Discussion Week 1

8/24/2018

1. Determine whether the following are True or False. Explain your answer.
    a. $\emptyset$ and $\{\emptyset\}$ have the same cardinality.
    b. Define $B = \{1, 2, 3, 2, 1\}$. (i) $|B| = 5$. (ii) $|P(B)| = 8$
    c. If T = {right-most digits of all positive integer multiples of 7} and S={0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, then S=T.
    d. For n, $a \in \boldsymbol{Z}$, define the set $n\boldsymbol{Z} + a = \{x \in \boldsymbol{Z}: x = nt + a \text{ for } t \in Z\}$. It is then the case that the sets $7\boldsymbol{Z} + 2$ and $7\boldsymbol{Z} + 16$ are equal.

2. Suppose $A = \{1, 2\}, B = \{a, b\}, C =$ Powerset(B), please answer the following questions and explain why.
    a. How many functions are there that map A to B?
    b. What is $C - B$?
    c. How many functions $f: A \to B$ are bijective?
    d. How many functions $f: A \to C$ are bijective?

3. Suppose the universal is all integers, set $A = \{all\ odd\ numbers\}$, $B = \{prime\ numbers\}, C = \{n \in Z: 0 \leq n \leq 100\}$, please answer the following questions.
    a. Draw the Venn diagram that includes all the sets described above.
    b. Use set notations to represent all even prime numbers and shade the corresponding area in the Venn diagram.
    c. Use set notations to represent the set of numbers that includes even numbers and all prime numbers smaller than or equal to 100. Then shade the corresponding area in the Venn diagram.

4. Indicate whether each of these functions is injective (one-to-one), surjective (onto), neither, or both (bijective). Explain your reasoning.
    a. f: Z $\to$ O, where f(x) = 2x+3
    b. f: O $\to$ $(N \cap E)$ where $f(x) = (x+5)^2$
    c. f: $Z^+ \to Z^+$ where $f(x) = x^2$
    d. Define **A** be the set of all 4-digit integers (0000 through 9999) labeled *abcd*.
    Define **B** be the set of all 2-digit integers (00 through 99) labeled *nm*.
    f: A $\to$ B, where $f(x) = 10n + m$;
    $n = (a + b)\ mod\ 10$; $m = (c + d)\ mod\ 10$
    For example, $x = 2098, n = (2 + 0)\ mod\ 10 = 2, m = (9 + 8)\ mod\ 10 = 17\ mod\ 10 = 7, f(x) = 27$

5. Define $S_n = \sum_{i=1}^{n} i^2$
   a. What are the first 5 terms $(S_1, S_2, S_3, S_4, S_5)$ in this sequence?
   b. Determine a recurrence relation for S_n.

6. Determine a_4 and the closed form of a_n if:
   a. $a_n = \frac{a_{n-1}^2}{2}$; $a_0 = 2$
   b. $a_{n+1} + 1 = 3a_n + 3$; $a_0 = 1$ Hint: $a_{n+1} + k = 3(a_n + k)$

7. Show that if four distinct integers are chosen between 1 and 60 inclusive, some two of them must differ by at most 19.