

### Example of competing topologies

0. Find best  $E_k$ .

a. Recipe for best  $E_k$

Optimal  $k^{th}$  critical energy. For  $E_1$  we have seen that the star is optimal. Now if we want to minimize  $E_k$ , we need to make  $E_k$  as small as possible.

b. Example

c. Colormap of  $E_1$  for those topologies//

d. Explicit formula to explain intersections.//

e. Generally if we consider larger  $k, N$ , then we can consider more complicated topologies that have more critical energy intersections. This motivates the idea of using a colormap with a large number of topologies. From analysis of simple topologies we can classify the set of networks that have the same behavior for low energies. The larger the  $k$ , the more possible optimal topologies. Interesting behaviors of  $E_s$

a. How to minimize a specific  $E_k$  that we are more interested in

b. Which of the possible topologies is the best for a given beta (environmental energy distribution)?

1. Analytically find when  $p$  values cross

2. Not reasonable to do this for large numbers of topologies

3.

Looking at small topologies (figure with star and line), we can already see that