Lab 1: The Apollo Missions

Francis Peria
University of Maryland, College Park
PHYS265
Peter Teuben and Brian Clark
9 March 2025

I. Introduction

The goal of this lab is to understand the gravitational potential of the system of the Earth and the Moon and to quantify the performance of the Saturn V, a new rocket which will carry the capsule in the Apollo program. The entire study is analyzed through Python. The code and its plots analyze the gravitational potential of the Earth as well as that of the Earth-Moon system and its gravitational force field and it calculates the altitude of the Saturn V rocket. This information will be useful in the Apollo program to compare against experimental results and help NASA develop a rocket designed for space travel to the Moon.

II. The gravitational potential of the Earth-Moon system

First, the gravitational potential of the Earth is determined using this equation:

$$\phi(r) = -\frac{GM}{r} \tag{1}$$

where G is the universal gravitational constant, M is the mass of an object (in this case, Earth) and r is the distance from Earth. For the purposes of the assignment, the Earth is approximated as a point particle, but since this creates a singularity where r may be zero and thus, causes a divide by zero, a large negative value is inserted for r when r is less than a certain value close to zero. This leads to Figs. 1 and 2, which plot the absolute value of the gravitational potential |Φ| as a function of distance from the surface of the Earth and a color-mesh plot of the potential Φ with the Earth at the origin.

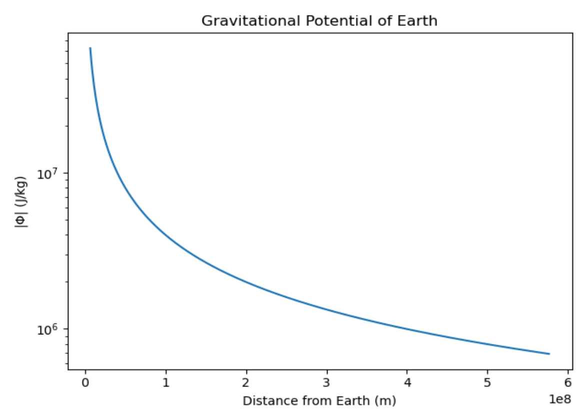

Fig. 1: 1D plot of absolute value of the gravitational potential of Earth

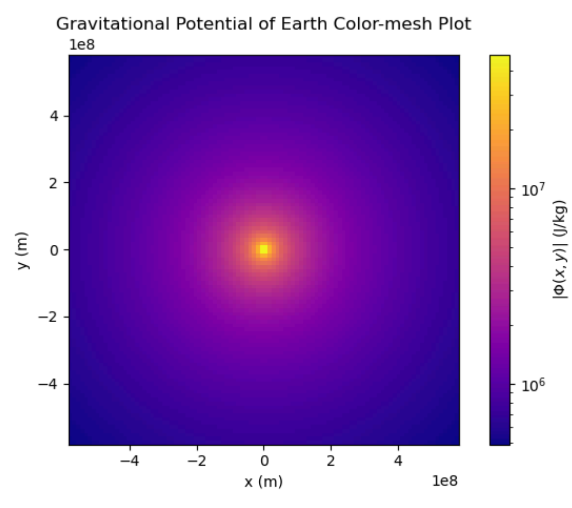

Fig 2: Color-mesh plot of the gravitational potential

Further analysis of the gravitational potential can be made to show the potential of the Earth-Moon system, as shown by Figs. 3 and 4, which show that in a color-mesh plot and a contour plot. The Earth is treated to be at the origin while the Moon is at a location on the plot that is analogous to its distance from the Earth.

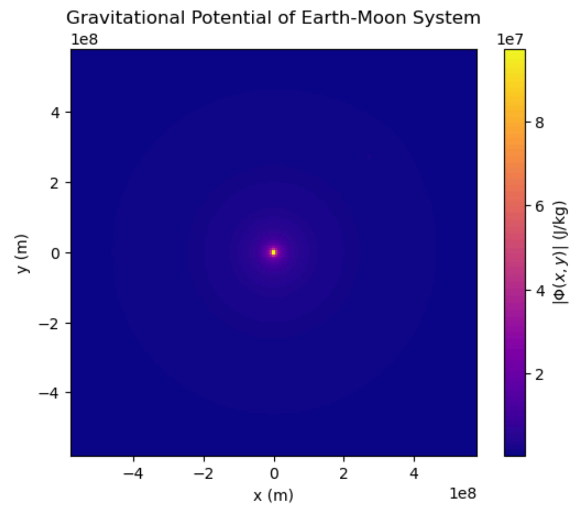

Fig. 3: 2D color-mesh plot of the gravitational potential of Earth-Moon system

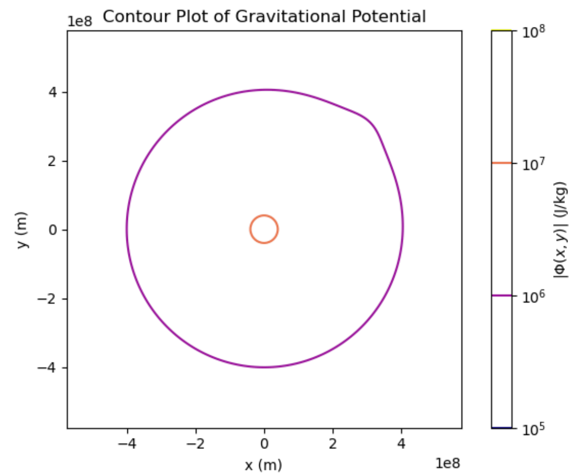

Fig. 4: 2D contour plot of the gravitational potential of Earth-Moon system

III. The gravitational force of the Earth-Moon system

To calculate the gravitational force of one mass on another mass, the following equation can be used:

$$F_{21} = -G\frac{M_1 m_2}{|r_{21}|^2}r_{21} \tag{2}$$

where G is the universal gravitational constant, M_1 and m_2 are the masses, and r_{21} is the distance between the masses. Plugging in the mass of the Earth, the mass of the Moon, and the distance between them, the results can be used to create Fig. 5, a streamplot of the gravitational force around the Earth-Moon system that would affect the Apollo 11 command module.

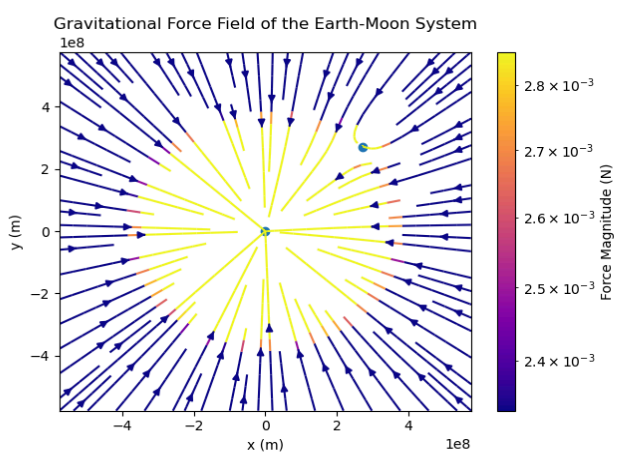

Fig. 5: A 2D streamplot plot of the gravitational force of the Earth-Moon system

IV. Projected performance of the Saturn V Stage 1

Rockets function by conservation of momentum. As fuel is ejected backwards, the rocket is propelled forwards. The Tsiolkovsky rocket equation shows the change in a rocket's velocity Δv as a function of time.

$$\Delta v(t) = v_e ln(\frac{m_0}{m(t)}) - gt \tag{3}$$

where m_0 is the initial wet mass, $m(t) = m_0 - \dot{m}t$ is the mass at time t, $\dot{m}$ is the fuel burn rate, v_e is the fuel exhaust velocity, and g is the gravitational acceleration. This can then be used to determine the altitude h of the rocket at burnout, or when the fuel has been depleted.

$$h = \int_0^T \Delta v(t)dt \tag{4}$$

Where $\Delta v(t)$ comes from eq. (3) and T is the total burn time of the rocket, which from conservation of momentum is given by:

$$T = \frac{m_0 - m_f}{\dot{m}} \tag{5}$$

where m_f is the final dry mass of the rocket after all the fuel is burned. Given that the exhaust velocity of Saturn V Stage 1 (S-1C) (v_e) is 2.4 × 103 m/s, the burn rate of S-1C ($\dot{m}$) is 1.3 × 104 kg/s, the wet mass of S-1C (m_0) is 2.8 × 106 kg, and the dry mass of S-1C (m_f) is 7.5 × 105 kg, the resulting burn time and final altitude is 157.69 s and 74.093 km, respectively.

V. Discussion and Future Work

In this lab, an important approximation was made during calculations by treating Earth as a point particle. This is significant because it means that there must be implementations to prevent singularities by creating a threshold of values for which r, the distance of a point from Earth, can be approximated to a very small number if r is too close to zero. To make calculations more accurate, future work could be done to consider the Earth as a larger body instead of a point particle, which means the gravitational potential would have to be calculated for the mass inside the Earth as well as outside. The resulting calculations for burn time and final altitude are 157.69 s and 74.093 km, but the experimental measurements for the first prototype showed that the burn time and final altitude were 160 s and 70 km. These values are very close to the calculations, but the slight differences could be due to neglected factors such as drag, which would have opposed the rocket's upward motion, causing it to fall slightly under the expected value, while the burn rate, while assumed to be constant, may actually fluctuate and cause a time discrepancy.