DIRECT FORM II

The signal flow graph for the Direct-Form-II (DF-II) realization of the second-order IIR filter section is shown in Fig.9.2.

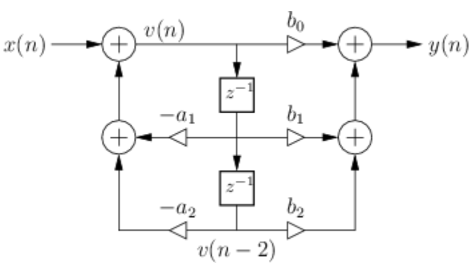

Figure 9.2: Direct-Form-II implementation of a 2nd-order digital filter.

The difference equation for the second-order DF-II structure can be written as

$$\begin{aligned} v(n) &= x(n) - a_1 v(n-1) - a_2 v(n-2) \\ y(n) &= b_0 v(n) + b_1 v(n-1) + b_2 v(n-2) \end{aligned}$$

which can be interpreted as a two-pole filter followed in series by a two-zero filter. This contrasts with the DF-I structure of the previous section (diagrammed in Fig.9.1) in which the two-zero FIR section precedes the two-pole recursive section in series. Since LTI filters in series *commute* (§6.7), we may reverse this ordering and implement an all-pole filter followed by an FIR filter in series. In other words, the zeros may come first, followed by the poles, without changing the transfer function. When this is done, it is easy to see that the delay elements in the two filter sections contain the same numbers (see Fig.5.1). As a result, a single delay line can be *shared* between the all-pole and all-zero (FIR) sections. This new combined structure is called ``direct form II'' [60, p. 153-155]. The second-order case is shown in Fig.9.2. It specifies exactly the same digital filter as shown in Fig.9.1 in the case of infinite-precision numerical computations.

In summary, the DF-II structure has the following properties:

1. It can be regarded as a two-pole filter section followed by a two-zero filter section.
2. It is *canonical with respect to delay*. This happens because delay elements associated with the two-pole and two-zero sections are *shared*.
3. In fixed-point arithmetic, overflow can occur at the delay-line input (output of the leftmost summer in Fig.9.2), unlike in the DF-I implementation.
4. As with all direct-form filter structures, the poles and zeros are sensitive to round-off errors in the coefficients a_i and b_i, especially for high transfer-function orders. Lower sensitivity is obtained using series low-order sections (*e.g.*, second order), or by using ladder or lattice filter structures [86].

MORE ABOUT POTENTIAL INTERNAL OVERFLOW OF DF-II

Since the poles come first in the DF-II realization of an IIR filter, the signal entering the state delay-line (see Fig.9.2) typically requires a larger dynamic range than the output signal $y(n)$. In other words, it is common for the feedback portion of a DF-II IIR filter to provide a large signal *boost* which is then compensated by *attenuation* in the feedforward portion (the zeros). As a result, if the input dynamic range is to remain unrestricted, the two delay elements may need to be implemented with high-order *guard bits* to accommodate an extended dynamic range. If the number of bits in the delay elements is doubled (which still does not guarantee impossibility of internal overflow), the benefit of halving the number of delays relative to the DF-I structure is approximately canceled. In other words, the DF-II structure, which is canonical with respect to delay, may require just as much or more memory as the DF-I structure, even though the DF-I uses twice as many addressable delay elements for the filter state memory.

Next Section:
Transposed Direct-Forms
Previous Section:
Direct-Form I