9

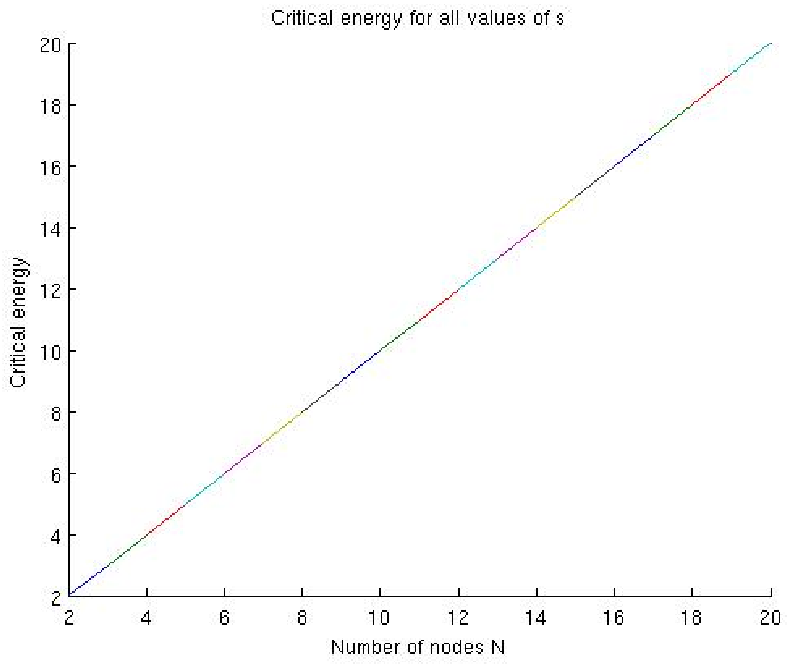


p=0.5

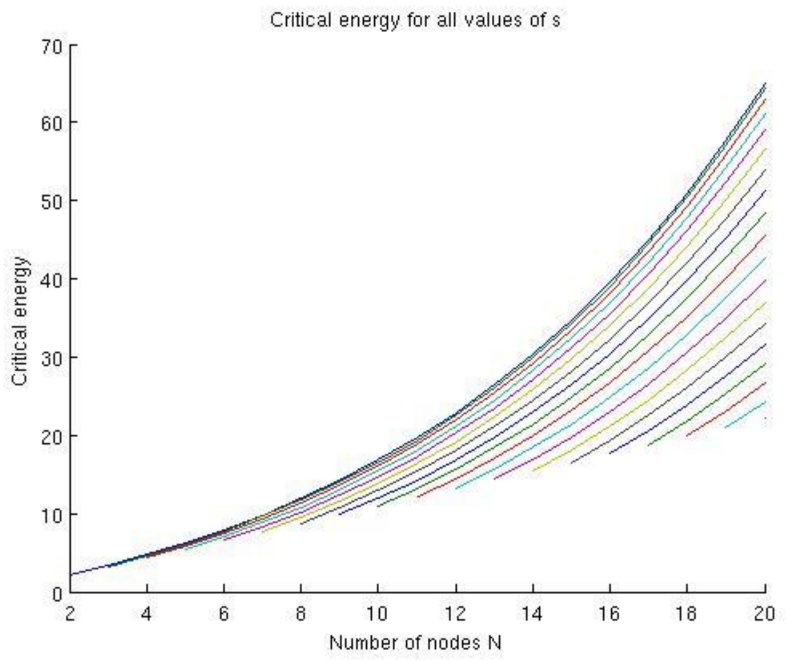


Plots of critical energy for fork varying N plotting S in the range 1 to N-1

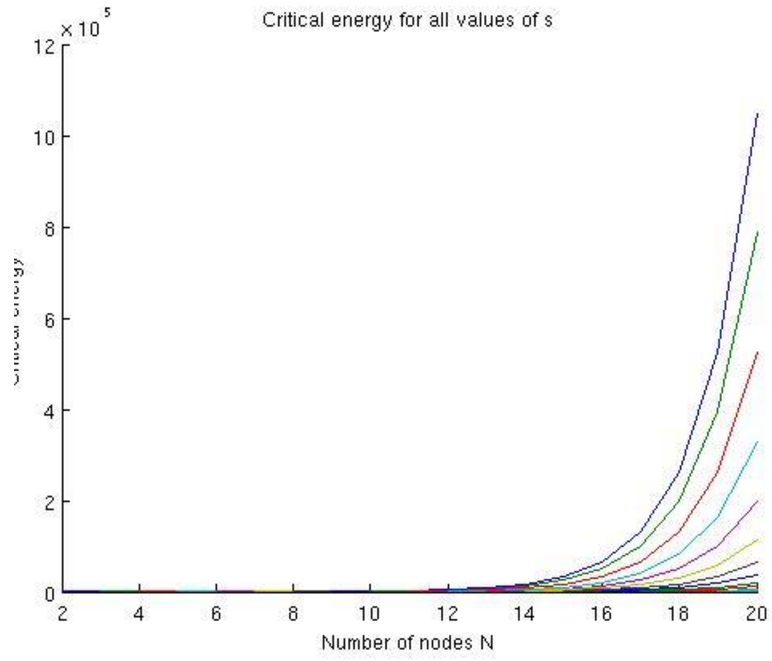
p=1



p=0.9

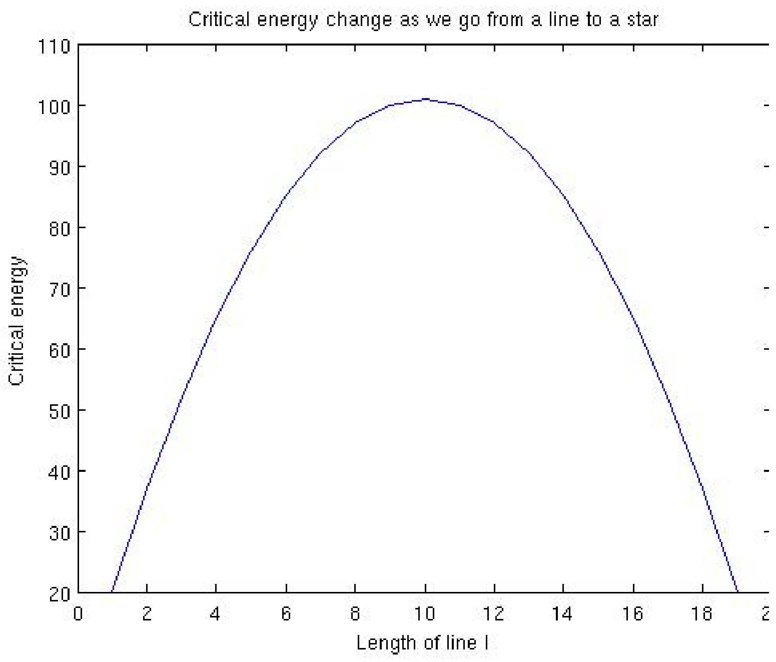


p=0.5

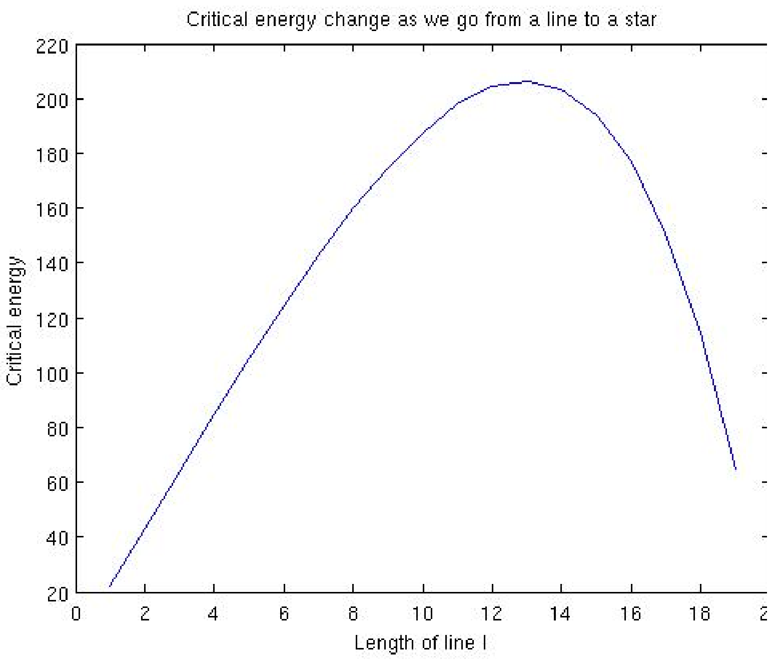


Critical energy for the stellar flare with constant N=20

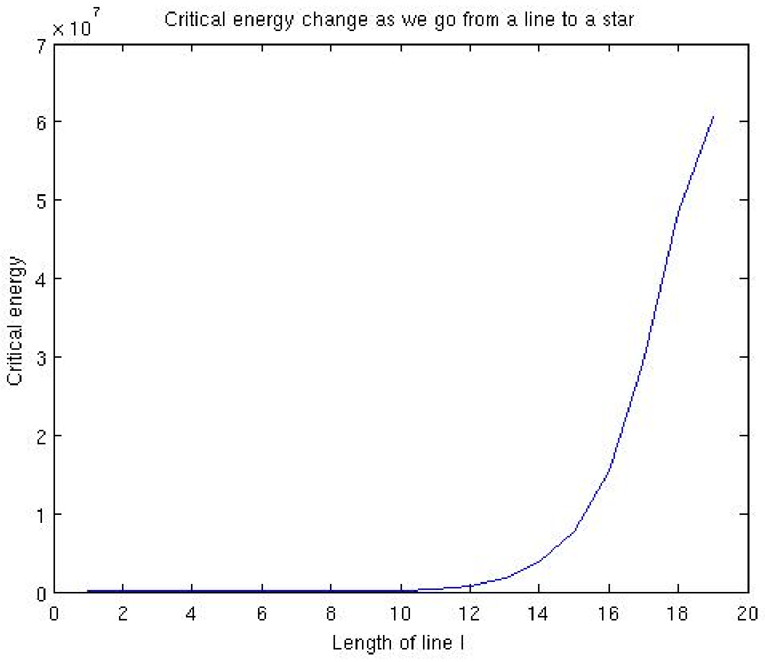
p=1



p=0.9

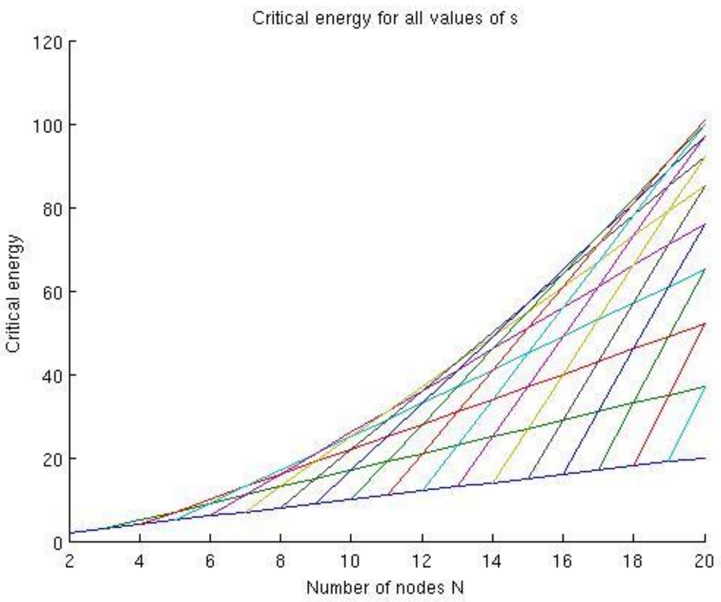


p=0.4

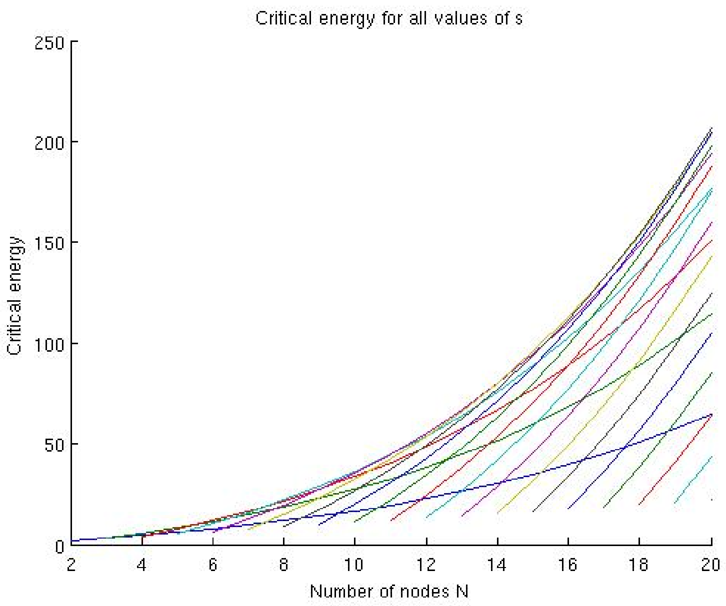


Critical energy of the stellar flare varying N and plotting S in the range 1 to N-1

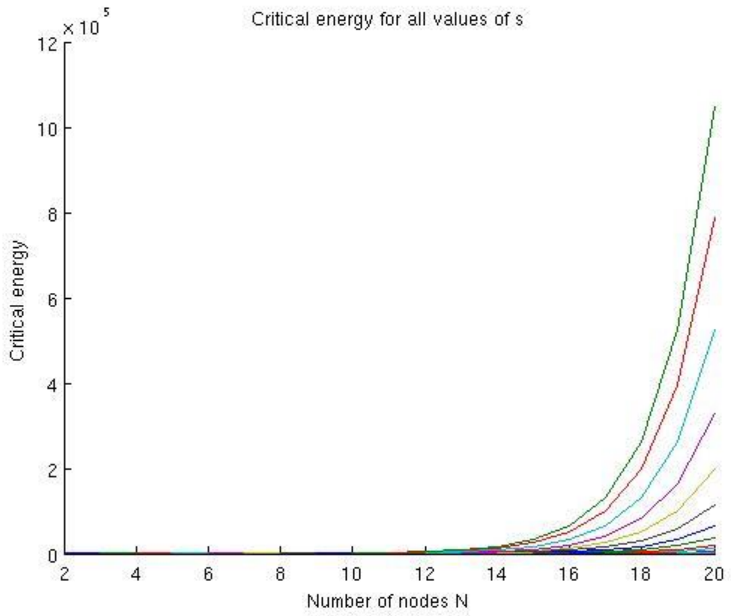
p=1



p=0.9



p=0.5

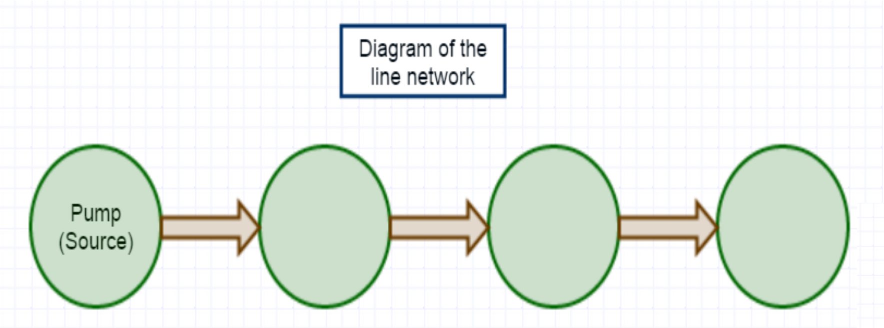


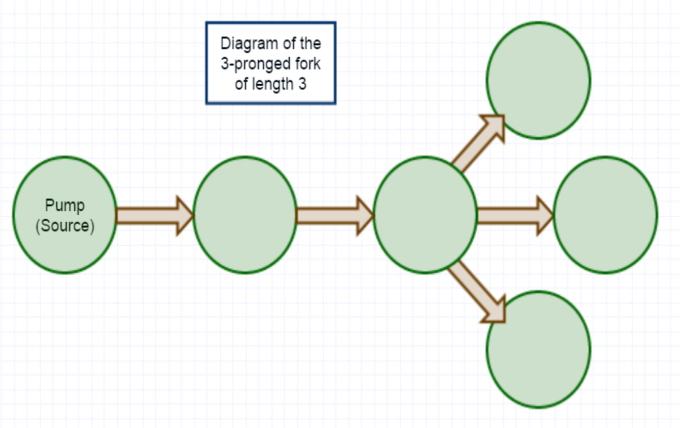
Robustness

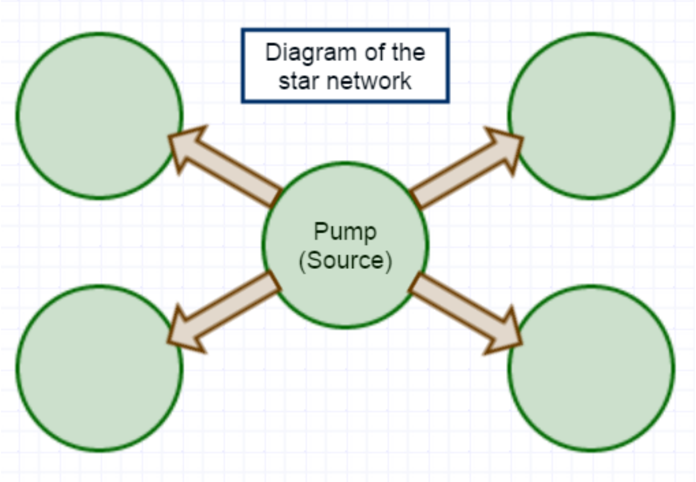
- A robust system is one that loses a small proportion of its total cells when it first drops below the critical energy.

More robust

Least robust







Recursive formula for critical energy (in progress)

- Idea: Develop a recursive formula dependent on the energy function and the branching factor at each node.

Future directions

- Additional topologies analyzed
- Verification of simulation
- Exploring other possible fitness metrics
- **Evolutionary simulations**

Thank you for your attention!