

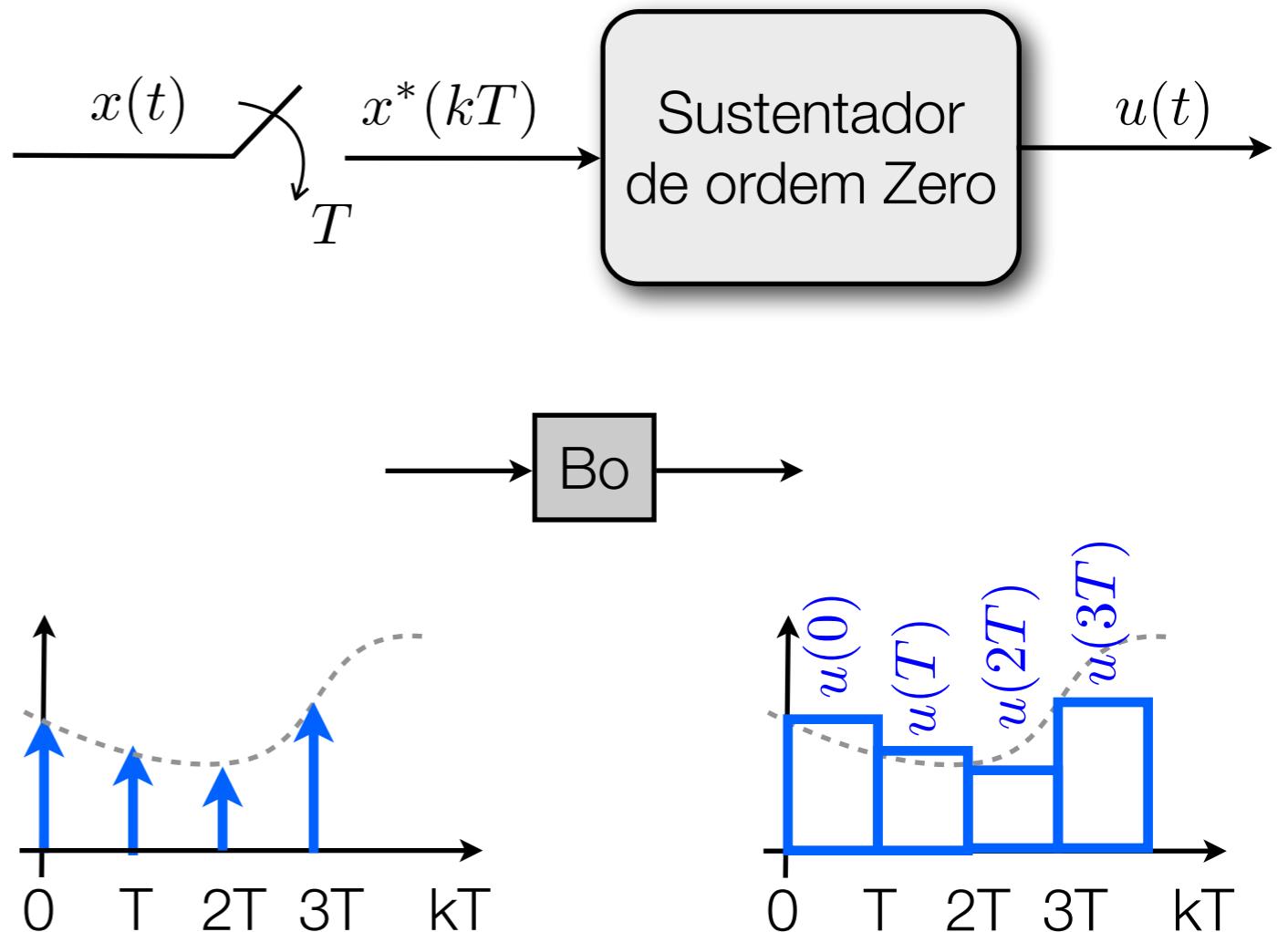
Sustentador de Ordem Zero

Cálculo de BoG(z)

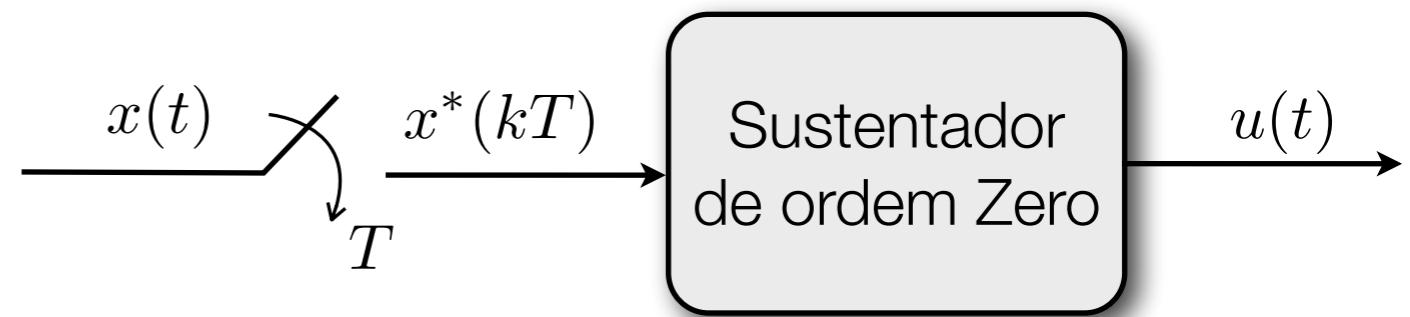
Prof. Fernando Passold

Sistemas com Bloqueador (ou Sustentador ou “Holder”)

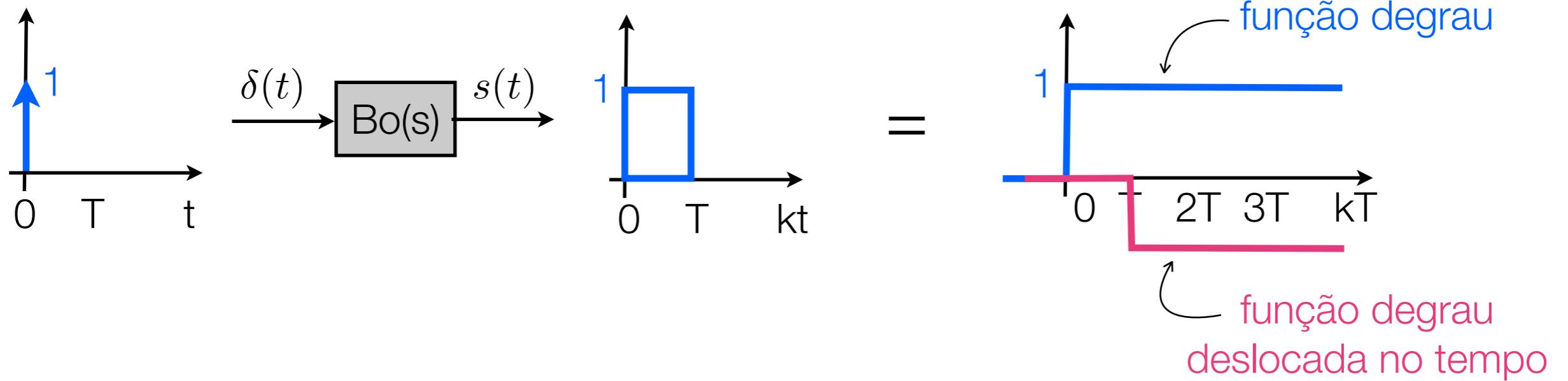
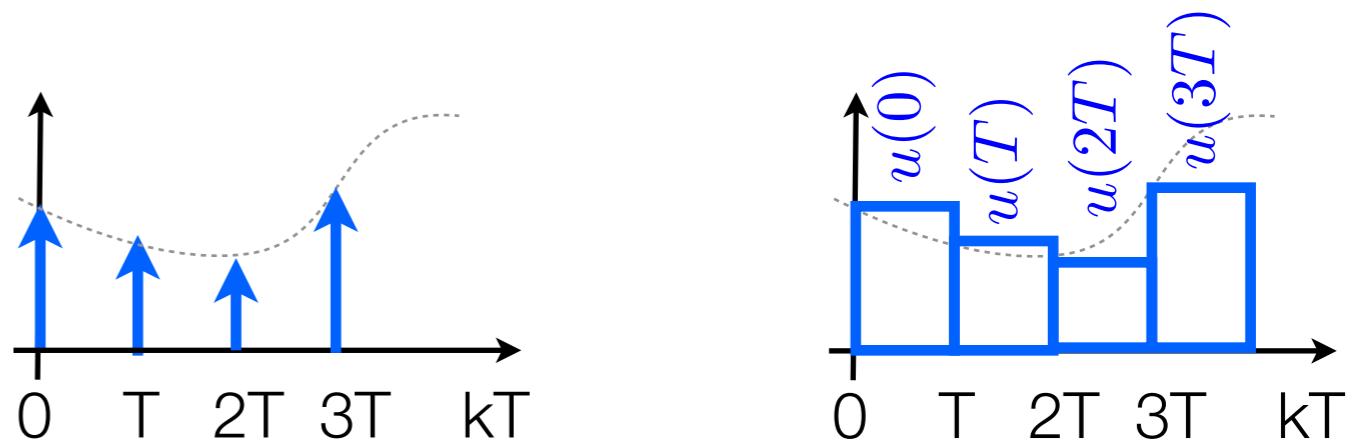
- Sistemas discretos no tempo --> sequencia de números;
- Processo para obter sequencia de números --> amostragem (“sampling”);
- Valores de $x(t)$ são tomados em pontos equidistantes de tempo: $\{x(kT)\}$, $k = 0, 1, 2, 3, \dots$ --> sinal discreto no tempo.
- O sinal discreto é obtido através de uma chave que “congela” este sinal até o próximo instante de amostragem --> “holder”.
- Amostragem é ideal se saída da chave amostradora (*sampler*) seja praticamente um degrau --> Bloqueador de ordem zero: Bo .



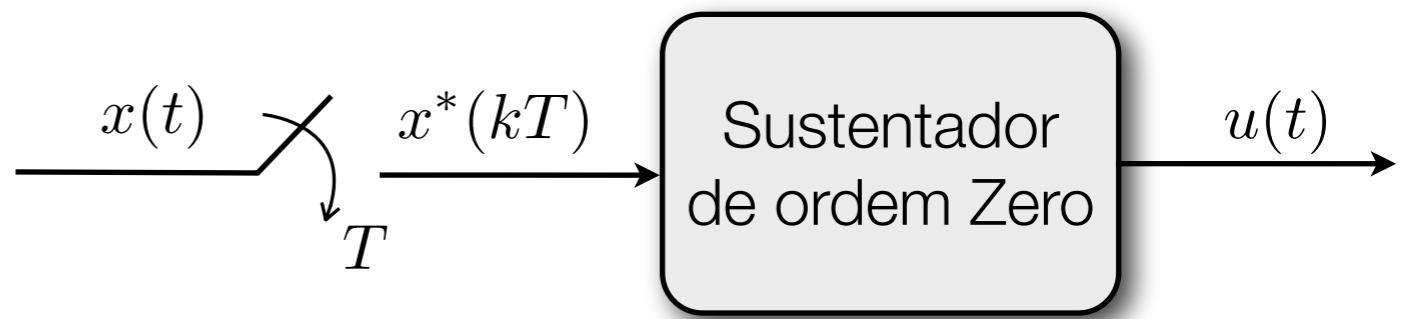
Sistemas com Bloqueador (ou Sustentador ou “Holder”)



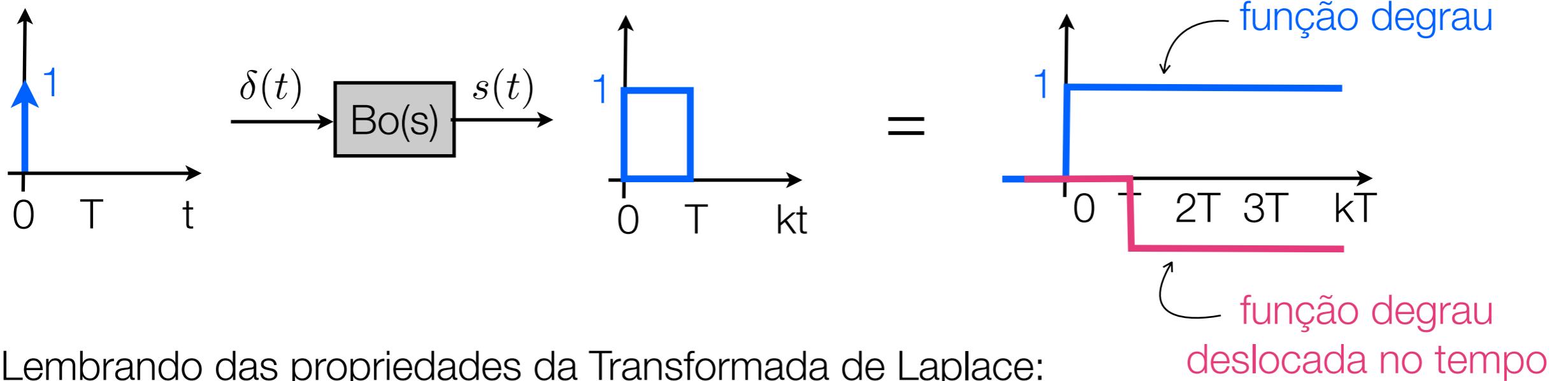
- Modelagem:



Sistemas com Bloqueador (ou Sustentador ou “Holder”)



- Modelagem:



Lembrando das propriedades da Transformada de Laplace:

- Deslocamento no tempo: $\mathcal{L}\{f(t - a) \cdot u(t - a)\} = e^{-as} \cdot F(s)$

- Função degrau: $\mathcal{L}\{u(t)\} = \frac{1}{s}$

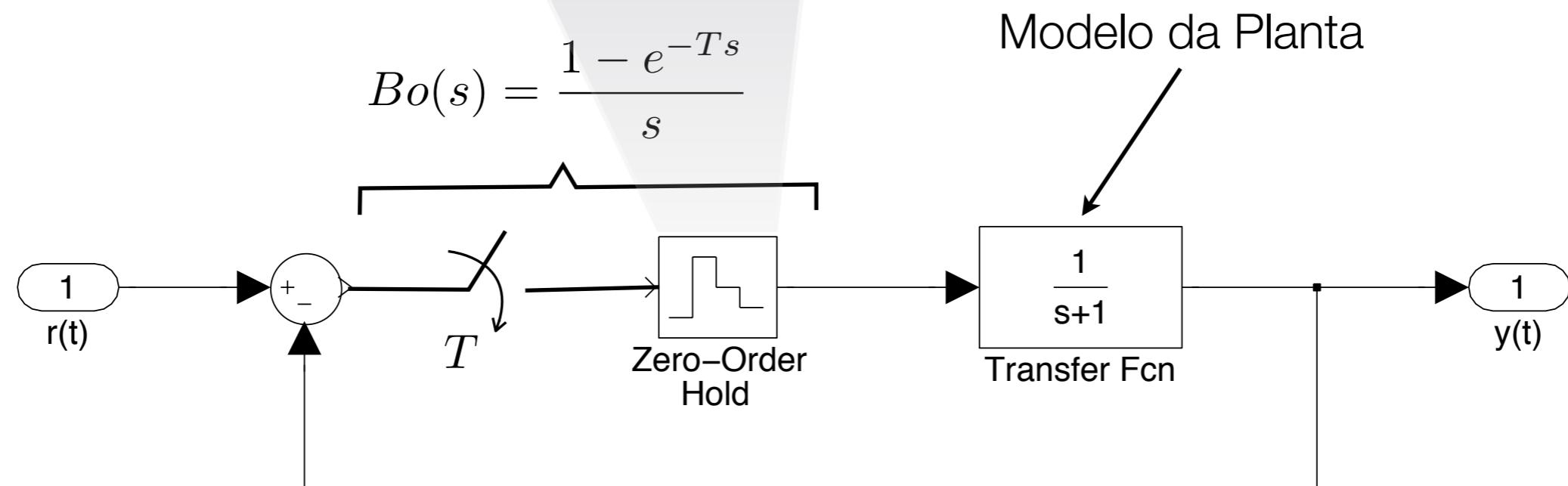
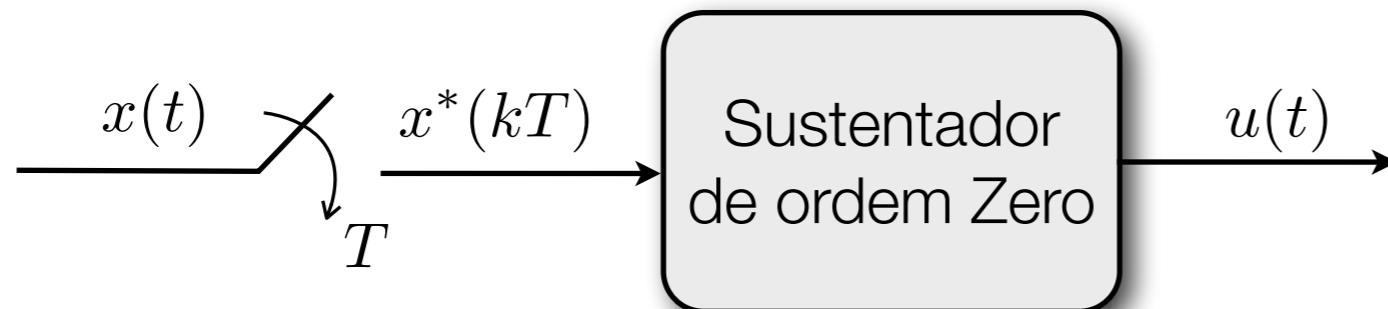
Temos então:

$$Bo(s) = \frac{S(s)}{E(s)}$$

$$\begin{aligned} e(t) &= \delta(t) \quad \therefore \quad E(s) = 1 \\ s(t) &= u(t) - u(t - T) \end{aligned}$$

$$Bo(s) = \frac{1}{s} - \frac{e^{-Ts}}{s}$$

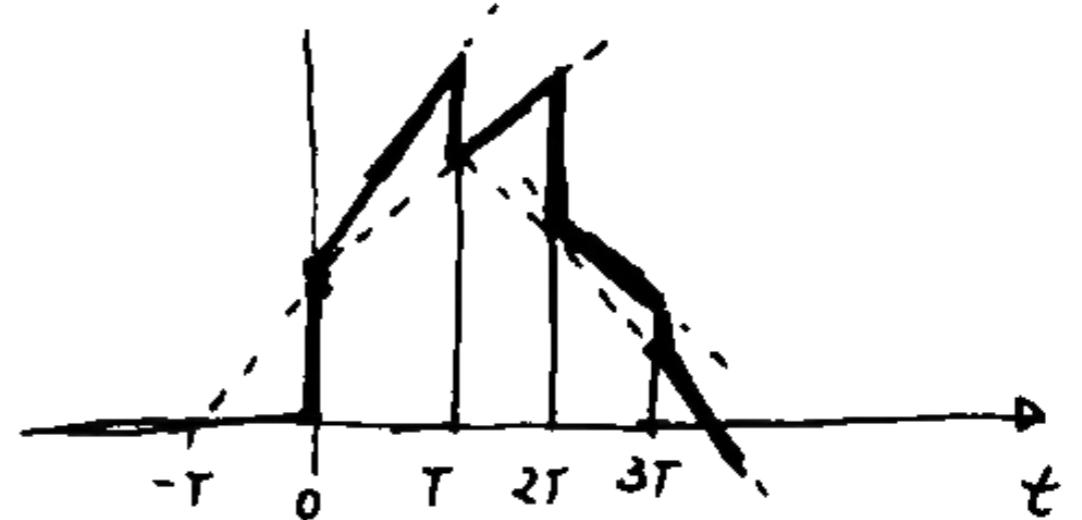
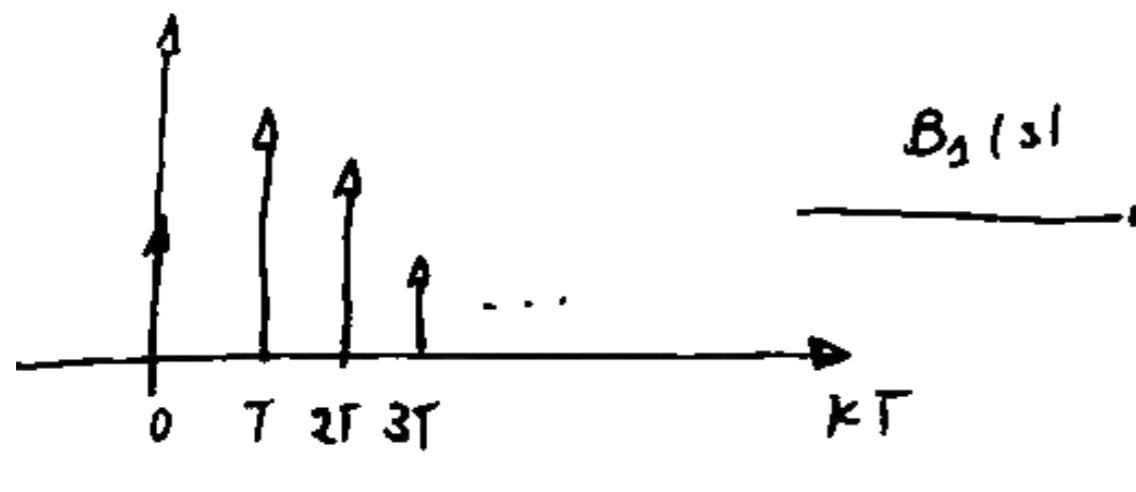
$$Bo(s) = \frac{1 - e^{-Ts}}{s}$$



Introdução do Sustentador
em uma planta

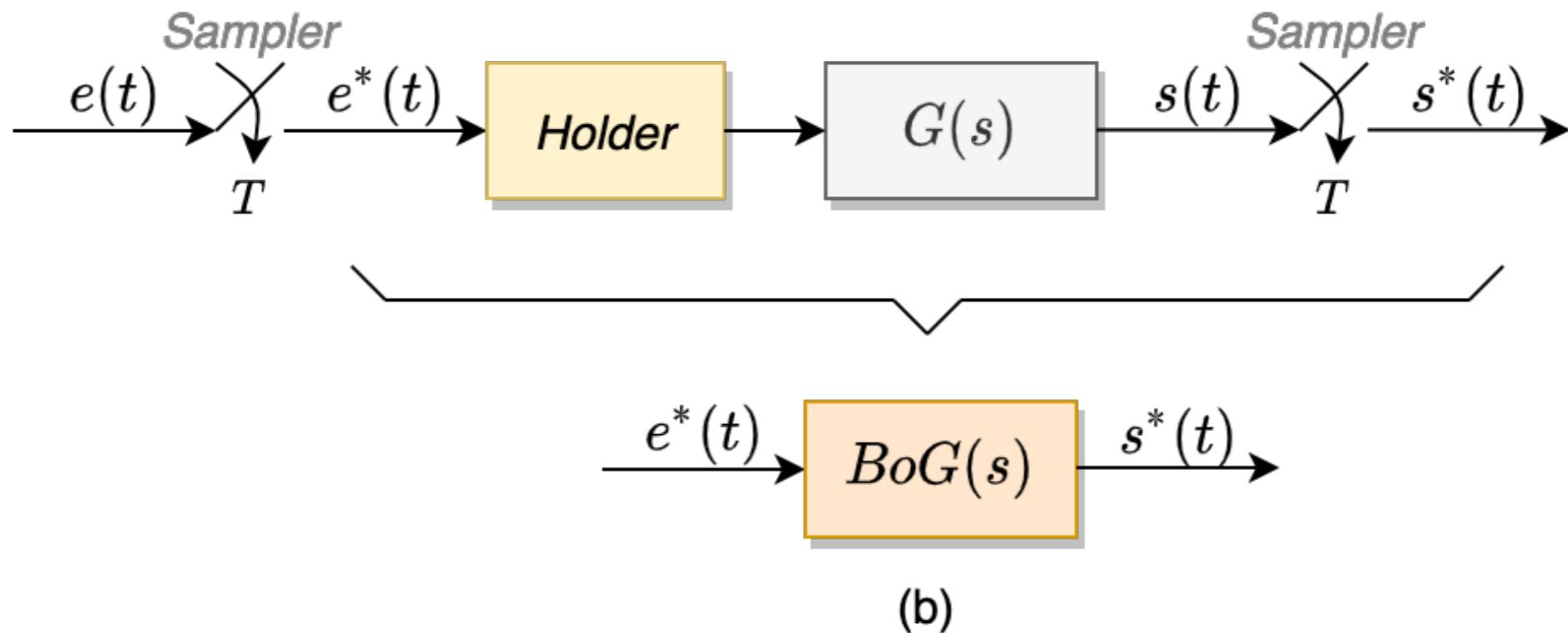
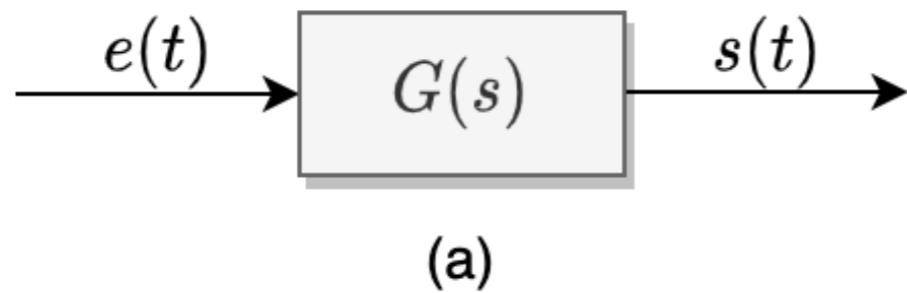
Modelo Final

Sustentador de 1^a-ordem:

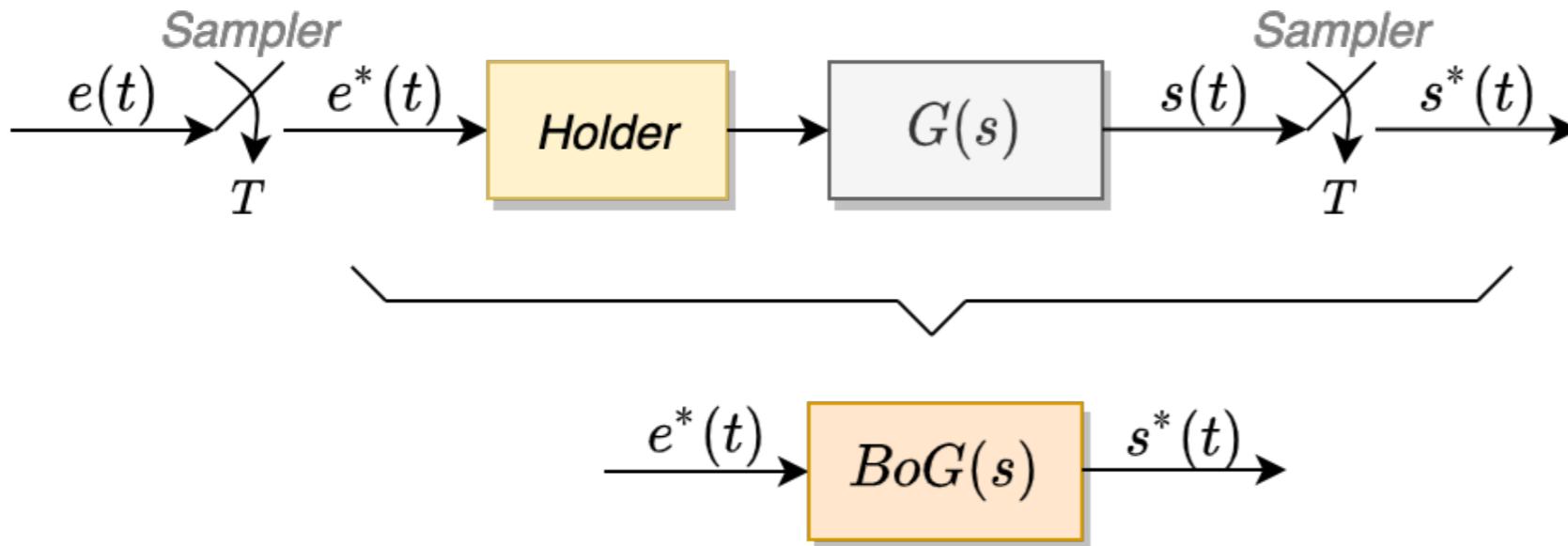


Outros Modelos de
Sustentadores

Associando o Bloqueador a um processo...



- Como o sistema (a) pode ser visualizado depois de amostrado (b)



Note que:

$$\frac{S(s)}{E(s)} = Bo(s) \cdot G(s) \text{ e que: } \frac{S(z)}{E(z)} = Bo \cdot G(z) = \mathbb{Z} \{ Bo(s) \cdot G(s) \}$$

$$\text{Então: } \mathbb{Z} \{ Bo(s) \cdot G(s) \} = \mathbb{Z} \left\{ \left(\frac{1 - e^{-Ts}}{s} \right) \cdot G(s) \right\} = \mathbb{Z} \left\{ \frac{G(s)}{s} \right\} - \mathbb{Z} \left\{ \frac{e^{-Ts} G(s)}{s} \right\}$$

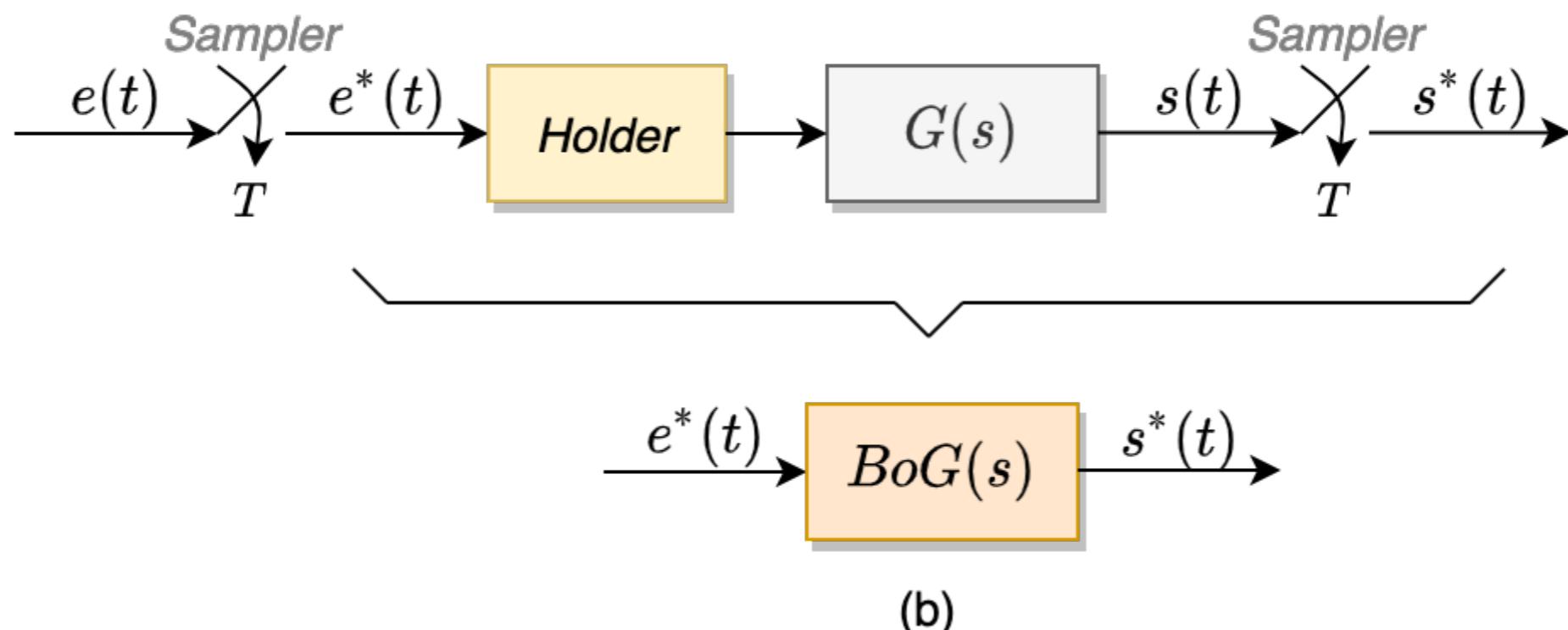
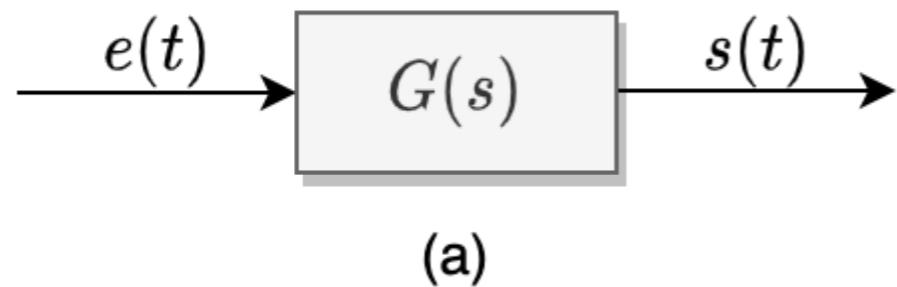
$$\text{Se } F(s) = \frac{G(s)}{s}, \quad \text{temos:} \quad \mathbb{Z} \{ Bo(s) \cdot G(s) \} = \mathbb{Z} \{ F(s) \} - \mathbb{Z} \{ e^{-Ts} F(s) \}$$

Como: $e^{-Ts} \cdot F(s)$ é equivalente a um atraso no tempo de um período de amostragem:
 $e^{-Ts} \cdot F(s) \Rightarrow f(t - T)$, então: $\mathbb{Z} \{ f[(k - 1)T] \} = z^{-1} \cdot F(s)$

E assim: $\mathbb{Z} \{ Bo(s) \cdot G(s) \} = \mathbb{Z} \{ F(s) \} - z^{-1} \cdot \mathbb{Z} \{ F(s) \}$ ou simplesmente:

$$BoG(z) = (1 - z^{-1}) \cdot \mathbb{Z} \left\{ \frac{G(s)}{s} \right\}$$

$$\mathcal{Z} \{ Bo(s) \cdot G(s) \} = \mathcal{Z} \left\{ \frac{1 - e^{-T s}}{s} \cdot G(s) \right\} = \mathcal{Z} \left\{ \frac{G(s)}{s} \right\} - \mathcal{Z} \left\{ \frac{e^{-T s} G(s)}{s} \right\}$$



$$BoG(z) = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{G(s)}{s} \right\}$$

Exemplo_1) Obter BoG(z) para o sistema abaixo:

Se $G(s) = \frac{K \cdot g}{(1 + \tau s)}$ (um sistema de 1a-ordem).

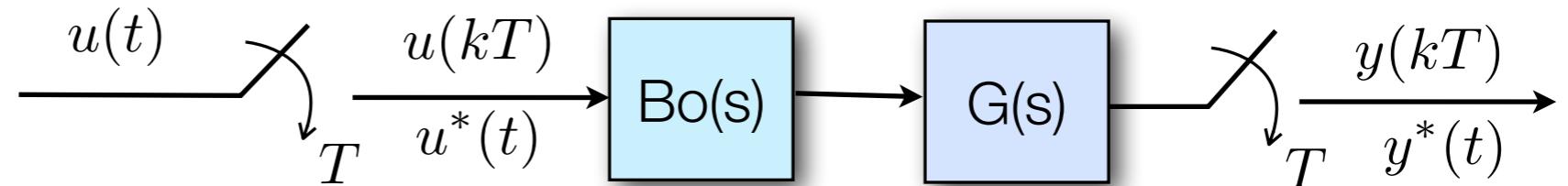
Aplicando: $BoG(z) = (1 - z^{-1}) \cdot \mathbb{Z} \left\{ \frac{G(s)}{s} \right\}$

Obtemos: $\mathbb{Z} \left\{ \frac{K(1 - e^{-Ts}) g}{s(1 + \tau s)} \right\} = K g (1 - z^{-1}) \cdot \mathbb{Z} \left\{ \frac{1}{s(1 + \tau s)} \right\}$

“Pausa” para Transformada Z...

Exemplo: BoG(z) de Sistema de 1a-Ordem...

- Seja o sistema:



Se $G(s) = \frac{K \cdot g}{(1 + \tau s)}$ (Um sistema de 1a-ordem)

Aplicando o que acabamos de verificar: $BoG(z) = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{G(s)}{s} \right\}$, obtemos:

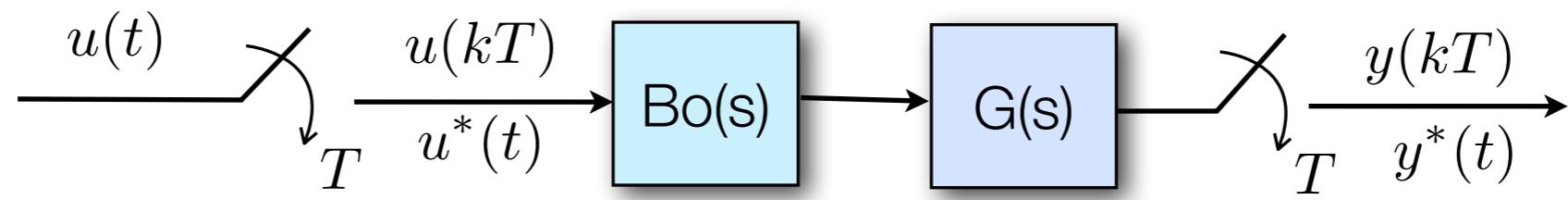
$$BoG(z) = \mathcal{Z} \left\{ \frac{K \cdot g \cdot (1 - e^{-Ts})}{s(1 + \tau s)} \right\} = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{K \cdot g}{s(1 + \tau s)} \right\} = K g (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{1}{s(1 + \tau s)} \right\}$$

$$\frac{1}{s(1 + \tau s)} = \frac{A}{s} + \frac{B}{(1 + \tau s)} = \frac{A(1 + \tau s) + Bs}{s(1 + \tau s)} = \frac{A + s(B + \tau A)}{s(1 + \tau s)}$$

$$A = \left. \frac{1}{(s + \tau s)} \right|_{s=0} = 1$$

$$B = \left. \frac{1}{s} \right|_{s=1/(-\tau)} = -\tau$$

$$\frac{1}{s(1 + \tau s)} = \frac{1}{s} - \frac{\tau}{(1 + \tau s)} = \frac{1}{s} - \frac{1}{1/\tau + s}$$



Se $G(s) = \frac{K \cdot g}{(1 + \tau s)}$ (Um sistema de 1a-ordem)

Aplicando o que acabamos de verificar: $BoG(z) = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{G(s)}{s} \right\}$, obtemos:

$$BoG(z) = \mathcal{Z} \left\{ \frac{K \cdot g \cdot (1 - e^{-Ts})}{s(1 + \tau s)} \right\} = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{K \cdot g}{s(1 + \tau s)} \right\} = K g (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{1}{s(1 + \tau s)} \right\}$$

$$\frac{1}{s(1 + \tau s)} = \frac{A}{s} + \frac{B}{(1 + \tau s)} = \frac{A(1 + \tau s) + Bs}{s(1 + \tau s)} = \frac{A + s(B + \tau A)}{s(1 + \tau s)}$$

$$A = \left. \frac{1}{(s + \tau s)} \right|_{s=0} = 1$$

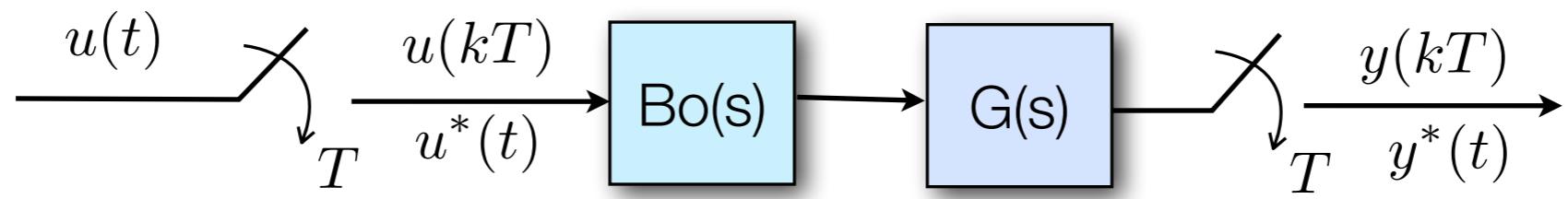
$$B = \left. \frac{1}{s} \right|_{s=1/(-\tau)} = -\tau$$

$$\frac{1}{s(1 + \tau s)} = \frac{1}{s} - \frac{\tau}{(1 + \tau s)} = \frac{1}{s} - \frac{1}{1/\tau + s}$$

Determinando a transformada Z:

$$\mathcal{Z} \left\{ \frac{1}{s(s + \tau s)} \right\} = \frac{z}{z - 1} - \frac{z}{z - e^{-T/\tau}} \quad \text{ou}$$

$$\mathcal{Z} \left\{ \frac{1}{s(s + \tau s)} \right\} = \frac{1}{1 - z^{-1}} - \frac{1}{1 - z^{-1} \cdot e^{-T/\tau}}$$



Se $G(s) = \frac{K \cdot g}{(1 + \tau s)}$ (Um sistema de 1a-ordem)

Aplicando o que acabamos de verificar: $BoG(z) = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{G(s)}{s} \right\}$, obtemos:

$$BoG(z) = \mathcal{Z} \left\{ \frac{K \cdot g \cdot (1 - e^{-Ts})}{s(1 + \tau s)} \right\} = (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{K \cdot g}{s(1 + \tau s)} \right\} = K g (1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{1}{s(1 + \tau s)} \right\}$$

$$\mathcal{Z} \left\{ \frac{1}{s(s + \tau s)} \right\} = \frac{z}{z - 1} - \frac{z}{z - e^{-T/\tau}} \quad \text{ou}$$

$$\mathcal{Z} \left\{ \frac{1}{s(s + \tau s)} \right\} = \frac{1}{1 - z^{-1}} - \frac{1}{1 - z^{-1} \cdot e^{-T/\tau}}$$

$$BoG(z) = \frac{K g (1 - z^{-1}) \cdot 1}{(1 - z^{-1})} - \frac{K g (1 - z^{-1}) \cdot 1}{(1 - z^{-1} \cdot e^{-T/\tau})}$$

$$BoG(z) = Kg - \frac{Kg(z - 1)}{(z - e^{-T/\tau})} = \frac{Kg(z - e^{-T/\tau}) - Kg(z - 1)}{(z - e^{-T/\tau})} = \frac{Kg z - Kg e^{-T/\tau} - Kg z + Kg}{(z - e^{-T/\tau})}$$

$$BoG(z) = \frac{K g (1 - e^{-T/\tau})}{(z - e^{-T/\tau})}$$

Note: 1^a-ordem com integrador

Problema

- Determine BoG(z) para:

$$BoG(z) = \frac{K}{s(s+a)}$$

$$BoG(z) = \mathcal{Z} \left\{ \frac{K(1 - e^{-Ts})}{s^2(s+a)} \right\} = K(1 - z^{-1}) \cdot \mathcal{Z} \left\{ \frac{1}{s^2(s+a)} \right\}$$

- Resposta:

$$BoG(z) = \frac{K [z(aT + e^{-aT} - 1) + (1 - aTe^{-aT} - e^{-aT})]}{a^2(z-1)(z-e^{-aT})}$$