Lab 2: Mine Crafting

Francis Peria
University of Maryland, College Park

PHYS265

Peter Teuben and Brian Clark

10 April 2025

I. Introduction

The goal of this lab is to understand the effects of different forces and complex situations regarding the operation of a vertical mine at the Earth's equator. The shaft is roughly 4 km deep. The study will analyze situations involving a 1 kg test mass affected by complex assumptions that adjust the conditions of each given situation. All analysis is performed using Python.

II. Calculation of fall time (including drag and variable g)

A projectile experiencing a constant gravitational force, plus a drag force, follows the second order differential equation:

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma} \quad (1)$$

where t is time, y is the height, g is the gravitational acceleration, α is the drag coefficient, and γ is the speed dependence of the drag. The analytical calculation for the time of a test mass to reach the bottom of a 4km shaft was 28.6 seconds, and this ended up being the same answer for the numerical calculation as well. However, drag should not be ignored and g should not be assumed to be constant. Approximating the mass of the Earth as distributed homogeneously, g depends on distance from the center of the Earth r in this relationship:

$$g(r) = g_0\left(\frac{r}{R_\oplus}\right) \quad (2)$$

where g_0 is gravity at the surface and $R_\oplus$ is the radius of the Earth. Including drag and variable g, the time became 83.5 seconds.

III. Feasibility of depth measurement approach (including Coriolis forces)

Other than drag forces and a variable g, there is also a Coriolis force on the test mass as it falls which is due to the fact that the Earth is rotating. This force is represented by:

$$\vec{F}_C = -2m(\vec{\Omega} \times \vec{v}) \quad (3)$$

where $\vec{\Omega}$ is the Earth's rotation rate for a vector along $\hat{z}$ and m is the mass of the object (which in this case is 1 kg for the test mass). Considering the $\hat{x}$-axis to point towards the East, the $\hat{y}$-axis into the shaft, and the $\hat{z}$-axis towards the North, then the components of the force are:

$$F_{C_x} = +2m\Omega v_y,\ F_{C_y} = -2m\Omega v_x,\ F_{c_z} = 0 \quad (4, 5, 6)$$

Using these equations, the transverse position of the object was plotted as a function of depth to show that the test mass will hit the wall before it hits the bottom of the shaft with or without drag, as in Figure 3 shown below.

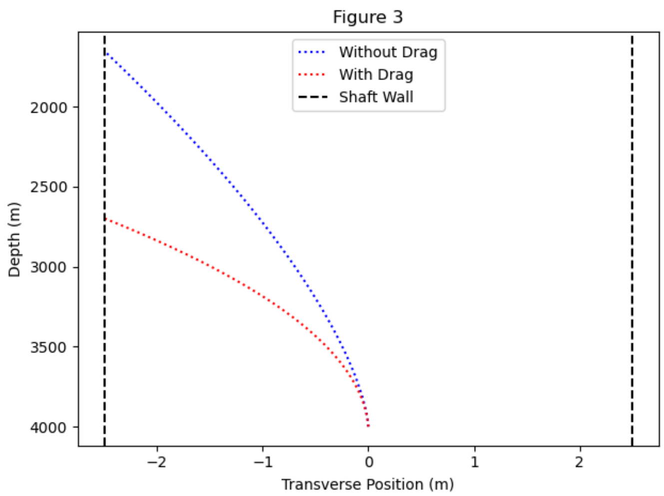
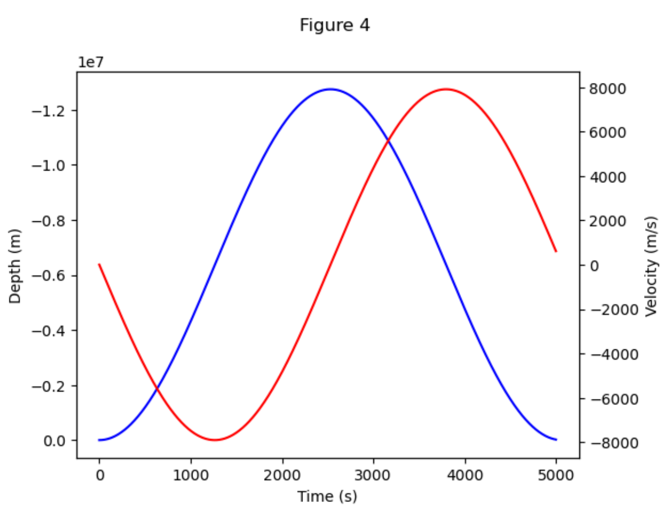


Fig. 3: Transverse position of test mass as a function of depth, with and without drag.

Without drag, the test mass is determined to hit the wall at 21.9 seconds while with drag, the test mass is determined to hit the wall at 29.6 seconds. This means that in either case, whether it is affected by drag or not, Coriolis forces prevent the test mass from reaching the bottom. This measurement technique would not be effective.

IV. Calculation of crossing times for homogeneous and non-homogeneous earth

In the case of an infinitely deep mine, or a tunnel that traverses the full diameter of the Earth, the Coriolis force and drag force can be neglected and that $\hat{\Omega} \times \hat{v} = 0$. Assuming a constant density Earth, the depth and velocity can be plotted as a function of time, as shown in Figure 4 below.



Fig, 4: Depth and velocity of the particle as a function of time.

This plot shows that the particle reaches the center of the Earth in exactly half the time it will take to reach the other side of the Earth. This "crossing time" of reaching the center of the Earth should be about the same as the orbital period, since this point would be where the object is in centripetal balance with the attractive force of gravity.

However, in the case of a non-uniform Earth, density increases towards the center, as represented by:

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n \tag{7}$$

where n is some exponent, and ρ_n is a normalizing constant. A constant density Earth would mean $n = 0$ while $n = 2$ is closer to the real value. The differences of two extreme cases of density concentrations is exhibited in Figure 7 below.

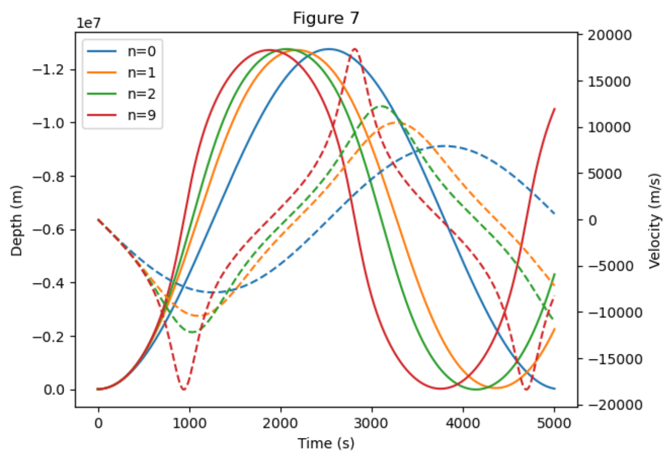


Fig. 7: Depth and velocity at different density concentrations as a function of time.

V. Discussion and Future Work

In this lab, important assumptions were made about the presence of drag, variable g, the presence of a Coriolis force, and uniform density of the Earth depending on the task.