

## Introduction: Biology and transport problems

### 1. Ecological networks

- Describe from previous work (suggestions for papers?)
- food web (Is a food web optimal?)(Sun  $\rightarrow$  Grass  $\rightarrow$  Herbivore  $\rightarrow$  Carnivore  $\rightarrow$  Bacteria(?) )
- Interested in statistical properties
- Decide how the 'p' level is done—keep constant
- Need to add more nodes (possibly adjust p,c) in this food web case to handle the complexity of interactions between different types of individuals
- Concern/Difficulty: Connecting to empirical data
- Check for connection weights

### 2. Network Optimization

- Empirical study of networks
- information transport ( $c \rightarrow 0$  (?) ) (see email from 12/21)
- Relation to evolution

### 3. Transport problems

## Description of the model

### 1. Discrete network model

- Pump
  - Other nodes
  - Connectivity rules
  - Consumption of energy
  - Cost of transport proportional to amount of transport
- ### 2. Determining energy at each node
- Given an energy that the pump receives, we can know the energy received by all other cells.
  - Simple examples (figure 1)
  - Critical energy - Knowing a point when a certain number of cells die, find the energy at the pump

### 3. Special Networks

- Line
- Star
- Importance of these motifs
- Simple generalizations
  - Fork
  - Stellar flare (re-name)
- Provide explicit formula of first critical energy
- Node to die is independent of c and p
- only one path back to the pump

## Recursive Formula

- 0. Motivation
  - a. Where to start?
  - b. Which node dies first? second, etc.
  - c. More paths back to the pump as the network grows
  - d. How do we handle loops (i.e. when there is no explicit formula)?
- 1. The formula itself
  - a. The special topologies don't span the space of allowable topologies
  - i. Topologies may depend on  $c$  and  $p$  (figure)
  - ii. What happens when a node dies in general?
  - iii. How many nodes die at a given event?
  - iv. The formula
  - v. Example of a simple topology with non-trivial death events (figure)
  - vi. Example of dependence on  $p$  even within a single topology
- b. Calculate critical energy for everyone, the maximum is the 1st critical energy (2nd largest - 2nd critical energy, 3rd, etc.)
- 2. (Aside/Example: Finding the stoichiometric coefficient of a chemical reaction)
- 3. Anything else goes into Supplementary Material

Example of competing topologies

- 0. Find best  $E_k$ .
  - a. How to minimize a specific  $E_k$  that we are more interested in
  - b. Which of the possible topologies is the best for a given beta (environmental energy distribution)?
- 1. Analytically find when  $p$  values cross
- 2. Not reasonable to do this for large numbers of topologies
- 3.

Colormap and Data Analysis (random and builder)

- 1. Cross-section
  - a.  $p$
  - b.  $E$
- 2. Mean and Standard Deviation
- 3. Low/Upper edge
- 4. Derivatives
- 5. Compare random and builder (Description of each in supplementary material)

Conclusions and Future Work

- 0. Connections to biological evolution
  - 1. Analysis of  $E_1$ ,  $E_2$ , and  $E_3$  in real vs. optimal networks
  - 2. Comparing result with Diffusion (flux?)