

# CS 397: Topics in Computer Science—Probability & Statistics

Dr. Francis Parisi

Pace University

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# Practice Problems

Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages.

If a customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John?

# Practice Problems

An allergist claims that 50% of the patients she tests are allergic to some type of weed. What is the probability that

1. Exactly 3 of her next 4 patients are allergic to weeds?
2. None of her next 4 patients is allergic to weeds?

# Practice Problems

If the joint probability distribution of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{x + y}{30}, \text{ for } x = 0, 1, 2, 3; y = 0, 1, 2,$$

find

1.  $P[X \leq 2, Y = 1]$
2.  $P[X > 2, Y \leq 1]$
3.  $P[X > Y]$
4.  $P[X + Y = 4]$

# Practice Problems

Consider the random variables  $X$  and  $Y$  with joint density function

$$\begin{cases} f(x, y) = x + y, & 0 \leq x, y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

1. Find the marginal distributions of  $X$  and  $Y$
2. Find  $P[X > 0.5, Y > 0.5]$

# Practice Problems

Find the covariance of random variables  $X$  and  $Y$  having the joint probability density function

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

# Practice Problems

Suppose it is known that the life  $X$  of a particular compressor, in hours, has the density function

$$f(x) = \begin{cases} = \frac{1}{900}e^{-x/900}, & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

1. Find the mean life of the compressor
2. Find  $\mathbb{E}[X^2]$
3. Find the variance and standard deviation of  $X$

# Practice Problems

To avoid detection at customs, a traveler places 6 narcotic tablets in a bottle containing 9 vitamin tablets that are similar in appearance. If the customs official selects 3 of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics?



# Practice Problems

On average 10 oil tankers arrive each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

# Practice Problems

Imperfections in computer circuit boards and computer chips lend themselves to statistical treatment. For a particular type of board, the probability of a diode failure is 0.03 and the board contains 200 diodes.

1. What is the mean number of failures among the diodes?
2. What is the variance?
3. The board will work if there are no defective diodes. What is the probability that a board will work?

# Practice Problems

A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

# Practice Problems

A manufacturer of car batteries guarantees that the batteries will last, on average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, should the manufacturer still be convinced that the batteries have a standard deviation of 1 year? Assume that the battery lifetime follows a normal distribution.

HINT: for normally distributed data,  $(n - 1)S^2/\sigma^2$  is distributed  $\chi^2$  with  $n - 1$  degrees of freedom.

# Practice Problems

Average power usage (dB per hour) for a particular company is studied and is known to have a lognormal distribution with parameters  $\mu = 4$  and  $\sigma = 2$ . What is the probability that the company uses more than 270 dB during any particular hour?