MEDIAN SELECTION OF 9 ELEMENTS IN 14 COMPARISONS

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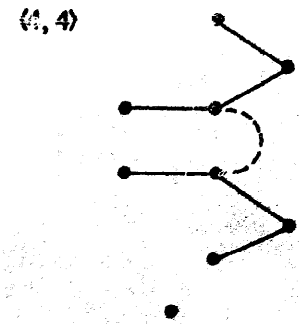
sorting median selection minimum-comparison

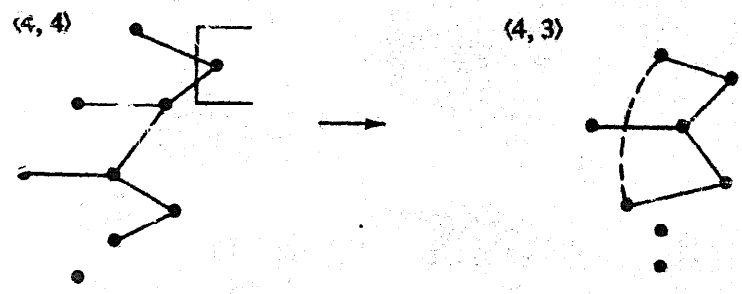
This note shows an improved upper bound for $V_5(9)$ in table 1 on page 215 of Knuth's book [1], i.e., $V_5(9) \leq 14$.

We shall specify an algorithm for selecting the median of nine elements by using a graphical representation as in [1], with the following additional conventions.
(1)] (or [) indicates an element smaller (or larger) than the median.
(2) $\langle s, l \rangle$: s (or l) indicates the number of elements not yet classified as being smaller (or larger) than the median.
(3) A dotted line (– – – – – –) indicates the pair for comparison in the next step.

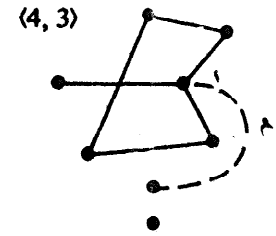
The algorithm reads as follows. Just after Step 6 (The first 6 steps are obvious):



Step 7:

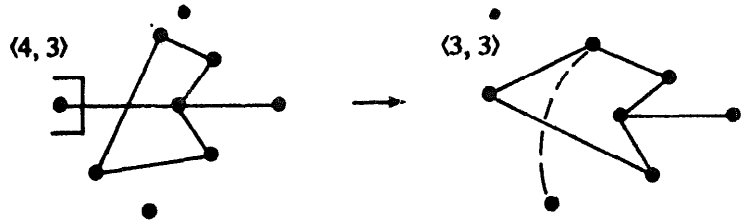


Step 8:

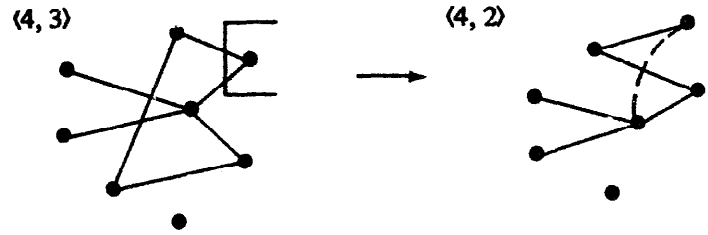


Step 9:

Case 1



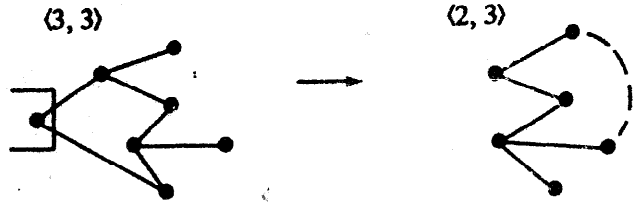
Case 2



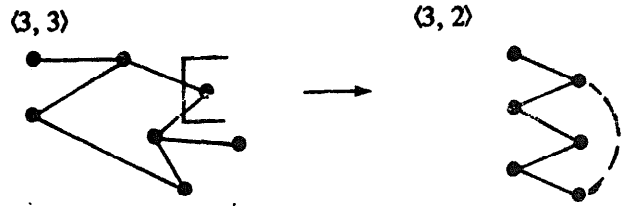
Case 1

Step 10:

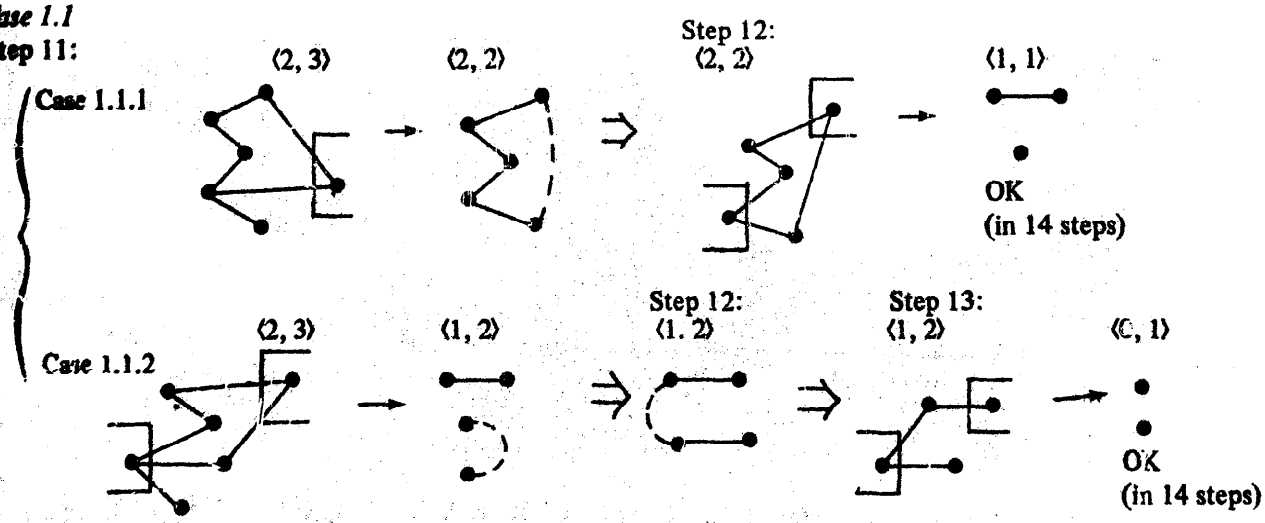
Case 1.1



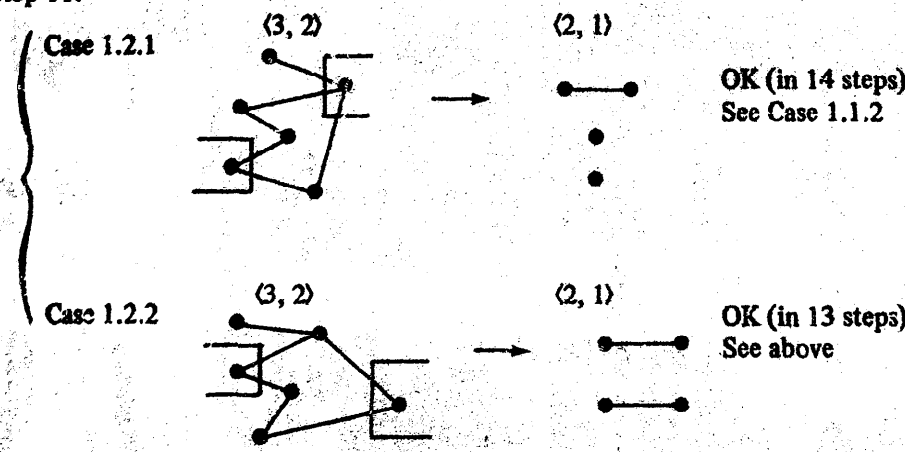
Case 1.2



Case 1.1
Step 11:



Case 1.2
Step 11:

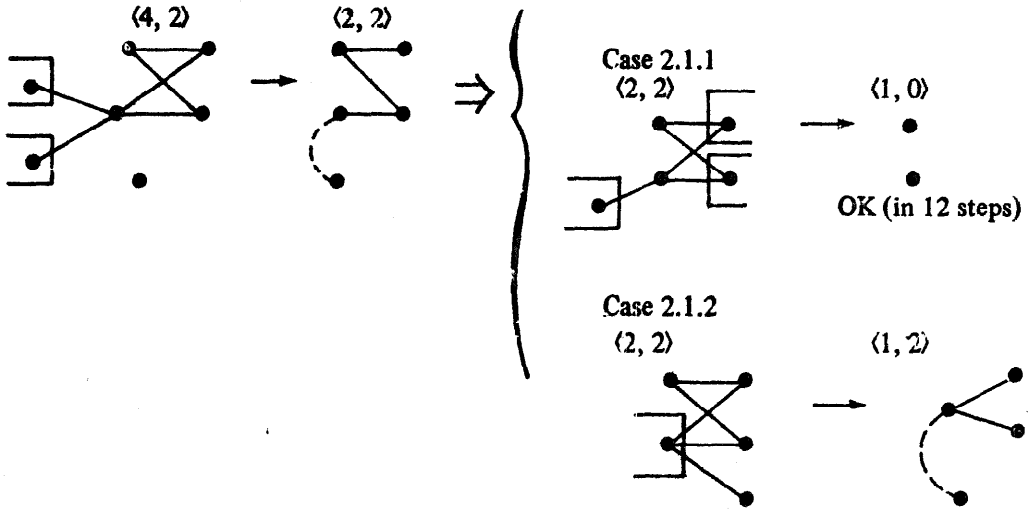


Case 2

Step 10:

Step 11:

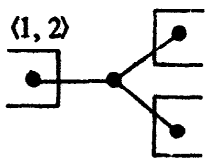
Case 2.1



Case 2.1.2

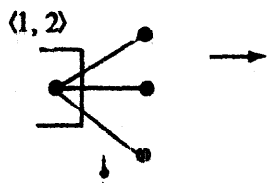
Step 12:

Case 2.1.2.1



OK (in 12 steps)

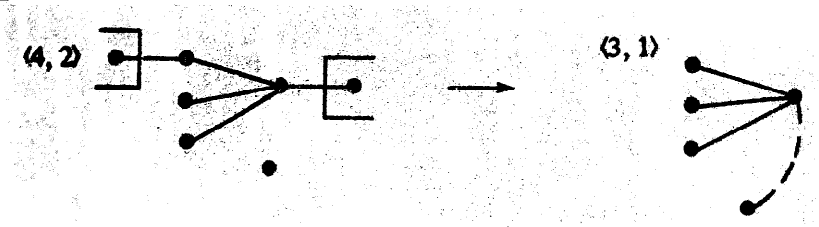
Case 2.1.2.2



OK (in 14 steps)

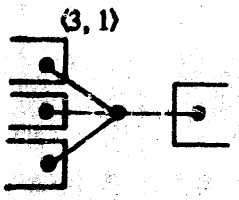
(Case 2, step 10)

Case 2.2



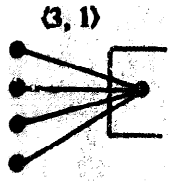
Step 11:

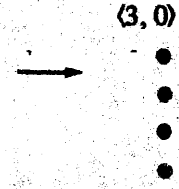
(Case 2.2.1



OK (in 11 steps)

(Case 2.2.2





OK (in 14 steps)

Combined with the answer to exercise 14 on page 636 [1], this result immediately implies the selection of the largest 3 elements of 10 in 15 comparisons, which has been quoted in Nozaki's paper [2].

I would like to thank Professor D.E. Knuth for refining this note.

References

[1] D.E. Knuth, The Art of Computer Programming, Vol. 3 (Sorting and Searching), Addison-Wesley (1973).
[2] A. Nozaki, J. Comp. Syst. Sc. 7 (1973) 615–621.